The new hybrid value at risk approach based on the extreme value theory*

El nuevo enfoque híbrido de value at risk basado en la teoría de valores extremos

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Abstract

In this paper the authors introduce a new hybrid approach based on the Extreme Value Theory (EVT) to joint estimation of Value at Risk (VaR) and Expected Shortfall (ES) for high quantiles of return distributions. The approach is suitable for measuring market risk in the emerging markets. It is designed to capture the empirical features of returns with emerging markets, such as leptokurtosis, asymmetry, autocorrelation and heteroscedasticity.

Key words: *Value at Risk; Extreme Value Theory; Expected Shortfall; emerging markets; market risk.*

JEL Classification: *G24, C22, C52, C53*

Resumen

Este trabajo introduce un nuevo enfoque híbrido basado en la teoría de valores extremos para la estimación conjunta de valor al riesgo (Value at Risk, VaR) y expectativa de caída (Expected Shortfall, ES) para cuantiles superiores de la distribución de retornos. Este enfoque es apropiado para medir el riesgo de mercado en mercados emergentes y está diseñado para capturar sus regularidades empíricas: leptokurtosis, asimetría, autocorrelación y heteroscedasticidad.

Palabras clave: *Value at Risk; Extreme Value Theory; Expected Shortfall; mercados emergentes.*

Clasificación JEL: *G24, C22, C52, C53*

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1. Introduction

Although there is a widespread agreement about the use of VaR as a general measure of market risk and the economic loss that banks and other financial institutions may suffer due to exposure to the market risk, there is no consensus on the preferred approach to VaR's calculation. The difficulties in obtaining reliable VaR estimates derive from the fact that all the existing approaches involve certain compromises and simplifications (Boudoukh, Richardson and Whitelaw, 1998). They are based on certain assumptions which greatly simplify the real market conditions. The assumptions on which these approaches are based represent a compromise between the efficiency of implementation, on one hand, and the statistical precision of market risk estimates on the other. Hence, determining the best approach to VaR estimation is an empirical question and a question of implementation. In other words, the choice of the optimal approach represents a decision between the efficiency of implementation and the statistic reliability of the approach. It may depend on the number and types of assets in portfolios and their sensitivity to changes in risk factors. The validity of the VaR models application primarily depends on a degree of compatibility between the characteristics of the real environment and the assumptions on which the models are based. Apart from that, the VaR estimates do not always fulfill all the characteristics of the coherent risk measures. The VaR estimates do not universally exhibit sub-additivity. The risk of a portfolio can be greater than the sum of the stand-alone risks of its components.

Despite the significant differences which derive from the differences in theoretical postulates on which approaches are based, a common feature of the most popular VaR approaches is their inability to be simultaneously effective in capturing leptokurtosis and strong time-varying volatility. In general, it can be said that nonparametric approaches are effective in capturing kurtosis and fat tails, but they will not be successful in capturing heteroscedasticity. On the negative side, nonparametric approaches depend too much on the historical data set, react slowly to changes in the market and are subject to predictable jumps in their forecasts of volatility (Zikovic, 2010). On the other hand, parametric approaches can be expected to be successful in capturing the dynamics of the time series of returns, but also quite unreliable when empirical distribution is deviates from the theoretical.

Since many empirical studies of the emerging markets show that the series of returns are characterized by excess kurtosis when compared to normal distribution (extreme financial returns are more likely than the normal distribution implies), they show a significant degree of autocorrelation and heteroscedasticity. It means that VaR approaches which are based on constant volatility, such as the Historical Simulation approach (HS), or VaR approaches that take simple techniques of modeling conditional volatility, such as equally weighted and exponentially weighted (e.g. RiskMetrics) models, will not be able to capture adequately the dynamics of returns in these markets (to be noticed that some authors, such as Schittenkopf, Lehar and Scheicher (2002), Harmantzis, Miao and Chien (2006) claim that complex techniques of modeling conditional volatility bring no significant improvement to VaR estimates in conditions of high volatility). This means that, in the context of emerging markets, some popular and most widely used VaR approaches are based on false assumptions. This is very indicative for risk management in banks and other financial institutions, because when elementary assumptions of most VaR approaches are not satisfied, VaR estimates will be unreliable and, at best, they can only provide unconditional coverage. The optimal approach for emerging markets is the one that can equally well capture both leptokurtosis (excess kurtosis) and time-varying volatility (heteroscedasticity). In order to capture successfully these specifics of emerging markets, it is necessary to design a sophisticated model of conditional volatility, which will also be easy to implement.

Considering these requirements, a new VaR approach for estimating market risk of the portfolio of banks, which operate in emerging markets, has been developed in this paper. The approach is designated as the new hybrid VaR approach based on the EVT. It is based on the solution proposed by McNeil and Fray (2000) in the application of EVT to estimate the market risk. Also, this approach incorporates the solutions proposed by Hull and White (1998) in improvement of HS approach and the solutions proposed by Zikovic (2010) in the development of semi-parametric model to VaR estimation. The approach is designed to combine the best features of HS approach, the application of EVT to VaR estimation and the advantages of GARCH (p,q) model to capture the heteroscedasticity. In other words, the approach is designed to capture successfully the two most conspicuous characteristics of financial asset returns with emerging markets, namely strong time varying volatility and excess kurtosis.

The paper is organized as follows: Section 1 contains the introduction. Section 2 gives an overview of the most significant, recent empirical researches in the area of VaR models. Section 3 presents a theoretical background of the new hybrid VaR approach based on the EVT. Section 4 provides a brief description of the analyzed data, the methodology used and the descriptive statistics of selected emerging markets. The backtesting results are presented in section 5. Since both backtesting tests used in the paper and the Kupiec's and Christoffersen's conditional coverage test are based on certain asymptotic assumptions and don't show the desired characteristics when performed on finite size samples, verification of their results was necessary. The Dufour (2006) Monte Carlo testing technique was used for that purpose. The final section summarizes the conclusions.

