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COVARIANCE STABILITY AND THE 2008 FINANCIAL CRISIS: THE IMPACT IN THE PORTFOLIO OF THE 10 BIGGEST COMPANIES IN BM&FBOVESPA BETWEEN 2004 E 2012

Leticia Naomi Ono Maeda
UNICAMP - Universidade Estadual de Campinas, Brazil
E-mail: leticiamaeda@gmail.com

Johan Hendrik Poker
UNICAMP - Universidade Estadual de Campinas, Brazil
E-mail: johan.poker@fca.unicamp.br

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ABSTRACT

This study's purpose is to analyze the influence of the covariance fluctuation between assets over the structure of a portfolio of investments. To accomplish that, the covariances between the daily returns of the 10 biggest participants of the BM&FBovespa stock market are analyzed, before, during and after the 2008 financial crisis. The procedure of this research includes: (1) collection of returns of the selected stocks between 2004 and 2012; (2) composition of the classical portfolio proposed by Markowitz's theory (1952); and (3) the measurement of the covariances instability effect between the 10 selected assets over the maintenance of a portfolio's risk and return, according to the preferences of a hypothetical investor. We discover that the asset's covariance vary over time and affect the correlations among the assets, especially in financial crisis periods. Consequently, both risk and return of the portfolio may change greatly if the asset's weights are not recalculated periodically. This supports the idea that portfolio theory might benefit from the development of stability weighted techniques.

Keywords: portfolio theory; asset risk management; financial crisis.

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1. INTRODUCTION

The ability to predict the future value of assets in the financial market was always desirable, and there are currently many ways to choose assets which compose a given investment portfolio, evaluating the assets characteristics such as their expected return, risk, investment period, liquidity, among others. One of the possible financial instruments that analyze the relationship between two of these characteristics – precisely, risk and return –to elect the best investment option is the Markowitz model (1952).

However, it is crucial to clarify that the covariance stability between the companies is assumed over time, so that the chosen investment portfolio according to the Markowitz model (1952) is maintained during the investment period. Thus, if the covariances are unstable, possible commitments related to the expected portfolios results may occur.

Since covariances are dynamic and dependent on economy variations in general and, specifically, on the financial market, this study is justified by the need to assess to what extent and in what kind of scenario it would be unwise to use the Markowitz model (1952) – especially in economic instability situations, as in the recent 2008 crisis – with no use of improvements, in order not to put at risk results expected from a portfolio.

2. THEORETICAL FOUNDATION

2.1. Markowitz model (1952)

According to the Markowitz model (1952), an investor tries to predict the future outcome of assets basically through the analysis of expected return and risk of the asset. The latter, in turn, is considered, according to Luenberg (1997), as random variables, since the asset can take different future values, each with a given probability of occurrence, considering that the future asset value is not known upon purchase.

Thus, mathematically speaking, the expected return is basically the sum of the possible asset returns x_i weighted by their probabilities p_i of occurence, whereas the risk is in the variance – or on the square of the variance (standard deviation) which is

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most routinely used – of the aforesaid return, that is, the calculation of how far a value is from its expected value. Both described in the following formulas,

$$\bar{x} = \sum_{i=1}^{m} x_i p_i \quad (1)$$

Where

- $\bar{x}$ = expected value of asset X;
- x_i = value of asset X in time i;
- p_i = probability of the value of asset X in time i.

$$\sigma_x = \sqrt{E[(x_i - \bar{x})^2] p_i} \quad (2)$$

Where

- σ_x = standard deviation of the asset X;
- x_i = value of asset X in the time i;
- $\bar{x}$ = expected value of the asset X;
- p_i = probability of the value of asset X in the time i.

Usually, however, one does not invest in a single asset but in a set of assets, named assets portfolio or investment portfolio. The preference for an investment portfolio to only one asset occurs due to the need to diminish the risk of an investment. According to Bodie *et al*., (2010), the risk may be classified as non-diversifiable risk and diversifiable risk. The first is the risk inherent in the market as a whole, whereas the second is closely related to one or more specific parts of the market and, therefore, may be minimized by diversifying assets, that is, an investment of a specific amount in different assets of the financial market. Markowitz (1952, p. 89) describes this phenomenon as follows:

> "In an attempt to reduce variance, investing in various assets is not enough. One must avoid that the investment is made in assets with high covariance between them. We must diversify investment among industries, particularly industries with different economic characteristics, since companies from different industries have lower covariance than companies in the same industry."

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In this respect, composing an assets portfolio decreases diversifiable risk significantly, increasing the probability of an asset to obtain a certain expected value, or in other words, reducing risk. However, we still need to understand how we should select some on them among the various assets available in the market, which can form what Markowitz (1952) named as efficient portfolio investment. A portfolio is effective for a given return, there is no other portfolio with less risk, or, similarly, for a given risk, there is no other portfolio with a higher return. This concept can also be interpreted by the Sharpe Dominance Principle (1965):

"An investor should choose their optimal portfolio from the set of portfolios that:

1. Offers maximum expected return for different levels of risk, and
2. Offers minimal risk for different levels of expected return."

Thus, in order to calculate the expected return and the risk of a portfolio, it is assumed that an investor distributes an amount X_0 between *n* assets, each with a weight w_i in the portfolio, whereas $\sum_{i=1}^{n} Wi = 1$ and $X_0 = \sum_{i=1}^{n} X_i$ where X_i is the amount invested in the *i* th asset, it follows that the total return of the portfolio is given by:

$$\mu_p = \sum_{i=1}^{n} = w_i \bar{x}_i \quad (3)$$

Where:

- μ_p = portfolio total expected return;
- w_i = weight of the asset i;
- $\bar{x}_i$ = total expected return of the asset i.

In order to calculate a portfolio variance, the covariance and correlation concept is necessary. Both the covariance and the correlation can be clarified as the interdependence of two random variables. With respect to correlation, it follows that:

- If $\rho(x,y)$ = 0, then X and Y are uncorrelated;
- If $\rho(x,y)$ = 1, then X and Y are perfectly correlated;
- If $\rho(x,y)$ = -1, then X and Y are negatively correlated.

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Furthermore, the covariance between two X and Y assets can be mathematically defined by:

$$Cov_{x,y} = \rho_{x,y}\, \sigma_x\, \sigma_y \qquad (4)$$

By knowing the covariance value between two variables it is possible to calculate the standard deviation (risk) of a portfolio of two assets, which is given by:

$$\sigma_p = \sqrt{(w_x^2\, \sigma_x^2) + (w_y^2\, \sigma_y^2) + 2\, w_x\, w_y\, Cov_{x,y}} \qquad (5)$$

Where:

- σ_p = portfolio standard deviation;
- w_x = weight of the asset X;
- w_y = weight of the asset Y;
- σ_x = standard deviation of asset X;
- σ_y= standard deviation of asset Y
- $Cov_{x,y}$ = covariance between assets X and Y.

However, if we wish to know the variance of a portfolio with more than two assets, we just need to use, according to Luenberger (1997), the formula:

$$\sigma_p^2 = \sum_{i,j=1}^{n} w_i w_j Cov_{ij} \qquad (6)$$

Where:

- σ_p^2 = portfolio total variance;
- w_i = weight of the asset i;
- w_j = weight of the asset j;
- Cov_{ij} = covariance between asset i and j.

Thus, we can reject that the variance of the portfolio is calculated from the covariance between pairs of assets. Recalling that $Cov_{ii} = \sigma_i^2$.

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Using these return and portfolio risk concepts we can relate them to a chart whose abscissa corresponds to the risk and the orderly, to the expected return. The points of the chart correspond to an investment portfolio involving certain assets. The points corresponding to these investment portfolios form return-risk curves. In this curve are presented investment portfolios composed of the same assets, however with different w_i weights for each portfolio (point) of the curve.

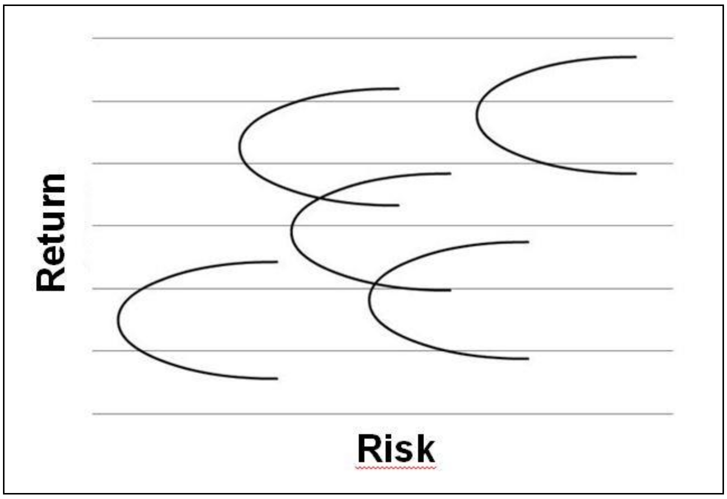

Figure 1: Return-Risk Curves.
Source: Adapted from Hieda and Oda (1998)

After choosing assets that will compose the portfolio, the corresponding Risk-Return curve to depreciated assets is found. Thus, we would obtain the following Risk-Return curve whose inner area is called feasible region. Both at the curved line as well as at the feasible region are all possible portfolios of the same assets, however with different w_i weights.

