Money and Price Theory

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I. Price determination and market mechanism. II. A theory of prices with a market mechanism and money: a simple illutration. III. Introduction to the market dynamics. References.

Price theory and money theory not develop separately. They are indeed the two faces of the same coin.

Modern price theory sums up in two central results, existence of general equilibrium prices and Pareto-optimality of competitive equilibrium. It does not succeed however in demonstrating that these prices are market prices. This is the major failure of this approach. Introducing a market mechanism is necessary but is possible only through assuming existence of money, as a general means of payment.

In the first section of this paper, relations between price determination and market mechanism are recalled. In the second section, a market mechanism -Cantillon's rule- is built in the basic pure exchange model and some consequences of this introduction are drawn. A third section is devoted interpretative remarks and to monetary control.

I. Price determination and market mechanism

In its more general meaning, market is a particular means of coordinating individual actions. The peculiarity is that individual actions

are descentralized, i.e. the individuals do not know the state of the economy in which they are acting. Besides, such a state is the result of individual actions and it is commonly admitted that it can be different from what market participants expected.

A *market mechanism* is an algorithm by which prices and allocations resulting from a given set of individual actions can be calculated. In a competitive economy these actions concern quantities and not prices. The latter are supposed to be determined by the anonymous market competition. This conception is common to the general competitive equilibrium theory as well to other price theories (productions prices, labor theory of value, etc...). All this can be expressed by the fact that the agents, when they make decisions, take prices as parameters (such prices can be either expected prices or Marx's "ideal prices" or they can be announced by an auctioneer). In the most advanced price theory - the general competitive equilibrium theory- there does not exist any algorithm which could be interpreted as a market mechanism. It is impossible to calculate market prices and allocations which correspond to individual actions taken for a given vector of parametric prices. Market outcomes are determined only for very particular vectors of parametric prices, i.e. prices such as there does no exist any positive market excess demand. In other terms, in the general competitive equilibrium theory, the mechanism is not defined except in equilibrium.

The existence of equilibrium positions are justified on the basis of the "law of demand and supply", and it was believed for a long time that, in accordance with this law, desequilibrium positions were transitory: adjustment forces of demand and supply will push the economy towards an equilibrium. But this belief has no longer been justified since the well know failure of the stability theory (Fisher 1983). Hence, it must be admitted that the general competitive equilibrium theory is not an acceptable theory of the competitive market.

This can be accounted for in the separation between the determination of equilibrium prices (by the system of simultaneous equations of market excess demand) and decentralized exchanges on markets. The exchange

activity, which is an essential one in a market economy, is never considered; it is not even mentioned in the standard price theory. As a reaction against this paradoxical separation we could note the attention paid to transaction costs and the development of theories such as search theory and strategic market games.

Some conclusions of these different researches are:

- Even in a general equilibrium situation, it is not generally possible to complete the desired transactions in a decentralized way (Ostroy and Starr, 1974). As a consequence, the individual equilibrium allocations may not be obtained. Negative consequences on welfare properties are obvious.

- The non-tâonnement processes without a general means of payment are arbitrary because the conditions of market efficiency are not fulfilled (Fisher, 1983).

- The theory of strategic market games shows that, without a unique means of payment, the prices which are obtained on different markets are incoherent (Sahi and Yao, 1989).

All these indications strongly suggest the necessity of introducing money in the general equilibrium theory. But this is a fruitless strategy. The existence of a generally accepted means of exchange, even if it were possible, could not modify the fundamental flaw of the general competitive equilibrium theory, i.e. the absence of a satisfactory market mechanism. The fact that the unique general means of exchange has a zero price at equilibrium, enables us to shorten the discussion on this point[1]. The central problem is that of the market mechanism. A simple principle can be found in the writings of the past, principle which we shall call "Cantillons rule" as a tribute to the first author who has presented it clearly (note that a similar rule is found Smith's work). This rule (Cantillon 1755) is as follows:

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1 On the inachievements of modern monetary theory, see M. Hellwig (1993).

Prices are set by proportion between commodities brougth to the markets and the money which is offered in order to buy them.

The same rule is adopted in many models of strategic market games (for a survey, see Shubik, 1990). It implies that money exists, and that individual actions concern quantities of goods as well as of money.

