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THE SPATIAL COHERENCE WAVELETS AND SECOND-ORDER CORRELATION

Juan González[*†], Román Castañeda [**]

[*] *Fundación Universidad Autónoma de Colombia, Calle 13 # 4-31, Bogotá.*
[**] *Universidad Nacional de Colombia, Sede Medellín, Calle 59ª # 63-20, Medellín.*

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Abstract: A description of the second-order spatial coherence based on the theory of spatial coherence wavelets is presented. Such description is performed in the classical context of optical fields and chaotic sources. The concepts of radiant and virtual point sources are introduced. This theory suggests that the second-order spatial coherence state of light can be described in terms of three layers of point sources; a strategy that can increase the performance of numerical algorithms. The modulation in coherence found is similar to that measured in Hanbury-Brown and Twiss effect for binary stars.

Keywords: Second-order spatial coherence, Hanbury-Brown and Twiss effect, Binary stars.

LAS ONDITAS DE COHERENCIA ESPACIAL Y CORRELACIÓN DE SEGUNDO ORDEN

Resumen: Se presenta una descripción de la coherencia espacial de segundo orden basada en la teoría de onditas de coherencia espacial. Tal descripción es realizada en el contexto de campos ópticos y fuentes caóticas. Se introducen los conceptos de fuentes puntuales radiantes y virtuales. Esta teoría sugiere que el estado de coherencia espacial de segundo orden puede ser descrito en términos de tres capas de fuentes puntuales; una estrategia que puede mejorar el rendimiento de los algoritmos numéricos. La modulación en la coherencia encontrada es similar a la medida en el efecto Hanbury-Brown y Twiss para estrellas binarias.

Palabras clave: Coherencia espacial de segundo orden, Efecto Hanbury-Brown y Twiss, Estrellas Binarias.

[†] Autor al que se le dirige la correspondencia:
Tel. (571) 3529986.
E-mail: jumgonzalezto@unal.edu.co (Juan González).

1. INTRODUCTION

The theory of spatial coherence wavelets provides a phase-space representation for classical optical phenomena within the scope of the first-order coherence (Castañeda, 2010a). In this theory, an optical field in any state of spatial coherence can be described by the emission of two types of point sources distributed in two different layers of space (Castañeda, 2010b).

The first type is called radiant point sources and they are located at first layer. These are responsible for the propagation of the radiant energy of the field, a definite positive quantity, independent of its spatial coherence state and it is recordable by the squared-modulus detectors.

The second type of sources is named virtual point sources and they are placed at second layer. Their energies can take positive and negative values and they are not recorded directly by detectors, but their energies are crucial to the description of the diffraction and interference because they modulate the radiant energy to be added to it; increasing and decreasing local value without altering the value of the total energy of the field. Such energies depend on its spatial coherence state. A virtual point source is turned at the midpoint of any pairs of radiant point sources, not necessarily consecutive, within the structured spatial coherence support centered at that position (Castañeda, 2010b), which implies that the set of radiant sources must be discrete. Because of this fact, these virtual sources are called first-order virtual sources. This modeling can completely describe the first-order spatial coherence properties of scalar wave fields.

Now, Young's experiments with first-order virtual sources are analyzed. The interference between contributions from these sources is a result of the state of second-order spatial coherence of the field. It leads to correlation between the spatial coherence wavelets, which also involves the correlation of the cross-spectral densities. Such result is compared with that obtained in the Hanbury-Brown and Twiss effect to measure the angular separation of binary stars systems.

At the present, the study in the spatial domain of this effect is vital to construction of modern intensities interferometers with Cherenkov telescopes (Le Bohec and Holder, 2006).

2. SPATIAL COHERENCE WAVELET

Spatial coherence wavelet is defined as the basic vehicle for simultaneous transport of information about the energy (power spectrum) and the state of the first-order spatial coherence of optical field (correlation between the complex amplitudes of the field at two different points of the space) from the aperture plane (*AP*) to the observation plane (*OP*) (Castañeda, 2010a). The planes are separated by a distance *z*. Center-difference coordinates in the *AP* (ξ_A,ξ_D) and in the *OP* $(\mathbf{r}_A,\mathbf{r}_D)$ are used in order to denote pairs of points at the positions $(\xi_A+\xi_D/2,\xi_A-\xi_D/2)$ and $(\mathbf{r}_A+\mathbf{r}_D/2,\mathbf{r}_A-\mathbf{r}_D/2)$, like is illustrated in Fig. 1. These points are simplified as $(\xi_A\pm\xi_D/2)$ and $(\mathbf{r}_A\pm\mathbf{r}_D/2)$, respectively.

