Taxes, compensations and renewable natural resources[*]

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Abstract

We start from a dynamic model of exploitation of renewable natural resources in which extinction is the expected outcome in the long run equilibrium. This paper introduces taxes on extraction to achieve a preservation standard. It also discusses how this standard can be achieved using compensations for non-extraction. In addition, it proposes combining taxes on extraction and compensations for non-extraction in a self-financing regulation: tax revenues revert to the natural resource, a lower tax is needed to achieve the preservation standard and the welfare of stakeholders is improved.

Keywords: taxes, compensations, extraction, tax revenue, compensation funding

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1

1 Introduction

The tax-subsidy approach of the Pigouvian tradition to control externalities has been studied extensively in the literature, especially in pollution-related externalities. After the work of R.Coase on social cost (Coase, 1960), the debate on the appropriateness of Pigouvian taxes or subsidies to address externalities enters its peak. Baumol (1972) addresses many of the issues raised by Coase that challenge the Pigouvian tradition such as the difficulty of estimating the social cost, the reciprocity inherent in social cost, the suitability of taxing/compensating victims of the externality rather than taxing/compensating those who generate the externality or the existence of multiple local maxima. To solve these problems, Baumol and Oates (1971) propose to combine the use of standards and taxes for the protection of the environment. They propose to select some maximal level of a pollutant that is considered satisfactory (say, the standard) and seek to determine a tax on inputs or outputs that cause the externality capable of achieving the chosen standard. This approach needs less information, has long lasting effects and achieves decreases in pollution at minimum cost to society.

Also the differences between taxes and subsidies have occupied much of the debate. Baumol and Oates (1975) find differences at the industry level showing that a Pigouvian tax will induce Pareto-optimal exit and entry decisions by all competitive firms whereas a Pigouvian subsidy will cause excessive entry. Then, pollution may increase above the levels that would have reached in the absence of subsidies.[1] Also the effectiveness of Pigovian taxes when the market structure is not perfect competition has been part of the debate (Buchanan, 1969).

In addition, an ever present question is how to get funding for subsidies or how to use the revenues form environmental taxes. This is related to a more recent literature that examines the Pigouvian tradition in the so called green tax reform. Here, one of the main points is the double dividend: whether an environmental tax reform yields not

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[1]Burrows (1979) shows that this can be the situation depending on the damage cost curves. He also show that a Pigouvian tax need not lead to a Pareto-optimal level of pollution by the industry.

only a cleaner environment but also non-environmental benefits. The environmental tax reform shifts the tax burden from economic 'goods' (employment, income, investment) to environmental 'bads' (pollution, resource depletion, waste) (Bosquet, 2000). But requiring environmental policy to serve not only environmental objectives but also other, non-environmental, goals may complicate the implementation of environmental policy by intensifying the distributional struggle about the property rights to the natural environment (Bovenberg, 1999).

This paper studies taxes and subsidies for environmental protection and includes the following aspects. First, and contrary to most of the papers mentioned above, the analysis is focused on the exploitation of renewable natural resources, that is, we move away from studies of pollution externalities. The aim is to improve the conservation of natural resources that are exploited under common property and, as far as possible, try to enhance the welfare of stakeholders. Second, we assume Baumol and Oates (1971) proposal and combine the use of standards and prices. The standard is a stock considered satisfactory from the point of view of natural resource conservation. A tax on extraction or a subsidy for non-extraction is determined accordingly. Third, achieving the conservation goal set by the standard is analyzed from a dynamic point of view. We construct the phase diagrams and discuss the change in the extraction dynamics and the stability of the results. The results show that the conservation standard can be achieved with both taxes on extraction or compensations for non-extraction. But, as noted above, the open question is how to use tax revenues or how to get the compensation fund. Fourth, our main proposal is to move away from the double dividend and focus only on environmental objectives. Therefore, we propose (i) a conservation standard, (ii) an extraction tax, (iii) the use of tax revenue to compensate non-extractors. The regulation system is self-sustaining and, in addition, improves the welfare of all stakeholders.

2 A simple model on resource extraction

Consider a natural resource with critical depensation whose growth function is[2]

$$G(X) = r (X - T) \left(1 - \frac{X}{K}\right) \tag{1}$$

where X is the resource biomass, r is the intrinsic growth rate, K is the carrying capacity and T is the biomass below which natural resource regeneration is not possible. The natural resource exhibits depensation if $X < X_D$ and compensation if $X > X_D$ where $X_D = \sqrt{KT}$. In addition, maximum sustainable yield is achieved whenever $X = X_{MSY}$ where $X_{MSY} = \frac{K+T}{2}$.

