

**Beneficial and Harmful Addictions:
Two sides of the same coin**

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Abstract: *In this paper we obtain an isomorphism between harmful and beneficial addictions in a discrete-time binary choice context. The equivalence thus established allows us to study both phenomena (harmful and beneficial addictions) as two sides of the same coin. Besides the theoretical insight it provides, this dualism is also particularly useful because it readily allows for the translation of the results obtained in the domain of harmful addictions to the domain of beneficial addictions. Once the dualism is established, we analyze addictions under both time-consistent and time-inconsistent preferences.*

Keywords: *addiction, self-control, negative internalities, habit formation, hyperbolic discounting, naïveté, sophistication, time inconsistency*

JEL classification: *A12, C79, D11, D60, D91*

Adicciones beneficiosas y adicciones perjudiciales: dos caras de la misma moneda

Resumen: *En este artículo obtenemos un isomorfismo entre las adicciones beneficiosas y las perjudiciales en un contexto de elección binaria en tiempo discreto. La equivalencia obtenida de este modo nos permite estudiar los dos fenómenos (los dos tipos de adicciones) como dos caras de la misma moneda. Además de la visión teórica que proporciona, esta dualidad también resulta particularmente útil porque permite trasladar fácilmente los resultados obtenidos en el campo de las adicciones perjudiciales al campo de las adicciones beneficiosas. Una vez establecido el dualismo, analizamos las adicciones bajo dos tipos de preferencias intertemporales: aquellas que son consistentes y aquellas que no lo son.*

Palabras clave: *adicción, autocontrol, internalidades negativas, formación de hábitos, descuento hiperbólico, ingenuidad, sofisticación, inconsistencia temporal*

Clasificación JEL: *A12, C79, D11, D60, D91*

Des addictions bénéfiques et des addictions néfastes: les deux faces de la même médaille

Résumé: *Dans cet article, on obtient un isomorphisme entre les addictions bénéfiques et les addictions néfastes dans un modèle avec choix binaire en temps discret. Nous obtenons une équivalence qui permet d'étudier les deux phénomènes (les deux types d'additions) comme deux faces d'une même médaille. Cette dualité est particulièrement utile car elle permet de traduire facilement les résultats obtenus dans le domaine des addictions néfastes, en résultats associés au domaine des addictions bénéfiques. Une fois le dualisme a été établi, nous analysons les addictions dans le cadre des préférences intertemporelles conformes et non conformes.*

Mots-clés: *additions, autocontrol, internalités négatives, formation des habitudes, escompte hyperbolique, ingéniosité, sophistication, incohérence temporelle*

Classification JEL : *A12, C79, D11, D60, D91*

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Introduction

Since the seminal work by Becker and Murphy (1988), there has been a growing body of literature devoted to the study of addiction. Even though the Becker-Murphy model accounted for both harmful addictions (e.g. drug consumption) and beneficial addictions (e.g. jogging), most of the recent research has focused solely on the former (see Orphanides & Zervos, 1995; Laibson, 2001; O'Donogue & Rabin, 2002; Bernheim & Rangel, 2004; Carrillo, 2005; López, 2006; Gul & Pesendorfer, 2007). However, there is a variety of economic contexts where beneficial addictions play an important role. People might get addicted to sports, living standards, work, high levels of human capital, etc. The key feature in an addictive product is that it generates *habit formation*:¹ past consumption of the product increases current desire for consumption. A habit-forming activity is a *harmful* addiction when it presents negative internalities, i.e. when it generates future costs or disutilities for the individual; whereas *beneficial* addictions imply positive internalities (i.e. futu-

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1 Becker and Murphy (1988) refer to this feature as adjacent complementarities.

re rewards). In general, the immediate effect of the activity could either be positive or negative in both harmful and beneficial addictions. However, the interesting cases are those where immediate and future effects go in opposite directions—when they go in the same direction, the individual behavior is trivial because there is no trade-off between present and future consequences of undertaking the activity.² For this reason, we will assume that harmful addictions generate immediate positive pleasure and beneficial addictions generate immediate negative pleasure.

In the present paper, we establish an isomorphism between harmful and beneficial addictions that allows us to study both phenomena as two sides of the same coin: any harmful addiction can be thought of as a beneficial addiction, and vice-versa, by simply taking the negation or inverse of the addictive activity from one realm to the other.

The above dualism holds for a specific context to which we refer as a binary activity choice accumulation problem (BACAP). In a BACAP, an individual faces the binary choice of whether to undertake or not an activity in each period; and his (instantaneous) payoff depends on his current choice as well as on the history of past choices, which is captured by a state variable. To illustrate the dualism, consider the following harmful addiction example. Suppose that the activity is “smoking one cigarette today” and that the history of past smoking is captured by the individual’s “nicotine level”. Negative internalities are captured by assuming that smoking today produces immediate pleasure but generates future costs (because it raises the nicotine level and therefore induces detrimental health effects). Habit formation is captured by assuming that higher nicotine levels induce greater desire for current smoking. Now consider the choice problem for the same individual, where the activity is defined as “avoiding smoking one cigarette today”. This problem presents positive internalities: by undertaking the activity, the individual renounces to the immediate pleasure (and thus incurs an immediate cost) but generates future benefits by reducing his addiction level. Moreover,

2 Imagine an activity that produces immediate pleasure and long-run positive effects; then, trivially, a rational individual will always undertake the activity. Similarly, an activity that implies both immediate and future negative effects will never be undertaken by a rational individual.

this problem also has the habit-forming feature: the more the individual has avoided smoking, the lower his nicotine levels and therefore the greater the desire to keep from smoking because its associated cost is lower. Therefore, the original harmful addiction problem may be seen as a beneficial addiction one; in fact, we show that both problems are equivalent.

