Some Aspects of Quantum Physics

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Abstract

I discuss some questions of quantum physics, for instance the validity and limitations of the basic language of set theory to deal with problems related to elementary particles. I also present a sketch of a formalization of a "metaphysics of structures", which might be useful for a kind of "ontic structural realism", and briefly review the concept of quasi-truth, which underlies my way of understanding scientific theories and the scientific activity.

1. Introduction

This paper constitutes a summary of some aspects of my work on the foundations of quantum physics, and especially results accomplished in the last three or four years. Most of it has not appeared in printed form. Many colleagues have helped me in this endeavor, particularly O. Bueno, I. Costa e Silva, F. A. Doria, D. Krause, A. A. M. Rodrigues, and A. Sant'Anna; to them go my warmest thanks.

I shall begin by stating a general principle, which underlies the whole discussion. The basic language $\mathcal{L}$ of set theory (although *not* necessarily specific systems of set theory like Zermelo-Fraenkel, von Neumann-Bernays-Gödel, ...) is a universal language. Scientific activity, and more generally rational activity, is essentially conceptual in nature, that is, it can be viewed as an interplay of concepts. But concepts have extensions, so in certain sense science is correlated with $\mathcal{L}$.

Of course, there is a vast *repertoire* of "interpretations" of $\mathcal{L}$; sets can be seen as extensions of concepts, as sets in the sense of von Neumann hierarchy, as Boolean valued constructs, as probabilistic items, as members of a forcing model, as structures or relations, etc. Sets may be employed with items which are not sets, for example physical bodies and collections of concrete objects. In $\mathcal{L}$ one even simulate category or topos theory. Indeed, any topos can be "translated" into a usual or strong set theory.

In spite of this wealth of systems and interpretations, most of the theoretical discussion about the foundations of physics can be developed inside relatively

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simple systems of set theory, like Zermelo-Fraenkel, perhaps reinforced by extra axioms or by new kinds of objects. At any rate, the basic notions of mathematics or of physics, at least theoretical physics, may be treated inside an appropriate set theory. It is in this sense that I shall assume that a set theoretical language underlies, in principle, all physical theories.

However, the simple picture just delineated will perhaps be challenged by the evolution of quantum physics, in particular in connection with quantum logic and the indiscernibility of elementary particles, topics to which I shall make reference below.

2. Quantum physics

I shall distinguish, from a theoretical standpoint, non-relativistic quantum mechanics (QM) from quantum field theory (QFT); QFT is the relativistic counterpart of QM. A good introduction to the first is Dirac 1958 and Sakurai 1985, and the second is treated in Cottinghan & Greenwood 2007 and Auyang 1995; these books contain extensive references to the pertinent literature.

In its usual formulation, the central mathematical tools of QM are Hilbert space and Schrödinger equation. The underlying space and time are classical, pre-relativistic. Among other restriction, there is no natural place for creation and annihilation of particles, and neither for spin; they are introduced in the theory in a somewhat *ad hoc* fashion. QM is basically a mechanics of particles applicable to electrons and, more generally, to fermions. As a mechanics of particles, the Lagrangian and the Hamiltonian formulations have important role, as it happens in analytical mechanics.

QM presents numerous philosophical problems concerning its interpretation, the foremost being the so called measurement problem (see Auyang 1995). It has, notwithstanding, a vast number of applications and empirical confirmations, successfully explaining many traits of molecular structure and Mendeleev periodic table of elements, as well as contributing in a fundamental way for the advance of electronic technology.

There are many alternative systematizations of quantum mechanics. However, the standard axiomatic procedure ia to assume certain general postulates, for instance Schrödinger equation, and to treat each particular quantum system (say the case of molecular structure) by the means of extra especial assumptions and approximate methods. In other words, the applications are handled via particular models or structures that have to obey the central postulates. The main point here is that it would not be appropriate to envisage an axiomatic systematization of QM as a set of sentences (of a given language) closed by logical inference rules. In fact, this is also true of classical mechanics and of general relativity. In the case of the latter theory, there are also some general postulates (say, the pseudo-riemannian structure of space-time and Einstein field equations), but the physical predictions are obtained with the help of specific models; all such models are equally *allowed* by the general theory, but their importance and validity vary according to the system under study and the physical approach adopted. Relativists even investigate theoretical spacetime geometries that seem to have little to do with the observed universe, like Gödel's model.