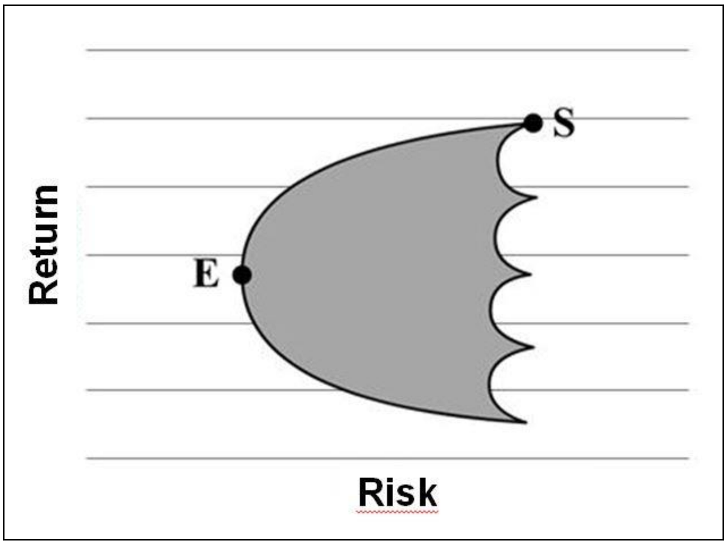

Figure 2: Feasible region
Source: Adapted from Luenberger (1997)

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However the only part of the Risk-Return curve that follows the Dominance principle, cited above, corresponds to the curved line that goes from point "E" of minimal risk to point "S" of maximum return.

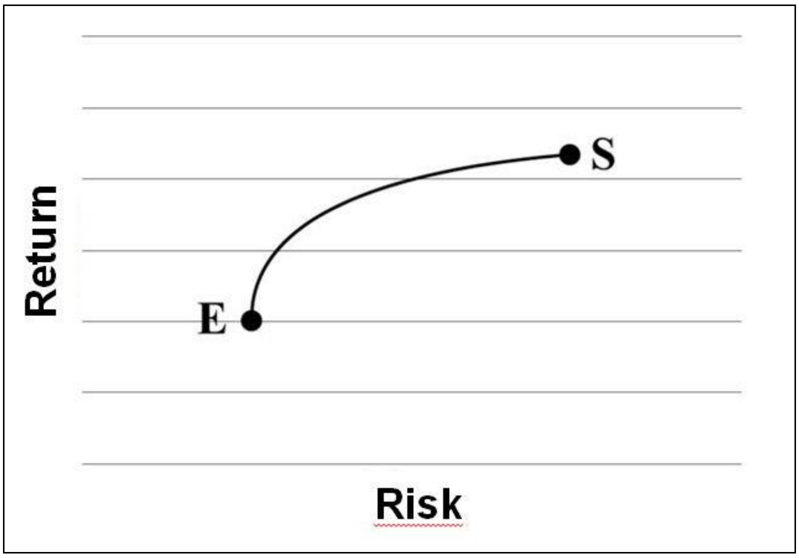

Figure 3: Efficient frontier
Source: Adapted from Hieda and Oda (1998)

The curve $\hat{E}S$ is called efficient frontier. Such boundary defines all the possible efficient portfolio investments, that is, those that for a given level of return have the minimum possible risk.

Finally, it is necessary to point out the relationship that the assets weights have with their correlation index. Assuming a portfolio composed of two X and Y assets, we form several X and Y combinations, each with a different correlation index among the same and different weights. Thus, short selling is not possible (*weight of X + weight of Y = 1*).

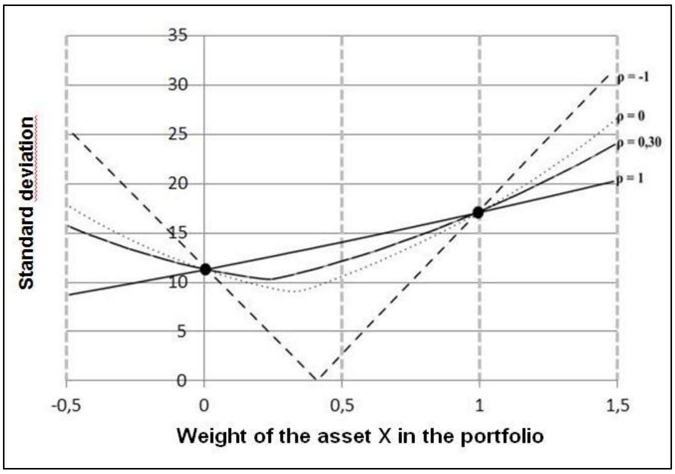

Chart 1: Correlation index
Source: Adapted from Bodie, Marcus and Kane (2010)

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In it there is the following information on the assets correlation influence on the diversification effects:

- when the correlation between assets is positively perfect ($\rho = 1$), there is no diversification effect of assets;

- when the correlation between assets is imperfect $-1 < \rho < 1$, there are imperfect effects of asset diversification;

- when the correlation between assets is negatively perfect ($\rho = -1$), there is a perfect effect of assets diversification shown by the scope of a risk equal to zero.

As the correlation between the two assets changes, the assets weights of the portfolio must also be changed in order to maintain a certain level of risk. For example, if it were necessary to maintain a minimum standard deviation, the asset X should correspond to approximately 25%, 37.5% and 43.75% of the total portfolio, if the correlations were, respectively, 0, 0.30 and -1 – assets of correlation perfect in this case would not reach the aforementioned level of risk.

2.2. Investor preferences

Although the efficient frontier points the best investment combination alternatives, there is nothing on which combination or which portfolio should be selected, since this decision is up to each investor according to their personal characteristics.

According to Danthine (2005), such preferences may take into account several variables: the wealth degree of the investor, uncertainty in the investment time, among others. However, a good instrument for assessing the preference of an investor regarding the choice between risk assets is the indifference curve.

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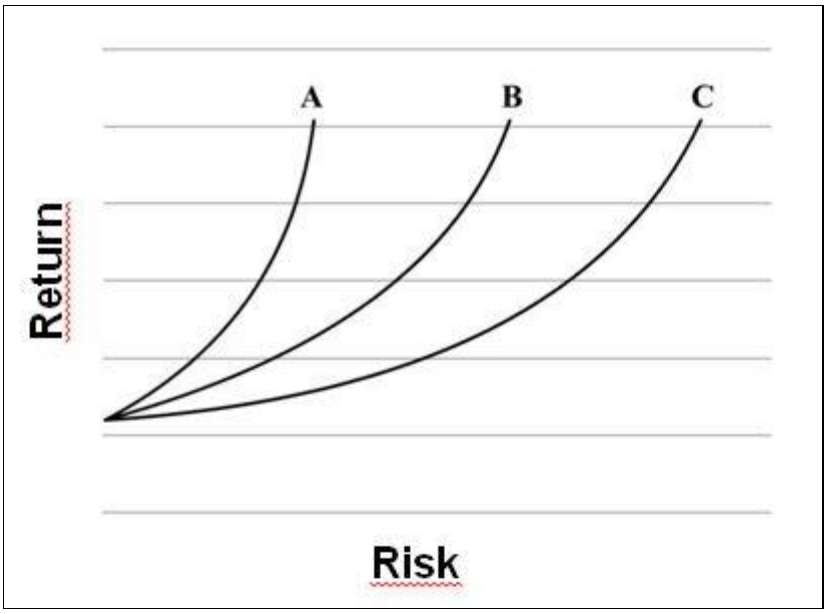

Figure 4: Risk aversion
Source: Adapted from Bodie, Marcus and Kane (2010)

The indifference curve measures the risk aversion degree of an investor, that is, the amount of additional return they need to accept one more risk unit. In the above, we observe three indifference curves. The steeper the curve, the greater the risk aversion degree is. Thus, the curves "A", "B" and "C" correspond to investors of, respectively, greater risk aversion, moderate risk aversion and lower risk aversion.

Similarly, risk aversion can also be calculated by the Sharpe index:

$$IS = \frac{x_p}{\sigma_x} \quad (7)$$

Where

– x_p = return of asset X;

– σ_x = standard deviation of asset X.

It measures how much more return is given for each additional unit of risk.

3. METHODOLOGY

3.1. General data

The methodology involves the quantitative model analysis, exploring the financial and statistical data from the top 10 companies of BM &, that is, companies with relatively large amounts of shares traded. This information was drawn from the

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company's Thomson Reuters Eikon database, a leader in collection and distribution of information on the business market.

To carry out this work we considered the following periods as pre-crisis, crisis and post-crisis scenarios.

Table 1: Analyzed periods

Period	Scenario
January/2004 to June/2007	Pre-crisis
July/2007 to June/2009	Crisis
July/2009 to December/2012	Post-crisis

Source: The author

The ten companies chosen for this study with their codes of their actions are listed in Table 2.

Table 2 – Analyzed companies

Action Code	Company
BBAS3.SA	Banco do Brasil
BBDC4.SA	Banco Bradesco
CCRO3.SA	Companhia de Concessões Rodoviárias
CMIG4.SA	Companhia Energética de Minas Gerais,
CSNA3.SA	Companhia Siderúrgica Nacional
EMBR3.SA	Embraer
GGB	Gerdau
ITUB.K	Itaú Unibanco Holding
PETR4.SA	Petrobrás
VALE5.SA	Vale

Source: The author

3.2. Analysis of covariance

In order to identify the covariance behavior over time we structured semiannual covariance matrices between the companies' share returns. Each matrix has the covariances of returns of the companies within a semester over eight years (2004-2012). Furthermore, for matrices calculation we used the "COVARIAÇÃO.S" tool from the Microsoft Excel program. The result of this formula is the deviation average of products of each pair of points in two datasets, in this case, two sets return of two different companies. Therefore, the matrix is composed of covariances of all possible pairwise combinations of the ten aforementioned companies.

