REDIMAT
Journal of Research in Mathematics Education

Hipatia Press
www.hipatiapress.com

Instructions for authors, subscriptions and further details:

http://redimat.hipatiapress.com

Investigating Pedagogical Content Knowledge-in-Action

Rukiye Didem Taylan[1] and João Pedro da Ponte[2]

1) MEF University, Turkey
2) Instituto de Educação, Universidade de Lisboa, Portugal

Date of publication: October 24th, 2016
Edition period: October 2016-February 2017

To cite this article: Taylan, R.D., & da Ponte, J.P. (2016). Investigating pedagogical content knowledge-in-action. *REDIMAT*, *5*(3), 212-234. doi: 10.4471/redimat.2016.2227

To link this article: http://dx.doi.org/10.4471/redimat.2016.2227

PLEASE SCROLL DOWN FOR ARTICLE

REDIMAT, Vol. 5 No. 3 October 2016 pp. 212-234

Investigating Pedagogical Content Knowledge-in-Action

Rukiye Didem Taylan
MEF University

João Pedro da Ponte
Universidade de Lisboa

(*Received: 25 August 2015; Accepted: 12 October 2016; Published: 24 October 2016)*

Abstract

This article investigates the pedagogical content knowledge (PCK) of a teacher educator who teaches 5th-grade mathematics in a school in the context of a university-school partnership project. PCK is analyzed in a qualitative way through video-taped classroom episodes with focus on interactions between the teacher and the students as well as the teacher's reflections and pedagogical reasoning on the interactions. The analyses indicate examples of PCK development during teaching, especially refinement in the domain of knowledge of instructional strategies and representations. This knowledge improved as a result of reflection on student questioning and analysis of students' misconceptions. Different roles of being teacher, teacher educator, and researcher afforded opportunities to gain insights on how to develop knowledge required for teaching and analyze it in order to facilitate future teachers' learning.

Keywords: Academic teacher-researcher, pedagogical content knowledge, knowledge of students, fractions

2016 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2016.2227

REDIMAT, Vol. 5 No. 3 Octubre 2016 pp. 212-234

Investigando Conocimiento Pedagógico del Contenido-en-Acción

Rukiye Didem Taylan
MEF University

João Pedro da Ponte
Universidade de Lisboa

(*Recibido: 25 Agosto 2016; Aceptado: 12 Octubre 2016; Publicado: 24 Octubre 2016)*

Resumen

Este artículo investiga el conocimiento pedagógico del contenido (PCK) de una formadora de maestros que enseña matemáticas de quinto grado en una escuela en el contexto de un proyecto de colaboración universidad-escuela. El PCK se analiza de forma cualitativa a través de episodios de aula grabados en vídeo con énfasis en las interacciones entre la maestra y los estudiantes así como reflexiones de la maestra y su razonamiento pedagógico sobre las interacciones. Los análisis muestran ejemplos del desarrollo del PCK durante la enseñanza, sobretodo de refinamiento en el dominio de conocimiento de estrategias y representaciones instruccionales. Este conocimiento mejoró como resultado de la reflexión sobre el cuestionamiento de los estudiantes y análisis de sus conceptos erróneos. Diferentes roles de ser maestro, formador e investigador ofrecen oportunidades para obtener ideas sobre cómo desarrollar el conocimiento necesario para la enseñanza y analizarlo con el fin de facilitar el aprendizaje de los futuros docentes.

Palabras clave: Profesor-investigador académico, conocimiento pedagógico del contenido, conocimiento dos estudiantes, fracciones

2016 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2016.2227

Hipatia Press
www.hipatiapress.com

An important change in teacher preparation occurred with a growing focus on school-based practices rather than relying solely on theoretical knowledge (Grossman, Hammerness, & McDonald, 2009). Another shift that is taking place is the role of one's own teaching experiences in becoming a teacher educator (Korthagen & Lunanberg, 2004; Koster et al., 2008). Like in other professions, reflection and analysis of one's own practices constitute an important part of professional development experiences for teacher educators (Association of Teacher Educators, 2008; Loughran, Berry, & Mulhall, 2012; Schön, 1996). Although studies investigating the nature and extent of teacher educators' qualifications, knowledge and professional learning have been growing in numbers, they mostly focus on teacher educators' learning in teaching at university or facilitating professional development programs for teachers (Abell et al., 2009; Demirdöğen, Aydın, & Tarkın, 2015; Faikhamta & Clarke, 2013). Özcan (2013) recommended that teacher educators go back to teaching in K-12 schools every few years in order to renew their teaching experience as part of their professional development. With growing focus on contextual aspects of professional knowledge, it is important that teacher educators experience teaching in similar contexts with that of pre and in-service teachers they work with. Just as teacher learning is considered life learning and professional development is considered essential for teachers' growth, the field needs to consider professional development opportunities for teacher educators, which may strengthen teacher education programs and facilitate teacher candidates' learning in better ways.