2. Literature Review

The recent studies about the applicability of VaR approaches in the emerging markets in terms of meeting the backtesting rules of Basel Committee, such as the studies conducted by Diamandis *et al.* (2011), Şener, Baronyana and Mengütürk (2012), Rossignolo, Fethib and Shaban (2012, 2013), Cui *et al.* (2013), Louzis, Xanthopoulos-Sisinis and Refenes (2014) and Del Brio, Mora-Valencia and Javier (2014), indicate the importance of developing the most appropriate VaR approach for measuring the market risk at the emerging markets. At the same time, they suggest that regulatory authorities should determine the use of approaches which can capture the heavy tails (particularly EVT approach) and discourage or prohibit the use of traditional VaR approaches, especially the Linear, HS and Filtered Historical Simulation (FHS) approaches. In other words, the results of these studies confirm the conclusions of earlier researches conducted by Lucas (2000), Berkowitz and O'Brien (2002), Wong, Cheng and Wong (2002), Gencay and Selcuk (2004), pointing out that neither the popular nonparametric, hybrid nor parametric approaches can provide reliable VaR estimates when the volatility is not constant over time and that they also cannot manage the extreme events and losses that fall at the end of the distribution tail. The first one to try to design the VaR approach adequate for emerging markets was Zikovic (2005). Zikovic's approach, known in literature as the Volatility and Time Weighted Historical Simulation approach (VTWHS), represents a simple combination of the two popular VaR approaches, Hull-White's and Hybrid approach, proposed by Boudoukh, Richardson and Whitelaw. Essentially, the approach represents an attempt to exploit the advantages of both approaches. The importance of this approach is reflected in the fact that it is a pioneer work in developing an adequate (optimal) approach to emerging markets. To improve the applicability of the Hybrid approach in emerging markets, Zikovic and Prohaska (2008) developed a procedure to determine the optimal decline factor. They tested the proposed procedure on the example of nine Mediterranean stock markets. The results were very poor. The main reason for this lies in the fact that the Hybrid approach reacts slowly to changes in the basic risk factors, despite its theoretical foundations. Zikovic (2010) suggested the Hybrid Historical Simulation approach (HHS). The HHS approach is based on a combination of a modified recursive bootstrap procedure proposed by Freedman and Peters (1984) and the parametric GARCH approach to volatility forecasting. The HHS approach is easy to implement. It operates with the observed data but is not free of distributional assumption, since the use of nonparametric bootstrapping requests that the observed returns should be identically and independently distributed (IID). The results of Zikovic's (2010) researches show that HHS approach adequately captures market risk in emerging markets of EU new member states. The main flaw of this approach is related to the use of re-sampling methods. An interesting nonparametric approach to VaR estimate has been proposed by Alemany, Bolancé and Guillén (2012). The approach is based on the double transformation of kernel estimation of the cumulative distribution function. The authors state that the approach is useful for large data sets and that it improves the quantile estimation compared to other methods in heavy tailed distributions (Alemany, Bolancé and Guillén. 2012). Unfortunately, the approach is more useful for measuring the operating rather than the market risk. Some interesting solutions in capturing the asymmetry of the basic data were also proposed by Şener, Baronyana, Mengütürk (2012) and Louzis, Xanthopoulos-Sisinis and Refenes (2014). Their solutions are based on the view that the validity of VaR approach depends not only on confidence level, as discussed by Beder (1995) and Christoffersen, Hahn and Inoue (2001), but also on the market conditions. They glorify the EVT approach for dealing with fat tails and extreme returns, which are otherwise typical for the emerging markets. For that reason, Şener, Baronyana and Mengütürk (2012) advocate that Conditional Autoregressive Value at Risk by regression quantiles (CAVaiR), proposed by Engle and Manganelli (2004), should be used combined with the EVT approach, but Louzis, Xanthopoulos-Sisinis and Refenes (2014) suggest that Asymmetric Heterogeneous Autoregressive (Asym. HAR) model, proposed by Louzis, Xanthopoulos-Sisinis and Refenes (2012) and Corsi and Reno (2012), should be used together with the EVT approach. The results of their researches show that the proposed solutions provide more statistically accurate VaR estimates, which minimize capital charges and allow more efficient capital allocations. However, when using these solutions, it should be considered that they are computationally more demanding when compared to the most commonly used approaches for VaR estimate.

In order to capture both characteristics of the financial data, heavy tails and heteroscedasticity, Bee (2012) presented the Dynamic Fat Tailed approach to VaR estimate. More precisely, he offered three dynamic VaR models: the Dynamic VaR model with Student t innovations, the Dynamic VaR model with Generalized Error Distribution (GED) innovations and the Dynamic Historical Simulation model (DHS). The first model is based on the use of a standardized Student t random variable as a model for the stochastic component of the GARCH process. The second model is based on modeling the residuals by means of the GED. DHS is semi-parametric. This model is very similar to the filtered HS model proposed by Fernandez (2003). The results of these models' validity tests testify in favor of DHS, which performs very-well, for an extremely high level of confidence. The research covers the developed markets, mainly the EU and the US, so there is no data on the application validity on the models in emerging markets.

3. The Theoretical Background of the New Hybrid VaR Approach Based on the EVT

As announced in the introduction, a new hybrid approach to the estimation of VaR and ES was developed in this paper. The proposed approach is based on the EVT. The starting point in the development of this approach was the fact that heteroscedasticity and the presence of autocorrelation are common features in series of financial data from the emerging markets, as well as the fact that extreme returns in these markets are more likely to appear than the presumption of elliptical distribution implies. Therefore, the basic idea on which the approach is based is that the dynamics of the returns of stock indexes in the emerging markets can successfully be captured by a simple AR(p)-GARCH(p, q) model.

The aim is to generate the standardized (IID) residuals, in order to obtain a stationary, or the IID series of returns which will be suitable for updating the volatility, according to the approach proposed by Hull and White (1998). The approach provides a coherent measure of risk. The final outcome of the approach is the ES-EVT. One disadvantage of the use of the most popular and widespread approaches to VaR estimation in the emerging markets is that their VaR estimates do not satisfy all the properties of a coherent risk measure. The reason for this is emphasized in the introduction. The final outcome of this approach is the ES-EVT, which is a coherent risk measure.

