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Feeding Large Econometric Models by a Mixed Approach of Classical Decomposition of Series and Dynamic Factor Analysis: Application to Wharton-UAM Model

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ABSTRACT

The aim of this article is to submit an applied methodology to design long term scenarios that, many times, are needed to feed large econometric models in their most classical approach of L.R.Klein's legacy. In our proposal we mix the classical decomposition of time series with dynamic factor analysis, which, in fact, is closely linked to most recent Klein's works on High Frequency Models. The methodological approach is illustrated through an application for the set of exogenous variables that shape the international environment of the Wharton-UAM model for the Spanish economy.

Keywords: Long-Term Forecast, World Growth, Structural Models, Dynamic Factor Analysis.

Alimentando grandes modelos econométricos mediante la combinación de la descomposición clásica de series y el análisis factorial dinámico: Una aplicación para el modelo Wharton-UAM

RESUMEN

El objetivo de este artículo es la presentación de un enfoque metodológico para diseñar escenarios a largo plazo que son necesarios para alimentar grandes modelos econométricos en su planteamiento más clásico recogido en la obra de L.R.Klein. En nuestra propuesta mezclamos la descomposición clásica de series con el análisis factorial dinámico, que enlaza nuevamente con los trabajos más recientes de Klein sobre los modelos de alta frecuencia. La propuesta metodológica se ilustra con una aplicación práctica de estimación a largo plazo del conjunto de variables exógenas que conforman el entorno internacional del modelo Wharton-UAM de la economía española.

Palabras clave: Predicción a largo plazo, crecimiento mundial, Modelos estructurales, Análisis factorial dinámico.

JEL Classification: C53, F01

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1. INTRODUCTION

In an ever-changing economy, forecasting has become an essential input for the performing of adequate scheduling activities, planning resource and the strategies for action. Certain activities that require long time of execution to come to fruition, such as infrastructure, the new power facilities, the social protection systems, environmental externalities, or even the long term planning in private business itself, call for the availability of quantified scenarios of evolution of the basic variables that define the volume of activity forecast for the future.

This need for quantitative forecasting faces serious limitations as regards the reliability of the forecasts, that tends to diminish as the forecast horizon is extended. The confidence intervals for forecasts widen progressively as one moves away from the observed period, both in causal models, as in univariate ones, at a rate that depends on the characteristics of the model, but that it is always highly positive correlated with the number of periods predicted.

The resulting loss of reliability in the forecasts leads to a conflict between the length of the forecast period and reliability, which is mostly resolved by reducing the forecast horizon to two or three years. For example, the IMF's World Economic Outlook in April 2012 provides forecasts for 2011, 2012 and 2013 in the paper edition, while the WEO database extends the horizon to 2017. As for the OECD, its 2012 Economic Outlook also limits its forecasts to 2013, like the IMF. However, the OECD from time to time gives longer-term forecasts, although not through the well-known Economic Outlook but through specific documents belonging to its OECD Economic Policy Papers series. These limitations, rather than suggesting a rejection of any forecast of more than four years in the future, warn of the increasing uncertainty in the forecasts the further we move away from the present.

The main goal of any forecast is to suggest the most likely future scenario according to today's available information, or making use of the Fame's terminology (1970) in his theory of efficient capital markets, the forecasts are efficient in a weak sense as they reflect the information available, together with the most likely future evolution, considered by the institution at the time that forecast are made. The simple occurrence of unanticipated factors, or those which by their nature cannot be anticipated, independently whether they are random or not, would alter the forecasts. The experience and professionalism of the forecasting institution is thus essential in such circumstances, because only these two qualities can ensure that all the relevant aspects are considered that may affect the scenario under which long-term forecasts are constructed.

In any event, despite the aforementioned limitations, the need to have such long-term scenarios does exist. It is thus necessary to have long-term forecast techniques for all the reference variables that conform the full scenario, that should be simple enough so that they can be implemented in the context of the specific models (it would have not sense to have a larger model for variables of framework than for the interest variables) and, at the same time, they must have the necessary congruence among the different variables used in each scenario.

The aim of this paper is to present some of these techniques applied in creating the benchmark international scenario that feeds the Wharton UAM model of the Spanish economy that has been used for over thirty years in The "L.R.Klein" Forecasting Institute at the Autonomous University of Madrid. To this end we will present, along with the methodological approaches, the main results and a brief analysis of them.

After this introduction, the second section explains different approaches to obtain long-term forecasts. In the third section, the methodological foundations of the techniques used are presented. The fourth section then describes the specific applications used. The fifth makes a preliminary analysis of the main results obtained, and the paper ends with a section about the main conclusions drawn.

2. ALTERNATIVE APPROACHES

There are currently several institutions, both public (IMF, UN, World Bank, OECD, European Commission), and private (Oxford Economics, Global Insight, WEFA, INFORUM), that regularly draw up and publish forecasts on the evolution of the main variables that define the international economic environment using different models and methodological approaches (e.g. Pulido, 2005). While it is true that, as we have said, these institutions regularly draw up and disseminate their forecasts about the evolution of the international environment's main variables, it is also true that these forecasts are generally restricted to a short-term environment, or at most a medium-term one.

The wide range of forecasting techniques provided by the applied literature ranges from strictly qualitative techniques to more complex variants of quantitative data extrapolation made using large databases of time series data. (Pulido and López, 1999).

Focusing on forecasting techniques that use statistical series to quantify the future, we can differentiate between those that only feed on the known data of the variables we want to provide, and that we can identify with a more or less evolved techniques of time-series analysis, both univariate or multivariate, and the others that use the causal relationships between variables that make up the system to be analysed, where we could include different variants of structural models (Econometric Models, Input-output type models or computable general equilibrium models). In turn, within the latter category we can differentiate between models that require exogenous information and those that incorporate all the variables of interest endogenously.

Obviously, in an ideal situation, any predictive model should endogenously involve all the variables of interest because in one way or another all the variables integrated in one system have a certain level of interdependence.

In practice, however, this approach would not work at all since, for instance, in order to forecast the demand for a particular product the predictive model would have to include the forecast evolution of the income of the potential clients of said product. This would mean that you should include every macroeconomic conditions specific to these clients' country joint with all of the conditions from the international environment of the country in question. (ie. you should develop a macroeconomic model for each microeconomic model constructed). For this reason, the usual situation is that the different predictive models focus on the subsystem closest to the phenomenon one intends to forecast and exogenously incorporate the future evolution of the variables that determine the most global environment in which the subsystem is located. This scheme of nested forecasts requires having forecasts of the variables' future that affect their referential environment with the same frequency and length of time with which we aim to make forecasts for our specific subsystem. So, if our needs fall in the range of long term annual macroeconomic forecast there is not too many sources to feed this kind of models.

Apart from the aforementioned IMF and OECD[1], through documents in its OECD Economic Policy Papers series, one of the few organizations that publish long-term forecasts, and the methodology used for them, is The Conference Board in its Global Economic Outlook (http://www.conference-board.org/data/globaloutlook.cfm), which gives forecasts up to 2025 in its 2012 edition. This institution, like the OECD, uses causal modelling of production in the medium and long term in a theoretical framework similar to the Solow model (extended with human capital and the public sector). In this, the forecast for GDP is made based on different forecasts of the factors included in this production function.

A brief description of the method used by this institution[2] will enable us to show the most relevant limitations found in this type of approach. Specifically, the Conference Board's method falls within the approximation based on growth accounting, according to which an economy's GDP (Y_{it}) can be represented by a function of production:

$$Y_{it} = A_{it} F(L_{it} Q_{it}, K_{it})$$

Where: Y_{it} is the GDP of a country i in year t; L_{it} is the total employment; Q_{it}

[1] OECD (2012). Looking to 2060: Long-term global growth prospects. A global for growth report. OCDE Economic Policy Papers, nº 3. November 2012

[2] Vivian Chen, Ben Cheng, Gad Levanon and Bart van Ark (2012). Projecting Economic Growth with Growth Accounting Techniques. The Conference Board Global Economic Outlook 2012. Sources and Methods

the composition of the labour force by educational level; K_{it} are the stock of capital; and A_{it} is the total factor productivity.

As we can clearly see, the long-term forecast for GDP necessarily involves forecasting employment levels, education level composition, capital investment, and the even more elusive total factor productivity. So, the uncertainty in forecasting the explanatory variables and the cost of the procedure increase with the number of variables used in modelling the target variable we wish to forecast, which, in this case, is the GDP.

With the macroeconomic models of a particular country, such as the Wharton-UAM model for the Spanish economy that will be used to illustrate the application we are presenting in this paper, the usual practice is to incorporate variables related to the world economy as a whole, whose forecasts are found from more or less detailed global models. These world economy variables define a global growth scenario that influences the set of domestic relationships included in the Wharton-UAM model for the Spanish economy.

The methodological approach adopted in defining this long-term global scenario, that will be developed in following sections, is based on the general scheme of decomposition of time series into their fundamental components linked to the trend, or supporting components of the series in the long term, and the cyclical component, which defines the fluctuations of the variable around the long-term trend support. Although this approach is methodologically complex to specify, it is operationally simple enough as it does not require any external information to be fed because the initial estimations only depend on deterministic trend variables. Additionally, the establishment of relationships between components of the different variables included in the full scenario ensures an adequate level of congruence among them.

