# Endogenous growth, education subsidies, and intergenerational transfers

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**Abstract**

In this paper we investigate the welfare effects of both education subsidies and intergenerational transfers along an arbitrary, non-optimal balanced growth path in an overlapping generations model with both physical and human capital. A lump-sum transfer from the working middle-aged to the elderly translates into a lower accumulation of both physical and human capital (and thus a smaller growth rate). However, it can increase or decrease welfare. A change in the rate of education subsidy can have either a positive or a negative effect on the accumulation of both physical and human capital, but conditions that guarantee a clear-cut sign of the effect of education subsidies on welfare are derived. We also study the comparative dynamics when departing from the laissez-faire balanced growth path, and show that the intergenerational effects of the optimal policy are equivalent to those of a pure pay-as-you-go social security system.

Keywords: endogenous growth, human capital, intergenerational transfers, education policy
JEL Classification: D90, H21, H52, H55

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# 1 Introduction

The welfare effects of education subsidies and the provision of old-age pensions have been the object of some interest in the literature of endogenous growth with overlapping generations. Docquier, Paddison and Pestieau (2007) identify the optimal education subsidy and argue that optimal pensions cannot be expected to be positive in general. Boldrin and Montes (2005) show that, in the absence of credit markets to finance education investment, two systems of independent intergenerational transfers, to the young from the middle-aged (education subsidies) and to the old from the same middle aged (pensions), can be used to replicate the laissez-faire equilibrium with credit markets. Del Rey and Lopez-Garcia (2013) find that the optimal education subsidy is negative (i.e., a tax) and that optimal pensions are positive when the repayment of education loans is subsidized in middle age.

Such different policy implications result naturally from different social objectives, each of them being subject to its own criticism. Docquier et al. (2007) posit that the objective of the social planner is to maximize a discounted sum of individual utilities defined over consumption levels per unit of natural labour. On the one hand, the choice of the discount rate under this approach is inherently arbitrary. On the other hand, and more importantly, the precise cardinalization of individual preferences affects the results in a crucial way, as different utility functions representing the same ordinal preferences give rise to different optimal resource allocations and optimal policies supporting them (Del Rey and Lopez-Garcia, 2012). The explicit purpose of Boldrin and Montes (2005) is to replicate the laissez-faire equilibrium balanced growth path with perfect credit markets when these are absent. This is done under the presumption that such a balanced growth path has some unspecified welfare properties. But, as explicitly recognized by them, this needs not be the case, and no explicit welfare analysis is provided. Finally, Del Rey and Lopez-Garcia (2013) embrace the counterpart in an endogenous growth setting of the Two Part Golden Rule criterion that has been widely used in exogenous growth models, e.g., Diamond (1965), Samuelson (1968, 1975a, 1975b). The social planner now selects the balanced growth path that maximizes the lifetime welfare of a representative generation subject to the constraint that everyone else's welfare is fixed at the same level. As productivity gains lead to consumption growth, this is done in the only sensible way in an endogenous growth framework, i.e., by considering a utility function whose arguments are individual consumptions per unit of *efficient* labour. As a consequence, the focus is on the choice between balanced growth paths, ignoring the initial conditions and the ensuing transitional periods.

In this paper we adopt the latter criterion, that is, a social valuation function whose arguments are individual consumptions per unit of efficient labour, but, instead of searching for the optimal policy, we investigate the comparative dynamics of modifying the tax parameters. This amounts to discussing, first, the consequences of changes in education subsidies and intergenerational transfers for the accumulation of both physical and human capital (and thus the economy's growth rate) along an arbitrary, non-optimal balanced growth path. And, second, evaluating the welfare effects of these changes. It is shown that lump-sum redistribution from the working middle-aged to the elderly reduces the accumulation of both physical and human capital (and thus implies a smaller growth rate). Such a redistribution, however, can either increase or decrease welfare depending on the interaction of two effects: (i) the relationship between the interest rate and the economy's growth rate, and (ii) the effect of education investment on the present value of the individual's lifetime resources. This second element is absent in life-cycle models with exogenous growth and is in the root of the general indeterminacy. A change in the rate of education subsidy can either have a positive or a negative effect on the accumulation of both physical and human capital. As for the effect of education subsidies on welfare, we have derived sufficient conditions (related to (i) and (ii) above) that guarantee that they are welfare increasing.

A situation that deserves particular attention is that where the starting point is the laissez-faire balanced growth path. We show that the introduction of a pay-as-you-go social security increases welfare whenever the growth rate of the economy is larger than the interest rate. However, nothing can be said with generality when the relationship between these rates is reversed. It is worth emphasizing that while the first result parallels its counterpart in overlapping generation models with exogenous growth, the second one is in open contradiction with it. As regarding the effects of education subsidies when the starting point is the laissez faire, provided that they foster physical capital accumulation, they will also entail a higher [resp. lower] welfare level whenever the economy's growth rate is lower [resp. higher] than the interest rate, and investing in education enhances [resp. reduces] the present value of individuals lifetime resources. Otherwise, no general statement can be advanced once again.

Finally, the optimal policy that allows the social planner to maximize social welfare is discussed. Since the social valuation function is the same as in Del Rey and Lopez-Garcia (2013), it is not surprising that the optimality conditions coincide. As stated above, they prove that the optimal education subsidy is negative (i.e., a tax) and that the optimal pensions are positive. However, we take here one step further and show that the intergenerational effects of the optimal tax policy are equivalent to those of a *pay-as-you-go* social security system where the middle-aged contribute and the old-aged retirees receive a pension benefit. And, interestingly, the pay-as-you-go nature of the optimal policy is independent of the characteristics of the laissez-faire balanced growth path. Once more, this is in sharp contrast with the results obtained from exogenous growth models, where the precise configuration of the optimal social security depends in a crucial way on the relationship between the interest rate and the growth rate in the laissez-faire equilibrium (Samuelson, 1975b).

The rest of the paper is organized as follows. Section 2 presents the model and discusses the decentralized equilibrium in the presence of government. Section 3 characterizes the balanced growth paths and provides expressions for the ratios of physical and human capital per unit of efficient labour as functions of the tax parameters. It also obtains the indirect utility function that is the basis of the welfare analysis. The comparative dynamics associated with changes in the education subsidies and the lump-sum tax levied on the working generation is analyzed in Section 4. Section 5 clarifies the pay-as-you-go nature of the intergenerational effects induced by the optimal policy. Section 6 concludes.