The approach is easy to understand and implement in practice. The number of parameters that have to be estimated is relatively small, and this number is determined by the GARCH specific structure and by the number of parameters of the extreme value distribution, because the approach is based on the assumption that extreme returns over a defined threshold (u) follow the Generalised Pareto Distribution (GPD) with the tail index (ξ) over 0. This assumption is relevant to the financial data because it suits fat tails.

The implementation of the approach begins from the basic specification of the autoregressive model that the return can be predicted by its past values and process innovations, which follows a white noise process:

$$r_i = \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{t-i} + \varepsilon_t \tag{1}$$

The first step in the implementation of the approach involves fitting the AR(p) model into a series of historical returns, in order to ensure IID residuals:

$$\begin{gathered} r_i = \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{t-i} + \varepsilon_t \\ \varepsilon_t = \eta_t \sqrt{\sigma_t^2} \eta_t \sim IID\ N(0,1) \end{gathered} \tag{2}$$

In the second step, a GARCH(p,q) model is fitted into the obtained residuals:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + i \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{3}$$

In the third step, the residuals (ε_t) obtained by applying the AR(p) model are divided by a corresponding conditional GARCH(p,q) volatility forecast (σ_t), to obtain the standardized residuals (z_t):

$$z_t = \frac{\varepsilon_t}{\sigma_t} \tag{4}$$

The next step implies that the standardized residuals (z_t) are multiplied with the latest GARCH volatility forecast $(\hat{\sigma}_{t+1})$ to obtain a series of historical residuals which have been updated for the latest volatility forecast in order to get a series of residuals which reflects the current market conditions $\{\hat{z}_{t+1}\}$.

$$\hat{z}_{t+1} = \left(z_t\right) \times (\hat{\sigma}_{t+1}) \tag{5}$$

After this, the simulated returns are obtained by using the updated historical residual $(\hat{z}_{t+1})$ in the Equation (1):

$$\hat{r}_{t+1} = \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{t-i+1} + \hat{z}_{t+1} \tag{6}$$

Thus obtained returns are suitable to be used for the VaR and ES estimation by applying the EVT.

In the final step, assuming that tail index (ξ) is less than 1, an ES-EVT estimate is obtained by using the following equation:

(7) $$ES - EVT = \frac{VaR_{cl}}{1-\xi} + \frac{\sigma - \xi u}{1-\xi}$$

noting that a VaR_{cl} estimate can be calculated as:

(8) $$VaR_{cl} = u + \frac{\sigma}{\xi}\left[\left(\frac{1-cl}{k/n}\right)^{-\xi} - 1\right]$$

where (*k*) represents the number of exceedings over the defined threshold (*u*), (σ) is the scale parameter and (*n*) the number of observations, or

(9) $$VaR_{cl} = x_{(n-k)}\left(\frac{n}{k}(1-cl)\right)^{-1/\hat{\alpha}^{H}}$$

when the tail index (ξ) is estimated by the Hill estimator.

Such a specified model provides a coherent market risk measure. The main advantage of this approach is its flexibility. It can be used to obtain the semi-parametric VaR estimates. In addition to this feature, the model flexibility can be seen through the possibility of applying various ARCH models. If the obtained standardized residuals, which are calculated in the third step, are not IID, some other ARCH model may be applied (i.e. IGARCH, EGARCH etc). The reason why the approach is based on the basic GARCH(p,q) model is that it is the simplest model able to capture the volatility of clusters and leptokurtosis in the data. The other models, despite the fact that they are able to capture the various empirical features of returns and generate reasonably accurate out-of-sample predictions of the entire distribution of future returns or just particular quantiles, as is needed for VaR forecasting, have the drawbacks because they require a relatively large number of parameters that cannot be solved in a closed, analytical form and can result in negative values, where both problems have a negative influence on the maximum likelihood estimate. The more volatility models get complex, the more estimated parameters become unstable, making such models vulnerable to parameter misspecification and model risk (Zikovic, 2010). A good example of this is the EGARCH(p,q) model suggested by Nelson (1991). Despite its numerous theoretical advantages over the basic GARCH(p,q) model, the EGARCH(p,q) model is known to be very problematic in practice, with the choice of starting values being extremely critical for successful likelihood maximization (Franses and van Dijk (1996), Johnson, 2001). Furthermore, some researches, such as the research conducted by Rossignolo, Fethib and Shaban (2012), showed that the EGARCH(p,q) model did not bring any significant advantage in volatility estimate over the GARCH model in emerging and frontier markets, pointing out that the density assumption was more important than the model specification itself.

The main flaw in the suggested approach comes from the limitations of the application of EVT to VaR estimation. When using the EVT model, we should be aware of its limitations, since it is developed by using the asymptotic arguments, which can create difficulties when applied to finite samples. The critical factor is the choice of the threshold. When applying the EVT, we should be careful about the size of the tail. The choice of the tail size can affect the VaR estimates through its effect on the tail index estimate. This is known as a compromise between the variance and the partiality. See details concerning this issue in the paper presented by Gonzalo and Olmo (2004).

4. Data and the Methodology of Analysis

The daily logarithmic returns of the stock indexes of a EU candidate (Serbia) and potential candidate countries (Bosnia and Herzegovina, Macedonia, Montenegro and Turkey) and a EU member (Croatia) were used for the performance analyses of the new hybrid approach. The tested stock indexes are the BIRS (Bosnia and Herzegovina), MONEX20 (Montenegro), MBI10 (Macedonia), BELEXline (Serbia), CROBEX (Croatia) and UX100 (Turkey). For the emerging markets, such as these, an extremely short history of securities trading and the phenomenon of non-synchronous trading can cause serious problems to a reliable statistical analysis. Therefore, to overcome the problem previously mentioned (namely, the short time series of returns of individual stocks and their highly variable liquidity), we use the stock indexes.