3. METHODOLOGICAL APPROACHES

The hypothesis of underlying components (SCH) is the starting point for the time series decomposition analysis. As the components are unobservable, a set of more or less complex techniques (based on different hypothesis) have been designed to distinguish and/or estimate each of the components. A description of some of the traditional methods for decomposition of series can be found in Uriel (1995), while in Fischer (1995) there is a brief review of the history of decomposition methods as well as an interesting description and comparison of the different procedures used by Eurostat to decompose time series. Other examples where the matter is reviewed are: Hylleberg (1992), and Bell and Hillmer (1992), although as this is a topic that has been dealt with regularly for a long time, there are many other works on the subject.

In this paper, the approach is pragmatic and strictly focused towards long-term prediction, letting outside other possible alternative aim. Its aim is applying techniques that enable estimates to be made of the time series components that can be easily extrapolated to the forecast period. Having this in mind, only the cyclical and trend components are of interest, since seasonality only appears in series observed more than once a year and irregular component is by nature unpredictable in the long run.

The conditions indicated imply that the most suitable method may be to apply the UCARIMA methodology (Unobserved Components in ARIMA models expressed in a reduced form) specified in Quilis (1997), Maravall (1984, 1986,1987,1988,1989), Bell & Hillmer (1992), Hillmer & Tiao (1982), Gómez & Maravall (1998), and others. The UCARIMA methodology ("unobserved ARIMA components model") assumes that both the observed series and the unobservable components respond to ARIMA models. The advantage provided by this method is tied to the estimate-prior specification of a model to the observed series which solves the problems of adapting the filtering to the nature of the series. Moreover, this method enables us to statistical measurements of confidence in the forecast obtained, and getting forecasts for each of the components, thus making it easier to obtain forecasts for the variable analysed for any period.

Despite the advantages of this method, its univariate nature can not collect relationships among different variables and components which may be decisive in the evolution of variables along the forecast period. Since the objective of this paper is to put forward an international macroeconomic scenario, for instance, it would be difficult to justify not using the information from countries in the same economic area when forecasting cyclical components. So, the proposed methodology combines univariate models for cyclical and trend components, with multivariate models for the cyclical component itself.

In a very stylized form, Table 1 summarizes the main steps composing the methodological proposal while in the following section shows in greater detail the procedure followed in each group of variables and the results from the forecast in each case. As it is shown in Table 1, the system proposed starts with the estimation of the trend components by using OLS estimation over one of the three alternative trend functions (linear, quadratic, or diffusion "S" type process).

We consider three alternative models for the trend component.

First, a linear trend of the type:

$$Y_{it} = a_i + b_i t + CI_{it} \qquad [1]$$

where t is a deterministic trend variable and CI is the Cycle-Irregular component. Second, a convex quadratic trend, whereas in the second period it was more like a concave quadratic trend such as:

$$Y_{it} = a_i + b_i t + c_i t^2 + CI_{it} \quad [2]$$

Finally, an "S- type" curve with an early period of slower growth, followed by strong expansion that tends to slow at the end of the forecast horizon, so that in the end it was decided to estimate a Bass-type model (1969). This responds to the expression shown below, where the growth rates tend to become more moderate the further we advance through the forecast horizon.

$$Y_{it} = a_i + b_i Y_{it-1} + c_i Y^2_{it-1} + CI_{it} \quad [3]$$

The estimated functions are used to obtain and forecast the trend components while in the second step, the cycle-irregular components are obtained through differences between original variables and its trends.

Once the cyclical components have been computed it is necessary to forecast each of them but having in mind that there are some interactions (correlations), among them.

Since the first applications developed by Stock y Watson (2002a) and (2002b), dynamic factor analysis offers us a good alternative to get these forecasts which keep the historical correlations into the future, as it is shown in more recent literature. Dahl *et al.*(2009), van Ruth (2014), Bańbura *et al.*(2015).

In a very stylized form the dynamic factor model[3] can be expressed as:

$$\begin{aligned} Y_{i,t} &= \Gamma F_{j,t} + u_{i,t} \\ F_{j,t} &= \frac{\Theta(L)}{\left[1-\Phi(L)\right]}\varepsilon_{j,t} \end{aligned} \quad [4]$$

Where the set of i variables of interest Yi can be obtained by linear combination of a set of j (j<i) common factors, each of one following a specific ARIMA data generation process.

Although there are several alternative procedures to estimate this kind of models we have opted for a simple eclectic approach with three steps (see steps 3 to 5 in table 1).

In the first step we obtain the historical values of unobserved common factors by mean of a Principal Components Analysis applied to the set of cyclical-irregular components, current and lagged.

$$F_{j,t} = \lambda Y_{i,m} \quad m = t, t-1, t-2, t-3 \quad [5]$$

Where the loading matrix λ is typically obtained in way where the subsequent factors retain the larger possible share of common variance and all are perfectly uncorrelated.

In the second step each of these common factors is forecasted using a speci-

[3] See Stock and Watson (2011) for s good survey on dynamic factor models.

fication that includes both deterministic periodic functions as well as ARMA components.

$$F_{j,t} = \sum_f \beta_{f,j} DP_{f,t} + \sum_p \phi_p Ar(p) + \sum_q \theta_q Ma(q) \quad [6]$$

Being $DP_{f,t}$ the deterministic periodical functions.

Finally in the third step, the forecast for cyclical components are obtained using forecasted common factors together with ARMA terms in a general specification like:

$$C_{i,t+r} = \alpha + \sum_{j=1}^{n\_f} \beta_j F_{j,t+r} + \sum_p \phi_p Ar(p) + \sum_q \theta_q Ma(q) \quad [7]$$

Determinist periodic functions DP_t are those that repeats its values each p observations and can be expressed as:

$$DP_t = A * cos\left(\frac{2\pi t}{p} + \theta\right) \quad [8]$$

Where A is the amplitude of oscillation, p is the length of the cycle, and θ is the gap (i.e. distance from first observation to first peak).

Alternatively, it can be expressed in a linear form that can be consistently estimated by OLS, like follows:

$$DP_t = a * cos(\varpi_o * p * t) + b * cos(\varpi_o * p * t) \quad [9]$$

Where ω_0 is the so-called Basic Frequency that is equal to:

$$\omega_o = \frac{2\pi}{N} \quad [10]$$

Being N the total number of observations.

At this point, the key problem is to identify the right set of deterministic functions to be included in each factor specification (i.e select the set of relevant p values)

To do that, we have used a discrete periodogram tool based on the Fourier Frequencies. This periodogram represents in the X-axis the set of all possible p_i Fourier frequencies that can be computed in an N size sample (i=1,2,….k, with k=N/2), and in Y-axis, the relative contribution to the total variance of the analysed time series of each p_i frequency $I(\omega_p)$ computed as:

$$I(\omega_p) = \frac{(a_p^2 + b_p^2)}{2\omega_0} \quad [11]$$

Where a_p and b_p are the coefficients linked to each frequency which can be estimated by OLS in a general model like:

$$Y_t = \sum_{p=1}^{k} a_p * cos\left(\varpi_o * p * t\right) + b_p * cos\left(\varpi_o * p * t\right) \qquad [12]$$

Finally, in the last sixth step shown in Table 1, the variables of interest are forecasted by adding the cyclical component to the forecasted trend.

Table 1
Summary of the methodological approach

Step	Target	Specification
1	Estimation and forecasting of Trend components:	
	Linear Trend	$Y_{it} = a_i + b_i t + CI_{it}$
	Quadratic Trend	$Y_{it} = a_i + b_i t + c_i t^2 + CI_{it}$
	Diffusion process	$Y_{it} = a_i + b_i Y_{it-1} + c_i Y^2_{it-1} + CI_{it}$
2	Obtaining the cyclical components	$CI_{i,t} = Y_{i,t} - T_{i,t}$
3	Extracting common cyclical factors	$F_{j,t} = f\left(CI_{i,t}, CI_{i,t-1},, CI_{i,t-2}, CI_{i,t-3}\right)$
4	Forecasting common cyclical factors	$F_{jt} = \sum_f \beta_f sen(2\pi f / Nt) + \beta'_f cos(2\pi f / Nt) + \sum_p \phi_p Ar(p) + \sum_q \theta_q Ma(q)$
5	Forecasting cyclical components	$C_{i,t} = \alpha + \sum_{j=1}^{n\_f} \beta_j F_{jt} + \sum_p \phi_p Ar(p) + \sum_q \theta_q Ma(q)$
6	Forecasting of the variables aggregated	$Y_{i,t} = T_{i,t} - C_{i,t}$

Source: Own elaboration.

4. ESTIMATION OF THE INTERNATIONAL BASELINE SCENARIO FROM THE WHARTON-UAM MODEL

Once the main methodological characteristics of the different techniques used have been presented, we will now present the specific application developed, which as we mentioned in the introduction, focuses on the construction of the benchmark international scenario for the Wharton-UAM model of the Spanish economy.