3.2.1. Constructing hypothetical portfolio

This section of the study primarily aims to quantify the influence of the covariances instability in a theoretical portfolio, by changing the assets weights over time.

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First of all, we identified, for each of the three periods studied (pre-crisis, crisis and post-crisis) their returns, standard deviations, and covariance matrices. From these variables, we built six hypothetical portfolios. Three portfolios have restrictions such as the preference of a Sharpe index of a hypothetical investor equal to 15% in the three periods. The three other portfolios must maintain constant their weights, to quantify the Sharpe index variation.

In order to find the returns, standard deviations and covariance matrices, we used formulas showed in the theoretical foundation of this article. Whereas the construction of portfolios that meet a Sharpe index of 15% were made by the SOLVER tool from Microsoft Excel, under the following restrictions:

- $Wi \geq 0$
- $\sum_{i=1}^{n} Wi = 1$

4. ANALYSIS OF COVARIANCE

The covariance matrices of the ten companies in the study are as follows.

Table 3: Covariance matrix of 1st semester of 2004

1º SEMESTRE 2004										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,13976	0,11425	0,00037	0,09142	0,15211	0,42324	0,04299	0,09565	0,16317	0,20200
BBDC4	0,11425	0,12120	- 0,00091	0,09135	0,13951	0,40071	0,03578	0,09630	0,15204	0,21505
CCRO3	0,00037	- 0,00091	0,01229	- 0,01088	- 0,02338	- 0,01371	0,00772	- 0,00540	- 0,01979	- 0,02362
CMIG4	0,09142	0,09135	- 0,01088	0,10475	0,15355	0,36308	0,03320	0,08241	0,16759	0,15618
CSNA3	0,15211	0,13951	- 0,02338	0,15355	0,37979	0,58122	0,06413	0,12466	0,28518	0,24623
EMBR3	0,42324	0,40071	- 0,01371	0,36308	0,58122	1,71749	0,14847	0,34412	0,61310	0,68965
GGBR4	0,04299	0,03578	0,00772	0,03320	0,06413	0,14847	0,03546	0,03228	0,05752	0,03202
ITSA4	0,09565	0,09630	- 0,00540	0,08241	0,12466	0,34412	0,03228	0,08837	0,14085	0,17904
PETR4	0,16317	0,15204	- 0,01979	0,16759	0,28518	0,61310	0,05752	0,14085	0,31301	0,27015
VALE5	0,20200	0,21505	- 0,02362	0,15618	0,24623	0,68965	0,03202	0,17904	0,27015	0,52373

Source: The author

Table 4: Covariance matrix of 2nd semester 2004

2º SEMESTRE 2004										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,64856	0,50188	0,28974	0,32087	0,20998	- 0,37874	0,14621	0,41379	0,50461	0,80898
BBDC4	0,50188	0,42866	0,23101	0,25949	0,15590	- 0,27209	0,10620	0,33990	0,39141	0,62594
CCRO3	0,28974	0,23101	0,14453	0,13471	0,11106	- 0,11970	0,07363	0,19748	0,21035	0,37403
CMIG4	0,32087	0,25949	0,13471	0,20333	0,10143	- 0,26803	0,08462	0,21442	0,30566	0,45203
CSNA3	0,20998	0,15590	0,11106	0,10143	0,14227	- 0,04589	0,09791	0,14553	0,15100	0,31666
EMBR3	- 0,37874	- 0,27209	- 0,11970	- 0,26803	- 0,04589	0,83688	- 0,10197	- 0,23487	- 0,52520	- 0,58181
GGBR4	0,14621	0,10620	0,07363	0,08462	0,09791	- 0,10197	0,10318	0,11659	0,15830	0,27617
ITSA4	0,41379	0,33990	0,19748	0,21442	0,14553	- 0,23487	0,11659	0,29874	0,35698	0,58470
PETR4	0,50461	0,39141	0,21035	0,30566	0,15100	- 0,52520	0,15830	0,35698	0,57686	0,80603
VALE5	0,80898	0,62594	0,37403	0,45203	0,31666	- 0,58181	0,27617	0,58470	0,80603	1,31350

Source: The author

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Table 5: Covariance matrix of 1st semester of 2005

1º SEMESTRE 2005										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,14786	-0,00854	0,00749	-0,02390	0,08254	0,14902	0,03161	0,00307	0,07713	0,11558
BBDC4	-0,00854	0,57346	-0,00155	0,19637	0,08807	0,14799	0,01299	0,27275	0,32895	0,10350
CCRO3	0,00749	-0,00155	0,01340	-0,00338	-0,00165	0,04050	0,01156	0,00508	0,02219	0,02741
CMIG4	-0,02390	0,19637	-0,00338	0,15862	-0,17742	-0,09033	-0,08779	0,11227	0,06689	-0,20827
CSNA3	0,08254	0,08807	-0,00165	-0,17742	1,07562	0,62375	0,32641	-0,08363	0,23824	1,10049
EMBR3	0,14902	0,14799	0,04050	-0,09033	0,62375	0,71186	0,23891	0,03237	0,33896	0,77870
GGBR4	0,03161	0,01299	0,01156	-0,08779	0,32641	0,23891	0,14799	-0,01627	0,07896	0,35271
ITSA4	0,00307	0,27275	0,00508	0,11227	-0,08363	0,03237	-0,01627	0,17005	0,15474	-0,06616
PETR4	0,07713	0,32895	0,02219	0,06689	0,23824	0,33896	0,07896	0,15474	0,38873	0,38035
VALE5	0,11558	0,10350	0,02741	-0,20827	1,10049	0,77870	0,35271	-0,06616	0,38035	1,34771

Source: The author

Table 6: Covariance matrix of 2nd semester of 2005

2º SEMESTRE 2005										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,99678	1,72126	0,23806	0,29896	0,21663	0,60271	0,62057	0,77516	1,07993	1,41330
BBDC4	1,72126	4,07016	0,59276	0,64877	0,36734	1,29271	1,25005	1,63812	2,17470	2,63051
CCRO3	0,23806	0,59276	0,10062	0,10011	0,05041	0,21639	0,19086	0,24514	0,32078	0,36927
CMIG4	0,29896	0,64877	0,10011	0,16006	0,13308	0,27138	0,23580	0,26584	0,47956	0,48064
CSNA3	0,21663	0,36734	0,05041	0,13308	0,18244	0,21190	0,18040	0,15971	0,42425	0,36817
EMBR3	0,60271	1,29271	0,21639	0,27138	0,21190	0,77816	0,50736	0,53947	0,91594	0,85563
GGBR4	0,62057	1,25005	0,19086	0,23580	0,18040	0,50736	0,48131	0,57144	0,81582	0,93038
ITSA4	0,77516	1,63812	0,24514	0,26584	0,15971	0,53947	0,57144	0,77098	0,93297	1,17970
PETR4	1,07993	2,17470	0,32078	0,47956	0,42425	0,91594	0,81582	0,93297	1,71862	1,70859
VALE5	1,41330	2,63051	0,36927	0,48064	0,36817	0,85563	0,93038	1,17970	1,70859	2,26387

Source: The author

Table 7: Covariance matrix of 1st semester of 2006

1º SEMESTRE 2006										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,91431	0,74643	0,04024	0,13731	0,33277	0,13474	0,47867	0,53313	0,58171	0,37023
BBDC4	0,74643	2,36171	0,29970	0,65302	0,16667	0,74102	0,38252	0,88887	0,90018	0,80541
CCRO3	0,04024	0,29970	0,09673	0,11935	-0,08782	0,18634	-0,07176	0,01374	0,07494	0,15332
CMIG4	0,13731	0,65302	0,11935	0,28975	-0,06071	0,25450	-0,07235	0,10324	0,24364	0,34343
CSNA3	0,33277	0,16667	-0,08782	-0,06071	0,55456	-0,14215	0,51322	0,36640	0,43826	0,23457
EMBR3	0,13474	0,74102	0,18634	0,25450	-0,14215	0,82973	-0,08886	0,11435	0,21765	0,22611
GGBR4	0,47867	0,38252	-0,07176	-0,07235	0,51322	-0,08886	0,84355	0,74880	0,38300	0,12264
ITSA4	0,53313	0,88887	0,01374	0,10324	0,36640	0,11435	0,74880	0,99294	0,41380	0,16951
PETR4	0,58171	0,90018	0,07494	0,24364	0,43826	0,21765	0,38300	0,41380	0,88353	0,78227
VALE5	0,37023	0,80541	0,15332	0,34343	0,23457	0,22611	0,12264	0,16951	0,78227	1,04537

Source: The author

Table 8: Covariance matrix of 2nd semester of 2006

2º SEMESTRE 2006										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	1,37130	0,74643	0,69311	0,36081	-0,06893	0,91140	0,23478	0,66043	0,83426	1,94974
BBDC4	1,17289	1,31008	0,67595	0,29520	-0,08279	1,22776	0,14248	0,72975	0,53635	1,69341
CCRO3	0,69311	0,67595	0,41101	0,16231	-0,08650	0,66275	0,10133	0,40131	0,31675	0,93522
CMIG4	0,36081	0,29520	0,16231	0,13126	0,03092	0,15555	0,06943	0,15723	0,29658	0,60819
CSNA3	-0,06893	-0,08279	-0,08650	0,03092	0,12142	-0,23705	-0,00244	-0,06338	0,11092	0,06248
EMBR3	0,91140	1,22776	0,66275	0,15555	-0,23705	2,00436	-0,06193	0,73338	-0,16400	1,11316
GGBR4	0,23478	0,14248	0,10133	0,06943	-0,00244	-0,06193	0,17395	0,16058	0,30398	0,36304
ITSA4	0,66043	0,72975	0,40131	0,15723	-0,06338	0,73338	0,16058	0,60894	0,30288	0,99574
PETR4	0,83426	0,53635	0,31675	0,29658	0,11092	-0,16400	0,30398	0,30288	1,04439	1,42515
VALE5	1,94974	1,69341	0,93522	0,60819	0,06248	1,11316	0,36304	0,99574	1,42515	3,21538