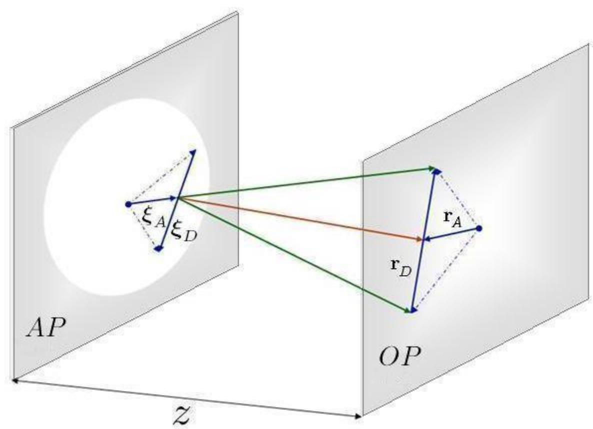

Fig. 1. Illustration of the center-difference coordinates at the aperture plane (ξ_A,ξ_D) and the observation plane. $(\mathbf{r}_A,\mathbf{r}_D)$.

The radiant and virtual point sources are located at the *AP*, each of these in the corresponding layer. ξ_D is the separation vector between radiant point sources. The wavelet is denoted as:

$$\mathbf{W}\left(\mathbf{r}_A\pm\frac{\mathbf{r}_D}{2};\xi_A\right)=\mathbf{S}(\xi_A,\mathbf{r}_A)\exp\left[-i\frac{k}{z}(\xi_A\cdot\mathbf{r}_D)\right], \quad (1)$$

with wave-number $k=2\pi/\lambda$, wave-length λ and $\mathbf{S}(\xi_A,\mathbf{r}_A)$ the marginal power spectrum. It is a

Wigner distribution function with energy units, defined as (Castañeda, 2010a):

$$\mathbf{S}(\xi_A, \mathbf{r}_A) = \int_{AP} W\left(\xi_A \pm \frac{\xi_D}{2}\right) \exp\left(i\frac{k}{z}\xi_A \cdot \xi_D\right) \times \exp\left(-i\frac{k}{z}\xi_D \cdot \mathbf{r}_A\right) d^2\xi. \quad (2)$$

The cross-spectral density W provides a measure of statistical similarity between light fluctuations at two points of space-time, which is a measure of the correlation between the complex amplitudes on the component frequency spectrum ω of light vibrations at these points (Mandel and Wolf, 1995). The superposition of spatial coherence wavelets produces the cross-spectral density of the field in the *OP*:

$$W\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}\right) = \left(\frac{1}{\lambda z}\right)^2 \exp\left[i\frac{k}{z}\mathbf{r}_A \cdot \mathbf{r}_D\right] \times \int_{AP} \mathbf{W}\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}; \xi_A\right) d^2\xi_A. \quad (3)$$

Interference terms between wavelets are not included in (3). It generates a moiré, which is called spatial coherence moiré (Castañeda, 2010a).

3. CORRELATION BETWEEN SPATIAL COHERENCE WAVELETS

The following equation determines the correlation between the cross-spectral densities of the field at the *OP* referred to the structured spatial coherence supports centered in the points $\mathbf{r}_A$ and $\mathbf{r}'_A$, respectively:

$$G\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}, \mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}\right) = \left\langle W\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}\right) W^*\left(\mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}\right)\right\rangle, \quad (4)$$

with $\langle\ \rangle$ denoting ensemble average. Thus (4) and (3) load to the correlation between spatial coherence wavelet:

$$G\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}, \mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}\right) = \left(\frac{1}{\lambda z}\right)^4 \exp\left(i\frac{k}{z}(\mathbf{r}_A \cdot \mathbf{r}_D - \mathbf{r}'_A \cdot \mathbf{r}'_D)\right) \times \int_{AP}\int_{AP} \left\langle \mathbf{W}\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}, \xi_A\right) \mathbf{W}^*\left(\mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}, \xi'_A\right)\right\rangle d^2\xi_A d^2\xi'_A. \quad (5)$$