Besides the theoretical insight it provides, this equivalence result is appealing because of its usefulness: it allows results to be obtained for both realms of addiction by focusing just on one. For instance, previous results obtained on harmful addictions (such as in O'Donogue & Rabin, 2002; and López, 2006) can be readily translated to the domain of beneficial addictions.

Once the dualism is established, we illustrate its attractiveness by analyzing beneficial addictions under two settings: time-consistent preferences (i.e. when the intertemporal utility presents exponential discounting) and time-inconsistent preferences (i.e. intertemporal utility with hyperbolic discounting). Under the first setting, we show that an individual's behavior depends crucially on the convexity of his instantaneous utility: when convexity holds, he either always undertakes the activity or always refrains; when convexity fails, we show that there might be other absorbing states. This result is worth mentioning because previous research (on harmful addiction) has assumed convexity and yet we believe that concavity might as well be of interest. Under the second setting, the individual can either be aware (he is *naïf*) or not (he is *sophisticated*) of his time inconsistency. When he is aware, he engages in an intrapersonal game. We show that the isomorphism preserves the equilibria of the induced game and then we analyze the implications of naïveté vs. sophistication. In particular, we show that the implications are the same whether we have immediate costs and delayed rewards (beneficial addictions) or immediate rewards and delayed costs (harmful addictions). These contrast with the implications of naïveté vs. sophistication in a context where the activity must be performed exactly once in a finite number of periods: as O'Donoghue and Rabin (1999) show, in the doing-it-once context sophistication exacerbates misbehavior under immediate costs while it mitigates it under immediate rewards. Our analysis shows that it would be misleading to extrapolate this conclusion to a full-fledged model of intertemporal decision making, i.e. one where actions are chosen repeatedly.

The paper proceeds as follows. In Section I, we formally define a BACAP and establish the isomorphism allowing us to show that a BACAP is equivalent to its dual. In Section II, we consider addictions under time consistency and show that concavity of the utility function may yield richer patterns of behavior as opposed to the cutoff rule implied by convexity. In Section III, we consider addictions under time inconsistency and show that the isomorphism is equilibria-preserving, so that the implications of naïveté vs. sophistication are the same under both realms (beneficial and harmful) of addiction. The final section concludes.

I. The Model

We will restrict ourselves to the following class of problems:

A. Binary Activity Choice Accumulation Problem (BACAP)

In a BACAP, an individual has to choose whether to undertake ($a_t = 1$) or not ($a_t = 0$) an activity in each period $t = 1, 2, \dots$. The instantaneous per-period payoff u_t depends on the period's action as well as on the history of past actions, which is assumed to be captured by a state variable k that evolves according to $k_{t+1} = \gamma k_t + a_t$ with $0 < \gamma < 1$. Notice that there is a maximum value for the state variable, $k^{\max} = \frac{1}{1-\gamma}$. The per-period payoff u_t is given by

$$u_t = u(k_t, a_t) = \begin{cases} \Psi(k_t) & \text{if } a_t = 1 \\ \Gamma(k_t) & \text{if } a_t = 0 \end{cases},$$

where $\Psi : [0, k^{\max}] \rightarrow \mathfrak{R}$ and $\Gamma : [0, k^{\max}] \rightarrow \mathfrak{R}$. Each period t , the individual aims at maximizing the intertemporal utility given by

$$U_t = U(u_t, u_{t+1}, \dots),$$

where $U : \mathfrak{R}^\infty \rightarrow \mathfrak{R}$. If $k_1 \in [0, k^{\max}]$ is the initial condition for the state variable k , a BACAP is completely characterized by $[\Psi, \Gamma, U, \gamma, k_1]$.

The main result of the following subsection is that for every BACAP $\mathbf{B} = [\Psi, \Gamma, U, \gamma, k_1]$ there is an equivalent BACAP where the activity may be viewed as the negation of the activity in \mathbf{B} .

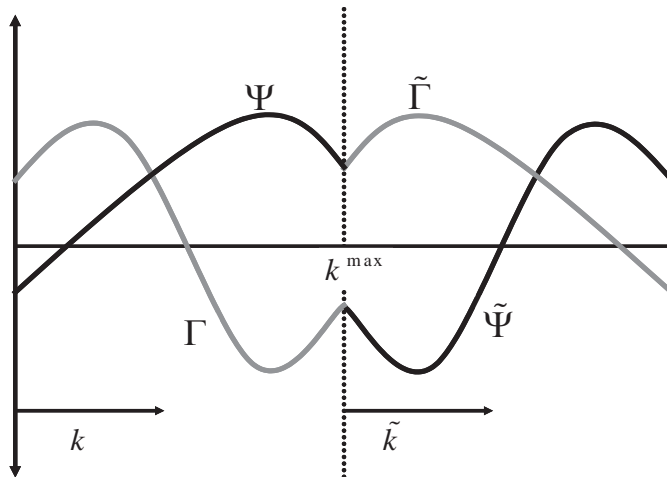
B. The DUAL of a BACAP

Given a BACAP $\mathbf{B} = [\Psi, \Gamma, U, \gamma, k_1]$, define $\tilde{\Psi} : [0, k^{\max}] \rightarrow \mathfrak{R}$ and $\tilde{\Gamma} : [0, k^{\max}] \rightarrow \mathfrak{R}$ as

$$\begin{aligned}\tilde{\Psi}(\tilde{k}) &= \Gamma(k^{\max} - \tilde{k}) \\ \tilde{\Gamma}(\tilde{k}) &= \Psi(k^{\max} - \tilde{k})\end{aligned}$$

so that $\tilde{\Psi}$ and $\tilde{\Gamma}$ are the reflections of Γ and Ψ with respect to k^{\max} as illustrated in Figure 1.