Source: The author

Table 9: Covariance matrix of 1st semester of 2007

1º SEMESTRE 2007										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	1,95447	1,55059	0,59248	0,73458	1,79245	0,36449	1,52807	1,44009	0,94343	2,95263
BBDC4	1,55059	1,71527	0,50276	0,55347	1,24199	-0,03416	1,04596	1,13766	0,91346	2,02145
CCRO3	0,59248	0,50276	0,23526	0,24036	0,59534	0,14763	0,50210	0,41877	0,34857	1,02566
CMIG4	0,73458	0,55347	0,24036	0,32861	0,68090	0,18940	0,63587	0,55590	0,44370	1,16171
CSNA3	1,79245	1,24199	0,59534	0,68090	2,41257	0,80712	1,59366	1,21309	0,73249	3,91059
EMBR3	0,36449	-0,03416	0,14763	0,18940	0,80712	0,68389	0,55657	0,26696	0,02220	1,50008
GGBR4	1,52807	1,04596	0,50210	0,63587	1,59366	0,55657	1,49018	1,23548	0,67336	2,64549
ITSA4	1,44009	1,13766	0,41877	0,55590	1,21309	0,26696	1,23548	1,29444	0,69618	1,95588
PETR4	0,94343	0,91346	0,34857	0,44370	0,73249	0,02220	0,67336	0,69618	1,04347	1,22648
VALE5	2,95263	2,02145	1,02566	1,16171	3,91059	1,50008	2,64549	1,95588	1,22648	6,97819

Source: The author

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Table 10: Covariance matrix of 2nd semester of 2007

2º SEMESTRE 2007										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	1,10353	1,30060	0,03283	0,12836	1,16506	0,16569	0,32025	0,40727	2,23637	2,69811
BBDC4	1,30060	2,91733	-0,33776	-0,07581	2,98669	-0,15428	1,24517	1,63211	6,63272	7,34219
CCRO3	0,03283	-0,33776	0,28072	0,18061	-0,60070	0,20966	-0,40561	-0,59232	-1,81161	-1,11298
CMIG4	0,12836	-0,07581	0,18061	0,22560	-0,35636	0,23671	-0,12199	-0,25331	-0,85223	-0,40863
CSNA3	1,16506	2,98669	-0,60070	-0,35636	3,90496	-0,50438	1,12836	1,80241	8,40760	8,86823
EMBR3	0,16569	-0,15428	0,20966	0,23671	-0,50438	0,82725	-0,23565	-0,24525	-1,36299	-1,31884
GGBR4	0,32025	1,24517	-0,40561	-0,12199	1,12836	-0,23565	1,75059	2,02726	4,09370	2,75780
ITSA4	0,40727	1,63211	-0,59232	-0,25331	1,80241	-0,24525	2,02726	2,67194	5,57878	4,13921
PETR4	2,23637	6,63272	-1,81161	-0,85223	8,40760	-1,36299	4,09370	5,57878	23,11169	19,53912
VALE5	2,69811	7,34219	-1,11298	-0,40863	8,86823	-1,31884	2,75780	4,13921	19,53912	24,52083

Source: The author

Table 11: Covariance matrix of 1st semester of 2008

1º SEMESTRE 2008										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	2,12778	0,82518	0,38739	0,57793	0,86635	-0,28203	0,97898	0,76947	2,72939	0,72102
BBDC4	0,82518	2,34019	0,52708	1,04348	4,20005	-0,89839	2,08556	0,32205	3,58118	3,63548
CCRO3	0,38739	0,52708	0,33101	0,46092	1,57699	-0,84673	1,61526	0,38926	1,47475	0,80937
CMIG4	0,57793	1,04348	0,46092	0,88828	2,63096	-1,53501	2,85297	0,81148	2,46916	1,48113
CSNA3	0,86635	4,20005	1,57699	2,63096	13,42529	-4,47486	7,75553	0,85962	9,58663	8,34521
EMBR3	-0,28203	-0,89839	-0,84673	-1,53501	-4,47486	4,26714	-6,86607	-1,89321	-3,33189	-0,72964
GGBR4	0,97898	2,08556	1,61526	2,85297	7,75553	-6,86607	12,76269	4,10880	6,71522	2,43176
ITSA4	0,76947	0,32205	0,38926	0,81148	0,85962	-1,89321	4,10880	2,36199	1,80052	0,00355
PETR4	2,72939	3,58118	1,47475	2,46916	9,58663	-3,33189	6,71522	1,80052	11,00839	6,69479
VALE5	0,72102	3,63548	0,80937	1,48113	8,34521	-0,72964	2,43176	0,00355	6,69479	8,21875

Source: The author

Table 12: Covariance matrix of 2nd semester of 2008

2º SEMESTRE 2008										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	7,72622	6,27814	1,88239	0,92553	13,64128	3,37084	18,27491	9,07395	15,15383	14,07742
BBDC4	6,27814	5,47312	1,56032	0,85640	11,15312	2,54475	14,60481	7,02226	12,34112	11,51012
CCRO3	1,88239	1,56032	0,57868	0,25522	3,48150	0,71373	4,35487	2,02395	3,54119	3,51304
CMIG4	0,92553	0,85640	0,25522	0,41635	1,73620	0,04110	2,18008	0,84860	1,92916	1,88641
CSNA3	13,64128	11,15312	3,48150	1,73620	26,20954	5,27812	34,50097	16,85438	28,25051	26,70725
EMBR3	3,37084	2,54475	0,71373	0,04110	5,27812	2,68585	7,62911	4,28697	5,87474	5,26867
GGBR4	18,27491	14,60481	4,35487	2,18008	34,50097	7,62911	50,75190	25,77196	38,51761	34,92815
ITSA4	9,07395	7,02226	2,02395	0,84860	16,85438	4,28697	25,77196	13,97273	19,20479	17,04780
PETR4	15,15383	12,34112	3,54119	1,92916	28,25051	5,87474	38,51761	19,20479	34,00734	29,82375
VALE5	14,07742	11,51012	3,51304	1,88641	26,70725	5,26867	34,92815	17,04780	29,82375	28,23413

Source: The author

Table 13: Covariance matrix of 1st semester of 2009

1º SEMESTRE 2009										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	4,97446	4,72829	1,23288	0,70893	4,64459	0,52488	0,68810	1,87534	6,15330	3,69955
BBDC4	4,72829	4,80855	1,26065	0,65862	4,63667	0,78386	0,69970	1,90940	5,88143	3,88590
CCRO3	1,23288	1,26065	0,41390	0,15382	1,27835	0,24503	0,40182	0,75307	1,41097	1,04435
CMIG4	0,70893	0,65862	0,15382	0,14061	0,62646	0,05766	0,06232	0,21036	0,87905	0,46476
CSNA3	4,64459	4,63667	1,27835	0,62646	5,00449	0,88044	0,97980	2,23040	5,84075	4,22374
EMBR3	0,52488	0,78386	0,24503	0,05766	0,88044	1,05649	-0,16319	0,42860	0,23845	0,80982
GGBR4	0,68810	0,69970	0,40182	0,06232	0,97980	-0,16319	1,38387	1,34977	0,79628	0,80861
ITSA4	1,87534	1,90940	0,75307	0,21036	2,23040	0,42860	1,34977	2,06206	1,89831	1,68284
PETR4	6,15330	5,88143	1,41097	0,87905	5,84075	0,23845	0,79628	1,89831	8,71967	5,09480
VALE5	3,69955	3,88590	1,04435	0,46476	4,22374	0,80982	0,80861	1,68284	5,09480	4,32041

Source: The author

Table 14: Covariance matrix of 2nd semester of 2009

2º SEMESTRE 2009										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	6,48752	5,23065	1,34217	0,96436	5,47145	0,70238	4,18699	5,05748	5,54585	9,15269
BBDC4	5,23065	4,77715	1,39568	1,02959	4,67192	0,29974	4,01282	4,67541	4,97745	8,29724
CCRO3	1,34217	1,39568	0,58866	0,40730	1,28176	-0,13812	1,61047	1,73529	1,60896	2,69423
CMIG4	0,96436	1,02959	0,40730	0,37875	0,93061	-0,01728	1,12373	1,16467	1,16014	1,87067
CSNA3	5,47145	4,67192	1,28176	0,93061	5,27897	0,45883	3,71879	4,49624	5,17922	8,66868
EMBR3	0,70238	0,29974	-0,13812	-0,01728	0,45883	0,62430	-0,29452	-0,10478	0,07070	0,18411
GGBR4	4,18699	4,01282	1,61047	1,12373	3,71879	-0,29452	5,31814	5,56190	4,86126	7,95266
ITSA4	5,05748	4,67541	1,73529	1,16467	4,49624	-0,10478	5,56190	6,24557	5,37819	9,15391
PETR4	5,54585	4,97745	1,60896	1,16014	5,17922	0,07070	4,86126	5,37819	5,90644	9,45604
VALE5	9,15269	8,29724	2,69423	1,87067	8,66868	0,18411	7,95266	9,15391	9,45604	15,97958