Thereby (1), (2) and (5) point out that the second-order spatial coherence state of the field at the *OP*, represented by $\langle W(\mathbf{r}_A \pm \mathbf{r}_D/2) W^*(\mathbf{r}'_A \pm \mathbf{r}'_D/2)\rangle$, results from contributions of the second-order spatial coherence state of the field at the *AP*, given by $\langle W(\xi_A \pm \xi_D/2) W^*(\xi'_A \pm \xi'_D/2)\rangle$, which are propagated by:

$$\begin{aligned}\mathbf{K}(\xi_A, \xi'_A, \mathbf{r}_A, \mathbf{r}'_A) &= \langle \mathbf{S}(\xi_A, \mathbf{r}_A)\mathbf{S}(\xi'_A, \mathbf{r}'_A)\rangle \\ &= \int_{AP}\int_{AP} \left\langle W\left(\xi_A \pm \frac{\xi_D}{2}\right) W^*\left(\xi'_A \pm \frac{\xi'_D}{2}\right)\right\rangle \\ &\times \exp\left[i\frac{k}{z}(\xi_A \cdot \xi_D - \xi'_A \cdot \xi'_D)\right] \\ &\times \exp\left[-i\frac{k}{z}(\xi_D \cdot \mathbf{r}_A - \xi'_D \cdot \mathbf{r}'_A)\right] d^2\xi_D d^2\xi'_D,\end{aligned} \quad (6)$$

such propagation is depicted in Fig. 2. Using (6), the new second-order spatial coherence wavelets that propagate the second-order spatial coherence state of the field can be expressed as:

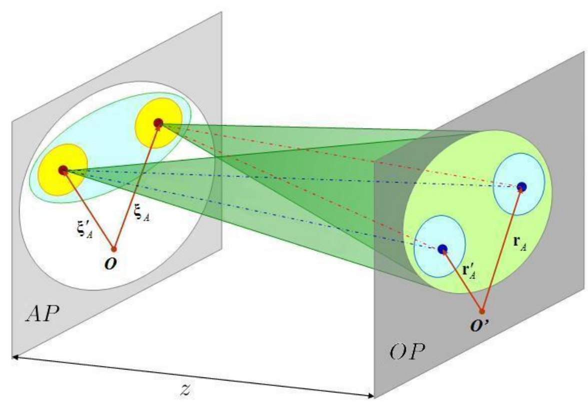

Fig. 2. The cones represent the propagation of $G(\mathbf{r}_A \pm \mathbf{r}_D/2, \mathbf{r}'_A \pm \mathbf{r}'_D/2)$

$$\begin{aligned}&\mathbf{G}\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}, \mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}; \xi_A, \xi'_A\right) = \\ &\left\langle \mathbf{W}\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}, \xi_A\right) \mathbf{W}^*\left(\mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}, \xi'_A\right)\right\rangle = \\ &\mathbf{K}(\xi_A, \xi'_A, \mathbf{r}_A, \mathbf{r}'_A) \exp\left[-i\frac{k}{z}(\xi_A \cdot \mathbf{r}_D - \xi'_A \cdot \mathbf{r}'_D)\right].\end{aligned} \quad (7)$$

With (7) in (5), the next equation is obtained:

$$G\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}, \mathbf{r}'_A \pm \frac{\mathbf{r}'_D}{2}; \xi_A, \xi'_A\right) = \left(\frac{1}{\lambda z}\right)^4 \exp\left[i\frac{k}{z}(\mathbf{r}_A \cdot \mathbf{r}_D - \mathbf{r}'_A \cdot \mathbf{r}'_D)\right] \times \int_{AP}\int_{AP} \mathbf{K}(\xi_A, \xi'_A, \mathbf{r}_A, \mathbf{r}'_A) \exp\left[-i\frac{k}{z}(\xi_A \cdot \mathbf{r}_D - \xi'_A \cdot \mathbf{r}'_D)\right] d^2\xi_A d^2\xi'_A. \quad (8)$$

The measurement of second-order correlation involves the combination in a correlator of the power spectrum values recorded simultaneously by two squared-modulus detectors placed at two different points of the *OP*. The correlator is an electronic device that receives the signals from both detectors and multiplies them (Hanbury-Brown and Twiss R, 1956). Such measuring strategy imposes the arrangement of the detectors are in the same structured support, i.e., $\mathbf{r}_A = \mathbf{r}'_A$ and $\mathbf{r}_D = \mathbf{r}'_D$, so that (8) becomes:

$$G^{(2)}\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2};\xi_A\right) = \left\langle \left| W\left(\mathbf{r}_A \pm \frac{\mathbf{r}_D}{2}\right)\right|^2 \right\rangle = \left(\frac{1}{\lambda z}\right)^4 \times \int_{AP}\int_{AP} \mathbf{K}(\xi_A,\xi'_A,\mathbf{r}_A)\exp\left[-i\frac{k}{z}(\xi_A-\xi'_A)\cdot\mathbf{r}_D\right] d^2\xi_A\, d^2\xi'_A \,. \qquad (9)$$

4. SECOND-ORDER YOUNG'S EXPERIMENT

The simplest configuration to consider second-order correlation of the field is one that involves a pair of virtual point sources at the second layer, each of them turned on by a specific pair of radiant sources on the first layer. This configuration contains four co-linear radiant point sources on one-dimensional mask. Such situation is illustrated in Fig. 3.

