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Students' Equation Understanding and Solving in Iran

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Abstract

The purpose of the present article is to investigate how 15-year-old Iranian students interpret the concept of equation, its solution, and studying the relation between the students' equation understanding and solving. Data from two equation-solving exercises are reported. Data analysis shows that there is a significant relationship between understanding and solving equation. The results indicate that students' understanding of equation has, basically, been shaped by their experiences in arithmetic and students usually have not any perception of equations and real world problems. Moreover, the study shows that students rarely paid any attention to the equality sign and the use of operators in both sides of the equation.

Keywords: Understanding equation, simplification in equation, standard forms, variable

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Comprensión y Resolución de Ecuaciones en Irán

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Resumen

El objetivo de este artículo es investigar cómo estudiantes iranís de 15 años interpretan el concepto de ecuación, su resolución y analizar la relación entre la comprensión que tienen los/as estudiantes de las ecuaciones y cómo las resuelven. Los datos proceden de dos ejercicios sobre ecuaciones. El análisis de los datos muestra que existe una relación significativa entre comprender y resolver las ecuaciones. Los resultados indican que la comprensión que tienen los/as estudiantes de las ecuaciones está, básicamente, formada por sus experiencias en aritmética y los estudiantes, habitualmente no tienen ninguna percepción de ecuaciones en problemas de la vida real. Es más, el estudio muestra que los/as estudiantes rara vez prestan atención al signo de igual y al uso de operadores en ambos lados de la ecuación.

Palabras clave: Comprensión de las ecuaciones, resolución de ecuación, simplificación en las ecuaciones, términos generales, variables

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Equation is one of the basic concepts of mathematics and is widely used in mathematics and other sciences. Algebraic equations play an important role in various branches of mathematics including algebra, trigonometry, linear programming and calculus (e.g., Hardy, Littlewood and Polya, 1934/1997).

However, research findings indicate that many students have problems and difficulties in understanding and solving equations. Teaching and learning algebra has long been seen as a source of difficulties in solving equations. Freitas (2002) categorized students' errors, in solving equations, in terms of misunderstanding algebraic rules. Previous researches in this field have shown that many difficulties in solving equation may be related to variables, algebraic symbolism, and equal sign.

Review of Literature

Based on some studies, the historical approach can play a valuable role in teaching and learning mathematics and it is a major issue of research in mathematics education, regarding all school levels (Heiede, 1996). Vaiyavutjamai (2004) and Lim (2000) reported that there has always been a strong emphasis on symbol manipulation, with less attention being given to the meaning of symbols. Tall and Thomas (1991) expressed one of the important themes that researches have focused on solution of equations and related problems. Bazzini and Tsamir (2004) suggest that students should understand the meaning of algebraic symbolism, expressions, equations, and also representation of the real world situations using those symbols. Kieran (1997) and Linchevsky and Sfard (1994) have indicated difficulties related to the use and meaning of the symbol of equality, while Kieran (1985) and Kuchemann (1981) found some results about misunderstandings in using and meaning of letters.

Some studies some students may be able to solve an equation with routine form using special rules. For example, Mayer and Hegarty (1996) reported that many students may know how to carry out basic mathematical procedures when problems are presented in symbolic and routine form, but they may not be able to apply these procedures to solve equations presented in other forms. In addition, the 1986 National Assessment of Educational Progress found that nearly all subjects could solve routine arithmetic problems, but almost all of them failed to solve none-routine problems.

Cortes and Pfaff (2000) found that the principles used by the students solving equations were all based on the movement of symbols from one side of equation to the other side as a routine procedure. They think that symbols are physical entities of equation that could be passed to the other side by changing the sign. These procedures were usually meaningless to students. Accordingly, Filloy and Rojano (1984) reported that many students, solving equation such as $x+5=x+x$, thought that the first x on the left side of the equation could take any value, but the second x on the right side had to be 5. Altogether, all these discussions reveal that it is necessary to emphasize the linear equation in algebra. Many errors observed in students' performance are common among the students. So by studying the causes of the errors, students could be helped to modify their understanding and also to modify the understanding of concepts that may cause errors in young students in lower grades.