Figure 1. *The dual of a BACAP*



Source: own elaboration.

Now consider the problem of an individual who faces the choice of undertaking ($\tilde{a}_t = 1$) or not ($\tilde{a}_t = 0$) an action in each period $t = 1, 2, \dots$, and where the instantaneous payoff \tilde{u}_t is given by

$$\tilde{u}_t = \tilde{u}(\tilde{k}_t, \tilde{a}_t) = \begin{cases} \tilde{\Psi}(\tilde{k}_t) & \text{if } \tilde{a}_t = 1 \\ \tilde{\Gamma}(\tilde{k}_t) & \text{if } \tilde{a}_t = 0 \end{cases},$$

where \tilde{k}_t is assumed to evolve according to $\tilde{k}_{t+1} = \gamma\tilde{k}_t + \tilde{a}_t$. Suppose that the individual aims each period t at maximizing the intertemporal utility

$$\tilde{U}_t = U(\tilde{u}_t, \tilde{u}_{t+1}, \dots).$$

Then, for a given initial condition \tilde{k}_1 , $[\tilde{\Psi}, \tilde{\Gamma}, U, \gamma, \tilde{k}_1]$ defines a BACAP.

Definition 1. Given a BACAP $\mathbf{B} = [\Psi, \Gamma, U, \gamma, k_1]$, its DUAL is the BACAP

$$D(\mathbf{B}) = [\tilde{\Psi}, \tilde{\Gamma}, U, \gamma, (k^{\max} - k_1)].$$

Definition 2. We say that a BACAP \mathbf{B} is equivalent to a BACAP \mathbf{B}' if

1. for any path $A = \{a_1, a_2, \dots\} \in \{0, 1\}^\infty$ in \mathbf{B} there is a path $A' = \{a'_1, a'_2, \dots\} \in \{0, 1\}^\infty$ in \mathbf{B}' that yields the same payoff; and
2. for any path $A' = \{a'_1, a'_2, \dots\} \in \{0, 1\}^\infty$ in \mathbf{B}' there is a path $A = \{a_1, a_2, \dots\} \in \{0, 1\}^\infty$ in \mathbf{B} that yields the same payoff.

Proposition 3. A BACAP \mathbf{B} is equivalent to its dual $D(\mathbf{B})$.

Proof. Let (a_t, k_t) and $(\tilde{a}_t, \tilde{k}_t)$ denote period- t actions and states in \mathbf{B} and $D(\mathbf{B})$, respectively. Consider the isomorphism $\Xi: \{0, 1\} \times [0, k^{\max}] \rightarrow \{0, 1\} \times [0, k^{\max}]$ that maps actions and states in \mathbf{B} to actions and states in $D(\mathbf{B})$ in the following way:

$$\Xi(a, k) = (1 - a, k^{\max} - k) = (\tilde{a}, \tilde{k}).$$

Then, if $a_t = 1$ we have $\tilde{a}_t = 0$ and

$$u(a_t, k_t) = \Psi(k_t) = \tilde{\Gamma}(\tilde{k}_t) = \tilde{u}(\tilde{a}_t, \tilde{k}_t) = \tilde{u}(\Xi(a_t, k_t)),$$

and if $a_t = 0$ we have $\tilde{a}_t = 1$ and

$$u(a_t, k_t) = \Gamma(k_t) = \tilde{\Psi}(\tilde{k}_t) = \tilde{u}(\tilde{a}_t, \tilde{k}_t) = \tilde{u}(\Xi(a_t, k_t)).$$

Thus, the per-period utilities associated to (a_t, k_t) and $\Xi(a_t, k_t)$ are the same. Moreover, according to the isomorphism the evolution of \tilde{k}_t is *consistent* with the evolution of k_t :

$$\begin{aligned}\tilde{k}_{t+1} &= \gamma\tilde{k}_t + \tilde{a}_t = \gamma(k^{\max} - k_t) + 1 - a_t \\ &= (\gamma k^{\max} + 1) - (\gamma k_t + a_t) = k^{\max} - k_{t+1} = \Xi(k_{t+1}).\end{aligned}$$

Since $\tilde{k}_1 = k^{\max} - k_1 = \Xi(k_1)$, so that the initial condition in $D(\mathbf{B})$ is the image of the initial condition of \mathbf{B} under Ξ , we conclude that any path $A \in \{0, 1\}^\infty$ in \mathbf{B} is payoff equivalent to path $\tilde{A} = \Xi(A)$ in $D(\mathbf{B})$. This shows part 1 of Definition 2. To see that part 2 is also satisfied, just notice that $D(D(\mathbf{B})) = \mathbf{B}$ i.e. “ \mathbf{B} is the dual of its dual”. \square

Definition 4. *We say that a BACAP has Positive Internalities if Ψ and Γ are increasing in k . That is, the more the activity has been undertaken in the past, the higher the present well-being.*

Definition 5. *We say that a BACAP has Negative Internalities if Ψ and Γ are decreasing in k . That is, the more the activity has been undertaken in the past, the lower the present well-being.*

Proposition 6. *If a BACAP \mathbf{B} has Positive (Negative) Internalities, its dual $D(\mathbf{B})$ has Negative (Positive) Internalities.*