Assume a Schaefer resource extraction function,[3]

$$H = qEX \tag{2}$$

where q is the capturability coefficient and resource extraction H depends positively on the effort level E and on the resource biomass X. Consider a well-defined community of users has the right to exploit the resource. The effort level is the sum of the effort extraction of each of the n potential resource users, $E = \sum_{i=1}^{n} e_i$. Potential resource users decide whether to extract the resource depending on the extraction payoff, $\pi_h = pH_i - ce_i$, where p is the resource market price, c is the opportunity cost of effort and H_i is the resource harvest of subject i. Whenever the extraction payoff is positive, there are incentives for extraction.

We assume that individual effort level is $e_i \in \{0, 1\}$ distinguishing between non-extractors, $e_i = 0$ and extractors $e_i = 1$ among the potential resource users and normalizing the effort level of extractors. Therefore, total effort level is $E = ns_h$ where s_h is the proportion of extractors among the potential resource users. Under these assumptions,

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[2]There are other logistic growth functions critical depensation. With any of them, a similar analysis to the one presented here can be done and the results are maintained.

[3]This extraction function is one of the most used in the literature of natural resources from the work of Schaefer (1954) and Gordon (1954).

the extraction payoff is equal for each extractor, $\pi_h = pqX - c$. We assume agents are not profit maximizers but adjust their strategies gradually towards those who get better result. To capture the incentives for resource extraction and to model the behavior of potential users, we assume a replicator dynamic (Taylor and Yonker, 1979).[4]

$$\frac{ds_h}{dt} = \dot{s}_h = s_h \left(1 - s_h\right) \left(pqX - c\right) \tag{3}$$

Observe that, whenever the extraction payoff is positive, $pqX - c > 0$, the proportion of extractors increases. On the contrary, whenever the extraction payoff is negative, $pqX - c < 0$, the proportion of extractors decreases. The incentives for resource extraction depend on the resource market price, the opportunity cost of effort, the capturability coefficient and the resource biomass. Let $X' = \frac{c}{pq}$, if $X < X'$ $(X > X')$ the extraction payoff is negative (positive) and the proportion of extractors decreases (increases).

The resource biomass is a variable whose evolution is given by the difference between natural growth and extraction

$$\frac{dX}{dt} = \dot{X} = r\left(X - T\right)\left(1 - \frac{X}{K}\right) - qns_h X \tag{4}$$

We assume that if $s_h = 1$ then $G(X) < H$ $\forall X$, that is, the community has overcapacity: if all potential users exercise their right of withdrawal, the long-run result is resource extinction.

Equations (3) and (4) form a dynamical system that is in equilibrium whenever $\dot{s}_h = 0$ and $\dot{X} = 0$.

Proposition 2.1 Resource extinction

a. *Open access among potential resource users leads to resource extinction when $X' < X_D$.*

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[4]The replicator dynamics has already been used in natural resource studies, see Sethi and Somanathan (1996) or Osés-Eraso and Viladrich-Grau (2007).

b. *Open access among potential resource users can surpass resource extinction when* $X' > X_D$ *as there exists an asymptotically locally stable equilibrium* (s_h^*, X^*), *with* $0 < s_h^* < 1$ *and* $X^* = X'$.
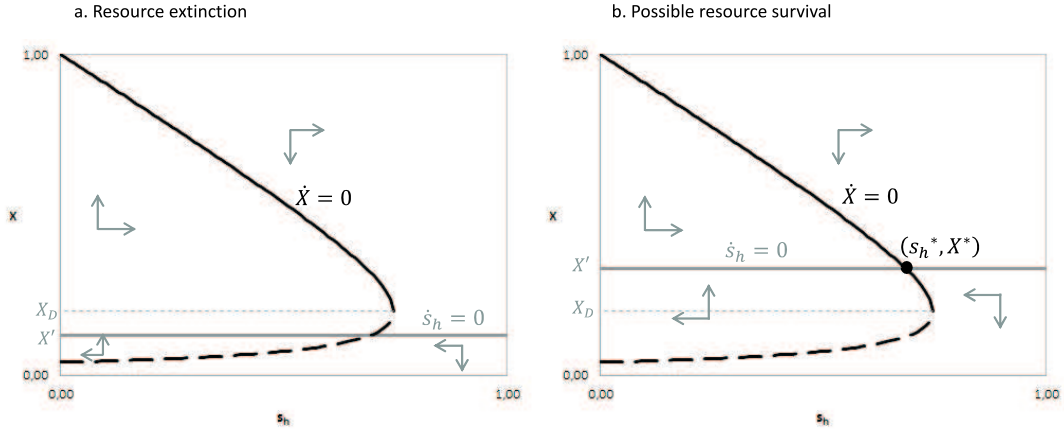

Figure 1: The possibility of resource extinction

Figure 1 shows the phase diagram of the above dynamic system. In figure 1a, $X' < X_D$ and extinction is a credible threat. In figure 1b, extinction is still possible but there exists an equilibrium point that guaranties resource survival. In such equilibrium, a welfare analysis shows that, in addition to resource preservation, extractors and non-extractors coexists and they have normal benefits, that is, zero benefit.