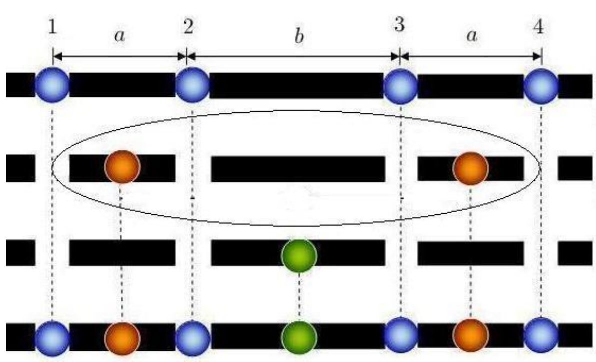

Fig. 3. Diagram of layers at AP for second-order Young's experiment with four co-linear radiant point sources.

The propagation occurs in Fraunhofer domain and degree of first-order spatial coherence $\mu(\xi_A \pm \xi_D/2)$ is real-valued function.

The correlated pairs are constituted by the first and the second sources; and the third and fourth sources, respectively. Two squared-modulus detectors placed at two positions with separation vector $r_D = x$ and the coordinate origin $r_A = 0$ at the *OP*. The first-order virtual sources of the second layer turn on a second-order virtual source at the third layer.

The first and second layers of *AP* are involved with radiant energy and modulated energy, respectively. The third layer is associated with modulated coherence. The fourth layer represented in Fig. 3 is the unified structure; this one contains the three types of sources.

5. RESULTS

The dimensionless functions $1 \equiv C\delta(\xi_D) + [1 - C\delta(\xi_D)]$ and $1 \equiv C'\delta(\xi'_D) + [1 - C'\delta(\xi'_D)]$ are introduced in order to separate the contributions of radiant and virtual sources in the equation (9). Since $W(\mathbf{r}_A \pm \mathbf{r}_D/2) = \mu(\mathbf{r}_A \pm \mathbf{r}_D/2)\sqrt{S(\mathbf{r}_A + \mathbf{r}_D/2)}\sqrt{S(\mathbf{r}_A - \mathbf{r}_D/2)}$, with S the power spectrum at the points $(\mathbf{r}_A + \mathbf{r}_D/2)$ and $(\mathbf{r}_A - \mathbf{r}_D/2)$ (Mandel and Wolf, 1995), (9) can be expressed like the sum of the terms:

$$G_{rad} = \left(\frac{1}{\lambda z}\right)^4 \left[\left\langle S^2(0)\right\rangle + \left\langle S^2(a)\right\rangle + \left\langle S^2(a+b)\right\rangle + \left\langle S^2(2a+b)\right\rangle\right], \qquad (10)$$

$$\begin{aligned} G_{vir}(x_D) = {} & \left(\frac{1}{\lambda z}\right)^4 \Big[\left\langle S(a)S(0)\right\rangle\left\langle \mu^2(a,0)\right\rangle \\ & + \left\langle S(b+2a)\,S(b+a)\right\rangle\left\langle \mu^2(b+2a,b+a)\right\rangle\Big] \\ & + \left(\frac{1}{\lambda z}\right)^4 2\left\langle \sqrt{S(a)}\sqrt{S(0)}\sqrt{S(b+2a)}\sqrt{S(b+a)}\right\rangle \\ & \times \left\langle \mu(a,0)\mu(b+2a,b+a)\right\rangle \cos\left[\frac{k}{z}(a+b)x_D\right]. \end{aligned} \qquad (11)$$

G_{rad} is the square of the power spectrum values recorded individually by the detectors; it is the contribution by the radiant point sources. G_{rad} does not depend on the coordinates of *OP*. The two first terms of G_{vir} are the contributions due to the first-order virtual sources while the last term is the contribution due to the second-order virtual source.

A pair of radiant point sources can be associated to the extremes of a star. Fig. 4 shows schematics of normalized G_{vir} when the pairs of radiant point sources (stars) are: a) correlated system of identical elements (same size and bright), b) correlated system with one element twice brighter than the other.