# 2 The Model and the Decentralized Equilibrium with Government

The framework of analysis is the overlapping generations model with both human and physical capital in Boldrin and Montes (2005), Docquier et al. (2007) and Del Rey and Lopez-Garcia (2013). At period $t$, $L_{t+1}$ individuals are born, and coexist with $L_t$ middle-aged and $L_{t-1}$ old-aged. Population grows at the exogenous rate $n$ so that $L_t = (1+n)L_{t-1}$ with $n > -1$. Agents are born with some level of human capital $h_{t-1}$, measured in units of efficient labour per unit of natural labour. Human capital in period $t$ results from the interaction of the amount of output invested in education $d_{t-1}$ and the inherited human capital $h_{t-1}$ according to the production function $h_t = E(d_{t-1}, h_{t-1})$. Assuming constant returns to scale, the production of human capital can be written in intensive terms as $h_t/h_{t-1} = e(\tilde{d}_{t-1})$, where $e(.)$ satisfies the Inada conditions and $\tilde{d}_{t-1} = d_{t-1}/h_{t-1}$ is the amount of output devoted to education per unit of inherited human capital. Therefore, the growth rate of productivity from period $t-1$ to period $t$, $g_t$, satisfies $h_t/h_{t-1} = e(\tilde{d}_{t-1}) = (1+g_t)$.

The economy is closed and produces a single good, $Y_t$, by means of physical capital $K_t$ and human capital $H_t$, according to a constant returns to scale production function $Y_t = F(K_t, H_t)$. Only the middle-aged work, supplying inelastically one unit of natural labour, so that $H_t = h_t L_t$. Physical capital fully depreciates each period. Letting $k_t = K_t/L_t$ be the physical capital per unit of natural labour ratio and $\tilde{k}_t = K_t/H_t = k_t/h_t$ the physical capital per unit of efficient labour ratio, the technology can be described as $Y_t/H_t = f(\tilde{k}_t)$, where $f(.)$ also satisfies the Inada conditions.

Factor prices are determined under perfect competition by their marginal products, so that, if $1+r_t$ and $w_t$ are respectively the interest factor and the wage rate per unit of efficient labour,

$$
\begin{aligned}
1 + r_t &= f'(\tilde{k}_t) & (1)\\
w_t &= f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t) & (2)
\end{aligned}
$$

The government has two policy instruments at its disposal: intergenerational transfers from the middle-aged to the elderly and subsidies to the repayment, in the second period of life, of the loans taken in the first one to pay for education. Let $z_t^m > 0$ [resp. $< 0$] be the lump-sum tax [transfer] the middle aged pay [receive], $z_t^o > 0$ [$< 0$] the lump-sum tax the old pay [the pension they receive] and let $\theta_t$ be the subsidy rate, all of them in period $t$. The government budget constraint is:

$$z_t^m L_t + z_t^o L_{t-1} = \theta_t (1 + r_t) d_{t-1} L_t \tag{3}$$

Notice that a subsidy to the repayment of loans is not the only way to model education subsidies. This approach, however, emphasizes the role of credit markets in financing human capital investments, and the interaction of this process with public policy.

Individuals behave as pure life-cyclers and only consume in their second and third period. The lifetime utility function of an individual born at period $t-1$ is $U_t = U(c_t^m, c_{t+1}^o)$, where $c_t^m$ and $c_{t+1}^o$ denote her consumption levels as middle-aged and old-aged respectively. This utility function is assumed to be strictly quasi-concave and homogeneous of degree $j > 0$.[1] In their first period, individuals born at $t-1$ borrow in perfect credit markets the amount they need to pay the education that maximizes their lifetime resources, $d_{t-1}$. In their second period they work, pay taxes $z_t^m$, pay back the loan less subsidies $(1 + r_t) d_{t-1}(1 - \theta_t)$, consume and save to finance consumption in their third period. In this third period, individuals consume and pay taxes $z_{t+1}^o$. Letting $s_t$ stand for savings of a middle-aged:

$$c_t^m = w_t h_t - (1 + r_t) d_{t-1}(1 - \theta_t) - z_t^m - s_t \tag{4}$$

$$c_{t+1}^o = (1 + r_{t+1}) s_t - z_{t+1}^o \tag{5}$$

Thus, the lifetime budget constraint of an individual born at period $t - 1$ is:

$$c_t^m + \frac{c_{t+1}^o}{(1 + r_{t+1})} = w_t h_t - (1 + r_t) d_{t-1}(1 - \theta_t) - z_t^m - \frac{z_{t+1}^o}{(1 + r_{t+1})} \tag{6}$$

The first order conditions associated with the individual decision variables, $d_{t-1}, c_t^m$ and $c_{t+1}^o$, are:

$$w_t e'(d_{t-1}/h_{t-1}) = (1 + r_t)(1 - \theta_t) \tag{7}$$

[1] The reason why we only impose strict quasi-concavity instead of strict concavity of the individual utility function is that, as it will be made clearer shortly, we are only interested in *ordinal* preferences.

$$\frac{\partial U(c_t^m, c_{t+1}^o)/\partial c_t^m}{\partial U(c_t^m, c_{t+1}^o)/\partial c_{t+1}^o} = (1 + r_{t+1}) \tag{8}$$

where use has been made of the homogeneity of degree one of the $E$ function, i.e., $h_t = e(d_{t-1}/h_{t-1})h_{t-1}$. Equation (7) shows that the individual will invest in education up to the point where the marginal benefit in terms of second period income equals the marginal cost of investing in human capital allowing for subsidies. Rewriting (7) as $e'(\tilde{d}_{t-1}) = (1 - \theta_t)$ $(1 + r(\tilde{k}_t))/w(\tilde{k}_t)$, this expression implicitly characterizes the optimal ratio $\tilde{d}_{t-1}$ as a function of $\tilde{k}_t$ and $\theta_t$, i.e., $\tilde{d}_{t-1} = \phi(\tilde{k}_t, \theta_t)$. Since $e'' < 0$ it can readily be shown that the greater $\tilde{k}_t$ and $\theta_t$, the greater $\tilde{d}_{t-1}$.

Using (3), (6) becomes

$$c_t^m + \frac{c_{t+1}^o}{(1 + r_{t+1})} = \omega_t \tag{9}$$

where $\omega_t$ is the present value of the net lifetime income of an individual born at $t - 1$:

$$\omega_t = w_t h_t - (1 + r_t) d_{t-1}(1 - \theta_t) - z_t^m - \frac{(1 + n)}{(1 + r_{t+1})}[\theta_{t+1}(1 + r_{t+1}) d_t - z_{t+1}^m] \tag{10}$$

The homogeneity assumption on preferences implies that the $c_{t+1}^o/c_t^m$ ratio is a function of $r_{t+1}$ only. This allows one to write consumption in the second period as $c_t^m = \pi(r_{t+1})\omega_t$, where the function $\pi(.)$ depends on the interest rate only. Equilibrium in the market for physical capital is achieved when the (physical) capital stock available in $t + 1$, $K_{t+1}$, equals gross savings made by the middle-aged in $t$, $s_t L_t$, minus the amount of output devoted to human capital investment by the young in $t$, $(1+n)d_t L_t$, i.e., when $K_{t+1} = s_t L_t - (1+n) d_t L_t$. This equilibrium condition can be expressed as

$$\tilde{k}_{t+1} = \frac{(1 - \pi(r_{t+1}))\,\tilde{\omega}_t}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))(1 + n)} - \frac{\tilde{z}_{t+1}^m}{(1 + r_{t+1})} - \frac{(1 - \theta_{t+1})\,\phi(\tilde{k}_{t+1}, \theta_{t+1})}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))} \tag{11}$$

where $\tilde{z}_{t+1}^m = z_{t+1}^m/h_{t+1}$ and $\tilde{\omega}_t = \omega_t/h_t$ is the present value of lifetime resources expressed in terms of output per unit of efficient labour. Taking into account (1) and (2), this expression implicitly provides $\tilde{k}_{t+1}$ as a function of $\tilde{k}_t$, $\tilde{z}_t^m$, $\tilde{z}_{t+1}^m$, $\theta_t$, and $\theta_{t+1}$, i.e., $\Psi(\tilde{k}_t; \tilde{z}_t^m, \tilde{z}_{t+1}^m, \theta_t, \theta_{t+1})$.