Observing this dynamic, it's easy to realize that changes in the market resource price, changes in the opportunity costs of effort or changes in the capturability coefficient can modify the equilibrium. For example, assume that for a given natural resource, $X' < X_D$, that is, extinction is a real threat on the long run. If the resource market price decreases, X' increases. If this increment is such that $X' > X_D$ there exist a possibility for resource survival and for a sustained resource exploitation. A similar analysis can be done for an increase in the opportunity cost of effort or for a decrease in the capturability coefficient.

The ability to modify these parameters to try to prevent resource extinction does not fall on the resource managers/users and it's beyond the reach of other authority levels. Intervention using other alternatives return necessary.

3 Taxes vs. compensations

Given the threat of extinction, various measures can be implemented to try to resolve the threat and preserve the resources in the long term. Two of the interventions studied in the literature are: (i) taxes on extraction (ii) taxes for non-extraction. We analyze the effects that each of these measures has on resource conservation and on welfare of stakeholders in the model presented above.

3.1 Taxes on extraction

Assume a tax t is imposed on extraction: each potential user who wants to exercise his right of withdrawal must pay the fee t. The imposition of this tax affects the dynamics described above as the extraction payoff must include payment of the tax. The new dynamic is described by the following equations, one for the dynamics of the potential users and one for the dynamics of the natural resource.

$$\frac{ds_h}{dt} = \dot{s}_h = s_h \left(1 - s_h\right) \left(pqX - c - t\right) \tag{5}$$

$$\frac{dX}{dt} = \dot{X} = r\left(X - T\right)\left(1 - \frac{X}{K}\right) - qns_hX \tag{6}$$

In this setting, there are incentives for extraction whenever $(pqX - c - t) > 0$. The stock level below which extraction payoffs are negative is now $X'_t = \frac{c+t}{pq}$. Introducing the tax reduces the situations in which there are incentives for resource extraction as $X'_t > X'$.

Proposition 3.1 Taxes, extinction and preservation standard

a. When $X' < X_D$, a <u>necessary</u> condition for an extraction tax to be effective in preventing extinction is $t > pq\sqrt{KT} - c$. With this tax, there exists an asymptotically locally stable equilibrium (s_h^*, X^*), with $0 < s_h^* < 1$ and $X^* = X_t'$.

b. When $X' > X_D$, any tax $t > 0$ increases the resource stock and reduces the proportion of extractors in the asymptotically locally stable equilibrium.

c. An asymptotically locally equilibrium with stock level (the preservation standard) $X^* > X_D$ is reachable with an extraction tax $t^* = pqX^* - c$.

d. The use of extraction taxes improves resource preservation but it does not modify stakeholders equilibrium welfare.

Observe that greater taxes are needed in order to avoid resource extinction the greater the resource market price, the greater the capturability coefficient or the smaller the opportunity cost of effort. Similarly, greater preservation goals or standards need greater taxes. Moreover, the greater (smaller) the resource market price (opportunity effort cost), the greater the extraction tax needed to get the same preservation standard. Similarly, the greater the capturability coefficient, the greater the extraction tax needed to get the same preservation standard.

The figure 2 shows how the dynamics of the system changes with the introduction of an extraction tax (to reach a conservation standard $X^* = X_t'$). Figure 2A shows the complete dynamics and figures 2B and 2C show the evolution over time of each of the main variables, the proportion of extractor (2B) and the resource stock (2C).

In the figure, $X' < X_D$ and , without taxes, the proportion of extractors increases which adversely affects natural resource whose stock declines to extinction. Assume we implement an extraction tax such that $X_t' > X_D$ (as stated in proposition 3.1a, $t > pq\sqrt{KT} - c$), it appears that the proportion of extractors increases more slowly thus slowing reduction of the resource stock. In the example shown, the dynamic changes in such a way that a new evolutionarily stable equilibrium is achieved where extractors and non-extractors coexists, preventing resource extinction.
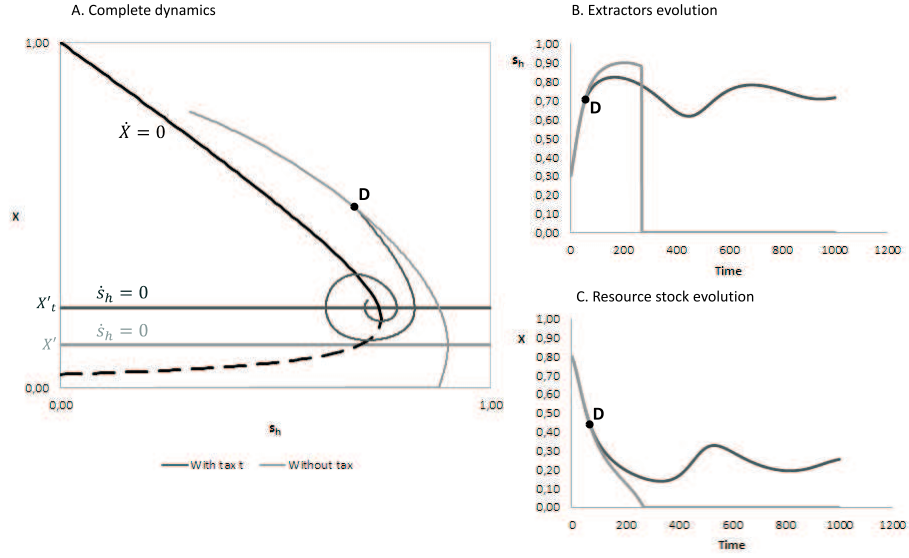
Figure 2: Changes in the dynamics introducing a tax