Proof. When Ψ and Γ are increasing (decreasing), $\tilde{\Psi}$ and $\tilde{\Gamma}$ are decreasing (increasing). \square

Definition 7. *We say that a BACAP exhibits Habit-Formation if $\Psi(k) - \Gamma(k)$ is increasing in k . That is, the more the activity has been undertaken in the past (as captured by k), the higher the marginal instantaneous utility of undertaking it in the present.*

Proposition 8. *If a BACAP \mathbf{B} exhibits Habit-Formation, its dual $D(\mathbf{B})$ exhibits Habit-Formation.*

Proof. If $\Psi(k) - \Gamma(k)$ is increasing, then $\tilde{\Psi}(k) - \tilde{\Gamma}(k)$ is increasing. \square

Definition 9. *We say that a BACAP presents **Harmful Addiction** if it exhibits Habit Formation and Negative Internalities. We say that a BACAP presents **Beneficial Addiction** if it exhibits Habit Formation and Positive Internalities.*

Proposition 10. *If a BACAP \mathbf{B} presents Harmful (Beneficial) Addiction, its dual $D(\mathbf{B})$ exhibits Beneficial (Harmful) Addiction.*

Proof. Direct from Propositions 6 and 8. \square

Let $\alpha : [0, k^{\max}] \rightarrow \{0, 1\}$ be a rule or strategy in a BACAP \mathbf{B} , that is, for each state level k , $\alpha(k)$ prescribes an admissible action. Define $\tilde{\alpha} : [0, k^{\max}] \rightarrow \{0, 1\}$ as the rule in $D(\mathbf{B})$ given by

$$\tilde{\alpha}(\tilde{k}) = 1 - \alpha(k^{\max} - \tilde{k}). \quad (1)$$

Proposition 11. *If α induces path A in a BACAP \mathbf{B} , then $\tilde{\alpha}$ induces path $\Xi(A)$ in $D(\mathbf{B})$.*

Proof. Suppose that at some period t , state levels are k_t and $\tilde{k}_t = (k^{\max} - k_t)$ in \mathbf{B} and $D(\mathbf{B})$ respectively. Then α prescribes in \mathbf{B} action $a_t = \alpha(k_t)$ and induces state level $k_{t+1} = \gamma k_t + a_t$; and $\tilde{\alpha}$ prescribes in $D(\mathbf{B})$ action $\tilde{a}_t = \tilde{\alpha}(\tilde{k}_t)$ and induces state level $\tilde{k}_{t+1} = \gamma \tilde{k}_t + \tilde{a}_t$. Now notice that

$$\tilde{a}_t = \tilde{\alpha}(\tilde{k}_t) = 1 - \alpha(k^{\max} - \tilde{k}_t) = 1 - \alpha(k_t) = 1 - a_t = \Xi(a_t)$$

and

$$\tilde{k}_{t+1} = \gamma \tilde{k}_t + \tilde{a}_t = \gamma(k^{\max} - k_t) + 1 - a_t = (\gamma k^{\max} + 1) - (\gamma k_t + a_t) = (k^{\max} - k_{t+1}).$$

Therefore, the proposition follows by induction and the fact that $\tilde{k}_1 = (k^{\max} - k_1)$. \square

The above proposition shows that, given a strategy α in a BACAP \mathbf{B} , strategy $\tilde{\alpha}$ as given by expression (1) has the natural interpretation of being the strategy in $D(\mathbf{B})$ induced by applying the isomorphism Ξ to α . Thus, with a slight abuse of notation we will refer to it as $\Xi(\alpha)$. Notice that, as is shown in the proof of Proposition 3, path A in \mathbf{B} is payoff equivalent to path $\Xi(A)$ in $D(\mathbf{B})$. Therefore, we readily obtain the following proposition:

Proposition 12. *Strategies α and $\Xi(\alpha)$ are payoff equivalent.*

Proof. Let A be the path generated by strategy α in \mathbf{B} . By definition, $\Xi(\alpha)$ is the strategy in $D(\mathbf{B})$ induced by applying the isomorphism Ξ to α and therefore the path it generates in $D(\mathbf{B})$ is precisely $\Xi(A)$. The proof of Pro-

position 3 shows that paths A and $\Xi(A)$ are payoff equivalent. Therefore, the strategies generating them are also payoff equivalent. \square

We conclude this section by giving examples that motivate the study of BACAPs presenting **Addiction**.

C. Some examples presenting Addiction

1. Drug consumption

Suppose that the activity is “smoking one cigarette” and k represents the individual’s addiction level. Let $f(k_t)$ be the utility associated to addiction level k_t irrespective of what the individual’s current choice is. Because of the detrimental effects of past smoking in current health, it is natural to assume that f is decreasing in k . Further, suppose that $x > 0$ reflects the intrinsic pleasure of smoking one cigarette and $w(k_t)$ is the withdrawal cost, which we assume increasing in k_t because the more the person has smoked in the past the harder for him to refrain from current smoking. Then, the instantaneous per-period utility is given by

$$u(k_t, a_t) = \begin{cases} \Psi(k_t) = x + f(k_t) & \text{if } a_t = 1 \\ \Gamma(k_t) = f(k_t) - w(k_t) & \text{if } a_t = 0 \end{cases}$$

and the (harmful) addictive properties of smoking are reflected by negative internalities (Ψ and Γ decreasing) and habit formation (w increasing). This is precisely the kind of model studied by O’Donoghue and Rabin (2002).