The returns are collected from the official stock exchange web sites of these countries for the period between February 2, 2009 and February 2, 2012. These data cover the periods of volatility patterns observed in the EU market. The daily returns of selected stocks are generated using the logarithmic approximation:

$$R_{i,t} = log\left(\frac{P_{i,t}}{P_{i,t-1}}\right) \tag{10}$$

where $P_{i,t}$ represents the closing price of asset i on the day t.

The VaR estimates were calculated for one day holding period and for the confidence level of 99 and 95%. The confidence levels were chosen taking into consideration the Basel Accord as well as the basic characteristics of the VaR calculation. The confidence level of 95% is appropriate for application in stable market conditions, while the 99% confidence level is appropriate for application in volatile market conditions. The VaR backtesting period is formed by taking out 253 of the latest observations from each stock index. The rest of the observations are used for volatility model calibration.

At the beginning of the analysis, the characteristics of selected markets for the entire observation period were analyzed. Table 1 gives a summary of the descriptive statistics and normality tests for the entire analyzed sample for all of the stock indexes. The descriptive statistics of the selected stock indexes confirm the results of the recent studies. The stock indexes show a great difference between their maximum and minimum returns. The standard deviations are also high, indicating a high level of fluctuations of the daily returns. The analysis of the selected stock indexes distribution shows that stock indexes have a significantly fatter distribution tails than assumed under normality, ranging from 2,4, in the case of the XU100 index to 9,3, in the case of the MBI10 index. In other words, all the analyzed stock indexes show a significant leptokurtosis. The skewness of all stock indexes is significantly different from zero, which indicates that the stock indexes have asymmetric returns. There is also evidence of negative skewness in the case of XU100, which means that the left tail is particularly extreme. In order to examine formally whether returns follow the normal distribution, we employed the Jarque-Bera test. The value of the Jarque-Bera test indicates that we should reject the null hypothesis of normality providing the evidence that the return series are not normally distributed.

TABLE 1
DESCRIPTIVE STATISTICS OF SELECTED EMERGING MARKETS

	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Mean	–0,0159	0,0008	–0,0080	0,0031	0,0073	0,0118
Standard Dev.	0,7492	1,4375	1,5478	1,0558	1,3245	1,1840
Sample Variance	0,5613	2,0665	2,3958	1,1146	1,7544	1,4019
Kurtosis	5,2436	7,6201	9,2880	3,7825	6,7776	2,4708
Skewness	0,1009	1,1441	0,7756	0,4264	0,4836	–0,1828
Jarque-Bera test	159,2	834	1312,5	42	477,1	12,9
p –value	0,0000	0,0000	0,0000	0,0000	0,0000	0,0015

Source: Authors’ calculations.

The presence of autocorrelation and the presence of conditional heteroscedasticity (the ARCH effect) in returns of the selected stock indexes are tested by their sample autocorrelation (ACF) and sample partial autocorrelation function (PAFC), calculating the Ljung-Box Q statistic and the Lagrange Multiplier test to test the presence of the ARCH effect. The results of these tests, for each of the selected stock indexes, are presented in Table 2 and Tables 3 and 3a in the appendix. As expected, the results of these tests confirm that there is a significant autocorrelation and the ARCH effect in returns. The aforesaid leads to the conclusion that classical VaR approaches couldn’t estimate the true risk level in these markets.

TABLE 2
THE ARCH EFFECTS

	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Lagrange Multiplier	24,443	106,861	26,997	218,104	28,789	84,656
p-value	0,000	0,000	0,000	0,000	0,000	0,000

Source: Authors’calculations.

Since the employed test discovers significant autocorrelation and heteroscedasticity in returns of the selected stock indexes, the original data should be transformed to the IID. As autocorrelation has been detected in both returns and squared returns, the returns should be modeled as an AR-GARCH process in order to deal successfully with both types of dependence. Assuming that the conditional volatility in these markets can be adequately captured by the simplest GARCH(1,1) model, the original data are transformed by applying an AR(p)-GARCH(1,1).

The parameters of the AR(p)-GARCH(1,1) model were rated by maximum likelihood estimation. The Levenberg-Marquardt algorithm was used to estimate the parameter of the AR(p) model. The maximum likelihood estimation is used to estimate the parameters of the GARCH(1,1) by choosing the parameters that maximize the Gaussian log-likelihood function:

$$LR = \sum_{i=1}^{T}\left[-\ln\sqrt{2\pi} - \frac{1}{2}\frac{\varepsilon_i^2}{\sigma_i^2(\alpha,\beta,\omega)} - \frac{1}{2}\ln\sigma_i^2(\alpha,\beta,\omega)\right] \tag{11}$$

The estimated AR(p)-GARCH(1,1) parameters for each of the selected stock indexes are given in Table 4. All estimated parameters are statistically significant.

TABLE 4

THE ESTIMATES OF OF AR(P)-GARCH(1,1) MODEL PARAMETERS

	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Parameters						
C	–	–	–	–	–	0,0003
AR(p)	0,1727	0,2986	0,1280	0,3896	0,1280	–0,0145
a	0,1666	0,1813	0,1786	0,2955	0,2088	0,1359
b	0,4367	0,7656	0,8200	0,6219	0,7639	0,0160
w	0,0000	0,0000	0,0000	0,0000	0,0000	0,0002

Source: Authors' calculations.

There are several methods for estimating the tail index of extreme value distribution from the empirical data. In this paper, we used the Hill estimator because it has more desirable properties than the other estimators:

$$\hat{\alpha}^H = \frac{1}{k}\sum_{i=1}^{k}\ln(x_{n-k+1}) - \ln(x_{n-k}) \tag{12}$$

The crucial step in estimating the tail index is the determination of a threshold (u). The threshold value for each index is determined by applying the rule of thumb for determining the threshold which was proposed by Christoffersen (2011). Christoffersen (2011) points out that, for big samples, a good rule of thumb is setting the threshold so that 5% of the greatest observations for estimating the tail index should be found in the distribution tail.

The threshold (*u*) will then simply be the 95th percentile of the data set. This instruction is applied in the paper. The value of thresholds and the maximum likelihood estimates of the tail index and maximum likelihood estimates of the sigma, for each stock index, are presented in Table 5.