Source: The author

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Table 15: Covariance matrix of 1st semester of 2010

	1º SEMESTRE 2010									
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	1,08944	0,73256	0,19359	0,00155	1,73424	0,17111	-0,46600	-0,21039	2,12857	2,34394
BBDC4	0,73256	0,85974	0,25670	0,04373	1,03349	0,07978	-0,14926	0,11200	2,01804	1,77293
CCRO3	0,19359	0,25670	0,14372	0,02394	0,27534	0,02265	-0,05263	0,06537	0,73855	0,55989
CMIG4	0,00155	0,04373	0,02394	0,07343	-0,03003	0,00449	0,02209	0,04939	-0,00338	-0,03164
CSNA3	1,73424	1,03349	0,27534	-0,03003	5,09103	0,57168	-1,97714	-1,37557	3,17656	5,58419
EMBR3	0,17111	0,07978	0,02265	0,00449	0,57168	0,09328	-0,21785	-0,15104	0,25836	0,58616
GGBR4	-0,46600	-0,14926	-0,05263	0,02209	-1,97714	-0,21785	1,80335	1,41815	-0,87970	-1,68629
ITSA4	-0,21039	0,11200	0,06537	0,04939	-1,37557	-0,15104	1,41815	1,31919	-0,18726	-0,92683
PETR4	2,12857	2,01804	0,73855	-0,00338	3,17656	0,25836	-0,87970	-0,18726	6,97530	4,96873
VALE5	2,34394	1,77293	0,55989	-0,03164	5,58419	0,58616	-1,68629	-0,92683	4,96873	7,34228

Source: The author

Table 16: Covariance matrix of 2nd semester of 2010

	2º SEMESTRE 2010									
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	3,40240	3,42213	1,39321	0,70688	0,32427	1,17502	0,00087	2,49599	-0,80192	5,41290
BBDC4	3,42213	3,66904	1,43975	0,74256	0,46996	1,31222	0,21248	2,59831	-0,68891	5,59098
CCRO3	1,39321	1,43975	0,73127	0,36650	-0,01583	0,58101	-0,01583	1,46768	-0,42569	2,59975
CMIG4	0,70688	0,74256	0,36650	0,25356	-0,04109	0,29455	0,02508	0,81171	-0,26463	1,34494
CSNA3	0,32427	0,46996	-0,01583	-0,04109	0,67079	0,14690	0,21385	-0,53972	0,43702	0,15393
EMBR3	1,17502	1,31222	0,58101	0,29455	0,14690	0,71667	0,09043	1,51817	-0,21939	2,51005
GGBR4	0,00087	0,21248	-0,01583	0,02508	0,21385	0,09043	0,51314	-0,09650	0,17555	-0,32831
ITSA4	2,49599	2,59831	1,46768	0,81171	-0,53972	1,51817	-0,09650	4,96337	-1,15739	6,28677
PETR4	-0,80192	-0,68891	-0,42569	-0,26463	0,43702	-0,21939	0,17555	-1,15739	0,97604	-1,54011
VALE5	5,41290	5,59098	2,59975	1,34494	0,15393	2,51005	-0,32831	6,28677	-1,54011	11,47598

Source: The author

Table 17: Covariance matrix of 1st semester of 2011

	1º SEMESTRE 2011									
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,79822	0,59042	-0,04161	-0,36680	1,46740	0,11869	0,25946	0,16578	0,78162	1,67941
BBDC4	0,59042	0,65341	0,00337	-0,05542	0,79986	-0,01671	0,15687	0,01190	0,59703	0,88418
CCRO3	-0,04161	0,00337	0,08936	0,17132	-0,19877	-0,04529	-0,04623	-0,04959	-0,15646	-0,32945
CMIG4	-0,36680	-0,05542	0,17132	0,70559	-1,37979	-0,28497	-0,45119	-0,17199	-0,85557	-1,58893
CSNA3	1,46740	0,79986	-0,19877	-1,37979	4,59986	0,75149	1,26877	0,06779	2,76336	4,44762
EMBR3	0,11869	-0,01671	-0,04529	-0,28497	0,75149	0,38654	0,43554	-0,09248	0,63768	0,58966
GGBR4	0,25946	0,15687	-0,04623	-0,45119	1,26877	0,43554	1,00068	-0,11815	1,21954	0,95741
ITSA4	0,16578	0,01190	-0,04959	-0,17199	0,06779	-0,09248	-0,11815	0,76527	-0,31299	0,49460
PETR4	0,78162	0,59703	-0,15646	-0,85557	2,76336	0,63768	1,21954	-0,31299	2,52832	2,47082
VALE5	1,67941	0,88418	-0,32945	-1,58893	4,44762	0,58966	0,95741	0,49460	2,47082	5,55003

Source: The author

Table 18: Covariance matrix of 2nd semester of 2011

	2º SEMESTRE 2011									
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	0,94184	0,32735	0,01359	0,06451	0,79585	0,01943	0,49539	0,83722	0,43950	1,24707
BBDC4	0,32735	1,84759	0,25530	0,73496	0,68553	0,77422	-0,29217	-0,91346	1,23619	1,11320
CCRO3	0,01359	0,25530	0,19628	0,04502	-0,07708	0,19559	-0,26257	-0,61149	-0,09010	-0,15141
CMIG4	0,06451	0,73496	0,04502	0,66208	0,30233	0,13216	0,09352	0,13238	0,83090	0,39456
CSNA3	0,79585	0,68553	-0,07708	0,30233	1,22668	0,31416	0,49297	0,68831	0,98532	2,03871
EMBR3	0,01943	0,77422	0,19559	0,13216	0,31416	0,78836	-0,47663	-1,19702	0,26121	0,74284
GGBR4	0,49539	-0,29217	-0,26257	0,09352	0,49297	-0,47663	1,30919	2,48687	0,34631	0,61319
ITSA4	0,83722	-0,91346	-0,61149	0,13238	0,68831	-1,19702	2,48687	5,25696	0,52274	0,69582
PETR4	0,43950	1,23619	-0,09010	0,83090	0,98532	0,26121	0,34631	0,52274	1,88634	1,84458
VALE5	1,24707	1,11320	-0,15141	0,39456	2,03871	0,74284	0,61319	0,69582	1,84458	4,26115

Source: The author

Table 19: Covariance matrix of 1st semester of 2012

	1º SEMESTRE 2012									
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	6,67060	2,04988	-1,84880	-3,13166	4,06216	-1,14158	-0,66529	2,02945	5,13989	3,17027
BBDC4	2,04988	1,18492	-0,71320	-1,18599	1,23358	-0,66608	-0,29070	0,18684	1,77561	1,08012
CCRO3	-1,84880	-0,71320	1,87426	2,91973	-1,26993	1,40975	1,07824	0,40327	-2,03368	-0,75057
CMIG4	-3,13166	-1,18599	2,91973	4,99766	-1,95533	2,59944	1,81474	0,91131	-3,37632	-0,97113
CSNA3	4,06216	1,23358	-1,26993	-1,95533	2,84367	-0,38658	-0,31901	1,70771	3,30038	2,18217
EMBR3	-1,14158	-0,66608	1,40975	2,59944	-0,38658	1,97924	1,27085	1,54689	-1,41576	-0,13378
GGBR4	-0,66529	-0,29070	1,07824	1,81474	-0,31901	1,27085	1,13765	1,22575	-0,96669	-0,38715
ITSA4	2,02945	0,18684	0,40327	0,91131	1,70771	1,54689	1,22575	3,18869	1,03238	1,01533
PETR4	5,13989	1,77561	-2,03368	-3,37632	3,30038	-1,41576	-0,96669	1,03238	4,51638	2,64995
VALE5	3,17027	1,08012	-0,75057	-0,97113	2,18217	-0,13378	-0,38715	1,01533	2,64995	3,18750

Source: The author

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Table 20: Covariance matrix of 2nd semester of 2012

2º SEMESTRE 2012										
	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGB	ITUB.K	PETR4	VALE5
BBAS3	3,28933	2,17496	0,58054	- 2,36866	0,74883	0,47601	0,34863	0,51171	1,68187	- 0,32253
BBDC4	2,17496	2,80084	0,87850	- 2,20863	0,47513	0,41649	0,30763	- 0,03131	0,40935	0,67816
CCRO3	0,58054	0,87850	0,77622	- 1,93930	- 0,00769	0,28620	0,39916	0,37758	0,01264	0,24862
CMIG4	- 2,36866	- 2,20863	- 1,93930	12,71010	- 0,94578	- 1,41416	- 2,00817	- 2,86852	- 1,24497	- 1,50049
CSNA3	0,74883	0,47513	- 0,00769	- 0,94578	0,55198	0,18891	0,08100	0,23511	0,54562	0,67921
EMBR3	0,47601	0,41649	0,28620	- 1,41416	0,18891	0,36323	0,23326	0,35360	0,30879	0,12144
GGBR4	0,34863	0,30763	0,39916	- 2,00817	0,08100	0,23326	0,47938	0,63210	0,23014	- 0,02634
ITSA4	0,51171	- 0,03131	0,37758	- 2,86852	0,23511	0,35360	0,63210	1,10605	0,65776	- 0,07442
PETR4	1,68187	0,40935	0,01264	- 1,24497	0,54562	0,30879	0,23014	0,65776	1,80913	- 0,81695
VALE5	- 0,32253	0,67816	0,24862	- 1,50049	0,67921	0,12144	- 0,02634	- 0,07442	- 0,81695	3,92926

Source: The author

Based on these matrices (Table 3 to 20), we can infer that the covariances are not stable over time, which would put at risk the maintenance over time of portfolios of investment according to the Markowitz model (1952). It is also important to note that these variabilities further increase in periods of crisis, when a significant increase in covariance is observed among the majority of shares in 2007 and, especially, in 2008.