As stated by Pulido & Pérez García (2001), the Wharton-UAM model is used for analysing and forecasting for the Spanish economy. It has been developing for about thirty years in the Institute for Economic Forecasting "L.R.Klein" at the University Autonomous of Madrid, and has been used since the eighties to provide forecasts and macro-economic consultancy to several tens of companies and public and private institutions that make up the Economic Forecasting Centre Association CEPREDE. This model has been re-estimated and updated continuously from its beginning to the present, and its actual version has a sample period spanning from 1970 to 2012 using homogeneous series from the National Accounts Database (2008 basis) , and it is generating forecasts up to 2030. To make these long-term predictions is required to have forecasts, duly quantified for the forecast period, for the full set of exogenous variables that are detailed below in Table 2.

Table 2
List of exogenous variables from the Wharton-UAM model

Area	Variables	Details
International environment		
Economic Activity	GDP volume indices	World total, Developed Economies; U.S.A., Japan, EU, Germany, Austria, Belgium, Denmark, Finland, France, Greece, Holland, Ireland, Portugal, Sweden, the United Kingdom, the Other Developed Economies, and Developing Economies: Africa, Eastern Europe and CIS, Asia, Latin America.
	World trade volume index	
Prices	C.P.I.	OECD, Monetary Union, USA, Japan, the UK.
	Commodity prices	Oil, Agricultural Products, Metal Products.
	Export prices	OECD, USA, Japan, European Union Total, Germany, Austria, Belgium, Denmark, Finland, France, Greece, Holland, Ireland, Portugal, Sweden, United Kingdom, Developing, Latin America.
	World trade deflator	
Exchange rates	Dollar references	Euro, Yen, Pound Sterling
Interest rates	Short-term interest rates	Monetary Union, USA, Japan, the UK.
	Long-term interest rates	Monetary Union, USA, Japan, the UK.
Domestic Environment		
Demographics	Population by age and sex	Total for men and women of 15 to 19 years of age, 20 to 25, 26 to 55, and 55 years and over.
Economic policy	Public Investment	Real Investment volume and rate of depreciation.
	Tax Rates	Income taxes, capital taxes, social security contributions; indirect taxes; taxes on imports.

Source: Own elaborations.

4.1. Forecasting the economic activity variables

As seen in Table 2, the set of exogenous variables that reflect the volume of economic activity is composed by the real GDP growth rates grouped into large geographic areas, together with the details of the main European countries and the world trade volume index.

As happened with the other variables in the international environment, the database used is the one provided by the IMF in April 2011, which contains historical data from 1980 to 2010 and forecasts until 2016.

Given that the Wharton-UAM model has been estimated on an annual database starting in 1970, it was first necessary to make a backward projection of the series till 1970, making use of the Historical database of the IMF itself.

In order to maintain the congruency of the short-term international scenario, the forecasts made by the IMF for the whole available horizon will be maintained, in this case until 2016. This also allows the short-term fluctuations caused by more recent events (debt crisis, fiscal adjustment etc.) could be collected properly.

Thus, for the practical purposes of our forecasting exercise we will consider as sample data the entire period between 1970 and 2016.

The process begins with the projection of the GDP volume indices for large geographic areas (U.S.A., Japan, European Union, other developed economies, Africa and the Middle East, Developing Asia, CIS and Eastern Europe, and Latin America and Caribean), using a classic series decomposition approach, in this case trend, cycle and irregular component.

For each of these regions, the best trend specification was identified using sample data available, finding that, as a general rule, the large developed economies (Japan, USA and the European Union) responded to a linear trend.

Eastern Europe & CIS and Latin America showed a structural change in the registered trends, in the former case as of the second half of the nineties as a result of the process of transition to market-oriented economies, and in the latter case as of the eighties, when large investment flows began to arrive towards Latin American countries.

Thus, during the first period, in Eastern Europe & CIS there was a convex quadratic trend, whereas in the second period it was more like a concave quadratic trend.

In Latin America, the first period showed a linear trend, whereas during the second period, as in other developing economies, the trends seem to follow an "S- type" curve with an early period of slower growth, followed by strong expansion that tends to slow at the end of the forecast horizon, so that in the end it was decided to estimate a Bass-type model (1969).

The results obtained in this first trend fitting are summarized in the Table 3.

In Table 3, together with the basic statistics from the estimated regression, one can see the unit root contrasts (ADF) for the residues from this regression. In general, they confirm that these residuals are stationary and thus the trend component has been completely eliminated. Given that our main objective is to project the trends, the relevant statistics are those that correspond to the R^2 determination coefficient, assuming that the non-significance associated with the coefficients δ of the Bass model estimates is caused by the high correlation between the explanatory variables used (Y_{it-1} and Y^2_{it-1}). Once the various equations have been estimated, the trend components are extrapolated using the estimated coefficients for the rest of the forecast period 2017-2030.

Table 3
Basic results from the trend setting

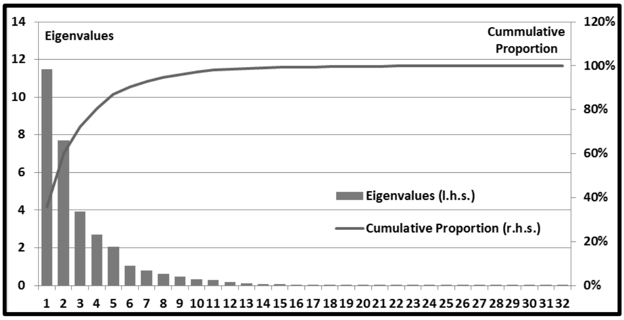
Regions	Period	Estimated coefficients			R^2	Dw	ADF
		a	b	c			
USA	1970-2016	24.95 (0.00)	1.85 (0.00)		0.984	0.19	-2.3 (0.02)
Japan	1970-2016	36.62 (0.00)	1.62 (0.00)		0.941	0.1	-1.92 (0.05)
EU	1970-2016	36.46 (0.00)	1.51 (0.00)		0.989	0.29	-2.51 (0.01)
Eastern Europe and CIS	1970-1995	23.14 (0.00)	4.44 (0.00)	-0.12 (0.00)	0.942	0.33	-2.07 (0.04)
	1996-2016	-15.49*(0.58)	1.78 (0.26)	0.02 (0.23)	0.983	0.71	-2.73 (0.01)
Latin America	1970-1982	22.87 (0.00)	2.03 (0.00)		0.994	1.09	-2.33 (0.03)
	1983-2016	-2.63 (0.49)	1.07 (0.00)	-0.0001 (0.09)	0.995	1.98	-2.08 (0.04)
Other developed countries	1970-2016	0.43 (0.59)	1.05 (0.00)	-0.0002 (0.81)	0.998	2.16	-1.99 (0.05)
Africa	1970-2016	-0.12 (0.89)	1.03 (0.00)	0.0001 (0.82)	0.999	0.92	-0.58 (0.45)
Asia	1970-2016	-0.43 (0.07)	1.09 (0.00)	-0.00009 (0.85)	0.999	1.26	2.18 (0.98)

Note: the probability associated with the test statistic (t-student and MacKinnon) is shown in brackets.

Source: Own elaboration.

Using the difference between the original series and the trend component, the cycle and irregular components are obtained for each of the variables for the sample period 1970-2016. The projection of these cycle-irregular components is made by applying the dynamic factor analysis, thereby ensuring the extrapolation into the future of the possible cyclical co-variations observed in the past.

Figure 1
PC analysis for cyclical components



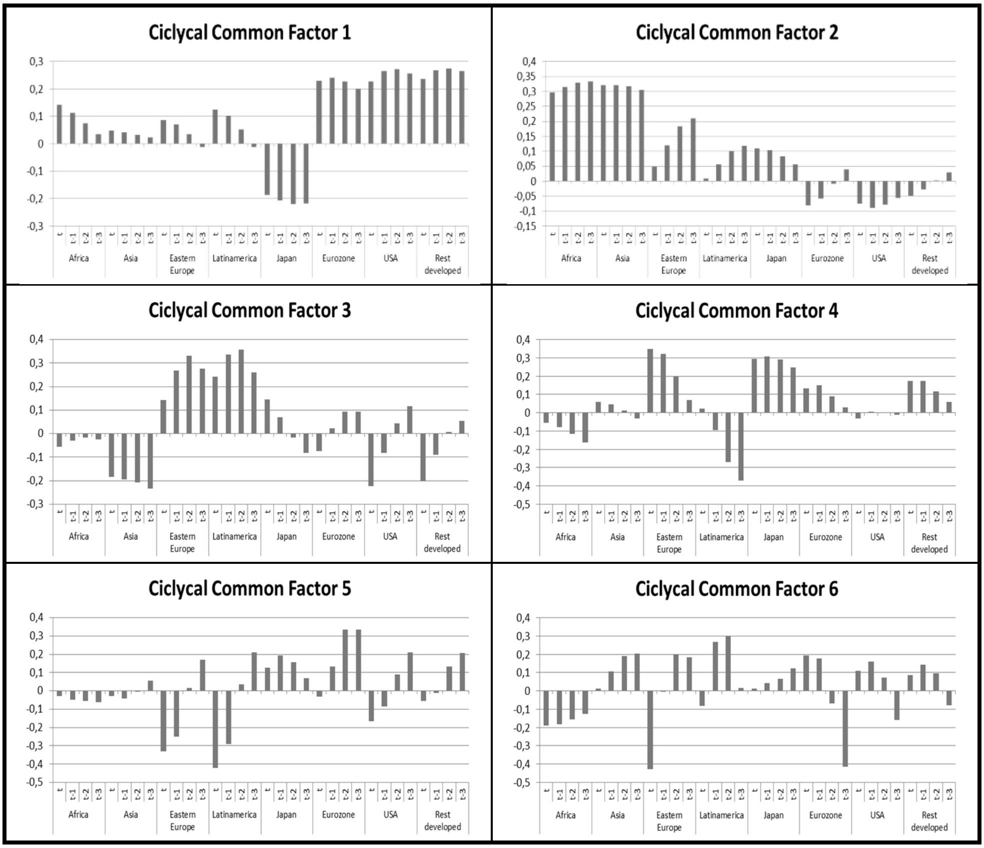
Source: Own elaboration.