QFT was born when Dirac obtained a relativistic version of Schrödinger equation, which was latter interpreted as a field equation. Dirac equation, together with the operation of quantization of the fields, originated quantum electrodynamics (QED), the first quantum field theory. The so-called problem of infinities, linked to the perturbation methods adopting to compute physical quantities, was systematically handled by the device of renormalization, developed mainly by Feynman, Tomonaga, Schwinger, and Freeman Dyson. Renormalization is not, however, a mathematically well-defined procedure. It is rather a set of thumb rules to deal with infinite quantities, their justification being on physical (empirical) grounds. Even Feynman and Dirac had their misgivings about its systematic use in quantum physics.

QED concerns the electromagnetic interaction. There are two other branches of QFT: one is devoted to the weak interaction (combined with electromagnetism) and another, called quantum chromodynamics (QCD), studies the strong interaction. Just like in other fundamental domains of physics, in QFT theory formulation *combines* basic axioms with the elaboration of special models or structures, in any case satisfying the axioms that are initially accepted.

QFT is the theoretical framework of the so-called *standard model* of particle physics (see Cottinghan & Greenwood 2007). This model, in fact, gives us a mathematical picture of the weak, strong and electromagnetic interactions between leptons and quarks, which, together with the gauge bosons, constitute its elementary particles.

The standard model is an extremely successful theory from the experimental viewpoint. From a theoretical perspective, however, it lacks the uniform formal elegance of general relativity, and the electroweak and strong interactions are treated as separated theories, although physicists are confident that these interactions ought to be different manifestations of a single underlying force. More importantly, despite many efforts for more than a half of century, gravity remains theoretically irreconcilable with the basic postulates of quantum physics; it is treated separately by general relativity, which is not a quantum theory. A quantum theory of gravity remains elusive.

One curious point is that one may have the impression that particles are the basic ingredients of QFT. Nonetheless, this is not so. For example, Auyang writes that,

> So far we have not introduced the notion of particles. Particles in field theories are nothing like tiny pebbles; they are normal modes or quanta of excitation for the field. To get some idea of them, consider the motion of a violin string with length L. Since the ends of the string are fixed, only those wavelengths of the vibration that are integral divisions of $2L$ are allowed. These wavelengths correspond to the harmonic frequencies . . ., each of which is a *normal mode* of the string. (Auyang 1995, p. 51)

Auyang notes that normal modes have properties that make them fundamental in the analysis of vibrations, waves, and fields. She also asserts that,

> Normal modes, field quanta, and particles are good concepts for describing continuous systems only when the coupling between then is negligible. The condition is not always satisfied . . . when quantum fields interact, quanta can be excited and deexcited easily so that the static picture of free fields depicted above no longer applies. That is why field theories say particles are epiphenomena and the concept of particle is not central to the description of fields. (1995, p. 53)

According to the prevailing interpretation of Dirac and Klein-Gordon equations, as well as other basic equations of QFT, they are field equations. It seems impossible to treat them as referring to free particles. In other words the ontology underlying QFT consists of fields. Notwithstanding this, the language of particles is relevant in many cases, particularly in connection with the description of some results of experiment and in the handling of quantum statistics; this is so because the regions of interaction of the fields are often very small (in a suitable sense), and outside of them, the description in terms of free fields (whose basic excitations are described as particles) gives a good picture of phenomena.

It is very important to insist on the fact that QFT has a more or less definite domain of application. Thus, when some questions of QED involve situations of very high energy, this causes some theoretical difficulties, for instance Landau ghost, which is an inconsistency in the mechanism of renormalization at that level of energy. The same occurs in other QFT's.

From a logical standpoint, there is no guarantee that QFT is consistent. Anyhow,if it is not so, taking into account its intrinsic relevance, we might consider changing its subjacent logic into a paraconsistent one. On the possible inconsistency of QED, Jaffe wrote that,

> Yet in spite of these great successes, we do not know if the equations of relativistic quantum electrodynamics make mathematical sense. In fact, the situation poses a dilemma. As a consequence of renormalization group theory, most physicists today believe that the equations of quantum electrodynamics in their simple form are inconsistent; in other words, we believe that the equations of electrodynamics have no solution at all!! (Jaffe 1999, p. 136)

In spite of these caveats, QFT is likely to remain a fundamental tool in the domain of physics to which it can be applied, even if inconsistent, similarly to the case of Bohr atom. I shall explore this topic below (see da Costa et al. 2007, and Appendix B).