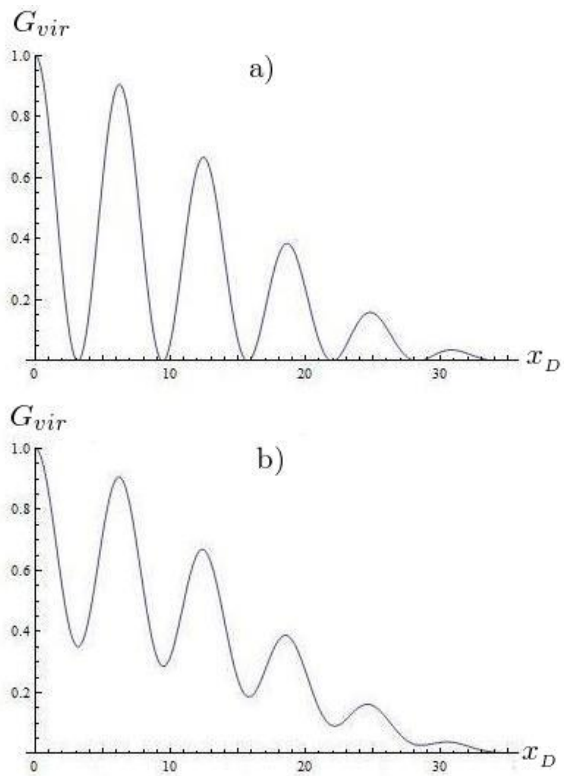

Fig. 4. Schematics of Gvir vs. xD obtained of two different instances of second-order Young's experiments.

6. HANBURY-BROWN AND TWISS EFFECT

The first measure of the degree of second-order spatial coherence was performed by Hanbury-Brown and Twiss (HB&T) with the intensity interferometer. This type of interferometer measured the correlation of fluctuations of the intensities recorded at two points (telescopes) at the same time. They wanted to measure the angular separation of binary stars systems which could not be solved by other methods. HB&T reported the following expression (Hanbury-Brown et al., 1967):

$$\Gamma^2(d)=\frac{1}{(I_1+I_2)^2}\times\left[I_1^2\Gamma_1^2+I_2^2\Gamma_2^2+2I_1I_2|\Gamma_1|\,|\Gamma_2|\cos\left\{\frac{2\pi\theta\, d}{\lambda}\cos\psi\right\}\right], \quad (12)$$

with Γ^2 as normalized degree of second-order spatial coherence, where d is the distance between the telescopes (called baseline), I_1 and I_2 are the recorded intensities, Γ_1 and Γ_2 are the degrees of first-order spatial coherence for each star, θ is the angular separation of binary stars system, λ is the chosen component of the light emitted by the stars and ψ is the angle between the line joining of stars and the baseline. The patterns given by (11) are similar to Fig. 4 (Le Bohec and Holder, 2006).

7. CONCLUSIONS

The second-order spatial coherence state of wave-fields can be analyzed and described in the framework of the classical wave picture, through the second-order spatial coherence wavelets. This description leads to correlation of power spectrums on the observation plane, which is product of Young's experiments with first-order virtual sources at the second layer. Second-order virtual point sources at a third layer are turned on because of correlations by pairs of the first-order virtual point sources. Such sources are responsible of modulation on the coherence.

It suggests that the second-order spatial coherence state of light can be described in terms of three layers of point sources; a strategy that can increase the performance of numerical algorithms. Such modeling leads to a degree of second-order spatial coherence which is closely related to the result obtained by HB&T for binary stars.

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ABOUT THE AUTORS

Juan González

Teacher of physics at Fundación Universidad Autónoma de Colombia, B.Sc. in Physics at the Universidad de Antioquia, MSc. in Physics of the Universidad Nacional de Colombia, sede Medellín. His area of interest is the second-order coherence of optical wave-fields.

Román Castañeda

Titular profesor for physics at the Universidad Nacional de Colombia, sede Medellín. He received his B.Sc. and M.Sc. in physics at the Universidad de Antioquia and his Dr. rer. nat. at the Technische Universitat Berlín (Germany). He is an active researcher on the theory of optical and electromagnetic coherence and the modeling of optical and electromagnetic wave-fields. He has been regular associate to the Abdus Salam International Centre for Theoretical Physics (Trieste, Italy) and is correspondent member of the Academia Colombiana de Ciencias Exactas, Fisicas y Naturales.