Finally, using factor prices in (1) and (2), the government budget constraint (3), the individual budget constraints in middle and old-age, (4) and (5), and equilibrium condition (11), one can find the aggregate feasibility constraint expressed in terms of output per unit of efficient labour:

$$\tilde{c}_t^m + \frac{\tilde{c}_t^o}{e(\tilde{d}_{t-1})(1 + n)} = f(\tilde{k}_t) - (1 + n)\tilde{d}_t - e(\tilde{d}_t)(1 + n)\tilde{k}_{t+1} \tag{12}$$

where $\tilde{c}_t^m = c_t^m/h_t$ and $\tilde{c}_t^o = c_t^o/h_{t-1}$ are, respectively, consumption of a middle-aged and of an old-aged in period $t$ per unit of labour efficiency.

# 3 Balanced Growth Paths

In the current framework, a balanced growth path is a situation where all variables expressed in terms of output per unit of natural labour grow at a constant rate. As a consequence, all variables per unit of efficient labour will remain constant over time, i.e., $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}$, $\tilde{z}_t^m = \tilde{z}_{t+1}^m = \tilde{z}^m$, and $\theta_t = \theta_{t+1} = \theta$. Along a balanced growth path, one can delete the time subscripts in (11) and write $\tilde{k} = \Psi(\tilde{k}; \tilde{z}^m, \theta)$. An equilibrium ratio of physical capital to labour in efficiency units along a balanced growth path in the presence of government intervention, $\tilde{k}_G$, will then be a fixed point of the $\Psi$ function, i.e., $\tilde{k}_G = \Psi(\tilde{k}_G; \tilde{z}^m, \theta)$. Such an equilibrium will be locally stable provided that $0 < \partial\Psi(\tilde{k}_G; \tilde{z}^m, \theta)/\partial\tilde{k} < 1$. In what follows, we will focus on situations where the equilibrium is unique and stable, so that the relationship between $\tilde{k}$ and the tax parameters can be written, with an obvious notation, as

$$\tilde{k} = \tilde{k}(\tilde{z}^m, \theta) \tag{13}$$

We can now turn to the determination of $\tilde{d}$ or, what is the same, the growth rate $g$ that satisfies $(1+g) = e(\tilde{d})$. The amount of output devoted to education per unit of inherited human capital along a balanced growth path will be governed by the relationship arising from the education decision (7), i.e., $\tilde{d} = \phi(\tilde{k}, \theta)$. Using (13) we can write $\tilde{d} = \phi\left(\tilde{k}(\tilde{z}^m, \theta), \theta\right)$ or, for short,

$$\tilde{d} = \tilde{d}(\tilde{z}^m, \theta) \tag{14}$$

Letting $g_G$ be the growth rate of any variable expressed in terms of output per unit of natural labour, we have $1 + g_G = e\left(\phi(\tilde{k}_G, \theta)\right)$. The growth rate of all variables expressed in absolute terms (physical capital, human capital and output) is $(1+g_G)(1+n)$.

Regarding consumer behaviour, the fact that the utility function is homothetic implies that the marginal rates of substitution in the $\left(c_t^m, c_{t+1}^o\right)$ and $(\tilde{c}^m, \tilde{c}^o)$ spaces will be the same. Thus, individual behavior along a balanced growth path in the presence of government intervention can be summarized by:

$$\frac{\partial U(\tilde{c}^m, \tilde{c}^o)/\partial\tilde{c}^m}{\partial U(\tilde{c}^m, \tilde{c}^o)/\partial\tilde{c}^o} = (1+r) \tag{15}$$

$$we'(\tilde{d}) = (1+r)(1-\theta) \tag{16}$$

$$\tilde{c}^m + \frac{\tilde{c}^o}{(1+r)} = \tilde{\omega} \tag{17}$$

where

$$\tilde{\omega} = w - \frac{(1+r)\tilde{d}}{(1+g)} - \frac{\left[\theta(1+r)\tilde{d} - (1+g)\tilde{z}^m\right]}{(1+r)(1+g)}\left[(1+g)(1+n) - (1+r)\right] \tag{18}$$

Eq. (15) can be helpful in illustrating the approach that is adopted in the subsequent section. Clearly, (15) states that, along a balanced growth path, the marginal rate of substitution between second and third period consumptions *per unit of efficient labour*, $\tilde{c}^m$ and $\tilde{c}^o$, has to be equal to the interest factor. To be sure, this is the same result as the one in (8). But one has to realize that the presence of productivity growth entails that along a balanced growth path consumptions per unit of natural labour, $c_t^m$ and $c_{t+1}^o$ will grow to infinity. It then follows that, for reasonable specifications of preferences, the utility level $U_t = U(c_t^m, c_{t+1}^o)$ will also grow without limit. The obvious consequence is that there is no scope for the evaluation of the welfare effects of modifying tax parameters along a balanced growth path. A way to sidestep this circumstance in order to analyze the comparative dynamics is to adopt the valuation function that underlies eq. (15). In other words, to evaluate the individual utility function using as arguments consumptions per unit of *efficient* labour. Along a balanced growth path $c_t^m$, $c_{t+1}^o$ and $h_t$ will be growing at the same rate $g$, so that the utility index $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$ converges and is well defined. Notice that this procedure entails a monotonic transformation of the individual utility function so that individual ordinal preferences are respected.[2]