Methodology

Purpose

In this article we have two main purposes:

- To study students' thinking about the meaning and solving of first and quadratic order equations.
- To study the relationship between the students' understanding and solving equation.

Participants and Research Design

This research was conducted to study the equation understanding and solving among Iranian students nearly based on Lima and Tall's study (2006) on 15-year-old Brazilian students. The research subjects were 100 students in two groups of 15 year old in high school. The first group consisted of 50 students from grade 9 and the second group 50 students from grade 10, from a public school in Hamedan[1]. Both groups had studied quadratic expressions and operations. The classes were taught by one of the researchers. The data were collected using a set of eleven carefully chosen questions based on research literature and authors' teaching experience in mathematics classes. Five questions in part 1were about understanding equation (see Figure 1), and six questions in part 2 about solving equation (see Figure 2). The questionnaire was conducted in two classes by the authors.

Explain your answer in each case. 1- What is an equation? 2- Give an example of an equation. 3- What does the solution of an equation mean? 4- Give an equation that doesn't have any answer? Is there any such equation? 5- Give an equation with answer *5*. Is it true to say $x=5$ is an answer?

Figure 1. Part 1. Understanding Equation

Solve the equations below and explain your solution. 1) 2m = 4m 2) 2x-1=4x+3 3) 5(t-1)=5 4) 4(t+1)=2 5) (y-2)(y-3)=0 6) (y-2)(y-3)=2 7) (n-1)(n-2)=(n-1)(n-3)

Figure 2. Part 2. Understanding Equation

Assessment Score for Data Analysis

The method used in this study for data analysis was assessed on a 0, 1, 2, 3, 4 basis where the scores were interpreted as showed in table 1.

Table 1
Classification of strategies

	Part 1: Understanding equation	Part 2: Solving equation
Score 0	The subject does not provide any answer.	The subject does not provide any answer.

Table 1 (…)
Classification of strategies

	Part 1: Understanding equation	Part 2: Solving equation
Score 1	The subject gives invalid answers.	The subject attempts but is unable to find appropriate procedure.
Score 2	The subject knows basic facts, but he/she only mentions the solution and cannot continue and, therefore, cannot give correct answer.	The subject finds appropriate procedure, but he/she cannot continue.
Score 3	The subject mentions the answer of question and he/she knows the meaning of the question, but he/she is unable to explain his/her answer correctly.	The subject uses an appropriate procedure and continues to the end, but he/she has error(s) in calculating.
Score 4	The subject gives correct answer and he/she is able to explain the concept of question correctly.	The subject gives correct answer by using a valid method.

Results

Both qualitative and quantitative methods were used in this research. The perpetuity of questionnaire was ensured through calculation of standardized Cronbach's alpha, which for the present work was calculated to be 0.92. Furthermore, straight and significant relationship between equation understanding and equation solving according to Pearson coefficient correlation ($r = 0.806$, $n = 100$, $p < 0.0005$, table 2) was observed.

According to Cohen (1988), because the value of coefficient correlation is more than 0.7, this relationship is strong. In addition, since Adjustment Coefficient has been 0.64 (Adj. $r^2 = 0.64$), 0.64% of score variation in equation solving was relevant to equation understanding (table 3). Now, we fit a Regression linear and assume "understanding equation" as independent variable and "solving equation" as dependent variable.

First, the researchers were ensured that the model is significant (table 4), the errors were normal, the variance was stable and average of errors was zero and the errors were independent (table 3). According to table 4, final equation turned out to be y= 0.852x where *y* meant *anticipation of score of understanding* equation, and *x* was *score of solving* equation.

By selecting "understanding equation" as dependent variable and "solving equation" as independent variable, attempts were made to fit another linear Regression. After the confirmation of required hypothesis for performing Regression, the result was as follows: y=0.93+ 0.76 x.