This article focuses on a mathematics teacher educator's professional learning from her own teaching experience in a K-12 setting by using the widely acknowledged theoretical construct of pedagogical content knowledge (PCK). Although PCK is composed of different components that are closely intertwined with each other, van Driel, Verloop, and de Vos (1998) argue that the components Knowledge of Students (KS) and Knowledge of Instructional Strategies and Representations (KISR) are central to the construct, helping teachers make decisions in the complex classroom environment as they interact with students (Alonzo, Kobarg, & Seidel, 2012; Borko, Roberts, & Shavelson, 2008). Our study addresses this relationship between a teacher's PCK (particularly KS and KISR) and her interaction with students during teaching.

The following research question guides this study: How did interaction with students influence a teacher researcher (TR) knowledge of students and knowledge of instructional strategies and representations as evident in observations of and reflections on her teaching of fractions over a sequence of four classroom episodes? Investigating how interactions with students may transform a teacher's PCK in detailed ways will help the field to understand teachers' decision-making processes in the moment of teaching. With a better understanding of such processes, teacher education contexts can potentially provide teacher educators with similar learning experiences, which can ultimately help them to make better instructional decisions and lead to improved prospective teachers' learning.

Theoretical Framework and Literature Review

PCK is one of the most frequently used theoretical constructs in describing knowledge needed for teaching. Shulman (1987) described it as "the special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8). PCK is essentially a practical knowledge, shaped by context:

> PCK is dynamic in so far as teaching practice and reflection-action allow the teachers to reconsider their knowledge, modifying or reaffirming part of the same. It only becomes visible through personal involvement, through reflection and observation, and requires teaching practice in the subject within a specific classroom context. (Blanco, 2004, pp. 33-34)

This is similar to the way Mason and Spence (1999) conceptualized teacher knowledge: "A snapshot of a state of knowing that is in constant flux according to prevailing personal and social conditions" (p. 135). In this study we investigate PCK from a situated perspective to provide insights for both teachers' and teacher educators' professional learning through experience.

A review of 60 studies on PCK in mathematics education revealed that none of the studies focused on teacher educators (Depaepe, Verschaffel, & Kelchtermans, 2013). There is a need to understand how different contexts afford opportunities for teacher educators' professional learning. How teacher educators can learn professionally in the context of K-12 teaching is not sufficiently known. In this manner, such experiences can be transferred to research and provide a road map for other teachers' learning.

PCK is often difficult to articulate and tacit to most teachers (Loughran, Berry, & Mulhall, 2012). Although there are many studies on the construct of PCK across teaching of different subject matter, only few of them focused on PCK-in action through classroom observations (Chan & Yung, 2015; Park & Oliver, 2008). How PCK may develop during the moment of teaching has been overlooked in the field of teacher learning, especially for teachers of mathematics. Since PCK is considered a practical knowledge, investigating PCK by video analysis allows researchers to unpack aspects of PCK that are hard to capture in other ways and help to build a knowledge base for teaching (Janik et al., 2009).

PCK is not only composed of results of educational research but also constructed in the moment of teaching in a way to encompass teachers' intuitive ways of understanding their students and interacting with them (Shulman, 1987). As teachers reflect on and analyze results of their decisions, they reorganize and expand on their knowledge to make better decisions in similar future situations. In their investigation of different science teachers' PCK by focusing on the classroom observations, Park and Oliver (2008) found that "PCK as knowledge-in-action became salient in situations where a teacher encountered an unexpectedly challenging moment in a given teaching circumstance" (p. 268). PCK was not only demonstrated as knowledge-in-action (how teachers demonstrated their knowledge in classroom practices) but also as knowledge-on-action (demonstrations of teacher knowledge in reflections and reports on teaching practices). When teachers reflected on their actions, they were likely to modify their PCK. Knowledge-in-action and knowledge-on-action aspects of PCK influenced each other through reflection both during class time and during teacher reflections (Park & Oliver, 2008). Just as PCK of a teacher influenced her instructional actions, teachers' instructional actions also influenced teachers' PCK, leading to a deeper and a more sophisticated forms of PCK. Another major factor that shaped PCK was students' misconceptions.