3. Quasi-truth

The traditional conception of truth as correspondence between sentences (of a fixed language) and reality is of the utmost importance. It has a long history and, from a certain perspective, can be said to have been systematized and formalized by Tarski in the 1930s (Tarski 1983). It is well known that Tarski's definition of truth has numerous applications in logic, mathematics, and the philosophy of science.

But it is difficult to adapt the Tarskian definition to all physical theories, especially to QFT. In fact, we know that a theory such classical mechanics can not correspond literally to reality However, rockets are sent to the Moon successfully through calculations carried out in this framework, so there is a sense in which it is *true*. The same happens with QM and QFT, that reflect a limited scope of reality or or our experience. The picture they present is partial and focussed on experience. On the other hand, as it is clear by now that theories may be inconsistent, and yet significant and rich. Thus, we need a new perspective on truth, not incompatible with the common one, for instance à la Tarski, but that will be more adapted to the present day state of physics (and of science in general).

The most basic feature of a theory is that it must save the appearances in given domains of experience and of our environment. Such conception of truth has been already developed and applied in several situations. It is called *quasi-truth* (or, sometimes, *pragmatic truth*; since I don't want to get into questions of exegeses, "quasi-truth" is preferable). Quasi-truth was heuristically motivated by the ideas of philosophers adept of the school of pragmatism, such as William James, Mead and specially Peirce. The first detailed exposition of the theory of quasi-truth appeared in Mikemberg, da Costa, & Chuaqui 1986, and, among the various works on the subject, I mention da Costa 1989, da Costa and French 2003, and da Costa & Bueno 1998. Quasi-truth generalizes the standard Tarskian definition of truth.

A good physical theory must be quasi-true in certain given domains of knowledge (see Appendix B). For example, classical mechanics can cope with moving bodies whose velocities are small as compared with the velocity of light and whose masses are not too large, having recourse to classical, non-relativistic concepts of space and time. QM, as I noted, has also its limitations and domains in which it is quasi-true. This is the case of all physical significant theories, hypothesis and laws.

What is then the objective of a theory? It is, above all, the systematization of part of our empirical knowledge of given domains, being quasi-true in these domains. Normally, we have a domain D and employ an appropriate mathematical species of structures $\mathcal{E}$ (Bourbaki 1968) to deal with D from a theoretical stance. Obviously, there must be some rules or processes with the help of which $\mathcal{E}$ can make contact with the empirical part of D. Sometimes these rules and processes reduce to paradigmatic examples of applications of $\mathcal{E}$ to D. Such procedure is ordinarily employed in science and may be clearly seen in classical celestial mechanics.

Concretely, D is usually investigated by the construction of models which satisfy the central principles of the theory. In general relativity, for example, we treat the geometries of the universe via special solutions of Einstein field equations. In this way one gets Friedmann-Robertson-Walter cosmology and Schwarzchild unique spherically-symmetric solution to the vacuum field equations (outside any source), which is static in its region (today employed in the description of geometrical aspects of certain class of black holes).

Sentences which are true in the Tarskian sense in a model M of the principles of a theory T, such that M reflects basic traits of D, are said to be *quasi-true* in D. M has to save the appearances in D up to some discrepancies (that are known and can, in principle, be controlled).

There may be distinct models M and M′ such that things occur in D as if M and M′ are both true versions of reality *in* D, up to a certain restrictions and approximations. So, there may exist sentences s and s' of the language by means of which we are talking about D, such that s and s' are true in the usual sense in M and M′, respectively, but s and s' are logically incompatible (one may be even the negation of the other). Since s and s' are said to be quasi-true in D if and only if they are true in the usual sense in M and M′, it follows that s and s' are quasi-true in D, even if each of them is equivalent to the negation of the other (s and s' compose a pair of incompatible propositions; in particular, they may be contradictory).

Taking into account the discussion of this section, we arrive at an informal definition and metalogical characterization of a physical theory in agreement with the present-day state of physics. A physical theory T, metalogically speaking, is a triple

$$\mathsf{T} = \langle \mathcal{E}, D, R \rangle,$$

where $\mathcal{E}$ is a mathematical species of structures (Bourbaki 1968), D is a domain or a family of domains of physics (composed by a class of physical systems), and R is a collection of rules that constitute the link between $\mathcal{E}$, D, and our experience (da Costa & Doria 2007). This link requires that some propositions (sentences) of the language L subjacent to T are quasi-true, in particular true in the Tarskian sense.

From the syntactical point of view, the relevant sentences of L are those which are quasi-true in D. Semantically, the quasi-truth of propositions of L depends on certain usual models (encoded in $\mathcal{E}$) that describe relevant aspects of D, leading to quasi-truths. R relates $\mathcal{E}$ and some of its models to empirical sentences, that is, to experience.