As it was stated in previous section, to apply this procedure, a principal component analysis is initially carried out on the entire cycle-irregular series of the different regions r (CI^r). This PC analysis is carried out using the current period, as well as the lagging series up to 3 periods ($CI^r{}_t$, $CI^r{}_{t-1}$, $CI^r{}_{t-2}$, $CI^r{}_{t-3}$), in order to better capture the effects of cyclic advance or delay among economies. From the full set of components (factors) obtained they are selected those that have eigenvalues greater than one, in this case 6, and which together account for 89% of the total variance.

If we look at the figures shown in Figure 1, we could retained just the first 5 components, because there seems to be a jump between the eigenvalues of factors 5 and 6, nevertheless we opted for the most common alternative of retain eigenvalues larger than 1.

In following figures we have represented the loading matrices for each factor that can help us to interpret them.

Figure 2
Loading matrices for cyclical components



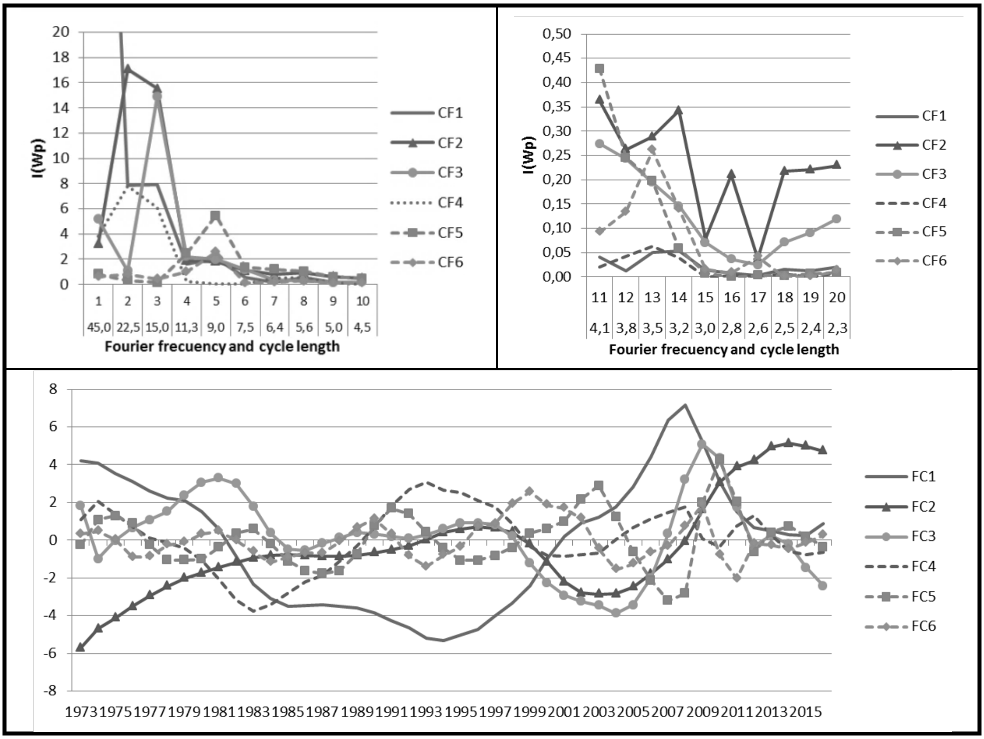
Source: Own elaboration.

As can be see easily, the first two factors are related to specific cycles of developed and emerging economies respectively, while the third seems to collect some fluctuations in the former “transition economies” and Latin America. Finally the last three factors are more undefined and must be related to other irregular factors.

Once the factors have been computed, an analysis is performed in the frequency domain of the six selected factors, identifying the most relevant frequencies contributing to the total variance of each of the factors through a discrete periodogram based on the Fourier frequencies.

The results of these analyses are summarized in following graphs where we have presented the discrete periodogram[4] (frequency domain representation) together with time series (time domain representation) for the six factors.

Figure 3
Selected cyclical components



Source: Own elaboration.

Finally, a regression adjustment was carried out for each of the factors, incorporating as explanatory variables the cyclical deterministic series with the

[4] To ease the visualization we have split the periodogram into two different graphs with low and high frequencies respectively because absolute values are much more larger in the first ones (see graph scales).

most relevant frequencies together with some autoregressive and moving average components to fit the irregular component

The results of these estimates are given in Table 4 below. From left to right, and for each factor (Fi) in rows, are shown the frequencies with a greater relative contribution to the total variance factor, the length (in years) of associated cycle with each of these frequencies and the relative contribution of each of them $I(w_p)$, as well as the order of the autoregressive and moving average processes identified together with the Total Determination Coefficient (R^2) of each regression[5].

Table 4
Basic Results from the cyclic factor adjustment

	Frequencies	Cycle duration	Contribution $I(w_p)$	AR	MA	R^2
F1	1;2;3	45.0;22.5;15.0	55.1;8.5;1.2	2		0.994
F2	1;2;3.5	45.0;22.5;15.0;9	9.7;14.9;16;1;3.0	2		0.968
F3	1;2;3.4;5	45.0;22.5;15.0;11.2;9	18.9;5.7;4.5;1.2;1.2	1	1	0.980
F4	2;3	22.5;15.0	6.1;6.6	1	1	0.932
F5	4;5	11.2;9	2.4;5.5	2	1	0.837
F6	5;13	9;3.5	2,5;0.26	1	1	0.745

Source: Own elaboration.

As we can see in the above table, the regressions show good levels of fit and, as seen in the set of graphs in the Annex[6], the first factors are generally associated with longer-term cyclical components (long cycles). Once the equations for each of the factors have been estimated, the specific cyclic components of each of the large regions being considered are then estimated using a mixed specification that includes the common factors as well as autoregressive and moving averages components.

Table 5 shows the basic results obtained in estimating the cyclical-irregular components of the large areas analysed. Together with the values of the main coefficients estimated and its p-values for individual significance, the order of the ARMA process estimated and the Total Determination Coefficient R^2 are also shown.

As for the trend component, the cyclical-irregular component is projected into the future by using the estimated equations together with the projection of the different common factors. Finally, by adding the trend component and the projected cyclical-irregular component can be obtained the corresponding esti-

[5] Factors are normalized variables, that's why we have not used error measures based on ratios between errors and levels because they have upside bias when average levels tend to cero and they are not too much informative.

[6] Only available in the electronic version of the paper. http://www.revista-eea.net/

mates for the future for the GDP volume indices of the different regions being considered.

Table 5
Basic results for the cyclical-irregular component adjustment

	Estimated coefficients							AR	MA	R^2
	α	β1	β 2	β 3	β 4	β 5	β 6			
USA	-0.44 (0.00)	0.27 (0.00)	0.84 (0.00)	0.28 (0.00)	-0.49 (0.00)	-0.73 (0.00)	-0.29 (0.05)	2		0.947
Japan	-0.59 (0.15)	-1.56 (0.00)	0.08 (0.59)	-0.16 (0.28)	0.88 (0.00)	-0.53 (0.00)	-0.92 (0.00)	1		0.971
EU	-0.19 (0.03)	0.28 (0.00)	0.65 (0.00)	0.06 (0.10)	-0.02 (0.72)	-0.44 (0.00)	-0.27 (0.01)	2		0.947
Other developed	-1.62 (0.00)	-0.46 (0.00)	0.32 (0.03)	0.25 (0.10)	-0.29 (0.08)	-0.19 (0.14)	-0.32 (0.14)	1		0.824
East Europe & CIS	-0.06 (0.82)	-0.02 (0.86	0.17 (0.12)	-0.10 (0.40)	0.86 (0.00)	-1.29 (0.00)	-1.01 (0.00)		1	0.865
Latin America	0.89 (0.00)	-0.00 (0.97)	-0.03 (0.52)	0.61 (0.00)	0.26 (0.00)	-0.20 (0.00)	0.87 (0.00)			0.885
Africa	3.69 (0.00)	0.85 (0.00)	0.09 (0.61)	-0.19 (0.30)	0.18 (0.31)	-0.30 (0.01)	-0.37 (0.02)	1	1	0.966
Asia	5.14 (0.00)	0.77 (0.00)	-0.33 (0.00)	1.67 (0.00)	0.40 (0.03)	-0.19 (0.01)	-0.73 (0.00)	2	4	0.989

Note: the probability associated with the test statistic (p-value) is shown in brackets.
Source: Own elaboration.