With the utility function $U(\tilde{c}^m, \tilde{c}^o)$ and the lifetime budget constraint (17)-(18) the demands for consumption can be written $\tilde{c}^m = \tilde{c}^m(\tilde{\omega}, r)$ and $\tilde{c}^o = \tilde{c}^o(\tilde{\omega}, r)$.[3] Therefore, an indirect utility function can be obtained

$$\tilde{U} = V(\tilde{\omega}, r) = U[\tilde{c}^m(\tilde{\omega}, r), \tilde{c}^o(\tilde{\omega}, r)] \tag{19}$$

providing the maximum level of $\tilde{U}$ as a function of the present value of lifetime resources and the relative price of old age and middle-age consumption. Observe that the present value lifetime income $\tilde{\omega}$, in spite of being a function of a number of variables, is taken as a parameter in the indirect utility function $V(\tilde{\omega}, r)$. For later use, we can obtain the partial derivatives of $V$ with respect to its arguments $\tilde{\omega}$ and $r$. Using (15):

$$\frac{\partial V}{\partial \tilde{\omega}} = (1+r)\frac{\partial U}{\partial \tilde{c}^o}\left(\frac{\partial \tilde{c}^m}{\partial \tilde{\omega}} + \frac{1}{(1+r)}\frac{\partial \tilde{c}^o}{\partial \tilde{\omega}}\right) = (1+r)\frac{\partial U}{\partial \tilde{c}^o} \tag{20}$$

$$\frac{\partial V}{\partial r} = (1+r)\frac{\partial U}{\partial \tilde{c}^o}\left(\frac{\partial \tilde{c}^m}{\partial r} + \frac{1}{(1+r)}\frac{\partial \tilde{c}^o}{\partial r}\right) = \frac{\partial U}{\partial \tilde{c}^o}\frac{\tilde{c}^o}{(1+r)} \tag{21}$$

[2] The homogeneity of degree $j$ of the utility function allows to write $\tilde{U}_t = U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) = U(c_t^m/h_t, c_{t+1}^o/h_t) = (1/h_t^j)U(c_t^m, c_{t+1}^o) = (1/h_t^j)U_t$. Notice that both $U(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ and $U(c_t^m, c_{t+1}^o)$ have the same functional form and are homogeneous of degree $j$. Thus, as stated above, the curvature and higher derivatives of indifference curves in $(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ will be the same as their counterparts in the $(c_t^m, c_{t+1}^o)$ space.

[3] Homogeneity of preferences implies that $c_t^m = \pi(r_{t+1})\omega_t$, so that, using (9), $c_{t+1}^o = (1 + r_{t+1})(1 - \pi(r_{t+1}))\omega_t$ and both consumption levels can be written as functions of $\omega_t$ and $r_{t+1}$. It then follows that, for any optimal choice of $d_{t-1}$, and thus of $h_t$, consumptions per unit of efficient labour become $\tilde{c}_t^m = \pi(r_{t+1})\tilde{\omega}_t$ and $\tilde{c}_{t+1}^o = (1 + r_{t+1})(1 - \pi(r_{t+1}))\tilde{\omega}_t$, i.e., functions of $\tilde{\omega}_t$ and $r_{t+1}$.

where the last equality follows in both cases from differentiation of the lifetime budget constraint (17).[4]

Finally, coming back to function $V(\tilde{\omega}, r)$, and taking into account that $\tilde{\omega} = \tilde{\omega}(\tilde{k}, \tilde{d}, \tilde{z}^m, \theta)$, $r = r(\tilde{k})$, $\tilde{k} = \tilde{k}(\tilde{z}^m, \theta)$ and $\tilde{d} = \tilde{d}(\tilde{z}^m, \theta)$, we can write a new function that, for a given value of $n$, depends only on $\tilde{z}^m$ and $\theta$, that is $\tilde{U} = V\left[\tilde{\omega}\left(\tilde{k}(\tilde{z}^m, \theta), \tilde{d}(\tilde{z}^m, \theta), \tilde{z}^m, \theta\right), r\left(\tilde{k}(\tilde{z}^m, \theta)\right)\right]$. As a consequence, we end up with a new indirect utility function,

$$\tilde{U} = \tilde{V}(\tilde{z}^m, \theta) \tag{22}$$

explicitly relating $\tilde{U}$ to the tax parameters $\tilde{z}^m$ and $\theta$. This, together with (13) and (14), are the relevant functions to undertake the comparative dynamics.

# 4 Comparative Dynamics

We are now in a position to discuss the comparative dynamics, i.e., the effects of variations in the level of intergenerational transfers and the tax parameter addressed to education decisions on both physical and human capital accumulation, and on welfare, when the economy follows an arbitrary balanced growth path.

As (13) is implicitly given by $\tilde{k} = \Psi(\tilde{k}; \tilde{z}^m, \theta)$, we have:

$$\frac{\partial \tilde{k}(\tilde{z}^m, \theta)}{\partial \tilde{z}^m} = \frac{\partial \Psi(.)}{\partial \tilde{k}} \frac{\partial \tilde{k}(\tilde{z}^m, \theta)}{\partial \tilde{z}^m} + \frac{\partial \Psi(.)}{\partial \tilde{z}^m} \tag{23}$$

with a similar expression for the changes in $\theta$. Thus, using Eq. (11) without time subscripts, we can write

$$\frac{\partial \tilde{k}(\tilde{z}^m, \theta)}{\partial \tilde{z}^m} = \frac{1}{\Omega}\left[\frac{-(1-\pi(r))}{e(\phi(.))(1+n)} - \frac{\pi(r)}{(1+r)}\right] < 0 \tag{24}$$

where $0 < \Omega = 1 - \partial \Psi(.)/\partial \tilde{k} < 1$ along a locally stable balanced growth path. In words, (24) implies that the larger are the taxes paid by the middle-aged, other things being the same, the lower is the $\tilde{k}$ ratio. Thus, the message emerging from exogenous growth models with a pure life-cycle saving motive continues to hold in the current framework: intergenerational transfers from the middle aged to the elderly depress savings and physical capital accumulation (as measured by $\tilde{k}$). Now, we can turn to expression $\tilde{d} = \phi\left(\tilde{k}(\tilde{z}^m, \theta), \theta\right)$, which implicitly characterizes (14). Since $\partial \phi/\partial \tilde{k} > 0$, and using (24), it follows that $\partial \tilde{d}(\tilde{z}^m, \theta)/\partial \tilde{z}^m = (\partial \phi(.)/\partial \tilde{k})(\partial \tilde{k}(.)/\partial \tilde{z}^m) < 0$. Therefore, the above result concerning physical capital can be extended to human capital: transfers from younger to older generations

[4]Clearly, $\tilde{\omega}$ is given at this stage. The effect of $r$ on the present value of lifetime resources $\tilde{\omega}$ will come forth when we differentiate $\tilde{\omega}$ with respect to $\tilde{k}$.

not only reduce the accumulation of physical capital (as measured by $\tilde{k}$) but also the accumulation of human capital (as measured by $\tilde{d}$) and thus the growth rate of the economy.

Clearly, the above discussion applies also to the situation where $\theta = 0$, so that the only role of the government is to provide pensions with an unfunded scheme, i.e., $-z_t^o = (1+n)z_t^m$ in (3). Therefore, the introduction or enlargement of a pure pay-as-you-go social security system will negatively affect the accumulation of both physical and human capital, and will depress the growth rate of the economy.