The adoption of this market mechanism deeply modifies the way of thinking the market theory. Market and money theories are one and the same theory. The question is not to show that a "thing", which does enter the utility functions and which is used as a general means of exchange, has a positive equilibrium price. In other words, contrary to a well-established tradition, the integration of money into value theory is an ill-formulated monetary problem, which would only makes sense if a price theory which excludes price formation by means of a market mechanism were acceptable. According to Cantillon's rule, money is the condition of the formation of a coherent system of market prices. It is important to note that this is true independently of the realization of a general equilibrium of the markets. The relationship between money and commodites which is indicated by Cantillon's rule, is not an equivalence relationship, or an exchange relationship. The exchange is realized only by the selling and buying considered together, as shown by Marx. As a medium of exchange, money is a means and not an end. The relationship between money and goods is defined by the market mechanism. (Note that is clearly seen by Law: *money is not the value for which, but by which, goods are exchanged*, and by Simmel -cf. his "teleological series"-.

The introduction of Cantillon's rule completely modifies the price theory. In this paper, we sketch a simple model of price formation, that is with a market mechanism and money in such a way that it can be compared with standard Walrasian general competitive equilibrium theory[2].

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2 Another possible example is Marx's theory of value (C. Benetti and J. Cartelier. 1994).

II. A theory of prices with a market mechanism and money: a simple illustration

In order to transform the general equilibrium basic model into a theory of market price formation, it is necessary to combine it with a market mechanism and with a system of payment.

Let us consider a pure exchange competitive economy with **L** commodities l = 1..... , L, and **H** individuals, h = 1, ... , H.

Individual h is described by:

- his / her initial endowment, $\overline{x}_h$
- his / her utility function, assumed to have all the "good" properties, $U_h(x_h)$, with x_h being individual h's allocation after the market.
- the amount of a general means of payment (not a commodity) M_h which is available to him/her at the beginning of the market and which must be paid back at the end.

Means of payment are nothing but pure intermediary of exchange and the do not enter utility functions. The total quantity of means of payment is $\sum M_h = M$

A. The market mechanism

The market mechanism is made of the following rules:

1. There exists an organized market for each commodity, -*i. e.* **L** markets, where individuals bring the quantities of commodities they wish to sell and spend the amount of means of payment corresponding to the expected value of the quantities of commodities the wish to buy $\mathbf{m_{hl}}$ under the constraint: $\sum_l m_{hl} \leq M_h$

2. The **L** markets open and close simultaneously; they last a uniform discrete period of time (t).

3. Market prices $\pi_l(t)$ are determined according to Cantillon's rule:

$$\pi_l(t) = \sum_h m_{hl(t)} / \sum_h | Z^-_{hl(t)} | \; \forall l \qquad (1)$$

where $Z^-_{hl(t)}$ is the excess-demand, if negative, of commodity **l** by individual **h**. Market prices are determined but as *monetary prices.*

4. Allocation of individual **h** of commodity **l** at the of the market is:

$$x_{hl(t)} = \bar{x}_{hl(t)} + z^-_{hl(t)} + m_{hl(t)} / \pi_{l(t)} \qquad (2)$$

and monetary balance of individual **h** is:

$$s_{h(t)} = \sum_l \pi_{l(t)} | z^-_{hl(t)} | - \sum_l m_{hl(t)} \qquad (3)$$

B. Money as payment system

The market mechanism described as Cantillon's ruled is very abstract. Means of payment are implied by this mechanism but they have not yet been defined. The market mechanism needs an institutional support. Here money enters the stage.

Money is not a "commodity" which must have a positive price in order to become an "economic thing". It is rather a specify way of organizing transactions.

There is a general agreement on the minimal properties of a market economy:

- individual decisions are decentralized, i. e. they depend on local and not on global conditions; as a consequence, prices generated by such decentralized transactions are generally not consistent; this prevents one from defining univocal individual budgetary constraints and deprives equivalence in exchange of any clear meaning.

- the overall outcome results from a market mechanism coordinating these individual decisions.