The structures encompassed under $\mathcal{E}$ may be partial structures, what contributes to give to T some power to accommodate or to overthrow most inconsistencies. For example, Bohr atom, even being inconsistent, constitutes a theory in the above sense. However, the natural tendency is to change or to eliminate theories as soon as they present inconsistencies (contradictions).

In QFT, $\mathcal{E}$ stands for some mathematical tools such as Lagrangeans, differential equations of fiber bundles, D denotes a family of fixed classes of physical systems, and R includes the standard methods of measurement, procedures of the theory of accidental errors, and technical devices of renormalization. (In fact, R can be rather complex and involves numerous mathematical and empirical techniques).

Implicit in the conception of quasi-truth is classical logic, above all in the characterization of the models employed to introduce quasi-truth in terms of the Tarskian truth. Nonetheless, classical logic can be replaced by other logics (for instance by a paraconsistent logic). Moreover, it is not difficult to perceive that the *logic of quasi-truth* is, strictly speaking, paraconsistent.

Bueno and the present author observed in da Costa & Bueno 1998 that (I insist on a question already referred to above):

> A remark on our terminology is important here. We call the kind of truth defined in this paper *pragmatic truth*, owing to its connections with the pragmatic conception of truth, as developed by philosophers like James, Dewey and particularly Peirce. (...) However, our piece is not exegetical. The sole point we wold like to emphasize is that our definition was heuristically inspired by some passages of pragmatics thinkers, such as Peirce, when he wrote that, 'consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object' (...)
>
> In our opinion, the definition of pragmatic truth studied in this paper captures, at least in part, the common concept of a theory *saving the appearances*, usually by means of partially fictitious constructions .... May be it would be better to call our kind of truth *quasi-truth* instead of pragmatic truth. (da Costa & Bueno 1998, p. 604)

4. Invariance and definability

The notion of invariance or symmetry belongs to the fundamental ideas of physics. Cottingham and Greenwood assert in their introductory book that,

> The construction of the Standard Model has been guided by principles of symmetry. The mathematics of symmetry is provided by group theory; groups of particular significance in the formulation of the Model are described ... The connection between symmetries and physics is deep. *Noether's theorem* states, essentially, that for every continuous symmetry of Nature there is a corresponding conservation law. For example, it

> follows from the presumed homogeneity of space and time that the Lagrangian of a closed system is invariant under uniform translations of the system in space and time. Such transformations are therefore symmetry operations on the system. It may be shown that they lead, respectively, to the laws of conservation of the momentum and conservation of energy. Symmetries, and symmetry breaking, will play a large part in this book. (Cottingham and Greenwood 2007, p. 3)

But the theory of invariance has a close relationship with the concept of *definability*. The generalized Galois theory (GGT) shows how intimate is the link between invariance and definability (see da Costa and Rodrigues 2007).

GGT constitutes an extension of common Galois theory and Klein's famous *Erlangen Program*. This program originally purported to systematize a good part of geometry through the concept of group of transformations. But many areas of mathematics may be conceived as the study of properties and relations invariant under certain groups of transformations, that is, their groups of automorphisms. Thus, topology deals with the group of homeomorphisms, projective geometry with the group of collineations and linear algebra with the linear automorphisms of a vector space.

An interesting point is that the mathematical structures considered in GGT are not first-order structures or models as in usual (first-order) model theory. The structures are higher-order constructs, like topological space, topological vector space, and differentiable manifolds. The mathematical structures in physics are, normally, higher-order arrangements of relations, including monadic relations.

We introduce the concept of definability by means of infinitary languages (the finitary languages are often too limited), and it is proved that definability and invariance are equivalent concepts, the two faces of the same coin. In reality, GGT generalizes first-order model theory, and one of the aims of some of my collaborators and of myself is to develop GGT to give rise to a convenient higher-order model theory, so that it contains, for instance, Bourbaki's theory of species of structures in a new setting.

GGT has inter-relations with *α-logic* and many of its results can be seen as belonging to this class of logics (on α-logic, see da Costa 1974, da Costa and Pinter 1976).