2. Jogging

Suppose the activity is “one hour of jogging”, so that k represents the fitness level. Let $g(k_t)$ be the utility derived from having fitness level k_t . This is a utility the individual enjoys at time t irrespective of what his current choice is; it includes the health benefits from past exercising and therefore is increasing in k_t . Also, let $e > 0$ be the effort or cost associated to “one hour of jogging” when the individual has never exercised before (i.e. $k = 0$). It seems natural to assume that this cost decreases as the fitness level increases. We capture this by letting the cost be $c(k_t) = e - h(k_t)$, with $h(k_t)$ increasing and $h(0) = 0$.

Notice that this allows for the jogging activity to even become “pleasurable” (i.e. when for some k , $h(k) > e$). Then, the instantaneous per-period utility is given by

$$u(k_t, a_t) = \begin{cases} \Psi(k_t) = g(k_t) - c(k_t) = -e + h(k_t) + g(k_t) & \text{if } a_t = 1 \\ \Gamma(k_t) = g(k_t) & \text{if } a_t = 0 \end{cases},$$

and the (beneficial) addictive properties of jogging are reflected by positive internalities (Ψ and Γ increasing) and habit formation (h increasing).

3. *Conspicuous consumption*

Suppose that the main source of utility for an individual is his living standard (or status), which we denote by k and can only be raised by the activity of conspicuous consumption (cf. Veblen). For instance, think of this activity ($a = 1$) as “offering a Great-Gatsby-type party”. Let $g(k)$ be the utility associated to status k that the individual enjoys irrespective of his current choice. Offering the party has an intrinsic instantaneous utility but it also produces a positive externality because it raises the host’s status for next period. Further, let v be the intrinsic utility of the party experienced by the host. We may assume that v is increasing in k (for instance, people attending the party are kinder to the host the higher his status is). If $e > 0$ represents the effort or cost of offering the party, the instantaneous per-period utility is given by

$$u(k_t, a_t) = \begin{cases} \Psi(k_t) = -e + v(k_t) + g(k_t) & \text{if } a_t = 1 \\ \Gamma(k_t) = g(k_t) & \text{if } a_t = 0 \end{cases},$$

and the (beneficial) addictive properties of conspicuous consumption are reflected by positive internalities (Ψ and Γ increasing) and habit formation (v increasing).

4. *Investment in Human Capital*

Suppose that the individual has a unit of free time and a unit of working time each period. The working time unit is exerted in a productive activity whose productivity $g(k)$ depends positively on the accumulated human

capital k . The individual may use his free time in either learning ($a = 1$), which might be interpreted as investment in human capital and therefore increases his stock k , or in an alternative activity ($a = 0$), which may be leisure or some other productive activity that does not affect k . When he decides to use his free time in the alternative activity, he obtains an instantaneous utility $e > 0$, which can be thought of as the pleasure derived from leisure or the profits from the alternative productive activity. If he devotes his free time to learning, he incurs a cost $c(k)$, which is decreasing in k and may eventually become negative (i.e. the individual “enjoys” learning). Therefore, the instantaneous per-period utility is given by

$$u(k_t, a_t) = \begin{cases} \Psi(k_t) = g(k_t) - c(k_t) & \text{if } a_t = 1 \\ \Gamma(k_t) = g(k_t) + e & \text{if } a_t = 0 \end{cases}$$

and the (beneficial) addictive properties of investment in human capital are reflected by positive internalities (Ψ and Γ increasing) and habit formation ($-c$ increasing).

II. Beneficial Addictions under Time Consistency

Consider a BACAP $\mathbf{B} = [\Psi, \Gamma, U, \gamma, k_1]$ presenting beneficial addiction and where the functional U takes the specific form

$$U(u_t, u_{t+1}, \dots) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u_{\tau}, \text{ with } 0 < \delta < 1, \quad (2)$$

i.e. where we have time separability and exponential discounting. Let $A \in \{0, 1\}^{\infty}$ denote a behavior path, that is, an infinite sequence of admissible actions. Keeping in mind the jogging example of the previous section, we will use E (from “exercising”) to denote the behavior path where the individual always undertakes the activity, that is, $E = (1, 1, \dots)$, and N (from “never exercising”) will denote the behavior path where the individual never undertakes the activity, that is, $N = (0, 0, \dots)$.

We will see that the optimal decision rule and long-run behavior depend crucially on the convexity of Ψ and Γ .

A. Ψ and Γ convex

From the previous section, we know that \mathbf{B} is equivalent to its dual $D(\mathbf{B}) = [\tilde{\Psi}, \tilde{\Gamma}, U, \gamma, (k^{\max} - k_1)]$. By Proposition 6, we know that $D(\mathbf{B})$ presents harmful addiction. Notice that $\tilde{\Psi}$ and $\tilde{\Gamma}$ are also convex. In O'Donoghue and Rabin (2002), it is shown that in a BACAP as $D(\mathbf{B})$ the optimal decision rule of the agent is of the cut-off type. That is, there exists a critical state level \tilde{k}^c such that the optimal action is $\tilde{a}_t = 0$ for levels below \tilde{k}^c and $\tilde{a}_t = 1$ for levels above \tilde{k}^c . Therefore, the optimal decision rule takes the form

$$\tilde{a}_t = \tilde{\alpha}(\tilde{k}_t) = \begin{cases} 0 & \text{if } \tilde{k}_t < \tilde{k}^c \\ 1 & \text{if } \tilde{k}_t \geq \tilde{k}^c \end{cases}.$$