Once the GDP volume indices of the large aggregates have been projected, and in order to obtain projections for the world total and for the aggregates of developed and developing economies, we estimate the weights for each of the regions over world GDP (Q^r_t) by means of a two-stage process. In the first stage, an initial weight ($I\_Q^r_t$) is estimated by using the dynamics observed in the GDP volume indices (GDP^r_t) and in the second stage the weights are re-scaled to ensure the sum unit of them by applying a formulation such as:

$$I\_Q^r{}_t = I\_Q^r{}_{t-1}(GDP^r_t / GDP^r_{t-1}) \quad [13]$$

$$Q^r{}_t = I\_Q^r{}_t / (\textstyle\sum_r I\_Q^r{}_t) \quad [14]$$

By applying these estimated weights to the growth rates of the GDP volume index of the different regions, the world GDP growth and that of the aggregated of developed and developing economies are finally obtained. To project the values of the GDP volume indices for the major economies of the European Union (Germany, Austria, Belgium, Denmark, Finland, France, Greece, Holland, Ireland, Italy, Portugal, Sweden and the UK), a factor analysis is used again.

The most relevant common factors observed in the GDP growth rates of each of these economies are extracted (this time three factors that account for

79.7% of the total variance). These common factors are then projected into the future using as explanatory variables, both the GDP growth rate of the European Union as a whole, as well as the common cyclical factors obtained in the previous stage together with some autoregressive components, as seen in the following table, where we present the estimated coefficients, with their p-values in brackets, and the Total Determination Coefficient (R2).

Table 6
Specification of the common factors for European economies

Factors	Specification	R^2
$F^{EU}1$	$-3.67 + 1.6 * \Delta\ GDP^{EU}_t + 0.39\ AR(1)$ (0.00) (0.00) (0.01)	0.971
$F^{EU}2$	$0.23 - 0.69 * F6_t\ -0.19 * F4_t$ (0.51) (0.05) (0.01) (includes dummy variables for 1976,1977 and 1991)	0.602
$F^{EU}3$	$-0.12 * F2_t\ -0.23*F3_t\ +0.23*F4_t + 047*F6_t$ (0.00) (0.00) (0.00) (0.00) (includes dummy variables for 1974 y 2010)	0.609

Source: Own elaboration.

Once the corresponding predictions of these three factors have been made with the estimated equations, the GDP growth rate for each of the country members is then estimated. To do so, we use a similar specification to [5] including the common factors as explanatory variables along with autoregressive and moving averages components, obtaining the results shown in Annex III[7].

To conclude this section on activity variables, we estimate a new equation to make a future projection for world trade volume (WTV). Here, the growth rate of this index is made to be dependent on a constant term, on the growth rates of the developed and developing economies as a whole (GDP^{DVD} and GDP^{DVG}) and a second-order autoregressive component, obtaining the results summarized below.

$$\Delta WTV_t = \underset{(0.00)}{-0.06} + \underset{(0.00)}{1.99}*\ \Delta GDP^{DVD}_t + \underset{(0.00)}{1.35}*\Delta GDP^{DVG}_t + \underset{(0.06)}{0.33}*AR(1) + \underset{(0.06)}{0.34}*AR(2)$$

R^2: 0.839

4.2. Forecasting price variables

This second group of variables includes the consumer price indices, commodity prices, export deflator and the deflator of world trade volume. To deal with the projection of this set of variables, we have followed a similar approach to the one used with the GDP volume indices. First of all, we have created an initial set that we call basic prices, which includes the prices of raw materials

[7] Ibid.

(oil, agricultural products, metal products, food and beverages), the world trade deflator, and export prices for developed and developing countries).A factor analysis is carried out on this basic price group, selecting the first three factors (which together account for over 98% of the total variance), and where the first one shows the general trend of prices, and the remaining two collect the cyclical fluctuations around this trend.

The next step is the forward projection of each of these common factors of basic prices by estimating of specific regressions for each of them, taking into account that, in this case, the sample period is limited to 1970-2011, given that the IMF does not publish detailed forecasts of the different price variables. To model each of these factors, we use a mixed specification that includes the following as explanatory variables: global activity levels, deterministic cyclical components and autoregressive and moving average elements, as shown in the following table:

Table 7
Specification of the common factors for basic prices

Factors	Activity	Cyclical elements	AR	MA	R^2
FPB1	GDP^{WOR}	Frequencies: 1 and 3 Cycles: 40.0 and 13.3		1	0.976
FPB2	ΔGDP^{DVD}; ΔGDP^{DVG}	Frequencies: 1 Cycles 40.0	1	2	0.903
FPB3		Frequencies: 1,2,4 and 5 Cycles: 40.0, 20,10,8	2	3	0.975

Source: Own elaboration.

According to the data presented in the above table and the graphs shown in Annex IV[8], the levels of fit for these factors for the basic prices would be quite suitable and they are the basis for making the future projection for these three factors, this time from 2012 to 2030. Based on the projections of these common factors of basic prices we may obtain the corresponding estimates for each of the price variables using regression analysis where the explanatory variables are the common factors together with autoregressive and moving average components. Details of the results can be seen in Annex V[9]. After projecting the basic price variables, the rest of prices indexes, grouped in two big sets, are estimated. The first group includes all the consumer price indices considered (OECD average, European Union, USA, Japan and the UK) while the second one contains, the major European economies' specific export prices (Germany, Austria, Belgium, Denmark, France, Greece, Holland, Ireland, Italy, Portugal, Sweden and the United Kingdom).

In each of these two groups, once again we apply the dynamic factor analysis to the annual growth rates, obtaining the most relevant factors (3 in both

[8] Ibid.
[9] Ibid.

cases). Once done, we specify a projection equation for each of them, using as explanatory variables: activity variables, basic price variables, deterministic cyclical components and stochastic components (processes autoregressive moving average), as shown in Table 8.

Using these estimates, whose results graphs are shown in Annex VI[10], each of the factors for consumer prices and for export prices are projected into the future. Afterwards, these factors are used as a basis for estimating and projecting the specific variables, following a general specification similar to previous cases ones, details of which are given in Annex VII[11].

Table 8

Specification of Common factors for consumer and export prices

Factors	Variables	Cyclical components	AR	MA	R^2
Consumer prices					
FPC1	FPB2, Trend		1	1	0.882
FPC2	FPB3;	Frequencies: 1,2,4 Cycles 40,20,10	2		0.809
FPC3		Frequencies: 4 Cycles: 10	1		0.546
European countries' export prices					
FXP1	ΔPX^{EU}		1		0.976
FXP2	FPB2; FPB3			1	0.480
FXP3	FPB1		3		0.221

Source: Own elaboration.

4.3. Forecasting interest rates and exchange rates

The last group of variables in the international environment of the Wharton-UAM model includes interest rates in the short and long term in the main developed economies (Monetary Union, USA, Japan and United Kingdom) and the exchange rates of the dollar against Euro, Yen and Pound Sterling. Once again, these two sets of variables are projected by using dynamic factor analysis, obtaining the most relevant factors from each of the two groups, projecting each of these factors into the future and obtaining the prediction in terms of the original variables based on said factors. For the group of interest rates, three factors are used which together account for 95% of the total variance, whereas for the exchange rates only the first two factors are selected accounting for over 96% of the total variance.

The equations for estimating the interest rate factors use as explanatory variables the common factors of consumer prices (FPCi) and the factors of the cyclical components of growth by region (Fi) together with a stochastic component. For the two factors from the exchange rates group, the factors extracted

[10] Ibid.

[11] Ibid.

from the interest rates group are included as explanatory variables in addition to the consumer price factors and cyclical components. The table that follows lists the basic specifications of the various factors, while Annex VIII[12] shows the graphical results of the forecast carried out.

As on previous occasions, once the various equations of the factors have been estimated, we can project them into the future. On this occasion, as happened with the price variables, the prediction horizon begins in 2012 because the IMF neither provides detailed forecasts of these variables.

Finally, forecasts are obtained for the original variables using a specification similar to the previous ones where the factors from each of the groups act as explanatory variables together with a stochastic component, as seen in Annex IX[13].

Table 9
Specification of the factors of interest rates and exchange rates

Factors	Prices	Activity	Interest rates	AR	MA	R^2
Interest rates						
FTI1	FPC1, FPC2; FPC3	F1,F2,F3,F4,F5			2	0.946
FTI2	FPC1, FPC2; FPC3	F1,F2,F3,F4,F5				0.731
FTI3	FPC1, FPC2; FPC3	F1,F2,F3,F4,F5				0.735
Exchange rates						
FTC1	FPC1	F4	FTI1		1	0.769
FTC2	FPC1, FPC2; FPC3	F1,F2,F4	FTI1, FTI3			0.816

Source: Own elaboration.

5. COMMENTS ON THE 2015-30 PROJECTIONS

This section discusses some of the main features of the forecasts carried out for purely descriptive purposes. We are aware that a detailed analysis of all the forecasts obtained would lead to an extension that would clearly go beyond the limits of this article. Starting with the aggregated total of world GDP, the final result from the scenario calculated would give an average growth rate of around 4%, similar to the one seen in recent years, with a cyclical peak around the year 2017. There would also be a general acceleration around 2020 for developing economies, especially in Africa, which would accelerate their growth rate while in the other emerging economies the growth rate would stabilize or fall slightly compared to the rates seen in recent years, (see Figure 4) while, the set of developed economies would present a progressive moderation in their average growth rates, which would fluctuate around 1.5%.