A welfare analysis shows that the implementation of an extraction tax enhances resource conservation and allows a sustainable resource exploitation. However, extractors and non-extractors still get zero benefits in the long run equilibrium.

Tax revenue in the equilibrium is $TR^* = ts_h^*n$. As the proportion of extractors is a decreasing function of t, an increase in the tax does not always lead to an increase in the collection. Figure 3 represents the increasing relationship between preservation standard, X, and the necessary tax to reach that standard together with the relationship between preservation standard and tax revenue in equilibrium. In this figure, equilibrium tax revenue is maximized for a preservation standard above X_{MSY}. This result can be generalized.

Proposition 3.2 Tax revenue

a. When $T = 0$, equilibrium tax revenue is maximized if $X^* = X_{MSY} + \frac{X'}{2}$, that is, for a preservation standard that avoids biological overexploitation.

b. When $T > 0$, equilibrium tax revenue is maximized if $X^* > X_{MSY}$, that is, for a preservation standard that avoids biological overexploitation. There is a positive
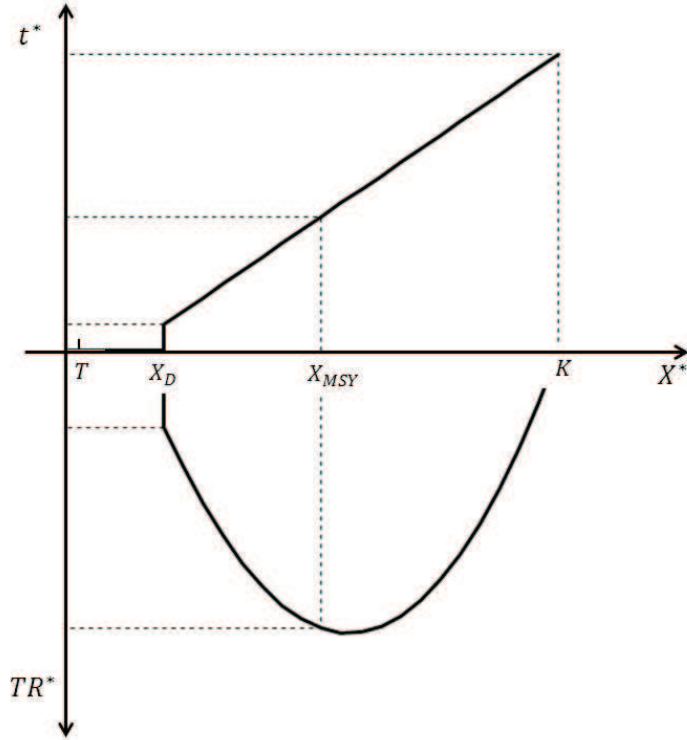
9

Figure 3: Preservation standard, taxes and tax revenue

relationship between T and the preservation standard that maximizes tax revenue but, the greater is T, the closer is the preservation standard that maximizes tax revenue to X_{MSY}.

The open question is how to use this tax revenue.

3.2 Compensations for non-extraction

Assume a compensation w is given to non-extractors: all potential users that decide not to extract the resource receive a compensation w. The dynamic system is now as follows:

$$\frac{ds_h}{dt} = \dot{s}_h = s_h \left(1 - s_h\right) \left(pqX - c - w\right) \tag{7}$$

$$\frac{dX}{dt} = \dot{X} = r\left(X - T\right)\left(1 - \frac{X}{K}\right) - qns_hX \tag{8}$$

In this setting, there are incentives for extraction whenever the extraction payoff is greater than the compensation, $\pi_h > w$. The stock level below which there are no incentives for resource extraction is $X'_w = \frac{c+w}{pq}$. As it happens with the introduction of taxes, the introduction of compensations for non-extraction reduces situations in which there are incentives for resource extraction. It's easy to observe that the phase diagram is the same for a compensation w and for a tax $t = w$ (see figure 2).