TABLE 5
THE MAXIMUM LIKELIHOOD ESTIMATES OF THE TAIL INDEX AND SIGMA, THRESHOLD VALUE

	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Parameters						
Threshold value	–1,3925	–2,1152	–1,5637	–1,4085	–1,3003	–1,6146
The tail index (ξ)	0,4632	0,5227	0,3801	0,3342	0,4010	0,3257
sigma	1,6978	2,7989	0,9137	1,6978	2,1843	0,8985

Source: Authors' calculations.

5. The Backtesting Results

In this section of the paper the backtesting results of the suggested approach are presented, analyzed and discussed. The approach is evaluated in terms of its accuracy in estimating VaR over the last 253 days of the observed period. The approach was tested as follows: first, the daily VaR estimates, which were obtained for confidence levels of 95% and 99%, were compared with the actual return movement that occurred in the backtesting period. In the case where the actual loss on a particular day exceeded the VaR estimate for that day, it was concluded that a VaR break had occurred. Then, the number/percentage of the VaR breaks over the backtesting period of the 253 days was established. According to Jorion (2006), in a good model the percentage of VaR breaks should be equal to one minus the level of confidence. In this case, it means that the number of VaR breaks mustn't exceed 3 at a 99% confidence level (1% of VaR estimates total number), i.e. not more than 13 VaR breaks at a 95% confidence level (5% of VaR estimates total number).

The number/percentage of VaR breaks at a 95% and 99% confidence levels over the backtesting period, separately for each of the selected stock indexes, are given in Table 6. As it can be seen in Table 6, percentages of VaR breaks are lower than the theoretical percentage values in all of the six emerging markets. The exception appears in the case of the MBI10 stock index at a 99% confidence level. The approach showed the best performances in the case of the BIRS index, since no VaR breaks were made at a 99% confidence level over the backtesting period of the 253 days and in the case of the CROBEX index at a 95% confidence level. In the case of the MBI10 stock index at a 99% confidence level, the number of breaks is equal to the expected frequen-

TABLE 6
THE NUMBER/PERCENTAGES OF VAR BREAKS AT A 95% AND 99% CONFIDENCE LEVELS

Stock index	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
No. of 95%VaR breaks	9	12	10	11	7	10
Percent of breaks 95%VaR breaks	3,56	4,74	3,95	4,35	2,77	3,95
No. of 99%VaR breaks	0	1	3	1	2	1
Percent of breaks 99%VaR breaks	0	0,4	1,19	0,4	0,79	0,4

Source: Authors' calculations.

cies of VaR breaks. For this reason, the percentage of breaks is higher that the theoretical value.

In order to determine whether the percentage of VaR breaks can be considered as equal to the theoretical value, we employed the unconditional coverage test introduced by Kupiec (1995). The Kupiec's test of the unconditional coverage represents the most widely used model for testing the VaR approach validity. The idea behind this test is that the frequency of VaR breaks should be statistically consistent with the probability level for which VaR is estimated (Samanta *et al.* 2010). In this paper we used the Kupiec's test at 5% significance level, because a significance level of this magnitude generates clear evidence about the validity of the approach and implies that a model should be rejected only if the evidence against it is reasonably strong. The following likelihood ratio test was employed to test null hypothesis:

$$LR_{uc} = 2\ln\left[(1-p)^{T-T_1}p^{T_1}\right] + 2\ln\left[(1-T_1/T)^{T-T_1}(T_1/T)^{T_1}\right] \tag{13}$$

where (p) is the tail probability (or the VaR coverage rate).

The backtesting results for VaR at the 95% confidence level are presented in Table 7.

TABLE 7
KUPIEC'S TEST BACKTESTING RESULTS AT 5% SIGNIFICANCE LEVEL FOR VAR AT THE 95% CONFIDENCE LEVEL

Stock index	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Kupiec's test (LR_{uc})	1,2274	0,0357	0,6277	0,2504	3,1473	0,6277
p-value	0,2679	0,8500	0,4282	0,6168	0,0761	0,4282

Source: Authors' calculations.

As can be seen from Table 7, the approach satisfied the Kupiec's test at the 5 percent significance level in all of the six emerging markets. Although it is very informative to look at VaR approach performance at different confidence levels, the true test of the VaR model acceptability to regulators is its performance at 99% confidence level, as prescribed by the Basel Committee. The backtesting results for VaR at the 99 % confidence level are presented in Table 8.

TABLE 8

KUPIEC'S TEST BACKTESTING RESULTS AT 5% SIGNIFICANCE LEVEL FOR VAR AT THE 99% CONFIDENCE LEVEL

Stock index	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Kupiec's test (LR_{uc})	–	1,2129	0,0823	1,2129	0,1208	1,2129
p-value	–	0,2708	0,773	0,2708	0,7281	0,2708

Source: Authors' calculations.

As can be seen from Table 8, in all of the six emerging markets, the approach satisfied the Kupiec's test at the 5 percent significance level.

However, one of the basic disadvantages of the Kupiec's test is that it considers only the number of VaR breaks and not the time when they occur. In other words, a shortcoming of this test is that it focuses exclusively on the unconditional coverage property of an adequate VaR measure and does not examine the extent to which the independence property is satisfied. The Kupiec's test is based on the assumption that the VaR estimates are efficient, which means that they incorporate all the information known at the time of the forecast. The history of VaR break does not give any information whether the VaR break will happen again or not. As a result it is expected that the probability of occurrence of a new VaR break after the previous one is the same as the probability of its occurrence after the days in which the VaR break did not occur. In other words, the test was based on the expectation that the VaR break would be evenly distributed over the backtesting period. This is equivalent to the assumption that risk forecasts will be independently distributed over time. That is why the time of the VaR break occurrence is not important and it focuses only on the unconditional coverage. The independence property of VaR breaks is nevertheless an essential property because any measure of risk must adapt automatically and immediately to any new information which entails a new evolution in the dynamics of asset returns. If the approach ignores such dynamics then the VaR will react slowly to changing market conditions and VaR breaks will appear clustered in time. The consequence of the exposure to the series of consecutive VaR breaks (clusters) can be just as problematic as the systematic incomplete reporting on exposure to market risks. The risk of bankruptcy is considerably greater than in the situation in which VaR breaks are evenly distributed over time. Hence, the perfect VaR approach needs to satisfy both properties.