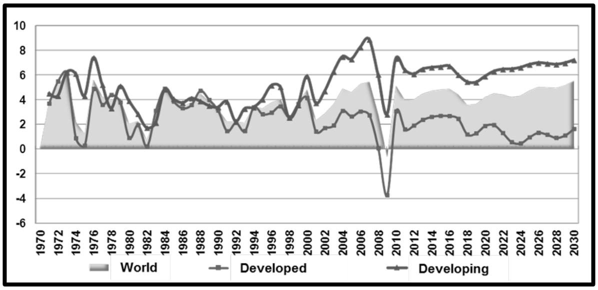
[12] Ibid.

[13] Ibid.

As a result of these GDP growth rates, the volume of world production valued in $ corrected for purchasing power parity (PPP), common terms used by the IMF, would show non-linear trend thus continuing the acceleration that began in the mid-nineties.

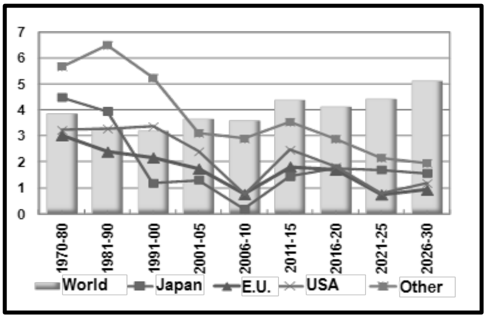
This acceleration in growth is induced by the developing economies, which together could outperform the developed economies in the very short term, and which by the end of the forecast horizon could generate nearly 70% of world production. This would mean that the relative proportions with which the 90s began would be inverted.

Figure 4
Growth rates of world GDP



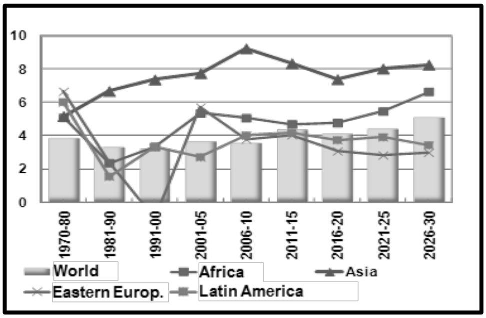
Source: Drawn up by the authors using IMF data.

Figure 5
Developed Economies



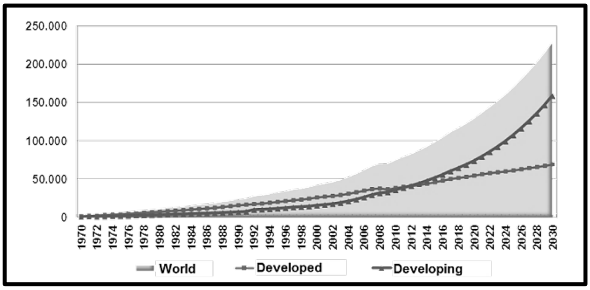
Source: Drawn up by the authors using IMF data.

Figure 6
Developing economies



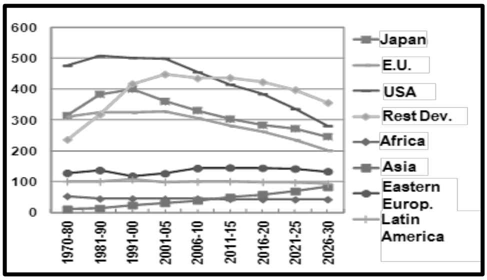
Source: Drawn up by the authors using IMF data.

Figure 7
World GDP in Thousands of $ PPP



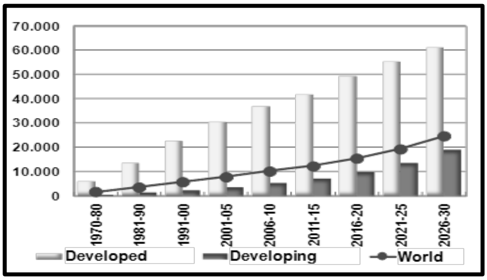
Source: Drawn up by the authors using IMF data.

Figure 8
Income pc Index
(World=100)



Source: Drawn up by the authors using IMF data.

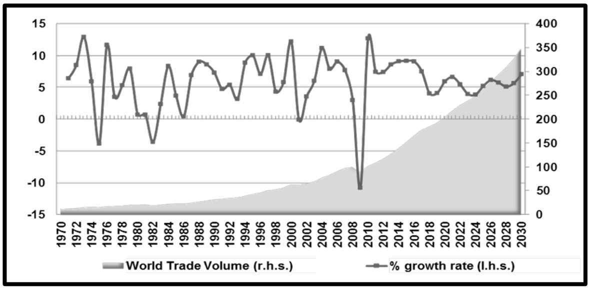
Figure 9
Income pc In $



Source: Drawn up by the authors using IMF data.

These dynamics of relatively higher growth in developing economies would generate a certain effect of convergence in per capita income levels in the different regions, because developed economies would progressively reduce their differential with respect to the global average while developing economies will exhibit modest progress as a result of their increased population growth. In fact, compared to the average situation in 2006-2010, only Asian economies will present significant progress in their per capita income related to the world average at the end of the forecast horizon. However, in absolute terms, the average income in the developed economies as a whole would still be more than three times higher than the one seen in the group of developing economies. As a result of this growth forecast for the world economy as a whole, total trade volume would grow at around 6%, with a dynamic slightly higher in the early years of the forecast horizon, gradually levelling out to average values of 5% in the last years. In addition, given the relative dynamics in different regions, the flow of trade will gradually shift from the developed economies to the developing economies. This would mean that the average growth for world trade would not be evenly distributed in the different geographical areas.

Figure 10
World trade volume

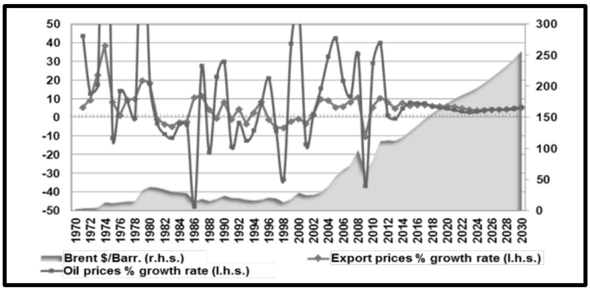


Source: Drawn up by the authors using IMF data.

For international export prices and raw material prices as a whole, the average growth is expected to be in the range of 5 to 7%, showing cyclical fluctuations that coincide with the periods of greatest expansion in world activity.

For the oil prices, this evolution would mean that the average price of a barrel of Brent at the end of the forecast horizon would exceed $ 300. (See Figure 11).

Figure 11
Evolution of International Prices

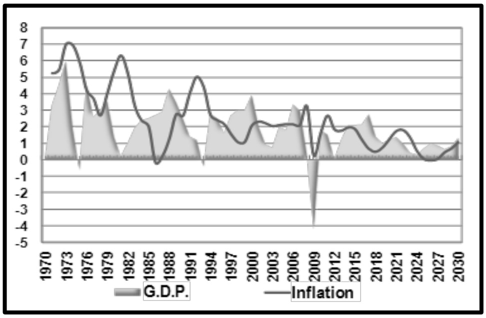


Source: Drawn up by the authors using IMF data.

We conclude this results section with a brief discussion of the projections obtained for the European Union as a whole in terms of growth and inflation, and the outlook for interest rates and exchange rate against the dollar.

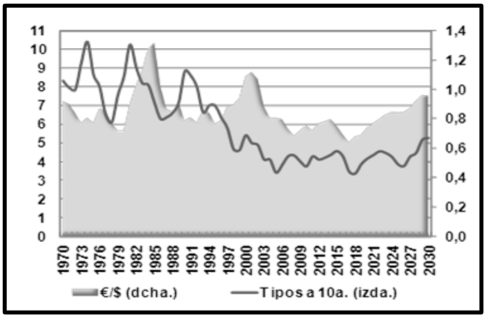
The European Union as a whole does not have a very high growth potential as befits a highly developed economy. Along the same lines, the inflation that may have cyclical fluctuations arising from changes in the international prices of raw materials, will tend to stabilize itself gradually in the long term at an average of around 1%.

Figure 12
Growth in GDP&Inflation in the European



Source: Drawn up by the authors using IMF data.

Figure 13
Exchange Rate €/$ and 10 Y. Interest Rate in the EU



Source: Drawn up by the authors using IMF data.

Meanwhile, interest rates will tend to stabilize in the range of 4.5% due to the reduced availability of international liquidity and increased competition for attracting global capital. At the same time, the exchange rate will tend to gradually approach a parity balance against the dollar.

5. CONCLUSIONS

The activity of planning raises information requirements regarding the future that sometimes go beyond the usual prediction horizon of one to two years. Given these requirements, and with all the provisos regarding the extension of the confidence intervals for the predictions as they move away from the sample period, we can opt for two alternative approaches.