As far as the effects of changes in the education subsidy are concerned, one would be tempted to conjecture that an increased $\theta$ will translate into a higher value of $\tilde{d}$. As $\partial\phi/\partial\theta > 0$ in (7), this is certainly the case when $\tilde{k}$ is held constant in the expression characterizing the individual's decision on education, $\tilde{d} = \phi(\tilde{k}, \theta)$. However, this is nothing else but a partial-equilibrium result that neglects the effects of the tax subsidy on $\tilde{k}$. As it will be seen shortly, when these effects are taken into account, it is impossible to say in general whether higher education subsidies will have a positive or a negative effect on the accumulation of human capital (as measured by $\tilde{d}$) and the economy's growth rate. The reason for this can be found in that

$$\begin{aligned}\frac{\partial\tilde{k}(\tilde{z}^m,\theta)}{\partial\theta} &= \frac{1}{\Omega}\left[\frac{(1-\pi)(1+r)\phi(.)}{e\left[\phi(.)\right]^2(1+n)} + \frac{\pi\phi(.)}{e\left[\phi(.)\right]}\right] \\ &\quad -\frac{1}{\Omega}\left[\frac{(1-\pi)e'(.)}{e\left[\phi(.)\right]^2(1+n)}\frac{\partial\phi(.)}{\partial\theta}\left(w - \frac{(1+r)(1-\theta)\phi(.)}{e\left[\phi(.)\right]} - \tilde{z}^m\right)\right] \\ &\quad -\frac{1}{\Omega}\left[\left(\frac{(1-\pi)(1+r)(1-\theta)}{e\left[\phi(.)\right]^2(1+n)} + \frac{(1-\pi\theta)}{e\left[\phi(.)\right]}\right)\frac{\partial\phi(.)}{\partial\theta}\left(1 - \frac{e'(.)}{e(.)/\phi(.)}\right)\right] \end{aligned} \tag{25}$$

The three terms in square brackets on the right hand side of (25) are positive.[5] However, the second and third terms are affected by a negative sign. Overall, this means that the sign of (25) is ambiguous. This indeterminacy reflects the complexity of the underlying interaction between the subsidy rate and the consumption and education decisions, and implies that the effect of the subsidy on the $\tilde{d}$ ratio,

$$\frac{\partial\tilde{d}(\tilde{z}^m,\theta)}{\partial\theta} = \frac{\partial\phi(.)}{\partial\tilde{k}}\frac{\partial\tilde{k}(\tilde{z}^m,\theta)}{\partial\theta} + \frac{\partial\phi(.)}{\partial\theta} \tag{26}$$

is also ambiguous.

The previous results can be summarized in the following proposition.

[5] Note that $\frac{\partial\phi(.)}{\partial\theta}$ in the second term on the right hand side of (25) is positive by the first order condition (7), and that the expression in brackets is also positive when $\tilde{k} > 0$. As for the third one, the concavity of the $e(\tilde{d})$ function ensures that the average product $e(.)/\phi(.)$ exceeds the marginal product $e'(.)$, and, on the other hand, $(1 - \pi\theta) > 0$ regardless of the sign of $\theta$.

**Proposition 1** *Along an arbitrary balanced growth path, a lump-sum transfer from the middle-aged to the elderly translates into a smaller accumulation of both physical and human capital (and thus a smaller growth rate). A change in the rate of education subsidy may have either a positive or negative effect on both.*

We can now focus on the welfare effects of changing the tax parameters along a balanced growth path. The conclusion obtained in overlapping generations models with exogenous growth is well known: when the marginal product of physical capital is lower [resp. higher] than the economy's growth rate (reflecting over [resp. under] accumulation of physical capital), a lump-sum transfer from [resp. to] younger to [resp. from] older generations provides a means to bring the economy closer to the Two Part Golden Rule. However, as our analysis will show, in the current model where human capital is the engine of growth, this condition is not sufficient any longer, and further requirements are to be met. Using the indirect utility function $\tilde{U} = V\left[\tilde{\omega}\left(\tilde{k}(\tilde{z}^m,\theta), \tilde{d}(\tilde{z}^m,\theta), \tilde{z}^m, \theta\right), r\left(\tilde{k}(\tilde{z}^m,\theta)\right)\right]$ in (22), we have:

$$\frac{\partial \tilde{V}(\tilde{z}^m,\theta)}{\partial \tilde{z}^m} = \frac{\partial V}{\partial \tilde{\omega}}\left(\frac{\partial \tilde{\omega}}{\partial \tilde{k}}\frac{\partial \tilde{k}}{\partial \tilde{z}^m} + \frac{\partial \tilde{\omega}}{\partial \tilde{d}}\frac{\partial \tilde{d}}{\partial \tilde{z}^m} + \frac{\partial \tilde{\omega}}{\partial \tilde{z}^m}\right) + \frac{\partial V}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}\tilde{k}}\frac{\partial \tilde{k}}{\partial \tilde{z}^m} \tag{27}$$

with a similar expression for the changes in $\theta$.

After some manipulation, eq. (27) can be rewritten, with the aid of (20), (21), the lifetime budget constraint and the equilibrium condition in the market for physical capital, as:

$$\frac{\partial \tilde{V}(\tilde{z}^m,\theta)}{\partial \tilde{z}^m} = \frac{\partial U}{\partial \tilde{c}^o}\left(J\left[(1+g)(1+n)-(1+r)\right] + M\frac{\partial \tilde{\omega}}{\partial \tilde{d}}\right)\frac{\partial \tilde{k}}{\partial \tilde{z}^m} + \frac{\partial U}{\partial \tilde{c}^o}\left[(1+g)(1+n)-(1+r)\right] \tag{28}$$

where $J = \tilde{k}f'' + \phi(.)(1-\theta)f''/(1+g)$ is negative, $M = (1+r)(\partial\phi(.)/\partial\tilde{k})$ is positive and

$$\frac{\partial \tilde{\omega}}{\partial \tilde{d}} = -\frac{(1+r)}{(1+g)} + \frac{(1+r)\phi(.)(1-\theta)e'(.)}{(1+g)^2} + \frac{\tilde{z}^m(1+n)e'(.)}{(1+r)} - \frac{\theta\left[(1+g)(1+n)-(1+r)\right]}{(1+g)} \tag{29}$$

In order to interpret (28), it is important to realize that $\partial\tilde{\omega}/\partial\tilde{d}$ captures the effect of a change in the accumulation of human capital (as measured by $\tilde{d}$) on the present value of the individual's lifetime resources for given values of $w$, $r$, $\tilde{z}^m$ and $\theta$. This can easily be checked by partial differentiation of (18). It can also be verified by mere inspection that this expression cannot be signed in general. Overall, the first term on the right hand side of (28) reflects the effect of the change in $\tilde{k}$ induced by the lump-sum tax on the middle aged, while the second term reflects the direct effect of $\tilde{z}^m$ on welfare. Clearly, if $[(1+g)(1+n)-(1+r)]$ and $\partial\tilde{\omega}/\partial\tilde{d}$ have opposite signs, (28) can be signed without ambiguity, and we can enunciate the following proposition.