This is why we employed the Christoffersen's conditional coverage test in the paper in addition to the Kupiec's test (Christoffersen, 2001):

$$LR_{cc} = LR_{uc} + LR_{ind} \tag{14}$$

where $LR_{ind} = -2\ln\left[(1-\pi)^{T_{00}+T_{11}}\,\pi^{T_{01}+T_{11}}\right] + 2\ln\left[(1-\pi_{01})^{T_{00}}\,\pi_{01}^{T_{01}}\,(1-\pi_{11})^{T_{01}}\,\pi_{11}^{T_{11}}\right]$

The number of days when after a no VaR break day occurred a no VaR break day (T_{00}), i.e. when after a no VaR break day occurred a VaR break day (T_{01}), i.e. when after a VaR break day occurred a no VaR break day (T_{10}), when after a VaR break day occurred a VaR break day (T_{11}), and their probabilities are given in Table 9 in the Appendix. The Christoffersen's conditional coverage test results for VaR at the 95 and 99 percent confidence levels are presented in Table 10. As can be seen from Table 10, in all of the six emerging markets the approach satisfied the Christoffersen's conditional coverage test at the 5 percent significance level.

TABLE 10

CHRISTOFFERSEN'S CONDITIONAL COVERAGE TEST BACKTESTING RESULTS AT 5% SIGNIFICANCE LEVEL FOR VAR AT THE 95% AND 99% CONFIDENCE LEVELS

Stock index	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
Christoffersen's conditional coverage test for 95%VaR (LR_{cc})	1,2274	0,0357	4,4835	0,8245	3,1473	0,6277
p-value	0,5414	0,9823	0,1063	0,6622	0,2073	0,7306
Christoffersen's conditional coverage test for 99%VaR (LR_{cc})	–	1,2022	5,4488	1,2022	0,1166	1,2022
p-value	–	0,5482	0,0656	0,5482	0,9433	0,5482

Notice: In the cases where the sample has $T_{11} = 0$ (there are no consecutive VaR breaks), an alternative formula was used in the paper to calculate the first-order Markov likelihood (see Brandolini and Colucci, 2013).

Source: Authors' calculations.

A significant disadvantage of these tests is reflected in the fact that they have a questionable statistical power for the sample size defined by the Basle Accord. Both of these tests are developed using asymptotic arguments, which can create difficulties when applied to finite samples. Namely, the LR_{uc} test is asymptotically distributed as $\chi 2$ with one degree of freedom under the null hypothesis that the tail probability *(p)* is the true probability. The LR_{cc} test is asymptotically distributed as $\chi 2$ with two degrees of freedom under the null hypothesis that the hit sequence is IID Bernoulli with the mean equal to the VaR coverage rate. Asymptotically, that is as the number of observations, T, goes to infinity, the LR_{uc} test will be distributed as a $\chi 2$ with one degree of freedom. It is the same with the LR_{cc} test. In large enough samples, the LR_{cc} test will be distributed as a $\chi 2$ with two degree of freedom. Many authors, such as, Christoffersen and Pelletier (2004), Hurlin *et al.* (2008), Berkowitz, Christoffersen and Pelletier (2008), Ziggel *et al.* (2013), have shown that when the number of VaR breaks is small, there are substantial differences between asymptotic probability distributions of the considered tests and their finite sample analogues. Hurlin *et al.* (2008) state that the use of asymptotic critical values based on a $\chi 2$ distribution induces important size distortions even for relatively large sample. Therefore they point out that in case of a small sample size, (as in sample size defined by Basle Accord), i.e. in case of a small number of VaR breaks (T_1), which are the informative observations, it is better to rely on Monte Carlo simulated p-values rather than on those from the $\chi 2$ distribution.

The differences between the finite sample critical values and the asymptotic critical values for both test statistics (the LR_{uc} and LR_{cc}) are shown in Table 11 in the Appendix. The finite sample critical values for the both test statistics for the lower 1 percent are based on 10.000 Monte Carlo simulations of sample size T = 253. The percentages shown in brackets represent quantiles that correspond to the asymptotic critical values under the finite sample distribution. When tests tend to be oversized in finite samples, it means their empirical distributions will be moved to the right off the theoretical shape; hence theoretical quantiles tend to be too small, translating into increased rejection rates. The opposite happens when the tests tend to be undersized in finite samples. In such case their empirical distributions will be moved to the left off the theoretical shape, which will give undersized rejection rates.

Due to the differences between the empirical and theoretical distribution quantiles, conclusions based on the results shown in Tables 7, 8 and 10 need to be checked. The Dufour (2006) Monte Carlo testing technique (see Appendix B)[1] was used for this purpose. Dufour (2006) proposed the Monte Carlo test procedure which allowed to obtain the null distribution of tests statistics in finite sample setting. The method has a great advantage of providing exact tests based on any statistics whose finite sample distribution is intractable but can be simulated (Malecka, 2014).

Following the Monte Carlo test procedure: first, 9.999 samples of random IID Bernoulli (*p*) variables were generated, where the sample size equals the actual sample. Based on these artificial samples, 9.999 simulated LR_{uc} tests were calculated and named $\left\{ L\tilde{R}_{uc}(i) \right\}_{i=1}^{9.999}$. Finally, the simulated p-values were calculated as the share of simulated LR_{uc} values which are larger than the actually obtained LR_{uc} test value:

$$p-value = \frac{1}{10.000}\left\{ 1 + \sum_{i=1}^{9.999} I\left(L\tilde{R}_{uc}(i) > LR_{uc} \right) \right\} \tag{15}$$

where I(·) takes on the value one if the argument is true and zero otherwise.