Money must be modeled in such a way that it allows the working of a market economy. To put it in a nutshell, it has to make transactions possible in disequilibrium as well as in equilibrium. This is the condition for transactions to be decentralized and for two individuals to conclude a transaction independently of other people. Money guarantees that market prices are consistent in contrast with barter.

To remind us that money is an institution, we shall use the term *payment system.*

Three elements are the minimal components of a system of payment.

1 *A nominal unit of account* is necessary to express prices in a monetary economy. Even if the unit of account is physically defined - e.g. by a gold weight - such an economy will differ from one in which prices are expressed in gold. In barter, gold is not accepted as a means of payment by agents who do not demand gold. Acceptance of money is not ruled by the same principles as the demand for goods. In a gold currency system, gold coins and not gold itself are the commonly accepted means of payment. When minting and melting are not free this makes a great difference.

It is not required that the unit of account should be physically defined. A dollar is a dollar. The unit of account is a language, the language used in the market.

2 *A minting process* is a necessary complement for the unit of account. The existence of a unit of account - say the dollar - only imposes that every means of payment has to be expressed in dollars.

In order to transact economic agents must have means of payment. Any given individual may obtain means of payment by selling something to others. But the questions is: how buyers may have means of payment? Clearly, if individuals may act in the market in a decentralized way,

they must obtain means of payment without waiting for the sales of their commodities. The minting process by which economic agents obtain means of payment independently of other people expenditures. Getting means of payment in that way allows one to act freely in the market, for instance to produce goods in view of selling them. Minting process and he so-called market division of labor are the one and same thing.

Several kinds of minting processes can be found in history. Availability in the means of payment is not the same in a strict gold currency system and in a complex banking system with credit and a Central Bank. In modern theory some examples of minting process can be found. The most familiar is the *cash-in advance hypothesis*. Obviously, there is a need for further elaboration in this field.

Since transactions take place in equilibrium as in disequilibrium, some individuals will experience at the end of the market that they spent more than they earned whereas others will discover that they earned more than they spent. In the aggregate, payments and receipts are necessarily equal but this is not true for individuals who have either monetary surpluses or monetary deficits. These monetary balances reveal that individual budgetary constrains and equivalence in exchange are not verified. Remember that a sale (or a purchase) is not an equivalence relationship.

3 Restoration of budgetary constraints requires a *principle of adjustment* (settlement of balances or postponement of payments through credit). As monetary surpluses and deficits are unavoidable, being inherent in market coordination where disequilibrium is the rule, it is not very sensible to think that high penalties for default would suffice to restore confidence. Neither individuals nor banks have sufficient knowledge to avoid disequilibrium situations.

The problem is not to preclude beforehand occurrence of disequilibrium but to make sure that disequilibria, if not too important:

- do not put the entire economy in danger.

- do not prevent economy from working as smoothly as possible so that confidence is self-enforcing.
- do act as signals inducing appropriate adaptive behavior from agents so that disequilibria are not cumulative.

A central Bank (or monetary authority) is the most common institutional device to get these outcomes.

C. Individual behavior

Nothing has been said so far about the way individuals calculate their desired transactions. As a matter of fact, the market mechanism is compatible with any decentralized behavior. The "rules of the game" is the framework in which individuals behave and it is to be defined logically prior to the behavioral assumptions. It is indeed here to adopt many different assumptions, be they founded on bounded or on Walrasian rationality hypothesis. To keep reasoning along general equilibrium tradition, we shall adopt the following assumptions:

1. The prices used by individuals to calculate their market plans are called *parametric prices* $\mathbf{P_{hl}}$. They are either announced prices $P\mu$ by an auctioneer or expected prices $\mathbf{P_{hl}}$. We shall consider thereafter only the case where parametric prices are identical for all individuals (a special case is when expected prices are equal to market prices of the proceeding period $P_l(t) = \pi_l(t-1)$

2. Individuals maximize utility functions respecting two constrains:

- the value -at parametric prices- of desired purchases shall not exceed that of expected sales

$$\sum_t P_{l(t)} Z_{hl(t)} \leq 0 \quad (4)$$

- the value of desired purchases ($z^+_{hl(t)}$ is the excess-demand of individual **h** for commodity l if positive) shall not exceed the amount of the means of payment available to individual **h**:

$$\sum_t P_{l(t)} Z^+_{hl(t)} \leq \mu_{h(t)} \quad (5)$$

3. The quanta of the means payment available to **h** is exogenously determined. It is also possible to assume a "monetary authority" controlling the $\mu_{h(t)}$'s according to the type of payment system which runs the market mechanism.