**Proposition 2** *Along an arbitrary balanced growth path, a lump-sum transfer from the middle-aged to the elderly entails a welfare increase [resp. decrease] provided that: (i) the economy's growth rate is greater [resp. less] than the interest rate, and (ii) investing in education reduces [resp. increases], at the margin, the present value of the individual's lifetime resources. Otherwise, the effects of intergenerational transfers on welfare along the balanced growth path are ambiguous.*

The intuition is simple and highlights the importance of both (i) and (ii) being simultaneously fulfilled. As for (i), this is actually what arises from an exogenous growth model when $\tilde{z}^m$ and $-\tilde{z}^o$ are respectively interpreted as the tax paid and the pension received in a pure pay-as-you-go social security system: if $(1+g)(1+n) > (1+r)$, so that the rate of return of "investing in future generations" exceeds that of investing in physical capital, an additional amount paid to social security will expand consumption possibilities and welfare. But, in an endogenous growth setting, the above condition must also be accompanied by (ii): since increasing $\tilde{z}^m$ depresses $\tilde{k}$ and thus discourages investments in $\tilde{d}$, welfare will unambiguously increase along the balanced growth path only if, in addition to $(1+g)(1+n) > (1+r)$, the reduced human capital accumulation translates into a greater lifetime income, i.e., $\partial\tilde{\omega}/\partial\tilde{d} < 0$. The argument should be accordingly reversed when the starting point entails $(1+g)(1+n) < (1+r)$ and $\partial\tilde{\omega}/\partial\tilde{d} > 0$. It is important to stress that, if $[(1+g)(1+n) - (1+r)]$ and $\partial\tilde{\omega}/\partial\tilde{d}$ have the same sign, the final effect is indeterminate.

The preceding proposition takes as a starting point for the tax change *any* balanced growth path, with arbitrary values of $\tilde{z}^m$ and $\theta$. We can now focus on the *laissez-faire* balanced growth path, i.e., when $\tilde{z}^m = \theta = 0$. In this case, (29) reduces to

$$\frac{\partial\tilde{\omega}_{LF}}{\partial\tilde{d}_{LF}} = -\frac{(1+r_{LF})}{(1+g_{LF})}\left(1 - \frac{e'(\tilde{d}_{LF})}{e(\tilde{d}_{LF})/\tilde{d}_{LF}}\right) < 0 \tag{30}$$

where the subscript $LF$ stands for laissez-faire and $\tilde{\omega}_{LF} = w_{LF} - (1+r_{LF})\tilde{d}_{LF}/(1+g_{LF})$. Clearly, the assumptions on the shape of the $e(.)$ function imply that (30) is negative: along the laissez-faire balanced growth path, a marginal increase in the resources devoted to education per unit of efficient labour translates into a reduction in the amount of output per unit of efficient labour available in adult life for consumption and saving. Going back to (28), we can evaluate $\partial\tilde{V}(0,0)/\partial\tilde{z}^m$ and state the following

**Proposition 3** *Starting from a laissez-faire equilibrium balanced growth path such that* $(1+g_{LF})(1+n) > (1+r_{LF})$*, the introduction of a pay-as-you-go social security increases welfare along the new balanced growth path. However, if* $(1+g_{LF})(1+n) < (1+r_{LF})$ *nothing can be said in general about the effect of a pay-as-you-go social security on welfare.*

This result is in contrast with the one arising in models with exogenous growth à la Diamond (1965). Indeed, in such a framework, there is an unambiguous relationship between, on the one hand, whether the laissez-faire interest rate exceeds or falls short of the economy's growth rate and, on the other, the desirability of introducing a pay-as-you-go social security system. Proposition 3 shows that, in the current framework, unfunded social security increases welfare along the new balanced growth path when $(1+g_{LF})(1+n) > (1+r_{LF})$ but it needs not reduce welfare when the rate of return on the payments made to social security in middle age is less than the opportunity cost (i.e., the return that would have been obtained had those payments been invested in physical capital).

Following the same steps, the effects of an increase in the education subsidy rate on welfare along a balanced growth path can be summarized in the following expression:

$$\begin{aligned}\frac{\partial \tilde{V}(\tilde{z}^m, \theta)}{\partial \theta} &= \frac{\partial U}{\partial \tilde{c}^o}\left(J\left[(1+g)(1+n)-(1+r)\right]+M\frac{\partial \tilde{\omega}}{\partial \tilde{d}}\right)\frac{\partial \tilde{k}}{\partial \theta} \\ &\quad -\frac{\partial U}{\partial \tilde{c}^o}N\left[(1+g)(1+n)-(1+r)\right]+\frac{\partial U}{\partial \tilde{c}^o}P\frac{\partial \tilde{\omega}}{\partial \tilde{d}} \qquad (31)\end{aligned}$$

where $N=(1+r)\phi(.)/(1+g)$ and $P=(1+r)(\partial\phi(.)/\partial\theta)$ are both positive. It is clear from mere inspection that the first terms on the right hand side of (28) and (31) have the same structure, i.e., they capture the effect on $\tilde{k}$ induced by a small change in a tax parameter. But this should not obscure the fact that, while $\partial\tilde{k}/\partial\tilde{z}^m$ can be signed, $\partial\tilde{k}/\partial\theta$ cannot. Indeed, only if (25) happens to be positive, (31) will also have a clear-cut sign whenever $[(1+g)(1+n)-(1+r)]$ and $\partial\tilde{\omega}/\partial\tilde{d}$ have opposite signs. In particular,

**Proposition 4** *Along an arbitrary balanced growth path, education subsidies will increase [resp. decrease] welfare provided that they foster physical capital accumulation, and: (i) the economy's growth rate is lower [resp. higher] than the interest rate, and (ii) investing in education increases [resp. decreases], at the margin, the present value of the individual's lifetime resources.*

The intuition underlying this result is similar to the one in Proposition 2. But in addition to the condition that $[(1+g)(1+n)-(1+r)]$ and $\partial\tilde{\omega}/\partial\tilde{d}$ have opposite signs, the requirement that $\partial\tilde{k}/\partial\theta > 0$ now becomes crucial. If, instead, $\partial\tilde{k}/\partial\theta < 0$, the sign of (31) is ambiguous and nothing can be said with generality. Otherwise, note also that the conditions for transfers from the middle-aged to the elderly to increase welfare imply that education subsidies decrease welfare and vice versa.