[1] See the advantages of applying simulation procedure over bootstrap method in Christoffersen and Pelletier (2004).

The same procedure was repeated with LR_{cc} test. The cases for which the tests were not feasible were rejected in the simulation. Average feasible rates of tests are from 0,868 and 0,872 for 99% VaR to 0,974 and 0,987 for 95% VaR, (for both tests) the LR_{uc} and LR_{cc}, respectively.

The backtesting results rely on the finite sample p-values, they are shown in table 12 and 13.

Based on the results presented in Table 12 and Table 13 we cannot dispute the use of the approach in the capital markets of EU candidate countries and potential candidate countries and Croatia, in terms of the backtesting rules of the Basel Committee. Particularly good results are gained in meeting the Kupiec's test of unconditional coverage. The explanation lies in the fact that the approach is designed in such a way that it can perfectly capture the dinamics in the series of stock returns.

TABLE 12
THE BACKTESTING RESULTS BASED ON THE MONTE CARLO P-VALUES
FOR THE LR_{UC} TEST

			95%VaR			
	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
p-value	0,1048	0,4499	0,3274	0,4715	0,1582	0,2714
			99%VaR			
	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
p-value	–	0,1798	0,5910	0,1022	0,3567	0,0987

Notice: Significante level of 5%. Samples where the test cannot be computed are omitted due to lack of VaR breaks.
Source: Authors' calculations.

TABLE 13
THE BACKTESTING RESULTS BASED ON THE MONTE CARLO P-VALUES
FOR THE LR_{CC} TEST

			95%VaR			
	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
p-value	0,3531	0,6221	0,0892	0,3542	0,1735	0,4115
			99%VaR			
	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
p-value	–	0,4588	0,0549	0,2588	0,1895	0,2588

Notice: Significante level of 5%. Samples where the test cannot be computed are omitted due to lack of VaR breaks.
Source: Authors' calculations.

6. Conclusion

Given the characteristics of the emerging markets, such as the capital markets of the EU candidate countries and potential candidate countries and Croatia, in this paper we developed, presented and tested a new hybrid approach based on EVT to estimate the market risk of the portfolio of banks and other financial institutions which were operating in these markets. The approach is designed to successfully capture the dynamics in the series of stock returns with emerging markets and to produce the innovations IID. It is based on the AR(p)-GARCH(1,1) model. At the same time, it recognizes the fact that the extreme returns with emerging markets are more likely than the assumption of normality implies.

The nonparametric part of the model enables us to capture successfully the leptokurtosis and the asymmetry, while the parametric part successfully captures the time changeable volatility. Despite the fact that it was designed to successfully capture the strong dynamics in emerging markets returns, this approach isn't computationally intensive as the other approaches which successfully capture the excess kurtosis and the time-varying volatility, and which are based on too many parameters that need to be estimated. The number of parameters which should be estimated in the model is relatively small.

The Kupiec's test of unconditional coverage and Christoffersen's test of conditional coverage were used for testing the validity of the approach. Despite the fact that the traffic light approach was assigned by Basel Accord, the Kupiec's test of unconditional coverage was chosen, because it is equally important for the bank whether the approach overestimates or underestimates the real level of market risk because in that case it additional capital is unnecessarily allocated which has negative impact on its profitability. The difference between the Kupiec's test and the traffic light approach is in the fact that the Kupiec's test is based on a two-sided test and the traffic light approach is based on a one-sided test. This is why we believe that the Kupiec's test is more suitable for the banks. Christoffersen's test of conditional coverage was chosen because it is a test which is simple and easy to implement but at the same time it tests both features which a perfect VaR approach must satisfy simultaneously (both unconditional coverage and independence).

Since these tests are based on certain asymptotic arguments, conclusions that were reached based on them need to be verified. This is why the Dufour (2006) Monte Carlo testing technique was used. Results of conducted simulations suggest that the VaR forecasts obtained by this approach can be trusted and that this approach can be reliably used in the emerging markets in terms of the Basel Committee's rules. Since in the case of market index BIRS for VaR estimates made for the level of trust of 99 % no exceedings were detected during the backtesting period it was not possible to conduct a simulation. This is why future researchers need to test the validity of the approach (once again) on this or on similar markets but for a different (or a longer) backtesting period. As the academic community insists on the use of AR(p)-Student-t-GARCH(1,1) model instead of the normal AR(p)-GARCH(1,1) model, particularly for more extreme (1% or less) VaR thresholds, future researchers are left to test the applicability of the specified (improved) approach.

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APPENDIX A

TABLE 3

ACF, PACF AND LJUNG-BOX Q TEST FOR THE DAILY LOG RETURNS FOR TESTED INDEXES IN THE PERIOD 02.02.2009 - 02.02.2012.

	BIRS index				MONEX20 index				MBI10 index			
lag	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value
1	0,1297	0,1297	12,7090	0,0004	0,1843	0,1843	25,6828	0,0000	0,1170	0,1170	10,3227	0,0013
5	0,0147	0,0113	18,6975	0,0022	0,0407	0,0004	58,4502	0,0000	0,0513	0,0533	19,2940	0,0017
10	0,0173	0,0135	26,8644	0,0027	0,0817	0,0685	70,9771	0,0000	0,0954	0,0936	32,0580	0,0004
15	-0,0221	-0,0415	35,3923	0,0022	0,0719	0,0621	83,2689	0,0000	0,0177	0,0097	45,3945	0,0001
20	0,0407	0,0219	37,3823	0,0105	0,0143	0,0085	84,1947	0,0000	0,0045	-0,0084	47,6616	0,0005
	BELEXline index				CROBEX index				XU100 index			
lag	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value
1	0,3440	0,3440	89,4756	0,0000	0,1214	0,1214	11,1445	0,0008	-0,1123	-0,1123	9,5369	0,0020
5	0,0743	0,0060	131,8199	0,0000	-0,0078	-0,0138	12,4370	0,0293	0,0221	0,0101	12,4585	0,0290
10	0,0291	-0,0236	145,8258	0,0000	0,0119	0,0077	14,7255	0,1424	-0,0120	-0,0249	16,4064	0,0886
15	0,0095	-0,0468	171,2537	0,0000	0,0740	0,0688	22,3823	0,0982	0,0322	0,0317	21,7823	0,1136
20	0,1088	0,0491	200,5026	0,0000	-0,0391	-0,0555	34,0363	0,0259	-0,0516	-0,0194	35,9534	0,0156

Source: Authors' calculations.