Obviously, theories and laws of physics must be invariant under the replacement of a structure by another isomorphic to it, since mathematical species of structures satisfy this condition (their axioms are always *transportable* according to Bourbaki). Moreover, there exists an interplay between Lagrangeans (or

Hamiltonians) of physical theories and invariance or symmetry. We determine, so to say, the great physical theories by the help of Lagrangeans (or of Hamiltonians). The most relevant symmetries are those of such mathematical structures. Here, Noether theorem and symmetry breaking are basic. The principles of covariance and the local gauge symmetries, involving Lie groups, can be investigated in GGT and perhaps by this way one clarifies and systematizes the invariance in physics. (In some cases we have to appeal to semi-groups, to pseudo-groups and to local species of structures).

Anyhow, we are allowed to affirm that physics, as a theory of dynamical physical systems, reduces in part to the investigation of Lagrangians or Hamiltonians and the corresponding differential equations, as well as their symmetries or invariances. (Thermodynamics is out of this characterization, since it does not get involved with the dynamical evolution of physical systems.)

Two papers of mine are devoted to GGT (da Costa 2007 and da Costa & Rodrigues 2007), where the interested reader will find numerous references, in particular on the works of the Portuguese mathematician José Sebastião e Silva and the Russian-French algebraist Mark Krasner, the creators of GGT.

5. Particles and fields

Some views of Schrödinger did impress me very much thirty years ago. He defended the thesis that the notion of equality or of identity didn't make sense in connection with elementary particles of the same kind (electrons, protons, etc.) (see Schrödinger 1952, pp.17–8). Then I tried to construct what I called non reflexive logic (specifically what I called *Schrödinger logic*), a logic in which the law of identity is not valid, in particular because equality has no meaning for certain objects (in consequence, these objects could not satisfy that law, also named principle of reflexivity of equality). It was clear for me that to build such logic would be a very difficult task: one would get into hard problems, including a profound revision of classical logic and the impossibility of employing natural language in the elaboration of an informal semantics for the new logical constants. (If objects *a* and *b* are out of the range or equality, how is it possible to say that they are distinct, i.e., not equal? What would the statement "*a* and *b* have distinct positions in space-time" mean? Is it meaningless to assert that two electrons, one in the North Pole and other in the South Pole, at a given time, are distinct?) Due to this fact, I suggested in da Costa 1980 (pp. 117ff) that a *theory of quasi-sets* should be constructed, encompassing the standard sets — of the usual theories of sets — as particular sets and, in such a theory, to build a semantics for my Schrödinger logic and, of course, for non-reflexive logics in general.

One of my former graduate students and today my colleague D. Krause begun to work in this subject under my guidance for his PhD in the eighties of the last century. He built a quasi-set theory, a theory which enables us to deal with collections of objects (quasi-sets) to which for some of its elements the standard notion of identity (equality), à la Schrödinger, does not hold, but there is a weaker notion of indiscernibility, or indistinguishability, instead (Krause 1992, French & Krause 2006). Krause has also extended Schödinger logic to a higher order system –simple theory of types– and to an intensional higher-order Schrödinger logic, on which a semantics founded in quasi-set theory was built and a weak completeness theorem presented –for these and other developments, see da Costa and Krause 1994, 1997, French & Krause 2006.

We have today formalisms to cope with the question of *non-individuals*, i.e., objects that infringe the classical theory of identity, which governs the *individuals*. However, we have to work to give to the logic of non-individuals a convenient physical semantic counterpart (a problema already discussed in French and Krause 2006). Solutions redefining the notion of equality are also possible, and have to be investigated in detail.

If one thinks of non-individuals (or *quasi-individuals*) as objects to which equality cannot be applied, one has to exclude from consideration space-time localization and properties depending essentially on space and time (in the case of elementary particles, certain quantum numbers). We, thus, should distinguish between properties intrinsic to a kind of particles (charge, rest mass, ...) and those that are extrinsic (space-time localization). If the principle of identity of the indiscernibles is assumed and only intrinsic properties are taken into account, then two electrons, for example, would be identical. However, identity could be conceived independently of the mentioned principle; in effect, there are several ways open to the elaboration of non reflexive logics, including stronger ones in which even the law of the indiscernibility of identicals in not, in general, valid.

The question then is to verify if QM possesses a formulation without equality, unless extrinsic properties get involved, when the evolution of quantum systems is described. Within certain limits this can be done. In QM, no elementary particle is distinguished from another one of the same species: any "permutation" of particles of the same species does not affect the validity of its laws and principles.

We normally introduce considerations related to space and time in QM.