4. Desired transactions are given by:

$$\text{MaxU}\ (x_{h(t)}) \qquad (6)$$

such that (4) and (5) are satisfied.

Desired transactions are thus continuos functions of parametric prices, initial endowments and amounts of available means of payment:

$$Z_{ht(l)} = Z_{hl}\ (P_{(t)}, \bar{x}_{h(t)}, \bar{\mu}_{h(t)}) \qquad (7)$$

D. Market outcomes

1. Price formation

For given P(t), $\bar{x}_{h(t)}$ and $\bar{\mu}_{h(t)}$, individual **h** spends $m_{hl(t)} = P_{l(t)}\, z^{+}_{hl(t)}$ on market **l**. Cantillon's rule (1) gives now:

$$\pi_{l(t)} = P_{l(t)} \sum_{h} z^{+}_{hl(t)} / \sum_{h} |\, z^{-}_{hl(t)} \,| \quad \forall\, l \qquad (8)$$

Market price is defined if $|\, z^{-}_{hl(t)} \,| > 0$ for at least an individual and positive if, besides, $z^{+}_{hl(t)} > 0$ for at least an individual.

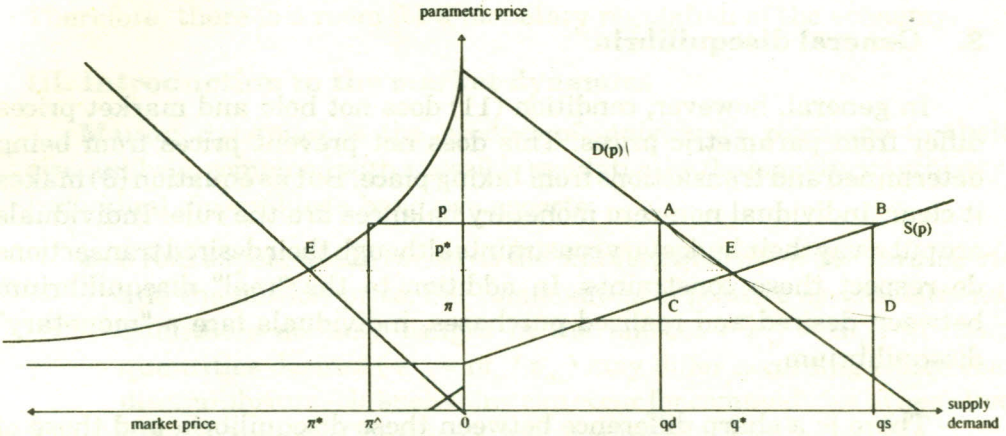
Relation (8) shows that formation of prices is decentralized market by market. General interdependence among markets is present however through some kind of Walras's law:

$$\sum_{t} (\pi_{l(t)} \sum_{h} |\, z^{-}_{hl(t)} \,|) \equiv \sum_{t} (P_{l(t)} \sum_{h} z^{+}_{hl(t)}) \qquad (9)$$

which amounts to:

$$\sum_{h(t)} (\sum_{t} \pi_{l(t)} |\, z^{-}_{hl(t)} \,| - \sum_{l} m_{hl(t)}) = \sum_{h(t)} s_{h(t)} \equiv 0 \ \forall (t) \qquad (10)$$

Relation (10) is the specific form of Walras's law in a monetary economy. Let us call it monetary identity. Monetary identity and Walras's law are expressions of equivalence in exchange but instead of putting emphasis on the interdependence among markets, relation (10) underlines interdependence among individuals. The diagram below describes the working of Cantillon's rule.