Proposition 3.3 Compensations, extinction and preservation standard

a. When $X' < X_D$, a _necessary_ condition for a non-extraction compensation to be effective in preventing extinction is $w > pq\sqrt{KT} - c$. With this compensation, there exists an asymptotically locally stable equilibrium (s_h^*, X^*), with $0 < s_h < 1$ and $X^* = X'_w$.

b. When $X' > X_D$, any compensation $w > 0$ increases the resource stock and reduces the proportion of extractors in the asymptotically locally equilibrium.

c. An asymptotically locally equilibrium with stock level (the preservation standard) $X^* > X_D$ is reachable with a compensation for non-extraction $w^* = pqX^* - c$.

d. The use of compensations for non-extraction improves resource preservation and increases stakeholders equilibrium welfare.

This result replicates the well-known literature on taxes and subsidies to correct externalities: with a compensation w for non-extraction we can get the same effect on the natural resource that implementing a tax $t(= w)$ on extractors. There is, however, a difference when we consider welfare of extractors and non-extractors. In the long-run equilibrium both of them get a positive payoff with compensations for non-extraction while they get zero payoff with extraction taxes. Observe that in this setting, the difference between taxes and subsidies observed by Baumol and Oates (1975) about the entry and exist of polluting firms is not relevant as the potential number of users is fixed.

Nevertheless, compensation regulation needs funding. Total funding in equilibrium is $F^* = w^*(1 - s_h^*)n$.

Proposition 3.4 Compensation funding

Total funding required at equilibrium increases with the amount of compensation, w^*, at an increasing rate.

The question that arises is how to get this funding.

4 Taxes and compensations

In public policy, tax revenue may be use for funding public spending, including several compensations or subsidies. In many cases, taxes that finance subsidies come from activities that have nothing to do with the activity that is going to be financed. The introduction of green taxes is a new source of revenue for public authorities. The fist goal of these green taxes is a better environment but the potential revenues may also be relevant. For example, according to the double dividend analysis, green taxes can substitute other distortionary taxes (Bovenberg, 1999; Bosquet, 2000). The proposal of same environmentalists is to use tax revenues from green taxes for environmental purposes. Thus, green taxes should revert on the environment: the tax solves an environmental problem and the tax revenue is used to solve another environmental problem. Why not use this tax collection to improve the same environmental problem?

Assume we implement both, taxes for extraction and compensations for non-extraction. In order for the regulation to be self-sustainable, we use tax revenue for funding compensations. We assume a fix tax t for extraction and a variable compensation $w(s_h)$ such that each year tax collection is equally distributed among non-extractors, that is, $w(s_h) = \frac{ts_h}{1-s_h}$. Observe that this compensation increases in s_h at an increasing rate.[5]

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[5] $\frac{dw(s_h)}{ds_h} > 0$ and $\frac{d^2w(s_h)}{ds_h^2} > 0$.

Consider the effect of this new regulation on the dynamics of the system.

$$\frac{ds_h}{dt} = \dot{s}_h = s_h\left(1 - s_h\right)\left(pqX - c - t - \frac{ts_h}{1 - s_h}\right) \tag{9}$$

$$\frac{dX}{dt} = \dot{X} = r\left(X - T\right)\left(1 - \frac{X}{K}\right) - qns_hX \tag{10}$$

Observe that the extraction payoff includes tax payment, $\pi_h = pqX - c - t$ and there are incentives for resource extraction whenever the extraction payoff exceeds the compensation, $w(s_h) = \frac{ts_h}{1-s_h}$. The system is in equilibrium whenever $\dot{s}_h = 0$ and $\dot{X} = 0$. The first condition, $\dot{s}_h = 0$, is fulfilled whenever $\pi_h = w(s_h)$.[6] Developing this condition we get the stock level for which extraction payoff equates compensation that is an increasing function of the proportion of extractors, $\widetilde{X}(s_h) = \frac{1}{pq}\left(\frac{t}{1-s_h} + c\right)$.

Proposition 4.1 Taxes and compensations

a. *When tax revenues from an extraction tax is equally divided between non-extractors, there is an asymptotically stable equilibrium (s_h^*, X^*) with $0 < s_h^* < 1$ and $X^* = \widetilde{\left(s_h^*\right)}$, whenever $X^* > X_D$.*

b. *Extraction taxes that are used to compensate non-extractors lead to better natural resource conservation.*

c. *A conservation standard can be achieved with lower extraction taxes if the tax revenue is used to compensate the non-extractors.*

Figure 4 shows the dynamics of resource extraction in three different settings: (i) without any regulation, (ii) regulation with tax on extraction (or compensation for non-extraction, same quantity) that is introduced in moment D, (iii) regulation with the previous tax on extraction, using the tax revenue to equally compensate no-extractors. In case (i), the result is resource extinction, $X = 0$ while in cases (ii) and (iii) there

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[6]The condition is also fulfilled whenever $s_h = 1$ or $s_h = 1$, there is no possibility to replicate other type of behavior.