Schrödinger equation determines the dynamical behaviour of a physical system. If extrinsic properties are introduced, as it really happens, then there is a weak version of equality in QM, and classical logic reappears. This move justifies the possibility of a logic of non-individuals (for technical details and philosophical discussion, see French and Krause 2006). So, there exists a version of QM based on a non-reflexive logic. But till now, it is fundamentally a philosophical proposal, since such approach didn't present an essential new empirical result.

However, in QFT there are various obstacles to the construction of a pure theory of particles, be as individuals or as non-individuals. This is so because QFT consists necessarily of a field theory, in which the proper concept of particle loses part of its relevance. Nonetheless, particles occur again in quantum statistical mechanics. Therefore, the best solution seems to maintain that the languages of fields and of particles are both fundamental, in a certain sense extending Bohr's thesis of complementarity. Perhaps in the future we shall get a particle version of QFT, including or not non-individuals.

There is another topic that will perhaps contribute new views on a set-theoretic approach to the foundations of quantum physics: quantum logics. If quantum logic is really necessary to underlie quantum theories, then the standard set-theoretic approach has to be revised (da Costa 1980).

All of us are at the beginning of our work. In this century we or other researchers surely will find the way to arrive at the solution to these questions.

Appendix A: A metaphysics of structures

I shall treat in the appendix some topics of a metaphysics of structures. This theme exemplifies very well what I asserted, in the Introduction, about the language of set theory and its relevance.

Some philosophers, such as S. French and J. Ladyman (Ladyman 1998) are trying to build a metaphysics of structures: the universe is composed by structures. I shall intend to show, in outline, that such conceptual construction is logically and formally possible, presenting a formalization of a *structural metaphysics* (and ontology)/ However, I don't want to get into exegetical questions, and don't argue that my formalization reflects precisely the views of the mentioned philosophers.

To begin with, let us reason on the basis of informal set theory, including relations and set theoretical structures (da Costa & Rodrigues 2007 and Bourbaki

1968). A relation R may be unary (set), binary, ..., n-ary ($0 < n < \omega$). If R is unary, then R is in fact a set; so, sets are relations. When R is n-ary, it consists of a set of n-tuples. In particular, the unit set formed by only one n-tuple $\langle a_1, \ldots, a_n \rangle$ is a relation. The empty set is also a relation. All relations are sets, that is, unary relations.

On the other hand, a mathematical structure $\mathcal{S}$ is an n-tuple of the form

$$\mathcal{S} = \langle D, r_1, \ldots, r_n \rangle$$

where D is a relation (set), called the domain of $\mathcal{S}$, and $r_1, \ldots, r_n$ are first-order relations defined on D (in particular, operations). I shall consider only first-order relations to simplify the exposition; but structures with higher-order relations are treated analogously.

Therefore, $\{\mathcal{S}\}$ is a unary relation (and also a unit set). We may then identify the structure $\mathcal{S}$ to the unary relation $\{\mathcal{S}\}$. In consequence, the construction of a metaphysics of structures is equivalent to the construction of a metaphysics of relations.

Any relation can be transformed into a structure. In effect, given the relation $R = R(r_1, \ldots, r_n)$ connecting other relations $r_1, \ldots, r_n$, it may be converted into a structure R^* as follows: we put

$$R^* = \{\langle K, r_1, \ldots, r_n \rangle\},$$

where $K = \bigcup_{i=1}^{n} \mathbf{field}(r_i)$, and $\mathbf{field}(r_i)$ is the union of the domain and the codomain of r_i; we then identity R with R^*. Clearly, R^* is a structure.

We postulate that any object is a structure. In addition, we suppose that any structure (relation), if not empty, is composed by structures, and that some sequences of structures, $s_1, s_2, \ldots,$, each of which, starting with s_2, is *part* of the preceding, may proceed to the infinite (the notion of part is easily definable). Then, a convenient formalization of the notion of structural metaphysics (or ontology) may be based on an appropriate system of set theory, without the axiom of regularity. A good option would be to take Zermelo-Fraenkel system minus regularity (see Fraenkel et al. 1973, Kuratowski & Mostowski 1968). Variables are interpreted as ranging on (unary) relations and the other relations and structures are introduced by definition. In synthesis, ZF gives us an abstract theory of structures according the above interpretation.

However, we need the structures that form the material universe and that, for this fact, are involved with space and time. We then add to ZF a new primitive unary predicate R, such that R(x) means that x is a *real* or *concrete* structure (for example, particles could be conceived as unary relations composed of relations –unary, binary, etc.). The resulting system will be denoted by ZF^{R}. Postulates governing the structures that satisfy R can be adjoining to those of ZF^{R}, in conformity with the view one has on the nature of the universe. Evidently, we may strengthen ZF^{R} in various distinct manners, for example to handle categories and toposes.