In the same way we proceeded with Proposition 3, we can now restrict the analysis even further and take the *laissez-faire* equilibrium as the starting point. In order to evaluate $\partial\tilde{V}(0,0)/\partial\theta$ we have to use (30). Under the assumption that $\partial\tilde{k}(.)/\partial\theta > 0$ we have

**Proposition 5** *Starting from a laissez-faire equilibrium balanced growth path such that* $(1+g_{LF})(1+n) > (1+r_{LF})$*, the introduction of education subsidies reduces welfare along the new balanced growth path, provided that they foster physical capital accumulation. However, nothing can be said in general in the case where* $(1+g_{LF})(1+n) < (1+r_{LF})$*.*

# 5 Optimal policy

The discussion in the preceding section raises the question of the characterization of the *optimal* public policy, i.e., the one that allows to decentralize the resource allocation that maximizes social welfare. As the indirect utility function $\tilde{U} = \tilde{V}(\tilde{z}^m, \theta)$ is obtained from the direct one $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$, the optimal values of the intergenerational transfers, $\tilde{z}^m_*$, and of the education subsidies, $\theta_*$, will be simultaneously determined by $\partial\tilde{V}(\tilde{z}^m_*, \theta_*)/\partial\tilde{z}^m = 0$ and $\partial\tilde{V}(\tilde{z}^m_*, \theta_*)/\partial\theta = 0$. From (28) and (31) it is clear that $(1+r_*) = (1+g_*)(1+n)$ and $\partial\tilde{\omega}_*/\partial\tilde{d} = 0$, both evaluated at the optimal physical and human capital ratios, $\tilde{k}_*$ and $\tilde{d}_*$, provide a solution to this system of equations. The first one is the equality of the marginal product of physical capital and the optimal growth rate of the economy,

$$f'(\tilde{k}_*) = e(\tilde{d}_*)(1+n) \tag{32}$$

The second one can be written as

$$\frac{e'(\tilde{d}_*)}{e(\tilde{d}_*)/\tilde{d}_*} - 1 = e'(\tilde{d}_*)\left(\frac{\tilde{d}_*\theta_*}{e(\tilde{d}_*)} - \frac{\tilde{z}^m_*}{e(\tilde{d}_*)(1+n)}\right) \tag{33}$$

which, using the individual budget constraint and the physical capital market equilibrium condition, reduces to

$$e'(\tilde{d}_*)\left(\frac{\tilde{c}^o_*}{\left[e(\tilde{d}_*)(1+n)\right]^2} - \tilde{k}_*\right) = 1 \tag{34}$$

In fact, as shown in Del Rey and Lopez-Garcia (2013), (32) and (34) are two of the optimality conditions of the problem where a social planner chooses the values $(\tilde{c}^m_*, \tilde{c}^o_*, \tilde{k}_*, \tilde{d}_*)$ that maximize the utility function $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$ subject to the balanced growth path version of the feasibility constraint (12) and the technological relationship $(1+g) = e(\tilde{d})$. This approach, which provides the highest welfare level that can be achieved by a representative individual subject to the resource constraint and the additional restriction that everyone else attains the same level, is reminiscent of the Two Part Golden Rule in life-cycle models with physical capital and exogenous growth discussed in Diamond (1965) and Samuelson (1968, 1975a and 1975b). Accordingly, it can be labelled as the *Golden Rule* balanced growth path in the presence of endogenous growth.

Decentralizing this Golden Rule entails: (i) positive pensions to the elderly, i.e., $\tilde{z}^o_* < 0$; and (ii) an education *tax* (instead of a subsidy) to the repayment of loans, i.e., $\theta_* < 0$ (see Propositions 2 and 3 in Del Rey and Lopez-Garcia, 2013). In order to understand these results and, in particular, the optimality of taxing education, it is worth recalling that the individual and the social planner pursue different objectives. These differences translate into different structures of marginal costs and benefits underlying their decisions. As becomes clear from (7) and (16), the individual is concerned with the increase in her *second period earnings* and the cost in terms of interest payments. The social planner, in contrast, weighs the advantages and disadvantages of increasing human capital in terms of *consumption* per unit of efficient labour. Clearly, there is no reason why these costs and benefits should coincide.

It is worth focusing at this point on the *intergenerational effects* of the optimal tax policy. To advance in the analysis, notice that along the Golden Rule balanced growth path, the individual's present value of lifetime resources in (18), $\tilde{\omega}_*$, is:

$$\tilde{\omega}_* = w_* - \frac{(1+r_*)\tilde{d}_*}{(1+g_*)} - \frac{\left[\theta_*(1+r_*)\tilde{d}_* - (1+g_*)\tilde{z}^m_*\right]}{(1+r_*)(1+g_*)}\left[(1+g_*)(1+n) - (1+r_*)\right] \tag{35}$$

where $w_* = f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)$ is the wage rate along the Golden Rule. Clearly, the third term on the right-hand side of (35) vanishes regardless of the sign of $\left[\theta_*(1+r_*)\tilde{d}_* - (1+g_*)\tilde{z}^m_*\right]$, and thus we end up with $\tilde{\omega}_* = w_* - (1+n)\tilde{d}_*$, which is precisely the right-hand side of the feasibility constraint evaluated along the Golden Rule. However, this should not obscure the crucial fact that from the government budget constraint (3) one gets $\left[\theta_*(1+r_*)\tilde{d}_* - (1+g_*)\tilde{z}^m_*\right] = \tilde{z}^o_*/(1+n)$. Since, as stated above, optimal pensions in old age are positive, i.e., $\tilde{z}^o_* < 0$, (35) can be rewritten as:

$$\tilde{\omega}_* = w_* - \frac{(1+r_*)\tilde{d}_*}{(1+g_*)} + \frac{(-\tilde{z}^o_*)}{\left[(1+g_*)(1+n)\right]^2}\left[(1+g_*)(1+n) - (1+r_*)\right] \tag{36}$$

Notice that (36) is exactly the individual's present value of lifetime income in the presence of a pure *pay-as-you-go social security* that forces individuals to contribute the amount $(-\tilde{z}^o_*)/(1+g_*)(1+n)$ in their middle age, entitling them to receive the pension benefit $(-\tilde{z}^o_*)$ in their old age. As individuals are actually obtaining the (counterpart in the present model of the) "biological interest rate" $(1+g_*)(1+n)$ through their "investment in shares of the future generations" and the opportunity cost is exactly the market interest rate $(1+r_*)$, both terms cancel in present value. This point can also be easily illustrated by means of the physical capital market equilibrium condition (11), that along the Golden Rule balanced growth path becomes:

$$(1+g_*)(1+n)\tilde{k}_* = w_* - \frac{(1+r_*)\tilde{d}_*}{(1+g_*)} - \frac{(-\tilde{z}^o_*)}{(1+g_*)(1+n)} - \tilde{c}^m_* - (1+n)\tilde{d}_* \tag{37}$$