Table 2
Correlations Equation solving vs. Equation understanding

Pearson correlation	.806**
Sig. (2-tailed)	.000
N	100

** Correlation is significant at the 0.01 level (2-tailed)

Table 3
Model summary

R	R Square	Adjusted R Square	Std. Error of the estimate	Durbin Watson
.806[a]	.649	.646	.53085	1.895

a. Predictors: (Constant), equation understanding; b. Dependent variable: Equation solving

Table 4
Signification of Regression Model[b]

	Sum of Squares	Df	Mean Square	F	Sig.
Regression	51.126	1	51.126	181.429	.000[a]
Residual	27.616	98	.282		
Total	78.742	99			

a. Predictors: (Constant), equation understanding; b. Dependent variable: Equation solving

Table 5
Coefficients of Regression Model

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	Beta	T	Sig.
(Constant)	.263	.212		1.242	.217
Equation understanding	.852	.063	.806	13.470	.000

a. Dependent Variable: Equation solving

The results from data analysis show that there is a significant relationship between "understanding equation" and "solving equation". Also, according to obtained coefficients we conclude that this relation is strong and the variations of one of them are dependent on the other straightly.

Analysis

Part 1: Understanding Equation

Question 1 was answered by most of the students as "it is a mathematical calculation" or "it is a calculation done to find out the answer, *x*." These responses suggest that most of the students consider an equation either as an arithmetic calculation or as a calculation to find unknown value, *x*. Similar results were found by Dreyfus and Hoch (2004). Some students referred to unknown *x* as an important feature of equations. However, as the analysis shows, grasping the concept of equation could be useful for students to solve it. Tall (2004) reported that it is necessary for students to give meaning to equation, and analysing data from conceptual maps designed by 14-16-year-old Brazilian students, Lima and Tall (2006) showed that the absence of meaning leads students to further difficulties on solving equations.

In question 2, (Give an example of an equation.), some students had responded correctly and their answers were like " $x+1=3$" *or* "$2x-1=1$" and the like. Ten of the subjects had answered " $x+y=3$" and five students "$x+y=y+x$" *or* "$x+3=3+x$ ". The interpretation was that they did not know the difference between equation and algebraic identity. The point observed in most of responses was that they used x as a variable. None of the subjects had been able to give an example from real world.

Question 3 asked the subjects about the meaning of the solution of an equation. Students' responses were as follows: ″ solution to a mathematical problem″ and ″ the unknown value ″. As Tall (2004) has also noted this point, an assessable expression was involved in all the cases and no equation was related to real world problems.

Question 4, asked the subjects to give an example of an equation that does not have any answer? Is there any such equation? In response to this question, most students believed that "there is not such an equation". Some had wondered: ″ Is it possible for an equation to have no answer? ″ Perhaps they were wrong in distinguishing between equation and algebraic identity and did not see any difference between them. Tall (1991) mentioned that we should not assume that all the problems have solutions. An equation could have several solutions or no solution. These points should be taught to the students and they could realize all of the possible positions.

In question 5 the subjects were asked to give an example of equation with answer 5. Is it true to say $x=5$ is an answer? The reason for asking this question was to put students in a condition that rarely occurs. Because usually an equation is given to students and they are asked to answer it. But in this question the students were asked to do the reverse. A number was given as an equation answer and the students were asked to make an equation with this answer. Some of the students gave the correct answer to this question. Most of their responses were, " $2\,x=10$″, ″ $x+5=10$ ", or the like.

Only one of the subjects had written " $x=5$ " and others had used string expressions. The remarkable point was that the other students did not accept this response and believed that " $x=5$ " is not an equation, because the value of x is known while other equations, after simplification, will be changed to $x=5$. They assumed that an equation should contain several terms and its solution should contain several stages, so they did not accept $x=5$. Five students had responded " $10x=2$ ". The observation that students solve equation $ax=b$ incorrectly with the use of method $x=a/b$ was made by Freitas (2002) and theorized by Linchevski and Sfard (1991). This is because of priority of procedure over concept.

Part 2: Solving Equation

In part 2 of questionnaire, (Solve the below equations) consisting of seven questions about equation solving**,** students were asked to solve the equations and explain their solutions. The most common and successful methods to solve them were the rules of “change side change sign”, and “move the coefficient of *x* to the other side of the equal sign or be divided by it.” To get the correct answer, such solutions contain symbol movement, or sign changing. Rote learning may easily occur and equation may be considered as meaningless action of shifting symbols and doing something else simultaneously. Such operations may be used improperly through the change of the sides but not the signs in an equation; or students may erroneously change the sign of coefficient of x as they shift it to the other side, or change the equation $ax = b$ into $x = a/ b$. Similarly Freitas (2002) pointed out these errors and Linchevski and Sfard (1991) theorized them as ‘pseudo-conceptual entities’.