There is a reasonable peak increase in the 2nd semester of 2007, followed by a slight drop in the 1st semester of 2008. And later, a substantially higher peak – approximately, 700% higher – in the 1st semester of 2008, reaching the maximum covariance of the nine years studied in this work. Thus, in general, the tables present a growing instability in the 1st semester of 2004 until the 2nd semester of 2007, when the summit is reached in 2008. Consecutively, from the 1st semester of 2009 until the 2nd semester of 2012, instabilities are perceived and they still exist, although decreasing.

Another secondary result is the stable relation of CMIG4 and CCRO3 have when compared to the others. Using a simple measurement of dispersion, namely the interval of variation, calculated by the difference between the maximum and minimum in the observation dataset, we found that CMIG4 and CCRO4 had 6,29% and 6,38% of variation respectively. As opposed to the GGB share with a very unstable variation of 45,38%.

In the following chart such a conclusion can be more visually observed.

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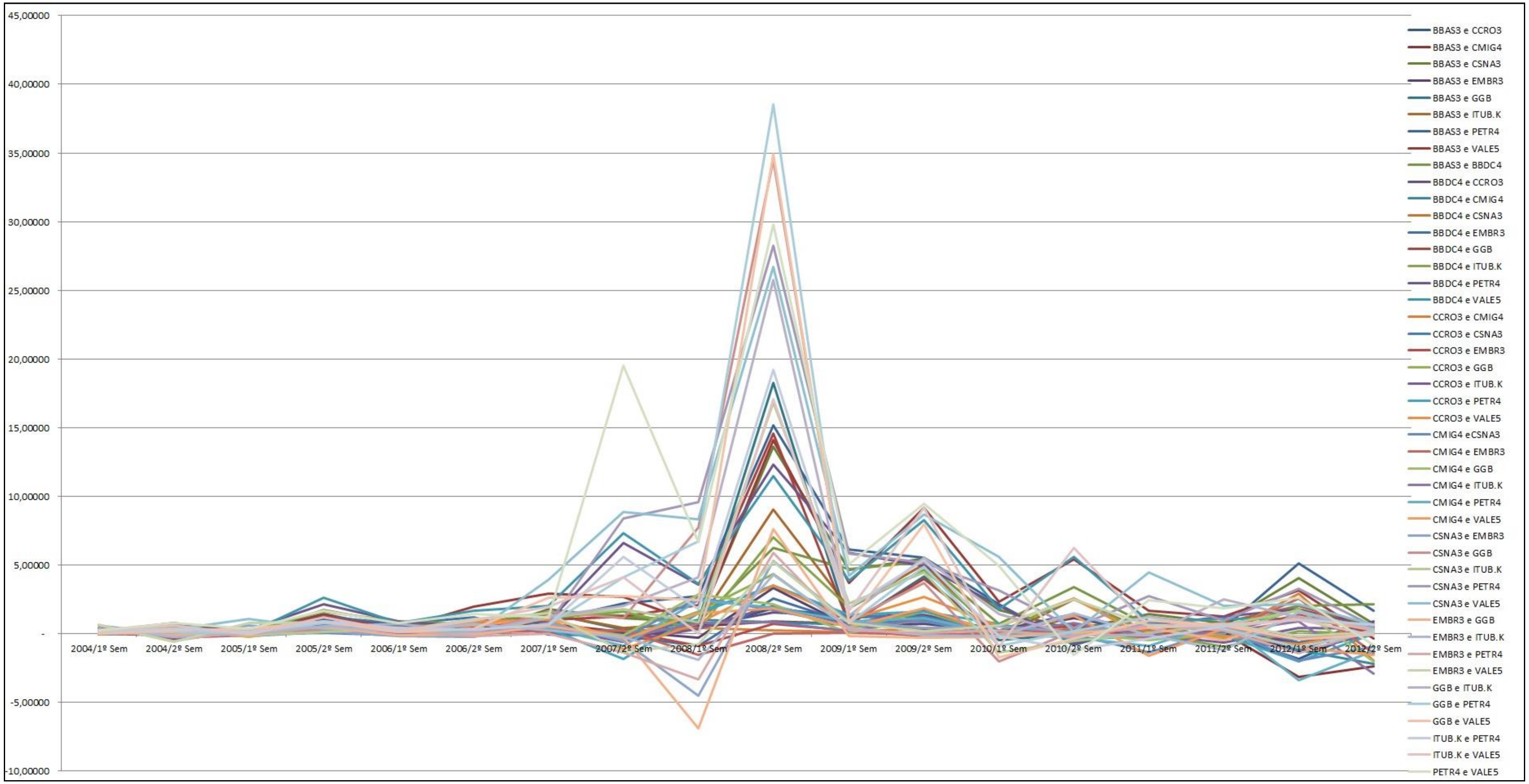

Chart 2: Behavior of covariance between shares returns over time
Source: The author

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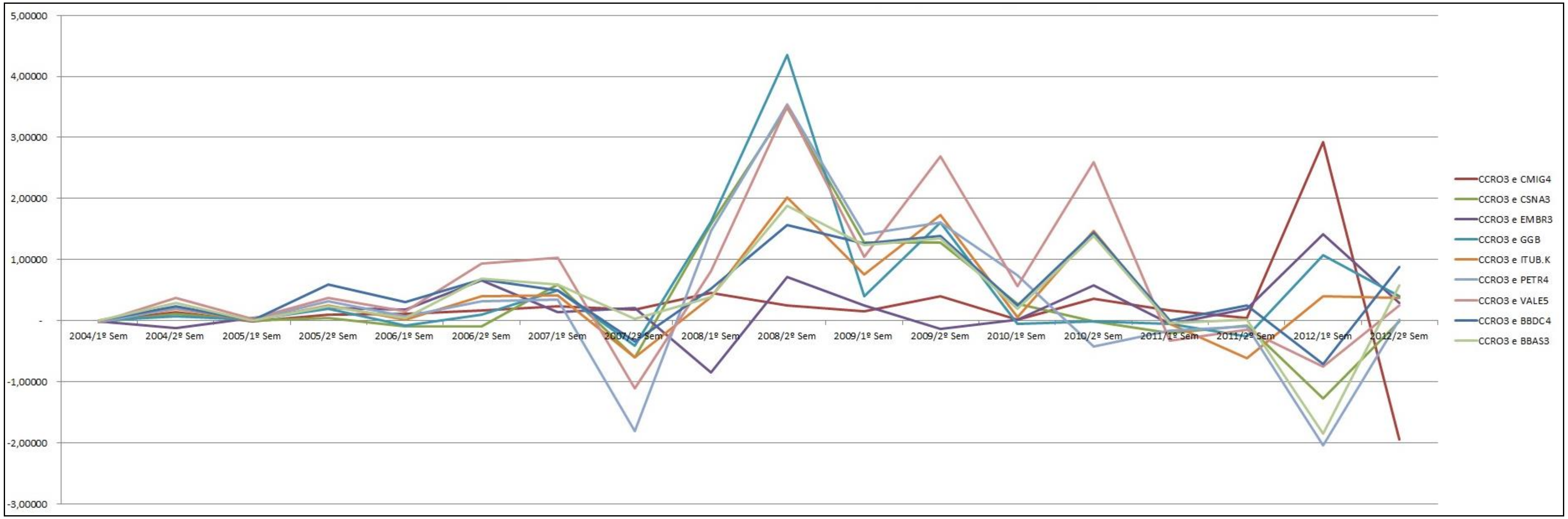

Chart 3: CCRO2 to other assets covariance in time
Source: The author

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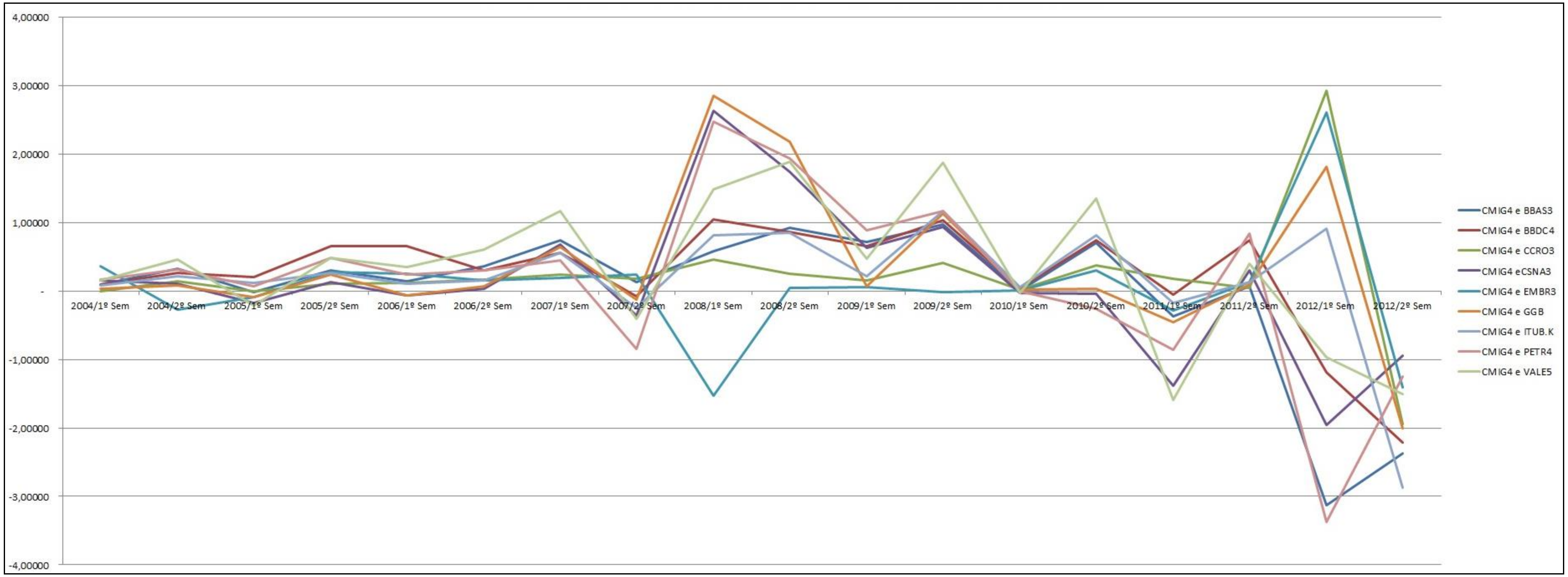