It is then clear that the optimal pay-as-you-go contribution will be given by

$$\frac{-\tilde{z}_*^o}{(1+g_*)(1+n)} = \frac{-\left[\theta_*(1+r_*)\tilde{d}_* - (1+g_*)\tilde{z}_*^m\right]}{(1+g_*)} > 0 \tag{38}$$

so that we can state the following

**Proposition 6** *The intergenerational effects of the optimal tax policy ($\tilde{z}_*^m$,$\tilde{z}_*^o$,$\theta_*$) are equivalent to those of a pay-as-you-go social security system where middle-aged individuals contribute the amount in (38) when middle-aged and receive a pension benefit ($-\tilde{z}_*^o$) when they retire.*

It should be stressed that it is the combination of the orthopaedics provided by the optimal values of $\tilde{z}_*^m$, $\tilde{z}_*^o$ and $\theta_*$, the one that allows the decentralized behaviour of individuals to replicate the Golden Rule balanced growth path. In other words, the *intergenerational income effects* associated with the scheme described in Proposition 6 have to be supplemented by the *price effects* in terms of investment in human capital induced by the tax on education. If individuals faced the optimal social security scheme but not the optimal education tax, they would fail to achieve the Golden Rule, and the same reasoning applies to the situation where the optimal education tax is in force but the intergenerational transfers are not optimally set. Notice that the optimal lump-sum tax on middle-aged, $\tilde{z}_*^m$, can either be positive or negative but, together with the revenue obtained by taxing the repayment of education loans, will generate a net contribution in middle age to finance optimal pensions to the elderly.

To conclude this section, it also worth emphasizing that, along the Golden Rule balanced growth path, the "intertemporal balance" of the individual vis-a-vis the government is zero. Indeed, from (6), the present value of net transfers from the government per unit of efficient labour along an arbitrary balanced growth path, $\tilde{b}$, is $\tilde{b} = \theta(1+r)\tilde{d}/(1+g) - \tilde{z}^m - \tilde{z}^o/(1+r)$. On the other hand, from the government budget constraint (3), $\tilde{z}^m + \tilde{z}^o/(1+g)(1+n) = \theta(1+r)\tilde{d}/(1+g)$. It then follows that when $(1+g_*)(1+n) = (1+r_*)$, i.e., along the Golden Rule, the individual net balance with the government in present value, $\tilde{b}_*$, will be zero.

# 6 Concluding comments

In this paper we have investigated the welfare effects of both education subsidies and intergenerational transfers along an arbitrary, non-optimal balanced growth path in an overlapping generations model with both physical and human capital. Because in our framework productivity gains translate into increases of consumption levels, the utility obtained by individuals from their consumption (measured in terms of output per unit of natural labour) will grow to infinity for reasonable specifications of the utility function. In this context, any attempt by a social planner to enlarge or maximize something that is inherently infinite becomes futile. In contrast, along a balanced growth path, all variables expressed in terms of output per unit of efficient labour, including of course consumption by individuals, will remain constant. When evaluating balanced growth paths, it is then natural for a social planner to use a social welfare function that treats all generations alike and, at the same time, is respectful with individual preferences. This can be achieved by defining social welfare as the utility obtained from consumption per unit of *efficient* labour. The new utility index so obtained converges and is well defined along a balanced growth path, and allows one to obtain an indirect utility function whose arguments are tax parameters. This is the basis of our analysis. Clearly, this approach is an alternative to the more conventional one where the optimal balanced growth path (and thus the optimal tax policy that supports it) is obtained maximizing a discounted sum of utilities.

We have analyzed the welfare effects along any arbitrary balanced growth path and also in the particularly relevant case associated with the laissez-faire. It has been shown that a lump-sum transfer from the middle-aged to the elderly translates into a lower accumulation of both physical and human capital (and thus a smaller growth rate). However, it can increase or decrease welfare depending on the interaction of two effects, namely, the relationship between the interest rate and the economy's growth rate on the one hand, and the effect of education investment on the present value of the individual's lifetime resources on the other. Concerning education subsidies, the results are even more ambiguous, since a change in the rate of education subsidy can have either a positive or a negative effect on the accumulation of both physical and human capital. Still, we have derived conditions that guarantee a clear-cut sign of the effect of education subsidies on welfare.

When the starting point is the laissez-faire balanced growth path, the introduction of a pay-as-you-go social security increases welfare whenever the growth rate of the economy is larger than the interest rate. However, nothing can be said with generality when the relationship between these rates is reversed. Notice that while the first result parallels its counterpart in overlapping generation models with exogenous growth, the second one is in open contradiction with it. As for the effects of education subsidies, provided that they foster physical capital accumulation, they will also entail an increased [resp. decreased] welfare level whenever (i) the economy's growth rate is lower [resp. higher] than the interest rate, and (ii) investing in education enhances [resp. reduces] the present value of individual's lifetime resources. Otherwise, and once again, no general statement can be advanced.

Finally, we have characterized the optimality conditions, i.e., the conditions that allow the social planner to maximize social welfare. Not surprisingly, these conditions coincide with those derived in Del Rey and Lopez-Garcia (2013). They show that the optimal pensions are positive and that the optimal education subsidy is negative, i.e., it should be a tax instead of a subsidy. However, we take here one step further and show that the intergenerational effects of the optimal tax policy are equivalent to those of a pay-as-you-go social security system where the middle aged contribute and the old-aged retirees receive a pension benefit. But one has to stress that these income effects need to be supplemented by the price effects that the optimal tax on education imposes in terms of human capital investments. Interestingly, the pay-as-you-go nature of the optimal policy is independent of the characteristics of the laissez-faire balanced growth path. Once again, this is in sharp contrast with the results obtained from exogenous growth models, where the precise configuration of the optimal social security depends in a crucial way of the relationship between the interest rate and the growth rate in the laissez-faire equilibrium.

A final comment seems in order. This paper has focused on two connected but different issues. The first one concerns the effects of variations in the tax parameters related to educations subsidies and intergenerational transfers on the ratios of physical and human capital per unit of efficient labour (and thus on the economy's growth rate) along a balanced growth path. This is a pure positive analysis. The second one, on normative grounds, has focused on the welfare effects of the above tax variations on a measure of social welfare that depends on individual's consumption levels per unit of efficient labour along a balanced growth path. Needless to say, one needs not adhere such a social valuation function. For that matter, neither need we support the standard approach that posits that social welfare is a discounted sum of individual utilities [e.g. Docquier et al. (2007)]. On the one hand, the choice of the discount rate is inherently arbitrary. On the other hand, and more importantly, the cardinalization of individual preferences affects the results in a crucial way [i.e., different utility functions representing the same preferences give rise to different optimal resource allocations and optimal policies supporting them, a point stressed in Del Rey and Lopez-Garcia (2012)]. The specification of an intertemporal objective function entails several non-trivial decisions, particularly concerning the dichotomies "discounting or not future utilities" and "maximizing representative individual or total utility". The literature on the axiomatic properties of different criteria for ordering infinite utility streams may be a promising avenue for future research, but this is beyond the scope of the current paper.

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