A. Complete dynamics

B. Extractors evolution

C. Resource stock evolution

Without regulation — taxes or compensations — Taxes and compensations
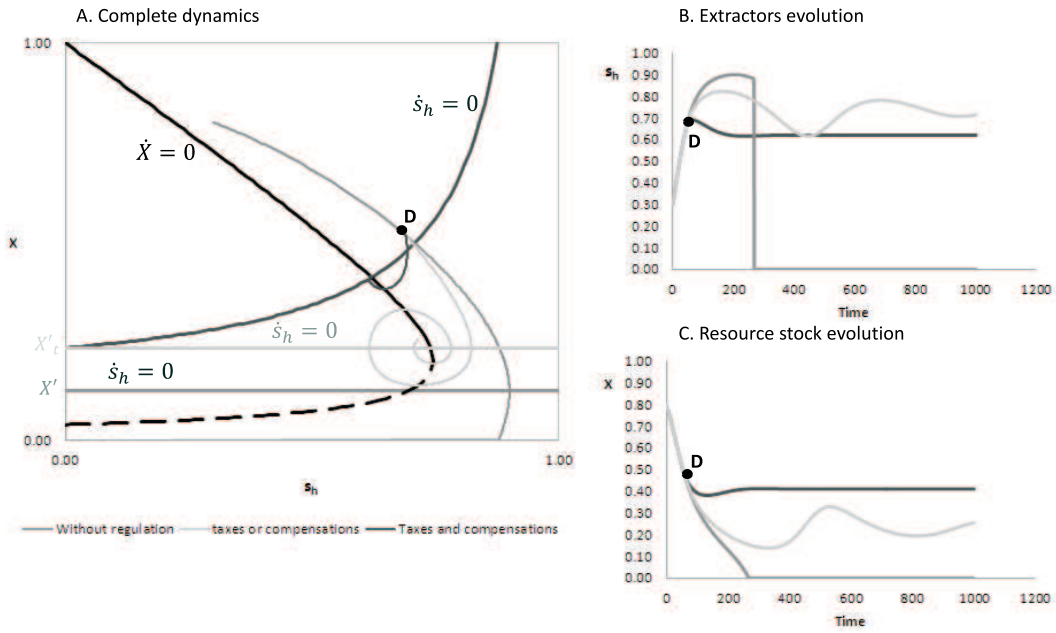
Figure 4: Taxes and compensations

exists an asymptotically locally equilibrium with $X > X_D$. In addition, although the extraction tax in the same in (ii) and (iii), resource stock is clearly greater in (iii) where tax revenue reverts on the resource through compensation for non-extraction.

An interesting point arises when we analyze welfare. On one hand, the use of extraction taxes to compensate the non-extractors can preserve the resource while it is exploited. Moreover, in the equilibrium non-extractors receive a positive compensation and extractors obtain a positive extraction payoff that equal to the compensation of non-extractors. A regulation with taxes and compensations manages to improve natural resource and also the parties involved get positive benefits with the advantage that the regulatory system is self-financing.

14

5   Conclusions

Based on a dynamic model of natural resource exploitation in which extinction is the expected outcome in the long term, we introduce a tax on extraction and we analyze how this tax changes the dynamics of resource exploitation. Resource preservation may be achieved with this extraction tax. A preservation standard that sets a resource stock level may be achieved by implementing the appropriate tax on extraction. The tax amount depends on economic variables such as the market prices of the natural resource and on biological variables such as the natural growth. The results also show that the higher tax revenues are obtained when preservation standards are set around the maximum sustainable yield. Moreover, maximum tax revenue is obtained when the preservation standard avoids biological overexploitation.

The preservation standard can also be achieved using compensations for non-extraction. The main advantage of this regulation system is that it improves agents' welfare, a result that is not achievable with a regulation by (only) extraction taxes. But the main disadvantage is that we need funding for compensations. Therefore, one of the most interesting contributions of this paper is to propose a regulation system that combines taxes and compensations to achieve a standard of conservation. It is a self-sustaining resource regulation: the revenue from the extraction tax is used to finance the compensation of those who are left without extracting the resource. This regulation allows reaching a certain preservation standard with lower taxes. Moreover, it maintains the main advantage of compensations as it improves the welfare of all stakeholders.