Chart 4: CMIG4 to other assets covariance in time
Source: The author

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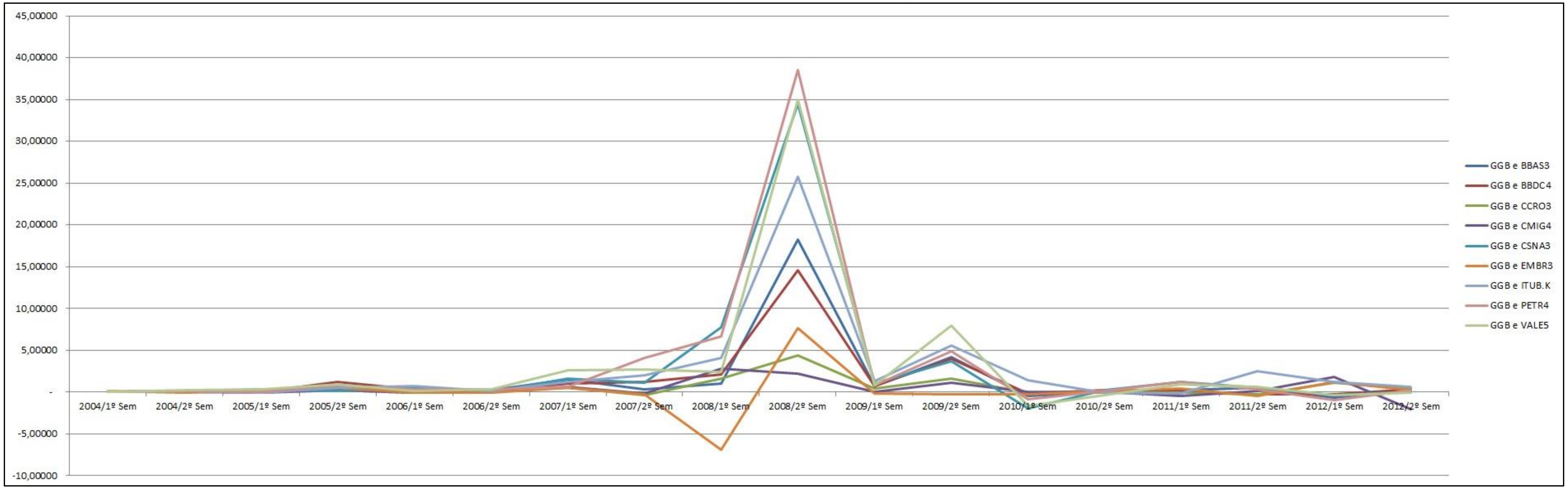

Chart 5: GGB to other assets covariance in time
Source: The author

5. CONSTRUCTING HYPOTHETICAL PORTFOLIO

After concluding that the instability of the covariances between the shares not only exist, but it is also significant, a more particular evaluation of these oscillations is necessary, from the construction of six hypothetical portfolios in relation to the periods of pre-crisis, crisis and post-crisis. These portfolios seek to identify the influence of covariance instability in the maintenance cost of the portfolios.

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We formed two species of hypothetical portfolios for each period aforementioned. Both types of portfolios do not allow short selling, that is, $\sum_{i=1}^{n} Wi = 1$. However, one of them has as a constraint, obtaining a Sharpe index

equivalent to 15% for its formation. The other portfolio genre should present constant asset weight over time, and it starts with a distribution that generates a Sharpe index also equivalent to 15%. In the following we present the data to obtain each portfolio – returns, standard deviations and covariance matrices – as well as the construction of portfolios with the respective weights of each asset.

5.1. Hypothetical portfolio pre-crisis: January/2004 to June/2007

Table 21: Data for individual assets in the pre-crisis period

Assets	μ	σ
BBAS3	253,20%	365,21%
BBDC4	270,12%	499,17%
CCRO3	416,51%	143,23%
CMIG4	129,92%	152,94%
CSNA3	150,99%	179,99%
EMBR3	20,23%	282,37%
GGBR4	410,43%	248,11%
ITUB.K	352,20%	339,76%
PETR4	160,81%	422,94%
VALE5	187,09%	538,25%

Source: The author

Table 22: Covariance between assets in the pre-crisis period

	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGBR4	ITUB.K	PETR4	VALE5
BBAS3	13.34	17.44	5.03	5.26	5.82	8.82	8.77	12.02	14.54	18.95
BBDC4	17.44	24.92	6.73	7.41	7.29	11.78	11.63	16.65	20.62	24.60
CCRO3	5.03	6.73	2.05	2.03	2.20	3.36	3.32	4.67	5.53	7.39
CMIG4	5.26	7.41	2.03	2.34	2.18	3.32	3.47	4.99	6.21	7.50
CSNA3	5.82	7.29	2.20	2.18	3.24	4.14	4.06	5.18	6.21	8.73
EMBR3	8.82	11.78	3.36	3.32	4.14	7.97	6.03	8.02	9.94	12.44
GGBR4	8.77	11.63	3.32	3.47	4.06	6.03	6.16	8.12	9.91	12.35
ITUB.K	12.02	16.65	4.67	4.99	5.18	8.02	8.12	11.54	13.86	17.15
PETR4	14.54	20.62	5.53	6.21	6.21	9.94	9.91	13.86	17.89	20.21
VALE5	18.95	24.60	7.39	7.50	8.73	12.44	12.35	17.15	20.21	28.97

Source: The author

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Table 23: Assets portfolios in the pre-crisis period

	IS = 15%	Constant weights
Assets	**Wi**	
BBAS3	0,00%	0,00%
BBDC4	0,00%	0,00%
CCRO3	0,00%	0,00%
CMIG4	0,00%	0,00%
CSNA3	0,00%	0,00%
EMBR3	83,83%	83,83%
GGBR4	0,00%	0,00%
ITUB.K	0,00%	0,00%
PETR4	4,14%	4,14%
VALE5	12,03%	12,03%
Σwi	100,00%	100,00%
μp	46,12%	46,12%
σp	307,46%	307,46%
IS	15,00%	15,00%

Source: The author

5.2. Crisis hypothetical portfolio: July/2007 to June/2009

Table 24: Data for individual assets in the pre-crisis period

Assets	μ	σ
BBAS3	20,72%	387,24%
BBDC4	25,78%	311,41%
CCRO3	22,57%	101,15%
CMIG4	5,70%	74,18%
CSNA3	86,07%	498,03%
EMBR3	-56,67%	432,06%
GGBR4	35,35%	511,16%
ITUB.K	33,09%	344,08%
PETR4	51,00%	570,36%
VALE5	22,14%	701,37%

Source: The author

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Table 25: Covariance between assets in the pre-crisis period

	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGBR4	ITUB.K	PETR4	VALE5
BBAS3	15,00	11,22	3,19	1,06	12,46	7,31	10,07	10,40	14,87	20,75
BBDC4	11,22	9,70	2,32	0,87	11,38	6,17	9,52	8,65	13,50	18,82
CCRO3	3,19	2,32	1,02	0,48	2,40	0,61	1,89	2,04	2,35	3,33
CMIG4	1,06	0,87	0,48	0,55	0,89	-0,76	1,11	0,72	0,71	0,41
CSNA3	12,46	11,38	2,40	0,89	24,80	1,15	16,62	11,14	26,07	24,47
EMBR3	7,31	6,17	0,61	-0,76	1,15	18,67	7,11	6,63	4,01	19,90
GGBR4	10,07	9,52	1,89	1,11	16,62	7,11	26,13	15,20	18,12	21,90
ITUB.K	10,40	8,65	2,04	0,72	11,14	6,63	15,20	11,84	13,22	18,07
PETR4	14,87	13,50	2,35	0,71	26,07	4,01	18,12	13,22	32,53	31,19
VALE5	20,75	18,82	3,33	0,41	24,47	19,90	21,90	18,07	31,19	49,19