In order to solve equations, students usually think of applying the rules while no one mentioned the idea of using the same operation in both sides, which was clearly shown in the equation solving exercise. In this part students’ solutions to each case are discussed separately.

2m=4m

The purpose of choosing this equation is to check how students simplify the coefficients and the variables. Some of the students (43 out of 100) solved it correctly and found out the answer through solving procedures such as:

$$4m=2m \rightarrow 4m-2m=0 \rightarrow 2m=0 \rightarrow m=0/2=0$$

One of the points that the researcher frequently observed in the classes was that many students used special algorithms for solving an equation. For example, in solving one-order equations, they shifted variables to the left side of equal sign and numbers to the right side. While solving this equation, finding they did not guess the answer and continued their algorithm to the end and they achieved $2m=0$. This point was also mentioned in Lima and Tall (2006). Seventeen students simplified the variable *m* and concluded that this equation does not have any answer. We will discuss this point later in equation: $(n-1).(n-2)=(n-1).(n-3)$. Thirteen students simplified the numbers first then continued their solution. However, some of the students simplified equations before solving those.

2x-1=4x+3

This equation was chosen based on the previous equation in which variable *m* was changed to variable x and two numbers were added to both sides. The reason of choosing this equation was that although it is longer than the previous one, its solution is more procurable.

The students used the method "variables in the same side and numbers in the same side" for solving it. Sixteen students solved it with the following method:

$$2x-4x=3+1 \rightarrow \ -2x=4 \rightarrow \ x=-2$$

There were two remarkable points in the solution of some students: first, they took variables to the left side of equal sign and numbers to the right side; second, some students shifted negative sign and coefficient to the other side separately:

$$-2x=4 \rightarrow 2x=-4 \rightarrow x=-2$$

In fact, they observed separate negative sign and coefficient. The TIMSS investigation of the late 1990s revealed that Grade 8 students do not solve equation in the form of $ax+b=cx+d$ very well. Vaiyavutjamai (2006) reported that in equation $12x-10=6x+32$, some of the students believed that the x's on the right side of the equation represent value. For instance, 5 on the left side and 3 on the right side are answers because if we replace them as values for x in equation, then equality is correct. In fact, they attended only to equality property.

5(t-1)=5

In this equation there are two equal numbers in two sides. The reason for selecting this was that the researcher observed that when some students simplify two equal numbers, they put zero at the other side instead of one as the following error in which 11 students simplified equation incorrectly:

$$5(t-1) = 5 \rightarrow t-1=0 \rightarrow t=1.$$

This error was observed by the researcher in the classroom and based on the interview with these students, the researcher noticed that they simplified the term *5t =5 to* t+*5=5*. Freitas (2002) claims that students have difficulties with multiplication and division involving zero and in this equation zero is involved. Thirty-one students multiplied coefficient by the entity inside the brackets and they achieved a correct response by solving a linear equation. Six students simplified equation correctly and found the answer:

$$5(t-1) = 5 \rightarrow t-1=1 \rightarrow t=2.$$

4(t+1)=2

This equation was selected for both groups based to the previous equation and the reason of choosing it was that there are two numbers in two sides of it that are not equal, but simplify each other. Some of the students (48 out of 100) solved this equation correctly and found true answer by producing coefficient in parenthesis. Six people made error in simplification and shifting the number to the other side, as follows:

$$4t+8=2 \rightarrow 4t=2+8$$

This error means, "change side without change sign" that is as Tall (2004) and Linchevski and Sfard (1991) mention, one of the frequent errors in solving equations. Seven students made mistake in shifting coefficient to other side such as the following:

$$4t+8=2 \rightarrow 4t=-6 \rightarrow t=-6/-4$$

In fact, they changed sign of coefficient in shifting it to the other side as theorized by Linchevski and Sfard (1991).