On the one hand, there is the causal modelling approach, whose models must be fed with predictions about the exogenous variables, while on the other, there are time series analysis techniques that allow us to extrapolate trend and cyclical patterns from the variables that must be predicted. The latter approach is the one presented in this article, supplemented with the use of dynamic factor analysis techniques in projecting the cycle, which enable us to use the co-movements observed in cyclical phases of economies located in the same geographic area. Using this approach, we have projected an international macroeconomic scenario with a maximum horizon until 2030, which includes a comprehensive set of indicators for different economies, whose main lines have been also presented.

The scenario projected by the method developed shows an average growth rate for world GDP of about 4%, with a cyclical peak around 2017, and some general acceleration as of 2020, for developing economies as a whole and especially for Africa. The main conclusion extracted has to do with the ability of the method proposed, that is based on the classical decomposition of time series together with dynamic factor analysis, to construct scenarios of long-term prediction, which keeps both the long term trend of the variables involved as well as their cyclical relationships.

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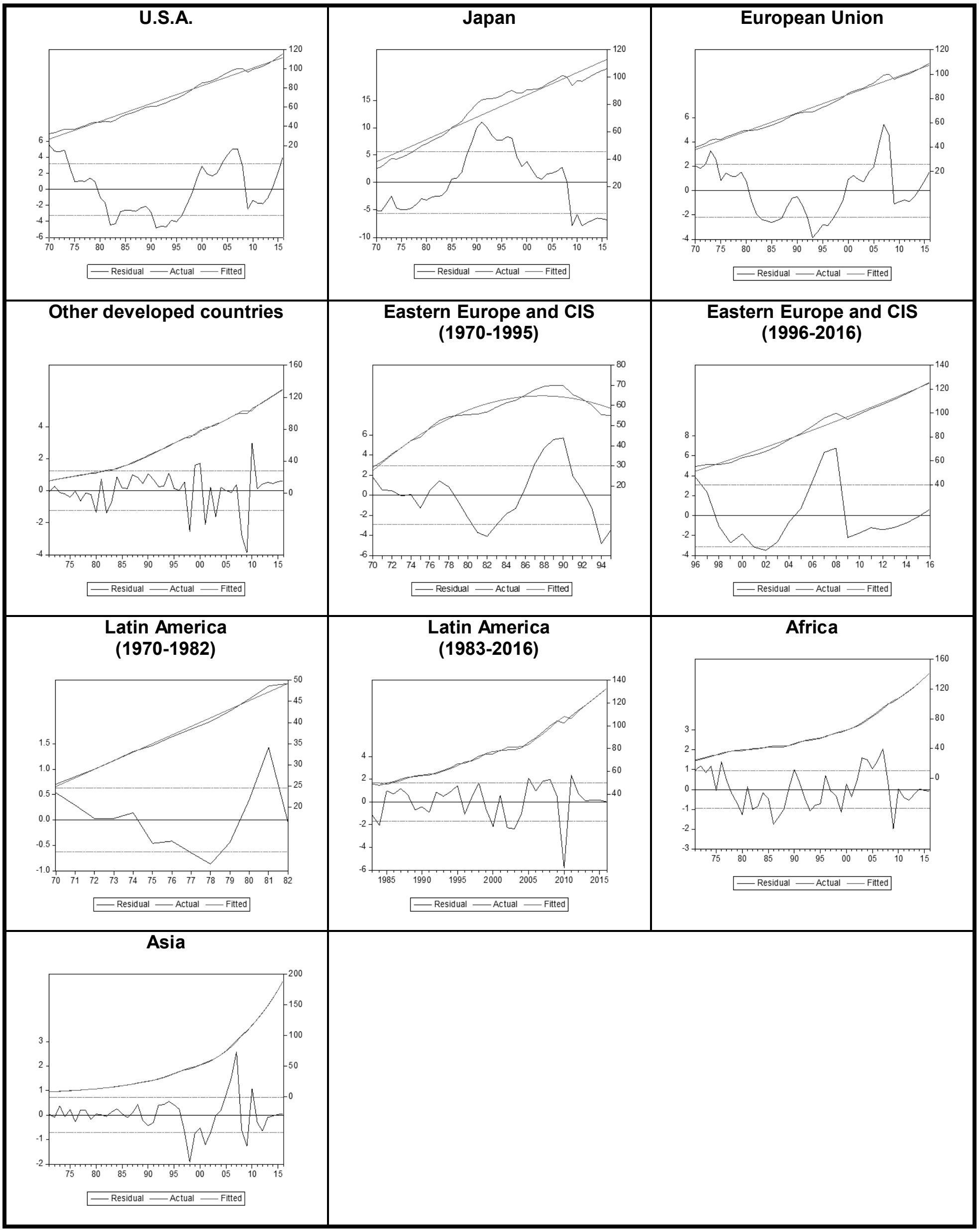
Annex I

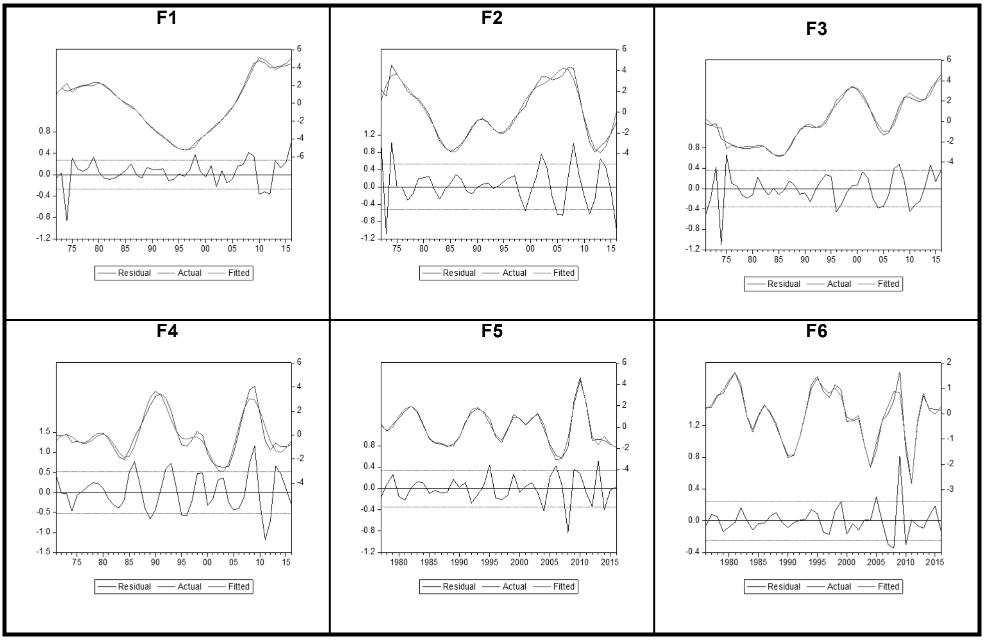
Trend Adjustments



Source: Drawn up by the authors using IMF data.

Annex II

Cyclic Factor Adjustments



Source: Drawn up by the authors using IMF data.

Annex III

Basic results of the growth adjustment by European countries

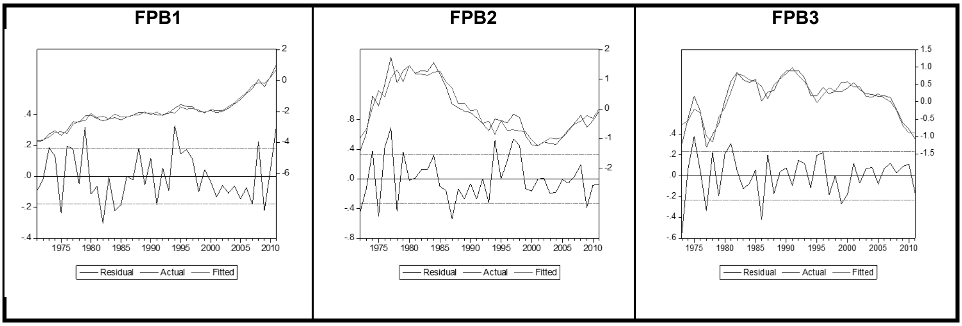
	α	β1	β2	β3	AR	MA	R^2
Germany	1.96 (0.00)	0.58 (0.00)	-0.36 (0.00)	-0.30 (0.00)			0.805
Austria	2.53 (0.00)	0.55 (0.00)	-0.48 (0.00)	-0.16 (0.25)	1		0.760
Belgium	2.26 (0.00)	0.56 (0.00)	-0.25 (0.01)	-0.18 (0.13)	3		0.827
Denmark	1.64 (0.00)	0.70 (0.00)	0.02 (0.90)	0.08 (0.70)		1	0.672
Finland	2.63 (0.00)	0.75 (0.00)	1.57 (0.00)	-0.37 (0.10)	1		0.860
France	2.22 (0.00)	0.49 (0.00)	-0.26 (0.00)	0.00 (0.97)			0.846
Greek	1.98 (0.00)	0.67 (0.00)	0.11 (0.64)	2.61 (0.00)	3		0.826
Holland	2.37 (0.00)	0.53 (0.00)	-0.15 (0.25)	-0.01 (0.57)			0.727
Ireland	4.41 (0.00)	0.76 (0.00)	1.06 (0.00)	1.35 (0.00)			0.617
Italy	2.07 (0.00)	0.69 (0.00)	-0.51 (0.00)	-0.54 (0.00)	1		0.895
Portugal	2.93 (0.00)	0.94 (0.00)	-1.15 (0.00)	0.50 (0.07)	3		0.721
Sweden	2.19 (0.00)	0.49 (0.00)	1.14 (0.00)	-0.75 (0.00)			0.926
United Kingdom	2.22 (0.00)	0.58 (0.00)	0.73 (0.00)	0.37 (0.05)			0.723

Coefficients β1, β2 and β3 correspond to each of the three estimated factors for the growth of European economies.