Source: The author

Table 26: Assets portfolios in the crisis period

	IS = 15%	Constant weights
Assets	Wi	
BBAS3	0,00%	0,00%
BBDC4	3,46%	0,00%
CCRO3	20,09%	0,00%
CMIG4	14,89%	0,00%
CSNA3	28,16%	0,00%
EMBR3	0,00%	83,83%
GGBR4	9,22%	0,00%
ITUB.K	13,46%	0,00%
PETR4	10,72%	4,14%
VALE5	0,00%	12,03%
Σwi	100,00%	100,00%
μp	43,69%	-42,73%
σp	291,27%	429,97%
IS	15,00%	-9,94%

Source: The author

5.3. Hypothetical portfolio post-crisis: July/2009 to December/2012

Table 27: Data for individual assets in the post-crisis period

Assets	μ	σ
BBAS3	-4,74%	214,79%
BBDC4	-24,48%	225,07%
CCRO3	-118,61%	296,50%
CMIG4	-47,94%	358,54%
CSNA3	50,82%	527,22%
EMBR3	-56,65%	174,90%
GGBR4	47,29%	289,20%
ITUB.K	31,06%	285,78%
PETR4	42,40%	360,04%
VALE5	-8,45%	300,85%

Source: The author

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Table 28: Covariance between assets in the post-crisis period

	BBAS3	BBDC4	CCRO3	CMIG4	CSNA3	EMBR3	GGBR4	ITUB.K	PETR4	VALE5
BBAS3	4,61	1,23	-2,49	-3,74	7,24	-0,54	2,65	3,65	3,70	4,23
BBDC4	1,23	5,07	4,64	3,48	-4,36	2,61	-2,67	-2,06	-2,73	0,82
CCRO3	-2,49	4,64	8,79	8,23	-12,63	4,07	-5,79	-5,54	-6,77	-2,95
CMIG4	-3,74	3,48	8,23	12,86	-13,52	4,38	-6,81	-5,61	-7,55	-3,43
CSNA3	7,24	-4,36	-12,63	-13,52	27,80	-4,83	13,09	10,35	15,87	8,90
EMBR3	-0,54	2,61	4,07	4,38	-4,83	3,06	-2,58	-1,42	-3,26	0,25
GGBR4	2,65	-2,67	-5,79	-6,81	13,09	-2,58	8,36	5,61	8,19	2,69
ITUB.K	3,65	-2,06	-5,54	-5,61	10,35	-1,42	5,61	8,17	4,75	5,10
PETR4	3,70	-2,73	-6,77	-7,55	15,87	-3,26	8,19	4,75	12,96	4,16
VALE5	4,23	0,82	-2,95	-3,43	8,90	0,25	2,69	5,10	4,16	9,05

Source: The author

Table 29: Assets portfolios in the post-crisis period

	IS = 15%	Constant weights
Assets	**Wi**	
BBAS3	0.00%	0.00%
BBDC4	0.00%	0.00%
CCRO3	0.00%	0.00%
CMIG4	0.00%	0.00%
CSNA3	0.34%	0.00%
EMBR3	0.00%	83.83%
GGBR4	39.48%	0.00%
ITUB.K	29.28%	0.00%
PETR4	30.90%	4.14%
VALE5	0.00%	12.03%
Σwi	100.00%	100.00%
μp	41.04%	-46.75%
σp	273.60%	147.23%
IS	15.00%	-31.75%

Source: The author

Where:

- μ = return of a given asset in the corresponding period;
- σ = standard deviation of a given asset in the corresponding period;
- SI = Sharpe index;
- Σwi = total sum of assets weights;
- μ_p = total expected return of the portfolio;
- σ_p = total standard deviation of a portfolio.

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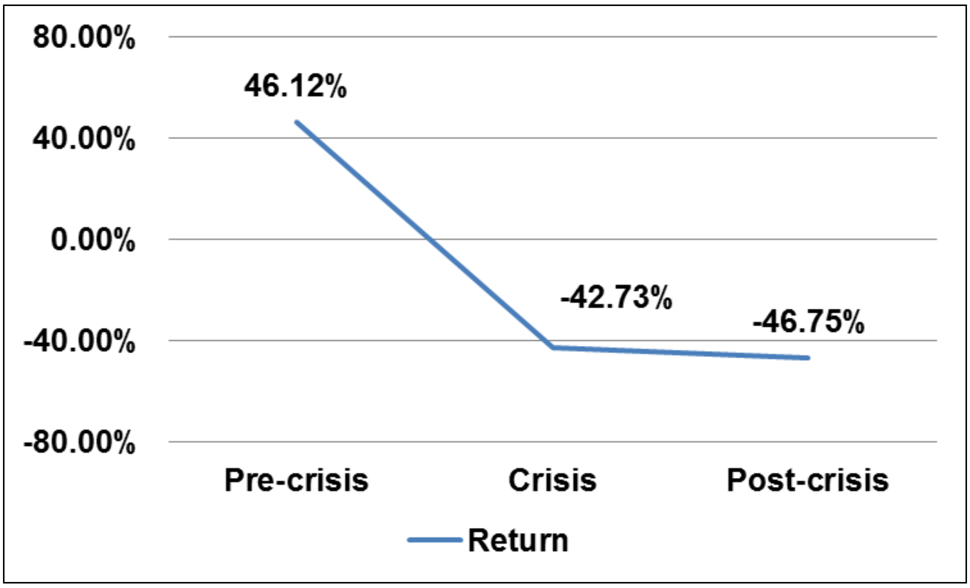
The preferences of return and risk of an investor are one of the most important factors to be considered in assembling portfolios, as seen in the theory. From this analysis it is evident that in order to maintain such preferences, in the case of a rate of beyond 15% of return for each additional unit of risk, it is necessary to change periodically the assets weights in the hypothetical portfolio. If the investor does not recalculate the assets weights of their portfolio as shown by the type of hypothetical portfolio of constant weights, their preference regarding return and risk expected is not met over time. Nevertheless, if the investor wishes to rescue the application in times of crisis or post-crisis, they will have a loss of -42.73% or -46.75%, respectively, of the initial investment made in January 2004 (pre-crisis). The behavior of returns and risks during the studied period can be verified according to the following charts.



Charts 3: Behavior of portfolio returns of constant weights over time
Source: The author

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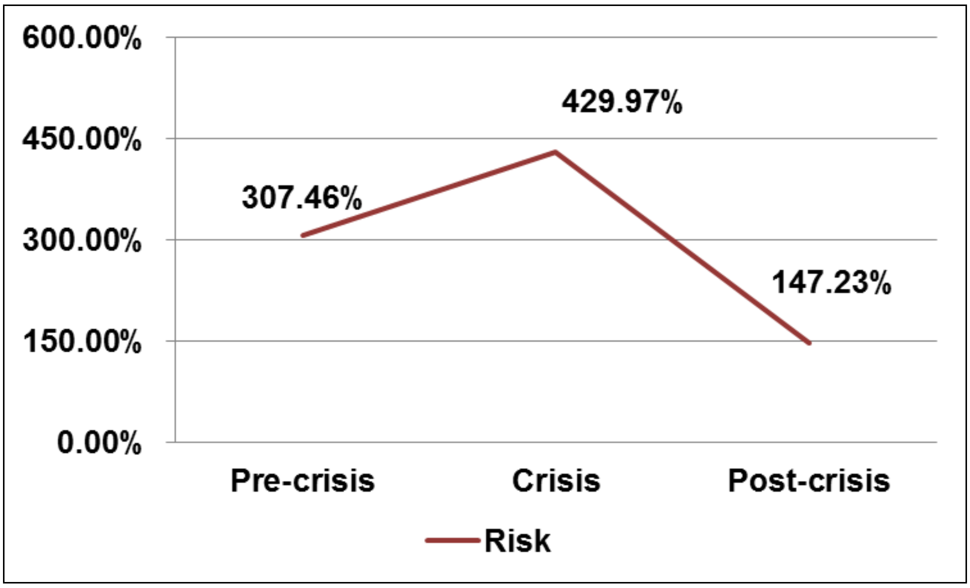

Chart 4: Behavior of portfolios risk of constant weights over time
Source: The author

It is interesting to note that the Petrobras share remains in the three hypothetical portfolios whose premise is a constant Sharpe index equal to 15%.

6. CONCLUSION

As we could observe in the theoretical foundation, the correlation index is a measure resulting from the ratio of the covariance and standard deviations of the analyzed elements. Thus, keeping everything else constant, as their covariance changes, the correlation between them also changes. Consequently, knowing that the risk-return curve has its curvature defined by the correlation itself, this curvature will depend on the changes undergone by the covariance statistics.

According to the charts we can clearly observe that the covariances, originated from the returns of the companies, are not stable over time, and in times of crisis they vary even more. These changes reflect in the risk-return curves, so to modify the possible sets of portfolios to be assembled and, therefore, the asset allocation within these portfolios. This observation is properly shown on chart 1.

This means that the portfolios assembly according to the Markowitz model works only for a period – which is lower or higher in accordance to the economic turmoil which oscillates the covariance between the returns of the companies – the portfolio must be constantly recomposed. In other words, given a specific set of shares, the holding of each share must be periodically recalculated, as demonstrated, in order to always adequate the preferences of a particular investor.

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As demonstrated in this research, share weights will change more in the portfolio in times of crisis, in which the covariances between companies change substantially.

Periodically recalculate the holding of each asset of the investment portfolio is a possible solution to the instability problem of covariance over time. However, it might increase the maintenance costs of portfolio which will be as costly as larger are the covariance volatilities. Another interesting solution to be explored in future studies is by identifying portfolios according to the degree of stability of their covariances which could be measured most precisely with the aid of a statistical hypothesis test.

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