If $S(p)$ and $D(p)$ denote respectively market supply and demand for commodity **l**, market price is given by $pD(p)/S(p)$. The relation between market price and parametric price is shown in the left part of the diagram. If $\hat{\mathbf{p}}$ is the parametric price, market price is then π. All the quantities brought to the market for $\hat{\mathbf{p}}$ are sold at π, determined by equality between area $O\hat{p}Aqd$ and area $0\ \hat{\pi}Dq_s$ (D and A are on the same branch of an hyperbola). If aggregate excess-demand market for commodity **l** is zero $(D(p) = S(p))$, market price of commodity **l** will be equal to the parametric price ($\pi^* = p^*$).

2. General equilibrium

If all aggregate excess-demand functions are simultaneously equal to zero, that is if:

$$\sum_h z^+_{hl(t)} = \sum_h |z^-_{hl(t)}| \quad \forall\, l \qquad (11)$$

market prices are general equilibrium prices.

It is also clear that if constraint (5) is not binding for any individual, such market prices will be also Walrasian equilibrium prices. If constraint (5) is binding for one individual at least, general equilibrium market prices differ from Walrasian prices[3].

3. General disequilibria

In general, however, condition (11) does not hold and market prices differ from parametric prices. This does not prevent prices from being determined and transactions from taking place. But as equation (3) makes it clear, individual non-zero monetary balances are the rule. Individuals are put away their budgetary constraints although their desired transactions do respect these constraints. In addition to the "real" disequilibrium between desired and realized purchases, individuals face a "monetary" disequilibrium.

There is a sharp difference between these disequilibria and those of different versions of the Walrasian theory.

In the traditional Walrasian model, individuals are always in equilibrium. Market disequilibrium is only virtual and can be know by a fictitious auctioneer only. In the scaled non-Walrasian price theory, individuals are always in constrained equilibrium, obtained by adding quantity signals to (fixed) price signals. Market disequilibrium prevents them from realizing their desired constrained transactions. Actual exchanges are determined by the "short side" of the market. *In every case, individuals are always in their budget set.*

In our model, the economic rationale of exchange is very different. Transactions are not the way of modifying endowments at given known prices. They are conceived as a social process by which prices will emerge

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3 For a rigourous treatement of this point, see Dubey and Shapley (1994).

by means of the market mechanism. As a consequence of Cantillon's rule, market work as a system by which commodities are reallocated among market participants. Thus, markets always clear[4], i. e. all commodities brought to the market are sold. It follows that disequilibrium appears only in the individual accounts as a difference between expected (or parametric) and market price. Such individual disequilibria are both real and monetary. Therefore, there is a room for a monetary regulation of the economy.

III. Introduction to the market dynamics

Market dynamics is the product of individual's reactions to their disequilibria, combined with a possible regulation by the monetary authority. Individual disequilibria have two aspects:

- a real aspect which concerns the difference between the parametric and the market price. For commodities in positive excess-demand, quantities actually bought in the market ($\tilde{z}^{+}_{hl} = m_{hl} / \pi_{l}$) and quantities desired ($z^{+}_{hl} = m_{hl} / p_{(t)}$) may differ according tithes real disequilibrium. No such thing can occur for commodities in negative excess-demand; Cantillon's rule ensures that $| \tilde{z}^{-}_{hl} | = | z^{-}_{hl} |$.

- a monetary aspect which concerns the difference between the value of actual sales and that of actual purchases. This monetary disequilibrium is nothing but the individual balance S_h (3).

  Note that this monetary balance differs from the value of real disequilibria which is the difference at market prices of desired and actual purchases.

The twofold aspect of individual disequilibrium implies two individual reaction to the gap between expected prices and market prices. The second one - that reaction to the monetary disequilibrium - is less familiar. It

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4 This implies that, in case of a negative market excess-demand, some individuals get quantities of commodity greater than desired. In order to avoid this situation, Cantillon's rule may be amended in assuming that quantities in excess supply are unsold. Some suppliers at least would accumulate non-desired inventories. The market value of these inventories would result in net positive monetary balances among other individuals. As Shapley and Shubik (1977) put it: "It is a matter of letting one's stomach rather thant one's purse absorb the fluctuations". p.(947).

expresses the way in which the reaction of the monetary authority modifies individual decisions.

- Starting from the observation of the market price at (t) the agent h expects a new price at (t + 1) according to:

$$P_{hl(t+1)} = \phi_{hl}(\pi_{l(t)}, \gamma) \qquad (12)$$

where γ is a parameter expressing the global reaction of the monetary authority.