(y-2)·(y-3)=0

This equation was given to the both groups. The reason of selecting it was that the researcher intended to see if the students used either the rule, "if the product of two numbers is zero, the one of them must be zero", or quadratic formula to solve the equation. Some of the students (21 out of 100) multiplied two parenthesis and they used quadratic formula after the simplification. Only seven students used the above rule as follows:

$$(y-2)\cdot(y-3)=0 \Rightarrow \begin{cases} y-2=0 \longrightarrow y=2 \\ y-3=0 \longrightarrow y=3 \end{cases}$$

Vaiyavutjamai and Clement (2006) reported that data related to solving quadratic equations that are soluble via null factor law," If $a{\cdot}b=0$, then $a=0$, or $b=0$, or both a and b are zero" have been reported by Vaiyavutjamai, Ellerton, Clement (2005) and Lim (2000), who found that when faced with an equation like $(x-3)(x-5)=0$, many secondary-level mathematics students, some university mathematics students, and even some teachers choose to write left side as $x^2 -8x+15$, then solve it with quadratic formula.

(y-2)·(y-3)=2

This equation was selected for both groups who knew quadric formula based on the previous equation and the reason for choosing it was studying the students' behaviour in dealing with the question that is similar to the question solved through method of solving instructed before. Usually students use similar procedures in dissimilar situations. This point was mentioned by Schoenfeld (1985) too. Some of the students (17 out of 100) solved this equation incorrectly and made mistake in using null factor law. They made the following error:

$$(y-2)\cdot(y-3)=2 \Rightarrow \begin{cases} y-2=2 \longrightarrow y=4 \\ y-3=2 \longrightarrow y=5 \end{cases}$$

And they did not check their answers. Other students solved this equation by quadratic formula.

(n-1)·(n-2)=(n-1)·(n-3)

The reason for selecting this equation was to study simplification method of algebraic expressions. Based on the researcher's observations many students, while facing an equation, think how they can simplify it. For example, to solve the equation $4x^2=6$ they simplify the numbers and conclude that $2x^2=3$. When they are not able to simplify numbers, they simplify variables. For example, for solving equation $3x^2=2x$ they simplify the variable *x* in two sides and the result would be $3x=2$. The equation $3x=2$ only has one root while the equation $3x^2=2x$ has two roots. This is exactly like the equation *2m=4m* where some students simplified the variable *m* and reached the conclusion that this equation does not have any answer. In fact, many students do not notice the differences between simplification of numbers and variables and they do not know that, while they simplify an algebraic expression from two sides of an equation, some of the roots may be eliminated and they do not realize that the omitted term should be equal to zero in order to find the eliminated roots.

Anyway, for this equation, the 13 students from first group and 18 students from second group multiplied parentheses and then solved it. Twenty-nine subjects from both groups simplified the term *(n-1)* and concluded that the equation had no answer at all.

$$(n-1).(n-2)=(n-1).(n-3) \rightarrow n-2=n-3 \rightarrow -2 \neq -3.$$

The students' responses to questions show that many of them have difficulties in both parts. Their perception of the concept of equation and its solution has basically been shaped by their experiences with variables and equal sign.

Conclusion

Data analysis shows that according to Pearson coefficient correlation ($r=0.806$, $n=100$, $p<0.0005$) there is a significant and strong relationship between understanding equations and solving them. Also, since Adjustment Coefficient has been equal to 0.64 (Adj. $r^2 = 0.64$). 0.64% (or more than half) of score variation in equation solving is relevant to equation understanding.

In addition, students' understanding of equation has basically been shaped by their experiences with arithmetic, and many of their errors in understanding equation are related to the concept of variables and algebraic expressions. The data collected from the questionnaire show that the students' responses to a question are related to other responses. For example, student who thought of equation as: "a set of algebraic expressions" in question 1, answered " $x+y=3$" to question 2, which was given as an example of an equation. Another subject who responded the solution of equation as: "simplification and finding x ", answered the question asking for an example of an equation that does not have any answer, as: "equation that has no answer does not exist ", or in another case, the student thought that solving equation is " calculating and finding x", in question 5 and, therefore, did not accept x=5 as an equation with answer 5.

The findings indicate that students usually find some rules through false inductions from previous examples and they try to solve the equations by those rules. They seem willing to use a formula (such as quadric formula) or certain method for solving an equation. For example, in linear equation $ax+b=0$, some of the students quickly answered: $x=-b/a$, so, it is helpful to give questions in different forms.

Notes

[1] A province in the West of Iran.

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