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Appendix: proofs

Proposition 2.1 Resource extinction

An equilibrium point of the system is (s_h^*, X^*) such that $X^* = \frac{c}{pq}$ and $s_h^* = \frac{r}{qn}\left(1 - \frac{T}{X^*}\right)\left(1 - \frac{X^*}{K}\right)$. The equilibrium point is asymptotically locally stable if the trace of the Jacobian matrix of the dynamic system evaluated at that point is negative and the determinant of such matrix is positive.

$$J_{11} + J_{22} = r\left(\frac{T}{X^*} - \frac{X^*}{K}\right) \tag{11}$$

$$J_{11}J_{22} - J_{21}J_{12} = qnXs_h^*(1 - s_h^*)pq \tag{12}$$

The determinant (equation (12)) is positive if $s_h^* \in (0,1)$ and the trace (equation (11)) in negative if $X^* > \sqrt{TK} = X_D$

Proposition 3.1 Taxes, extinction and preservation standard

a. From proposition 2.1, extraction without taxes leads to extinction if $X' < X_D$, that is, if $\frac{c}{pq} < \sqrt{KT}$. A similar analysis shows that, with a tax t an asymptotically locally stable equilibrium exists if $X_t' > X_D$, that is, if $\frac{c+t}{pq} > \sqrt{KT}$. This last inequality is fulfilled if $t > pq\sqrt{KT} - c$.

b. From proposition 2.1, with $X' > X_D$ and $t = 0$, the asymptotically locally stable equilibrium point of the dynamic system is (s_h^*, X^*), with $0 < s_h^* < 1$ and $X^* = X' = \frac{c}{pq}$. With $X' > X_D$ and $t > 0$, the asymptotically locally stable equilibrium point of the dynamic system is (s_h^*, X^*), with $0 < s_h^* < 1$ and $X^* = X_t' = \frac{c+t}{pq}$. And it is straight forward that $X' < X_t'$, $\forall t > 0$.

In both cases, $s_h^* = \frac{r}{qn}\left(1 - \frac{T}{X^*}\right)\left(1 - \frac{X^*}{K}\right)$ and $\frac{\partial s_h^*}{\partial X^*} = \frac{r}{qn}\left(\frac{KT - X^{*2}}{X^{*2}K}\right) < 0$ as $X^* > X_D$, that is, $X^* > \sqrt{KT}$ and then $X^{*2} > KT$. Therefore, s_h^* is smaller with $t > 0$ as X^* is greater.

c. An asymptotically locally equilibrium is characterized by $X^* = X_t' = \frac{c+t}{pq}$. Assume an extraction tax $t^* = pqX^* - c$. The corresponding equilibrium is $X_{t^*}' = \frac{c+t^*}{pq} = \frac{c+pqX^*-c}{pq} = X^*$.

d. The asymptotically stable equilibrium point is reached when $\dot{s}_h = 0$, that is, when $\pi_h = 0$. The stakeholders have normal benefits.

Proposition 3.2 Tax revenue

Equilibrium tax revenue is $TR^* = tns_h^*$ where $s_h^* = \frac{r}{qn}\left(1 - \frac{T}{X^*}\right)\left(1 - \frac{X^*}{K}\right)$. Assume the preservation standard is X^*, according to proposition 3.1c, the extraction tax should be $t^* = pqX^* - c$. Therefore, equilibrium tax revenue is

$$TR = (pqX^* - c)\frac{r}{q}\left(1 - \frac{T}{X^*}\right)\left(1 - \frac{X^*}{K}\right)$$

a. Assume $T = 0$, then $TR = (pqX^* - c)\frac{r}{q}\left(1 - \frac{X^*}{K}\right)$. To maximize tax revenue, the first order condition implies,

$$\frac{\partial TR}{\partial X^*} = \frac{r}{q}\left(pq + \frac{c}{K} - \frac{2pq}{K}X^*\right) = 0$$

The first order condition is fulfilled when $X^* = \frac{K}{2} + \frac{c}{2pq}$. As $X_{MSY} = \frac{K}{2}$ when $T = 0$ and $X' = \frac{c}{pq}$, $X^* = X_{MSY} + \frac{X'}{2}$.

For the second order condition, the second derivative is negative, $\frac{\partial^2 TC}{\partial X^{*2}} = -\frac{2rp}{K} < 0$ and $X^* = \frac{K}{2} + \frac{c}{2pq}$ is a maximum.

A natural resource with logistic growth function is said to be biologically over-exploited if $X < X_{MSY}$. Therefore, for $X^* = \frac{K}{2} + \frac{c}{2pq}$ there is no biological overexploitation.

b. Assume $T > 0$, then $TC = (pqX^* - c)\frac{r}{q}\left(1 - \frac{T}{X^*}\right)\left(1 - \frac{X^*}{K}\right)$ and the first order condition implies,

$$2pqX^3 - (pqK + pqT + c)X^2 + cTK = 0$$

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18

Proposition 3.3 Compensations, extinction and preservation standard

a. From proposition 2.1, extraction without an unregulated resource reaches extinction if $X' < X_D$, that is, if $\frac{c}{pqK} < \sqrt{KT}$. A similar analysis shows that, with a compensation for non-extraction t, an asymptotically locally stable equilibrium exists if $X'_w > X_D$, that is if $\frac{c+w}{pq} > \sqrt{KT}$. This inequality is fulfilled if $w > pq\sqrt{KT} - c$.

b. From proposition 2.1, with $X' > X_D$ and $w = 0$, the asymptotically locally stable equilibrium point of the dynamic system is (s_h^*, X^*), with $0 < s_h^* < 1$ and $X^* = X' = \frac{c}{pq}$. With $X' > X_D$ and $w > 0$, the asymptotically locally stable equilibrium point of the dynamic system is (s_h^*, X^*), with $0 < s_h^* < 1$ and $X^* = X'_w = \frac{c+w}{pq}$. And it is straight forward that $X' < X'_w$, $\forall w > 0$.