TABLE 3A

ACF, PACF AND LJUNG-BOX Q TEST FOR THE DAILY LOG SQUARED RETURNS FOR TESTED INDEXES IN THE PERIOD 02.02.2009 - 02.02.2012.

	BIRS index				MONEX20 index				MBI10 index			
lag	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value
1	0,0647	0,0647	3,1683	0,0751	0,3132	0,3132	74,1755	0,0000	0,1173	0,1173	10,3980	0,0013
5	0,0927	0,0884	14,0959	0,0150	0,2753	0,1925	229,9905	0,0000	0,1756	0,1529	78,3206	0,0000
10	0,0389	0,0362	16,0515	0,0982	0,1328	0,0524	290,9288	0,0000	0,0502	0,0027	148,9301	0,0000
15	0,0908	0,0671	47,6294	0,0000	0,0369	-0,0315	313,5787	0,0000	0,0590	0,0736	202,7552	0,0000
20	-0,0157	-0,0238	55,0095	0,0000	-0,0072	-0,0025	314,7851	0,0000	0,0203	0,0325	216,8705	0,0000
	BELEXline index				CROBEX index				XU100 index			
lag	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value	ACF	PACF	Q-stat.	p-value
1	0,4585	0,4585	158,9550	0,0000	0,1033	0,1033	8,0741	0,0045	0,1863	0,1863	26,2504	0,0000
5	0,0936	0,0355	284,7290	0,0000	0,1584	0,0951	91,0099	0,0000	-0,0015	-0,0221	36,7695	0,0000
10	0,0762	-0,0160	334,5536	0,0000	0,1853	0,1256	154,9989	0,0000	0,0181	0,0164	42,8916	0,0000
15	0,0757	0,0530	351,0855	0,0000	0,0994	0,0144	191,3901	0,0000	-0,0459	-0,0455	45,0271	0,0001
20	0,1461	0,0902	386,6475	0,0000	0,1193	0,0286	252,0330	0,0000	-0,0092	-0,0297	52,4803	0,0001

Source: Authors' calculations.

TABLE 9
THE HIT SEQUENCE OF VAR BREAKS

	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
	95%VaR					
T_0	244	241	243	243	246	243
T_1	9	12	10	11	7	10
T_{00}	235	229	235	232	239	233
T_{01}	9	12	8	10	7	10
T_{10}	9	12	8	10	7	10
T_{11}	0	0	2	1	0	0
p	0,0356	0,0474	0,0395	0,0433	0,0277	0,0395
p_{01}	0,0369	0,0498	0,0329	0,0412	0,0285	0,0412
p_{11}	0	0	0,2	0,0909	0	0
	BIRS	MONEX20	MBI10	BELEXline	CROBEX	XU100
	99%VaR					
T_0	253	252	249	251	250	251
T_1	0	1	3	1	2	1
T_{00}	253	251	247	250	248	250
T_{01}	0	1	2	1	2	1
T_{10}	0	1	2	1	2	1
T_{11}	0	0	1	0	0	0
π	0	0,004	0,0119	0,0040	0,0079	0,0040
π_{01}	0	0,004	0,0080	0,0040	0,0080	0,0040
π_{11}	/	0	0,3333	0	0	0

Source: Authors' calculations.

TABLE 11
THE DIFFERENCES BETWEEN THE FINITE SAMPLE CRITICAL VALUES AND THE ASYMPTOTIC CRITICAL VALUES FOR THE LR_{UC} AND THE LR_{CC} TEST STATISTICS

	Significance levels		
	1%	5%	10%
		LR_{uc} Statistic	
Asymptotic $\chi2(1)$	6,6348	3,8414	2,7055
Finite-sample	5,497	5,025	3,555
	(0,49%)	(9,49%)	(12,19%)
		LR_{cc} Statistic	
Asymptotic $\chi2(2)$	9,21	5,9915	4,605
Finite-sample	6,007	5,015	5,005
	(0,20%)	(1,10%)	(11,79%)

Note: The finite sample critical values for the both test statistics for the lower 1 percent are based on 10.000 Monte Carlo simulations of sample size T = 253. The percentages shown in the brackets represent quantiles that correspond to the asymptotic critical values under the finite sample distribution.

APPENDIX B:
The Dufour (2006) Monte Carlo Testing Technique

Lets take (*S*) a statistic of a given test of continuous survival function *G(.)* such as $Prob[S_i = S_j] = 0$. Theoretical p-value *G(.)* can be approximated by its empirical counterpart: $\hat{G}_M(x) = 1/M\sum_{i=1}^{M} I(S_i \geq x)$ where *I*(.) is the indicator function. (S_i) is the test statistic for a sample simulated under the null hypothesis. Dufour (2006) demonstrated that if (*M*) is big enough, whatever the value of (S_0), theoretical critical region $G(S_0) < \alpha$, with (*a*), the asymptotic nominal size, is equivalent to the critical region $\hat{p}_M(S_0) \leq \alpha_1$, with $\hat{p}_M(S_0) = \dfrac{M\hat{G}_M(S_0)+1}{M+1}$ and this $\forall\alpha_1$. When $Prob[S_i = S_j] \neq 0$, or when it is possible for a given simulation of the test statistic (under null hypothesis) to find the same value of (*S*) for two or more times, the empirical survival function can be written as follows: $\tilde{G}_M(S_0) = 1/M\sum_{i=1}^{M} I(S_i \leq S_0) + \dfrac{1}{M}\sum_{i=1}^{M} I(S_1 = S_0) \times I(U_i \geq U_0)$, where ($U_i$), $i = 0,1\ldots, M$ correspond to realizations of a uniform *[0,1]* variable.