- The second is the reaction of the monetary authority to individual disequilibrium:

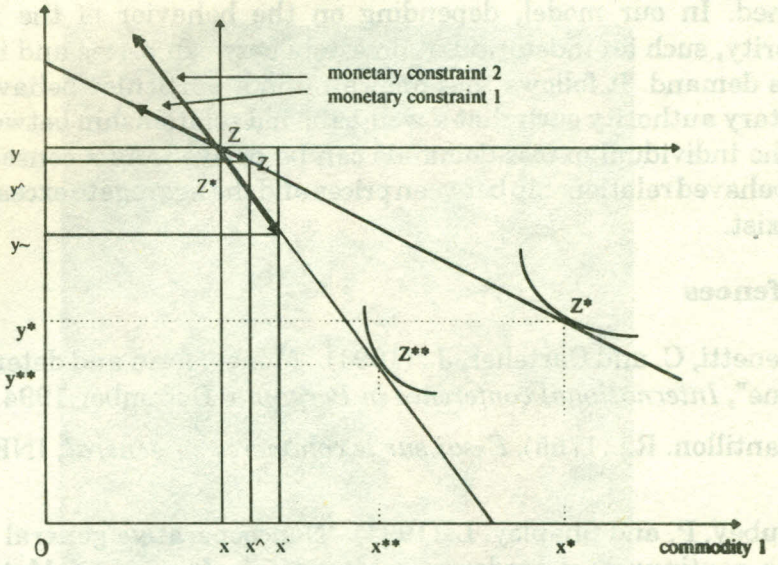
$$\pi_{h(l+1)} = \pi_{h(l)} + \alpha_h(S_{h(t)}, \gamma) \qquad (13)$$

Both functions is the reaction of the monetary authority to individual disequilibrium, hence the market dynamics.

From a general point view, two remarks are important:

(a) The change of prices is endogenous to the model since it is induced by the change of individual decisions as a consequence of individual disequilibria. Price changes because individuals change their decisions. This is in sharp constrast with the Walrasian rule according to which one must introduce a fictitious agent, the well-known auctioneer, in order to change prices. We conclude that, contrary to a well-established tradition, the auctioneer is not a necessary consequence of the competitive hypothesis, i.e. the price-taker hypothesis. In our model, the agents calculate their own actions. Nevertheless, the auctioneer is absent. His presence in the Walrasian general equilibrium theory must be related to the failure of this theory to explain the price formation.

(b) The market dynamics can a priori take any form, due to the variety of the reaction functions on which it depends. Accordingly, monetary regulation will play a crucial role. This point is being investigated on in order to verify if, for a general class of the reaction functions, a viable trajectory exists, i.e. which can be attained by the economy thanks to an appropriate monetary regulation. The following diagram illustrates this point.



For a given price and endowment **z**, agent h would have an allocation **z*** but, as a result of monetary constraint **1**, can only try $\hat{y}$ to get **z**. Let us suppose however that age **h** succeeds in selling commodity 2 but not in buying commodity 1 because market for commodity 1 is in excess demand. Agent **h** is located at **z'**, away from his/her budgetary constraint. Having buy back commodity 2 thanks to his/her surplus, agent **h** is again at **z**.

In the next market, price of commodity 1 raises relatively to price of commodity 2. This induces a decrease in notional demand for commodity 1, which is now **x****. But at the same time, monetary constraint changes. As a consequences of his/her preceding surplus, agent **h** is a able now to spend more. Monetary constraint 2 allows him/her to buy $\bar{\mathbf{x}}$ instead of $\hat{\mathbf{x}}$.

This very simple example shows that monetary adjustment may shape arbitrary individual behaviors.

This is an interesting property by comparison with general equilibrium. Sonenschein has shown that, even if the utility functions are well-behaved, any relationship between prices and aggregate excess demand may be

obtained. In our model, depending on the behavior of the monetary authority, such an indetermination exists between prices and individual excess demand. It follows that one can find a particular behavior of the monetary authority such that a well-behaved relationship between prices and the individual excess-demands can be derived. As a consequence, a well-behaved relationship between prices and the aggregate excess demand will exist.

Rerefences

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