The reason for this is that the time consistency implied by the intertemporal utility (2) allows for the use of a value function $V(\tilde{k}_t)$ that happens to be convex when $\tilde{\Psi}$ and $\tilde{\Gamma}$ are convex. Therefore, at any time t the agent chooses $\tilde{a}_t = 1$ if and only if

$$\tilde{\Psi}(\tilde{k}_t) - \tilde{\Gamma}(\tilde{k}_t) \geq \delta [V(\gamma\tilde{k}_t) - V(\gamma\tilde{k}_t + 1)], \quad (3)$$

i.e. if and only if the marginal instantaneous benefits $\tilde{\Psi}(\tilde{k}_t) - \tilde{\Gamma}(\tilde{k}_t)$ offset the marginal future costs $\delta [V(\gamma\tilde{k}_t) - V(\gamma\tilde{k}_t + 1)]$ of the action. Now, because of the habit-forming feature, the LHS of expression (3) is increasing, while the convexity of the value function V implies that the RHS of the same expression is decreasing, and therefore we get the existence of \tilde{k}^c .

Now, because of the equivalence between \mathbf{B} and $D(\mathbf{B})$, we readily obtain

Proposition 13. *In a BACAP $\mathbf{B} = [\Psi, \Gamma, U, \gamma, k_1]$ presenting beneficial addiction with Ψ and Γ convex and intertemporal utility U given by (2), the optimal decision rule is of the cutoff type. That is, there exists a critical state level $k^c \in [0, k^{\max}]$ such that*

$$a_t = \alpha(k_t) = \begin{cases} 0 & \text{if } k_t < k^c \\ 1 & \text{if } k_t \geq k^c \end{cases}.$$

Remark 14. *Notice that when Ψ and Γ are convex, long-run behavior depends totally on the initial condition k_1 : if $k_1 < k^c$, the individual will never undertake the activity*

(his realized behavior path is N), while when $k_1 \geq k^c$ he will undertake it forever (his realized behavior path is E). Therefore, there are only two absorbing states: 0 (when $k_1 < k^c$) and k^{\max} (when $k_1 \geq k^c$). This changes dramatically if we relax the convexity assumption as the following subsection shows.

B. Ψ and Γ concave

Suppose now that Ψ and Γ are concave. Even though we still may make use of a value function by virtue of intertemporal utility (2), this value function may not be convex nor concave. So the type of analysis used in the previous subsection is of no use. The concavity assumption makes the analysis much more complicated, but it also allows for a variety of pattern behaviors: the optimal decision rule may no longer be of the cutoff type; long-run behavior may be independent of initial conditions; and there might be other absorbing states. In particular, it is possible to observe non-monotonic behavior where the individual alternatively switches from undertaking the activity to refraining.³

C. Assuming Convexity

As we have seen, characterization of behavior is very simple under the convexity (of Ψ and Γ) assumption while it may be a very complicated task when we relax it. Assuming convexity is appealing because of its tractability, but it may be unsatisfactory for modeling some addictions. In essence, what the convexity assumption posits is that there are increasing marginal returns in the state variable. However, we believe that there are some realms of addiction where the opposite would hold. Consider, for instance, the jogging example provided in the previous section: perhaps the most reasonable assumption is decreasing marginal returns to the fitness level (i.e. Ψ and Γ concave). The same goes for the investment in human capital example: the standard would be to assume decreasing marginal returns to the accumulated human capital. In their harmful addiction model, O'Donoghue and Rabin

3 We have constructed examples for each of these cases. Because of their tedious technicalities, they have been omitted from the paper. However, they may be obtained upon request.

(2002) posit the convexity assumption arguing that “the more addicted the person becomes, the less a given increase in k hurts his instantaneous utility, and therefore the less harm hitting does to future utility”. This is debatable:⁴ It could be a reasonable assumption for some drugs, say “soft” drugs, but it may not be too realistic for “hard” drugs such as heroin.

We believe that the concavity assumption deserves attention since it may be appropriate for some addiction contexts.

III. Beneficial Addictions under Time Inconsistency

Suppose now that the functional U takes the specific form⁵

$$U(u_t, u_{t+1}, \dots) = u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_{\tau}, \text{ with } \beta \in (0, 1). \quad (4)$$

The present bias reflected in the parameter β induces the time inconsistency: because of the greater taste for immediate gratification, an optimal behavior path at some date t may no longer be optimal at a further date. Therefore, an optimal behavior path may not be implementable because future selves may have incentives to deviate (here, of course, we are not allowing for commitment possibilities). When the individual is fully unaware of his time-inconsistency (i.e. he is *naïf*), he chooses his action according to the behavior path that maximizes (4) believing (wrongly) that his future selves will stick to it. When the individual is fully aware of his time-inconsistency (i.e. he is *sophisticated*), he engages in an intrapersonal game: he plays against his future selves by maximizing (4) *subject to* the intended behavior path to be followed. In the infinite horizon case, this intrapersonal game will normally present multiple Markov Perfect equilibria, which is the natural solution concept to be called upon. In what follows, it will be useful to distinguish between a

4 We believe that they are aware of this since they state that “most results hold even if Ψ and Γ are a little concave, and some do not rely at all on them being convex”.

5 This functional form was first introduced by Phelps and Pollack (1968). Because of its simplicity and tractability, it has been widely used to model self-control problems since the work of Laibson (1994).

desired behavior path (DBP), that is, an infinite sequence of actions solving expression (4); and a *realized behavior path* (RBP), that is, a path actually followed by all selves.