We are allowed to found quantum physics on an ontology of structures. (Fields and particles constitute, really, structures, etc.) It is not difficult to perceive the philosophical simplicity that an ontology of structures would bring to the philosophy of physics.

A subtle point is the following: although ZF^{R} was motivated by set-theoretic considerations, it has other interpretations, even essentially non set-theoretical; for example, one purely structural, centered around the intuition of structuralist philosophers.

Appendix B: Quasi-truth

I present here a sketch of the theory of quasi-truth (for details, consult da Costa 1989, da Costa & Bueno 1998, da Costa & French 2003). It was conceived to formalize the informal and intuitive view according to which a good theory T (proposition, sentence, or hypothesis) save the appearances in a domain of knowledge D. Or, in other words, when things occur in D as if T were strictly true in D (in the Tarskian sense). Quasi-truth constitutes, in fact, a generalization of Tarski's conception of truth. In what follows, I reproduce an excerpt of da Costa et al. 2007, pp. 851–3:

> Let us suppose that we are interested in studying a certain domain of knowledge Δ in the field of empirical sciences, for instance, particle physics. We are, then, concerned with certain real objects (in particle physics, with some configurations in a Wilson chamber, some spectral lines, etc.). Let us denote the set of these objects by A_1. Among the objects of A_1, there are some relations that interest us, and that we model as partial relations R_i, $i \in I$ (every relation having a fixed arity). The relations R_i are partial relations, that is, each R_i, supposed of arity r_i, is not necessarily defined for all r_i-tuples of elements of A_1. More formally, an n-place partial relation R can be viewed as a triple $\langle R_1, R_2, R_3 \rangle$, where

R_1, R_2, and R_3 are mutually disjoint sets, with $R_1 \cup R_2 \cup R_3 = D^n$, and such that R_1 is the set of n-tuples that (we know) belong to R; R_2 the set of n-tuples that (we know) do not belong to R; and finally R_3 of those n-tuples for which it is not defined whether they belong or not to R. (Note that when R_3 is empty, R is a usual n-place relation that can be identified with R_1.)

The reason for using partial relations is that they are supposed to express what we do know, or what we accept as true, about the actual relations among the elements of A_1. Thus, the partial structure $\langle A_1, R_i \rangle_{i \in I}$ encompasses, so to say, what we know, or accept as true, about the actual structure of Δ. However, to systematize our knowledge of Δ, it is convenient to introduce in our structure $\langle A_1, R_i \rangle_{i \in I}$ some *ideal* objects. (In particle physics, quarks would be an example.) The set of these new objects will be denoted by A_2. It is understood that $A_1 \cap A_2 = \emptyset$, and we stipulate that $A = A_1 \cup A_2$. In this way, the modeling of Δ involves new partial relations R_j, $j \in J$, some of which extend the relations R_i, $i \in I$. Furthermore , there are some sentences (closed formulas) of the language L, in which we talk about the structure $\langle A, R_k \rangle_{k \in I \cup J}$ ($I \cap J = \emptyset$) that we accept as true, or that are true (in the sense of the correspondence theory of truth). This occurs, for instance, with sentences expressing true decidable propositions (a proposition whose truth or falsehood can be *decided*), and with some general sentences that express laws or theories already accepted as true. Let us denote the set of such sentences, dubbed *primary*, by $\mathcal{P}$ (this set may be empty).

Given this informal discussion, we suggest that what we call a *simple pragmatic structure* (sps) be regarded as a set-theoretic structure of the form:

$$\mathfrak{A} = \langle A_1, A_2, R_i, R_j, \mathcal{P} \rangle_{i \in I, j \in J},$$

"where the elements in question satisfy the conditions above. Alternatively, we can simply write:

$$\mathfrak{A} = \langle A, R_k, \mathcal{P} \rangle_{k \in K}$$

"for a sps, where $A = A_1 \cup A_2$ and the R_k are partial relations defined on A, and $\mathcal{P}$ is a set of sentences of the language L of the same similarity type as that of $\mathfrak{A}$, and which is interpreted in $\mathfrak{A}$. Note that for some k, R_k may be empty.