In both cases, $s_h^* = \frac{r}{qn}\left(1 - \frac{T}{X^*}\right)\left(1 - \frac{X^*}{K}\right)$ where $X^* = \frac{c+w}{pq}$ which implies $\frac{\partial s_h^*}{\partial w} = \frac{r}{qn}\left(\frac{TKp^2q^2 - (c+w)^2}{Kpq(c+w)^2}\right)$. As $X_D < X'$, $\sqrt{TK} < \frac{c+w}{pq}$ which implies $TKp^2q^2 < (c+w)^2$, that is, $\frac{\partial s_h^*}{\partial w} < 0$. The proportion of extractor in the asymptotically locally equilibrium is a decreasing function of the compensation for non-extraction.

c. An asymptotically locally equilibrium is characterized by $X^* = X'_w = \frac{c+w}{pq}$. Assume compensation for non-extraction $w^* = pqX^* - c$. The corresponding equilibrium is $X'_{w^*} = \frac{c+w^*}{pq} = \frac{c+pqX^*-c}{pq} = X^*$.

d. The resource stock in the asymptotically locally equilibrium is higher with than without compensations for resource extraction, $X'_w > X'$, that is, compensations for non-extraction improve resource preservation. In addition, in the asymptotically locally equilibrium, $\dot{s}_h = 0$ as $\pi_h = w$, the extraction payoff is positive and equal to the compensation for non-extractors. Both extractors and non-extractors get a positive payoff, increasing their welfare.

Proposition 3.4 Compensation funding

Funding is $F^* = w^*(1 - s_h^*)n$. We have already seen that the proportion of extractors is a decreasing function of w at the equilibrium, $\frac{\partial s_h^*}{\partial w} > 0$ (see proposition 3.3b). Therefore, $\frac{\partial F^*}{\partial w} = n\left((1 - s_h^*) - \frac{\partial s_h^*}{\partial w}\right) > 0$ and total funding increases with w^*. To see that total funding increases at an increasing rate, we calculate $\frac{\partial^2 F^*}{\partial w^2} = -2n\frac{\partial^2 s_h}{\partial w^2}$. Calculating $\frac{\partial^2 s_h}{\partial w^2} = -\frac{r}{qn}\frac{2TPq}{(c+w)^3} < 0$ we get $\frac{\partial^2 F^*}{\partial w^2} > 0$, that is, total funding increases at an increasing rate.

Proposition 4.1 Taxes and compensations

a. The equilibrium point is asymptotically locally stable if the trace of the Jacobian matrix of the dynamic system evaluated at that point is negative and the determinant of such matrix is positive. The trace is:

$$J_{11} + J_{22} = \frac{-ts_h^*}{1 - s_h^*} + \frac{r}{K}(K - 2X^* + T) - qns_h^*$$

where $\frac{-ts_h^*}{1-s_h^*} < 0$ and $\frac{r}{K}(K - 2X^* + T) - qns_h^* < 0$ if $X^* > X_D$. Therefore, the trace is negative whenever $X^* > X_D$.

The determinant is:

$$J_{11}J_{22} - J_{12}J_{21} = \frac{-ts_h^*}{1 - s_h^*}\left(\frac{r}{K}(K - 2X^* + T) - qns_h^*\right) + qnX^*s_h^*(1 - s_h^*)pq$$

where the last term is positive and the first one is positive provided $X^* > X_D$. That is the determinant is positive whenever $X^* > X_D$.

Therefore, the equilibrium point (s_h^*, X^*) with $0 < s_h^* < 1$ and $X^* = \tilde{}(s_h^*)$ is asymptotically locally stable whenever $X^* > X_D$.

b. Assume an extraction tax t. The resource stock in the asymptotically locally equilibrium is $X_1^* = \frac{c+t}{pq}$. If the tax revenue is used to equally compensate non-extractors, the resource stock in the asymptotically locally equilibrium is $X_2^* = \frac{1}{pq}\left(\frac{t}{1-s_h} + c\right)$ and it is straight forward that $X_2^* > X_1^*$.

20

c. From the previous statement, if the same t leads to greater resource stock when the tax revenue is used to compensate non-extractors, the same preservation standard, X^* is reached for a lower t when tax revenue is used to compensate non-extractors.