The implications of naïveté vs. sophistication have been studied in O'Donoghue and Rabin (2002) and in López (2006) for a BACAP presenting harmful addiction under time inconsistency as given by expression (4). We may now apply the duality established in Subsection I. B to translate their results into BACAPs with beneficial addictions. It is important to note that we will consider only the case where Ψ and Γ are convex. We do so because those studies presented this assumption.

A. Naïf behavior

O'Donoghue and Rabin (2002) show that in a BACAP \mathbf{B} presenting harmful addiction a naïf follows a cutoff rule α^n , that is, there is a critical state level k^n such that

$$\alpha^n(k) = \begin{cases} 0 & \text{if } k < k^n \\ 1 & \text{if } k \geq k^n \end{cases}.$$

Moreover, it happens that for levels below k^n the DBP is $R = 0000\dots$, while for levels above k^n the DBP is either $O = 1000\dots$ or $H = 1111\dots$. A naïf has self-control problems (or time inconsistency) whenever O is the DBP: Believing that he is able to follow it, he will undertake the activity; but, by doing so, he raises his state level for next period and therefore he does not stick to O . In fact, the RBP is either R (for any level below k^n) or H (for any level above k^n).

Because of the equivalence between \mathbf{B} and $D(\mathbf{B})$, (in particular because of Propositions 11 and 12) strategy $\tilde{\alpha}^n = \Xi(\alpha^n)$ describes naïf behavior in $D(\mathbf{B})$. Therefore,

$$\tilde{\alpha}^n(\tilde{k}) = 1 - \alpha^n(k^{\max} - \tilde{k}) = 1 - \begin{cases} 0 & \text{if } k^{\max} - \tilde{k} < k^n \\ 1 & \text{if } k^{\max} - \tilde{k} \geq k^n \end{cases} = \begin{cases} 1 & \text{if } k^{\max} - \tilde{k} < k^n \\ 0 & \text{if } k^{\max} - \tilde{k} \geq k^n \end{cases},$$

which is equivalent to:

$$\tilde{\alpha}^n(\tilde{k}) = \begin{cases} 0 & \text{if } \tilde{k} < \tilde{k}^n \\ 1 & \text{if } \tilde{k} \geq \tilde{k}^n \end{cases}, \text{ where } \tilde{k}^n = k^{\max} - k^n, \quad (5)$$

and thus the RBP of a naïf in $D(\mathbf{B})$ is either $\Xi(R) = 1111\dots$ (for any level above \tilde{k}^n) or $\Xi(H) = 0000\dots$ (for any level below \tilde{k}^n).

Since any BACAP presenting beneficial addiction can be thought of as the dual of some BACAP presenting harmful addiction, naïf behavior for beneficial addictions is completely characterized by expression (5).

B. Sophisticated behavior

When there is MPE multiplicity (as it is normally the case with an infinite horizon) in the induced intrapersonal game, sophisticated behavior would depend on the particular equilibrium selection. Rather than characterizing sophisticated behavior in a BACAP \mathbf{B} ⁶, what we want to point out is that the isomorphism Ξ is equilibrium-preserving:

Proposition 15. *If strategy α constitutes a MPE of a BACAP \mathbf{B} , then strategy $\tilde{\alpha} = \Xi(\alpha)$ constitutes a MPE of $D(\mathbf{B})$.*

Proof. Because of Propositions 11 and 12, if there were a profitable deviation from $\tilde{\alpha}$ in $D(\mathbf{B})$ then there would be a profitable deviation from α in \mathbf{B} : just apply the isomorphism Ξ to the deviation. \square

Therefore, the dualism also applies for sophisticated behavior: if some strategy α describes sophisticated behavior in a BACAP \mathbf{B} , then strategy $\tilde{\alpha} = \Xi(\alpha)$ describes sophisticated behavior in $D(\mathbf{B})$. We turn now to compare sophisticated versus naïf behavior.

C. Naïveté vs. sophistication

We will use the following notation: For any strategy $\alpha: [0, k^{\max}] \rightarrow \{0, 1\}$ and any level $k \in [0, k^{\max}]$, let $V(k; \alpha)$ denote the value (i.e. the maximal intertemporal utility) of following strategy α with starting state level k . For any

6 This would imply characterization of MPE of the induced game and, as O’Donoghue and Rabin (2002) point out, this can be a very complicated task.

behavior path $A \in \{0,1\}^\infty$ and any level $k \in [0, k^{\max}]$, let $V(k; A)$ denote the value of following path A with starting state level k .

Suppose that strategy α^s describes sophisticated behavior in a BACAP \mathbf{B} (i.e. α^s is the selected MPE of the induced game). Then, for a given initial state level k a sophisticated would be better off than a naïf if and only if

$$V(k; \alpha^s) \geq V(k; \alpha^n)$$

From the previous subsections, we know that in $D(\mathbf{B})$ naïf and sophisticated behaviors are described by strategies $\tilde{\alpha}^n = \Xi(\alpha^n)$ and $\tilde{\alpha}^s = \Xi(\alpha^s)$, respectively. Proposition 12 implies that $V(k; \alpha^s) = V(\tilde{k}; \tilde{\alpha}^s)$ and $V(k; \alpha^n) = V(\tilde{k}; \tilde{\alpha}^n)$, where $\tilde{k} = k^{\max} - k$, and therefore we obtain

Proposition 16. *If for some level k a sophisticated is better off than a naïf in a BACAP \mathbf{B} , for level $\tilde{k} = k^{\max} - k$ a sophisticated is better off than a naïf in $D(\mathbf{B})$.*

Definition 17. *We say that sophistication **mitigates (exacerbates) misbehavior** in a BACAP \mathbf{B} , if*

1. *for all state levels a sophisticated is better off (worse off) than a naïf; and*
2. *there are state levels for which a sophisticated is strictly better off (worse off) than a naïf.*

The above proposition directly implies

Proposition 18. *If sophistication **mitigates (exacerbates) misbehavior** in a BACAP, it **mitigates (exacerbates) misbehavior** in $D(\mathbf{B})$.*

Notice that the above proposition implies that the implications of sophistication vs. naïveté are the same for harmful and beneficial addictions since one may be seen as the dual of the other.