Let L be a first-order language with identity, but without function symbols. The symbols of L are logical symbols (connectives, individual variables, quantifiers, and the identity symbol), auxiliary symbols (parentheses), a collection of individual constants, and a collection of predicate symbols. To interpret L in a sps $\mathfrak{A}$ is to associate to each individual constant of L an element of A (the universe of $\mathfrak{A}$), and to each n-ary predicate symbol of L a relation R_k, $k \in K$, of the same arity. It is supposed that every predicate of the family R_k, $k \in K$, is associated with a predicate symbol.

Definition 5.1. *Let L and $\mathfrak{A} = \langle A, R_k, \mathcal{P} \rangle_{k \in K}$ be, respectively, a language and a sps in which L is interpreted. Let $\mathfrak{B}$ be a* total structure, *that is, a usual structure whose n-ary relations are defined for all n-tuples of elements of its universe. And suppose that L is also interpreted in $\mathfrak{B}$. Then, $\mathfrak{B}$ is said to be* $\mathfrak{A}$-normal *if the following conditions are met:*

1. *(1) The universe of $\mathfrak{B}$ is A.*
2. *(2) The (total) relations of $\mathfrak{B}$ extend the corresponding partial relations of $\mathfrak{A}$.*
3. *(3) If c is an individual constant of L, then in both $\mathfrak{A}$ and $\mathfrak{B}$, c is interpreted by the same element.*
4. *(4) If $\alpha \in \mathcal{P}$, then $\mathfrak{B} \models \alpha$.*

Given a pragmatic structure $\mathfrak{A}$, it may happen that there are no $\mathfrak{A}$-normal structures. It is possible, however, to provide a system of necessary and sufficient conditions for the existence of such structures (see Mikenberg et al. 1986). One condition of this system is the following: For each partial relation R_k in $\mathfrak{A}$, we construct a set M_k of atomic sentences and negations of atomic sentences such that the former correspond to n-tuples that satisfy R_k, and the latter to n-tuples that do not satisfy R_k (such sentences correspond to n-tuples in the 'anti-extension' of R_k). Let M be the set $\bigcup_{k \in K} M_k$. Therefore, a sps $\mathfrak{A}$ admits an $\mathfrak{A}$-normal structure only if the set $M \cup \mathcal{P}$ is consistent.

In what follows, we will always suppose that our sps satisfies the relevant conditions; in other words, given any sps $\mathfrak{A}$, the set of $\mathfrak{A}$-normal structures is not empty.

Definition 5.2. *Let L and $\mathfrak{A}$ be, respectively, a language and a sps in which L is interpreted. We say that a sentence α of L is* pragmatically true, *or* partially true *in the sps $\mathfrak{A}$, according to $\mathfrak{B}$, if*

1. *(1) $\mathfrak{B}$ is an $\mathfrak{A}$-normal structure, and*

2. *(2) $\mathfrak{B} \models \alpha$, that is, α is true in $\mathfrak{B}$ in accordance with the Tarskian definition of truth.*

In other words, we say that α is *pragmatically (or partially) true* in the sps $\mathfrak{A}$ if there exists an $\mathfrak{A}$-normal structure $\mathfrak{B}$ in which α is true in the standard Tarskian sense. If α is not pragmatically (partially) true in the sps $\mathfrak{A}$ according to $\mathfrak{B}$ (α is not pragmatically (partially) true in the sps $\mathfrak{A}$), we say that α is *pragmatically (partially) false* in the sps $\mathfrak{A}$ according to $\mathfrak{B}$ (α is pragmatically (partially) false in the sps $\mathfrak{A}$).

Combining the definitions of quasi-truth and of physical theory (delineated in the body of the paper), one completes the precise definition of the latter concept. Moreover, there exists a logic of quasi-truth, logic that is paraconsistent, since that in the same domain *D* inconsistent theories (sentences) may be quasi-truth (da Costa et al. 2007, pp. 853ff).

Classical logic was employed, as a base, in the definition of quasi-truth, but other logics could be employed instead; in effect, quasi-truth is defined up to a logic.

A general exposition of the concept of species of structures (Suppes' predicates) and structures in physics is found in da Costa & Doria 1991, and da Costa and Rodrigues 2007.

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Resumo

Discuto algumas questões da física quântica, por exemplo a validade e limitações da linguagem básica da teoria de conjuntos para tratar de problemas relacionados às partículas elementares. Também apresento um esboço de certa formalização de uma espécie de "metafísica de estruturas" que pode ser útil para prover uma fundamentação matemática para determinado tipo de "realismo estrutural ôntico", e brevemente delineio minha maneira de compreender as teorias científicas e a atividade científica.

Palavras-chave
Física quântica, teoria quântica de campos, quase-verdade, partículas e campos.