Source: Drawn up by the authors using IMF data.

Annex IV

Adjustments for basic price factors



Source: Drawn up by the authors using IMF data.

Annex V

Results from the adjustment of the variables in basic prices

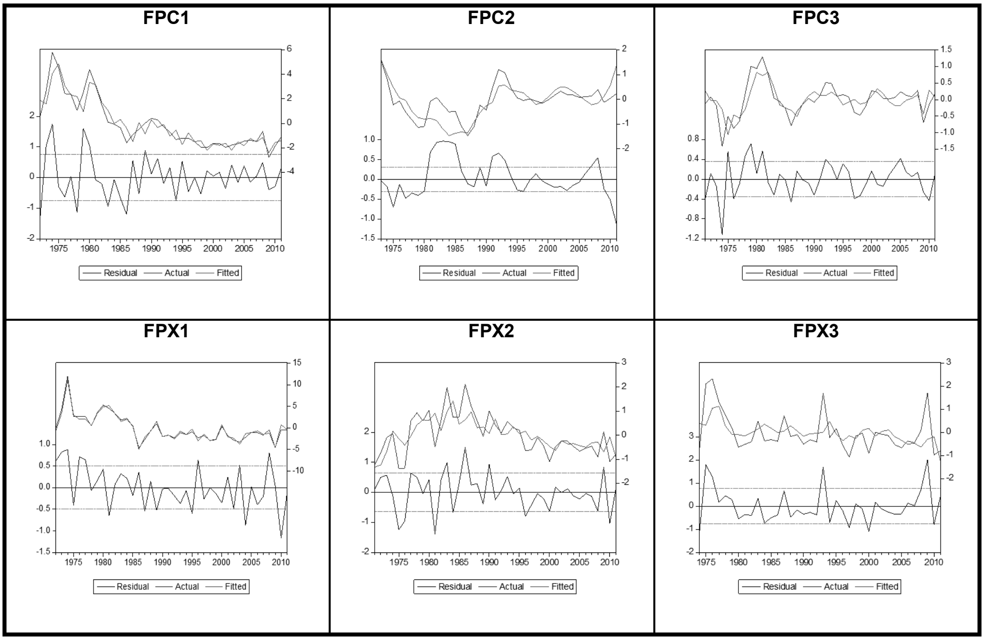
	α	β1	β 2	β 3	AR	MA	R^2
Raw Materials							
Oil (Brent)	30.23 (0.00)	7.37 (0.00)	5.56 (0.00)	-5.65 (0.00)			0.919
Agricultural Products	77.12 (0.00)	7.62 (0.00)	-3.49 (0.01)	1.22 (0.54)		1	0.921
Metal Products	45.34 (0.00)	8.71 (0.00)	4.45 (0.00)	-8.18 (0.00)		1	0.971
Food Products	66.68 (0.00)	3.97 (0.00)	6.10 (0.00)	-0.43 (0.72)		1	0.913
Drinks	80.52 (0.00)	2.09 (0.03)	18.46 (0.00)	2.63 (0.31)		1	0.866
Deflators							
World Trade	60.90 (0.00)	6.99 (0.00)	-2.58 (0.00)	1.98 (0.00)	3	1	0.991
Export. Developed	67.72 (0.00)	6.73 (0.00)	-4.72 (0.00)	3.32 (0.00)		1	0.986
Export. European Union	98.68 (0.00)	1.84 (0.00)	0.05 (0.91)	1.67 (0.06)	1	3	0.994
Export. U.S.A.	76.17 (0.00)	5.48 (0.00)	-2.68 (0.00)	5.30 (0.00)		1	0.995
Export. Japan	116.37 (0.00)	-2.33 (0.00)	5.89 (0.00)	15.35 (0.00)		1	0.967
Export. Developing	50.46 (0.00)	8.05 (0.00)	-1.74 (0.11)	5.25 (0.00)	1		0.986
Export. Latin America	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975

Coefficients β1, β2 and β3 correspond to each of the three estimated factors for basic prices.

Source: Drawn up by the authors using IMF data.

Annex VI

Adjustments for the consumer price factors and export prices



Source: Drawn up by the authors using IMF data.

Annex VII

Results from the adjustment of consumer price variables and export variables

	α	β1	β 2	β 3	AR	MA	R^2
Consumer Prices							
OECD average	30.23 (0.00)	7.37 (0.00)	5.56 (0.00)	-5.65 (0.00)			0.919
European Union	77.12 (0.00)	7.62 (0.00)	-3.49 (0.01)	1.22 (0.54)		1	0.921
U.S.A.	45.34 (0.00)	8.71 (0.00)	4.45 (0.00)	-8.18 (0.00)		1	0.971
Japan	66.68 (0.00)	3.97 (0.00)	6.10 (0.00)	-0.43 (0.72)		1	0.913
United Kingdom	80.52 (0.00)	2.09 (0.03)	18.46 (0.00)	2.63 (0.31)		1	0.866
Export prices							
Germany	60.90 (0.00)	6.99 (0.00)	-2.58 (0.00)	1.98 (0.00)	3	1	0.991
Austria	67.72 (0.00)	6.73 (0.00)	-4.72 (0.00)	3.32 (0.00)		1	0.986
Belgium	98.68 (0.00)	1.84 (0.00)	0.05 (0.91)	1.67 (0.06)	1	3	0.994
Denmark	76.17 (0.00)	5.48 (0.00)	-2.68 (0.00)	5.30 (0.00)		1	0.995
France	116.37 (0.00)	-2.33 (0.00)	5.89 (0.00)	15.35 (0.00)		1	0.967
Greece	50.46 (0.00)	8.05 (0.00)	-1.74 (0.11)	5.25 (0.00)	1		0.986
Holland	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975
Ireland	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975
Italy	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975
Portugal	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975
Sweden	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975
United Kingdom	51.00 (0.00)	6.66 (0.00)	1.91 (0.01)	-3.32 (0.00)		1	0.975

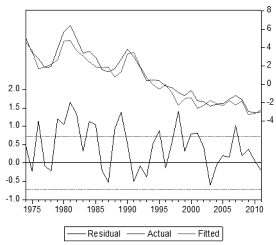
Coefficients β1, β2 and β3 correspond to each of the three estimated factors for basic prices.

Source: Drawn up by the authors using IMF data.

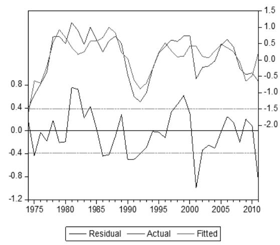
Annex VIII

Adjustments to the consumer price factors and export prices

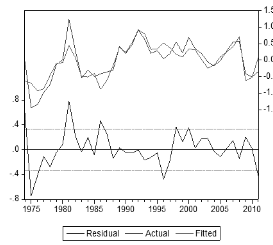
FTI1



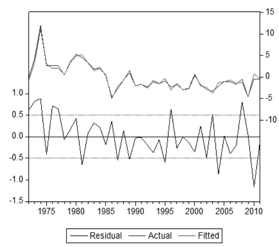
FTI2



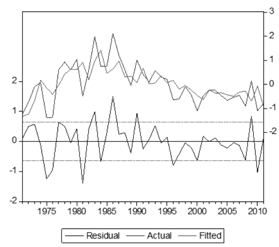
FTI3



FTC1



FTC2



Source: Drawn up by the authors using IMF data.

Annex IX

Results from the adjustment to the interest rate and exchange rate variables (*)

	α	β1	β 2	β 3	AR	MA	R^2
Interest rates							
ST U.S.A		1.42 (0.00)	2.33 (0.00)	0.86 (0.00)			0.883
ST Japan		1.45 (0.00)	0.00 (0.98)	-1.45 (0.00)			0.497
ST Eurozone		0.60 (0.00)	0.32 (0.19)	2.50 (0.00)			0.815
ST United Kingdom		1.58 (0.00)	0.58 (0.21)	-0.26 (0.54)	2		0.644
LT U.S.A.		0.77 (0.00)	1.03 (0.00)	-0.23 (0.26)			0.648
LT Japan		0.66 (0.00)	-0.00 (0.98)	-0.71 (0.00)			0.605
LT Eurozone		0.70 (0.00)	0.13 (0.34)	0.01 (0.95)			0.670
LT United Kingdom		1.08 (0.00)	0.49 (0.00)	-0.62 (0.00)			0.725
Exchange rates							
Yen / Dollar		-21.65 (0.00)	51.85 (0.00)				0.838
Pound / Dollar		0.07 (0.00)	-0.14 (0.02)				0.938
Euro / Dollar		0.06 (0.00)	0.10 (0.00)				0.966

(*) All the equations have been estimated as first order difference equations.

Source: Drawn up by the authors using IMF data.