We conclude this section by discussing a result obtained by O'Donoghue and Rabin (2002), namely, there are BACAPs presenting harmful addiction where sophistication exacerbates misbehavior. It is important to notice that this result depends not only on the characteristics of the BACAP, but also on the particular equilibrium selection describing sophisticated behavior. Regarding the characteristics, the BACAP must satisfy a condition that they refer

to as the Inevitability Condition (IC). In López (2006), it is shown that IC is equivalent to

$$V(0; H) \geq V(0; A) \text{ for } A = 011111\dots \quad (6)$$

that is, for an un-addicted individual ($k = 0$) the value of always hitting is higher than the value of refraining today and hitting from tomorrow ever after. It is also shown that expression (6) implies

$$V(k; H) \geq V(k; A) \text{ for } A = 011111\dots \quad \forall k. \quad (7)$$

The term Inevitability Condition is due to the following interpretation of expression (7): If sophisticated individuals “perceive that addiction is inevitable in the sense that no matter what they do today their future selves will hit forever after”, they might as well start hitting today.

Regarding the equilibrium selection, they propose the equilibrium corresponding to the limit of the unique finite-horizon MPE as the horizon becomes long—we will refer to this equilibrium as the ORE. O’Donoghue and Rabin (2002) show that when IC holds the ORE is characterized by strategy

$$\alpha^{ORE}(k) = 1 \quad \forall k,$$

and therefore sophistication exacerbates misbehavior: A sophisticated is worse off than a naïf for all state levels, and strictly worse off for any level below k^n (the threshold level characterizing naïf behavior).

Proposition 18 directly implies the following:

Proposition 19. *There are BACAPs presenting beneficial addiction where sophistication exacerbates misbehavior.*

Proof. Just consider the dual of a BACAP satisfying (7) and as the selected equilibrium describing sophisticated behavior strategy $\Xi(\alpha^{ORE})$. \square

We believe that the above result is worth mentioning for at least two reasons. First, it challenges the general view that sophistication mitigates misbehavior when costs are immediate and rewards are delayed, as it is the case of beneficial addictions. O’Donoghue and Rabin (1999) show that this view is correct in a context where the activity has to be undertaken **exactly once** and there is a finite number of periods where the individual can do it. However,

Proposition 19 shows that extrapolating it to the domain of BACAPs, where the activity may be repeated over time, is misleading.

Second, it relies on the selected equilibrium being the ORE, that is, the limit of the unique finite-horizon MPE as the horizon becomes long. In López (2006), other MPEs are provided such that, for BACAPs presenting harmful addiction and satisfying IC, sophistication mitigates misbehavior.

Concluding remarks

We have constructed an isomorphism that establishes a dualism between harmful and beneficial addictions: both phenomena are just two sides of the same coin. The dualism holds for the context of BACAPs, where an individual faces the binary choice of undertaking or not an activity in each period; and his payoff depends on his current choice as well as on the history of past behavior.

From a theoretical perspective, the dualism is appealing because it allows us to give insights for both realms of addiction by analyzing either one. We have shown that in a time consistent setting (i.e. when the intertemporal utility exhibits exponential discounting), whether the instantaneous utility function is convex or concave has very different implications: While under convexity the individual follows a cut-off rule, and therefore for a given initial state either always undertakes the activity or always refrains, assuming concavity may yield richer patterns of behavior. Since for some addiction contexts concavity might be the appropriate assumption, we believe that our results deserve attention and motivate further research.

We have also considered a time inconsistent setting, where the intertemporal utility function exhibits hyperbolic discounting, to study the implications of sophistication (i.e. when the individual is fully aware of his time inconsistent preferences) versus naïveté (i.e. when the individual is unaware of his time inconsistency). The isomorphism allows us to state that, whether sophistication hurts or benefits, the individual does not depend on whether costs are immediate and rewards delayed (beneficial addiction) or rewards are immediate and costs delayed (harmful addiction). This is worth mentioning

because in a doing-it-once context, where the activity must be performed exactly once in a finite number of periods, sophistication benefits the individual in the former while it hurts him in the latter case, as has been shown by O'Donoghue and Rabin (1999). Therefore, extrapolating this result to the domain of addictions would be misleading. To illustrate this, we make use of the dualism to show that there are BACAPs presenting beneficial addiction where sophistication hurts the individual. Nevertheless, we also point out that to obtain this result, which in fact is a translation of a result obtained by O'Donoghue and Rabin (2002) for harmful addictions, the particular equilibrium selection for the sophisticated's intrapersonal game plays a crucial role. If we allow other dominating MPE, as the ones provided in López (2006), then sophistication would mitigate misbehavior under both harmful and beneficial addictions.

Finally, we would like to point out that the isomorphism holds for arbitrary instantaneous payoff functions (Ψ and Γ) and arbitrary intertemporal utility function (U); however, the specific evolution of the state variable we have assumed in a BACAP (i.e. $k_{t+1} = \gamma k_t + a_t$) plays an important role in establishing it. Therefore, it would be of interest to consider whether the dualism still holds under more general conditions for the evolution of the state variable.

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