A growing number of scholars have speculated that PCK may develop during teaching (Chan & Yung, 2015; van Driel, Jong, & Verloop, 2002; Hashweh, 2005). Van Driel et al. (2002) found that teaching experience was a major component which helped to improve pre-service science teachers' PCK, especially in the area of knowledge of learners. Specifically, they found that student questions, correcting student written answers and analyzing student responses and observations of students shared by mentors or peers helped pre-service teachers to improve their knowledge of learners in science. Chan and Yung (2015) identified three steps for PCK on site development in their study of analyzing four experienced science teachers' practice: stimulus, integration and response. The stimuli that triggered reconstruction of PCK were unanticipated student questions and misconceptions during the moment of teaching. The analysis of observations and reflections on teaching revealed that teachers' subject matter knowledge of the topic, their general pedagogical knowledge and knowledge of learners were the main factors that shaped the integration process and led to developing of new instructional strategies, hence the reconstruction of PCK.

An investigation by Alonzo, Kobarg and Seidel (2012) was another rare study which focused on PCK-in-action by using observations as evidence of teacher PCK. Alonzo et al. investigated to what extent the teachers had flexible, rich use of content during instruction and demonstrated learner-centered instruction. The authors argued that flexible use of content allowed "teachers to listen and respond to specific ideas from their own students" (p. 1232), which revealed spontaneous forms of PCK development. Such complex forms of PCK may be hard to assess by paper and pencil tests or only through reflections.

In this study, we investigate a teacher-researcher PCK by holding the assumption that it is not static but dependent on the topic and the interaction with students. This article builds on definitions by Park and Oliver (2008) and Lannin et al. (2013) in portraying transformation of PCK during teaching. Park and Oliver indicate that "[to] employ PCK effectively, teachers must have knowledge about what students know about a topic and areas of likely difficulty" (p. 266). And they define knowledge of students (KS) as including "knowledge of students' conceptions of particular topics, learning difficulties, motivation, and diversity in ability, learning style, interest, developmental level, and need" (Park and Oliver, 2008, p. 266). And Lannin et al. (2013) define knowledge of instructional strategies and representations (KISR) as knowing "how to organize instruction, specific actions that the teacher can take during instruction, activities to use for specific mathematical content, what materials are needed for instruction, what representations are best for particular content" (p. 411)

Methodology

Context

This study is part of a larger research program that investigated the collaboration between a faculty of education and a K-12 school in a large city in Turkey. The main focus of the larger project was to build bridges between wisdom gained from school experience and theoretical knowledge developed in the university in order to improve teacher education as well as enhancing the quality of partner schools (Özcan, 2013).

During the year 2014-2015, the first author taught mathematics in a 5th grade classroom as part of a research team within the project 'University within School'. The professional development experience through weekly planning and reflection meetings and enhanced student learning in fractions as a result of the project were reported elsewhere (Aydın et al., 2016; Tunç-Pekkan et al., 2016). In teaching fractions, developing conceptual understanding of fractions on the number line, one of the most difficult representations for students (Tunc-Pekkan, 2015), was an important instructional goal. In this article, the focus is on one's teaching through a researcher perspective and analysis of a sequence of lessons on fractions on the number lines based on the construct of PCK-in-action.

The classroom consisted of 33 students with an average age of 11, a majority of who came from low socioeconomic levels and large households. Verbal and physical bullying, and incidences of violence were quite common among students, not only at the classroom but also at the school. Considering this challenging context, the TR was aware of the importance of implementing socio mathematical norms within the classroom in the beginning of the semester. The teacher aimed to create a learning environment for students that is recommended by research which may be summarized in the following way:

> Developing mathematical understanding requires that students have the opportunity to present problem solutions, make conjectures, talk about a variety of mathematical representations, explain their solution processes, prove why solutions work, and make explicit generalizations. (Franke, Kazemi & Battey, 2007, p. 230)

Raised in an authoritarian Turkish culture (Yılmaz, 2007), the students had a hard time in assuming individual responsibility in both their own and their peers' learning, providing respectful and meaningful contributions to the classroom learning environment and engaging in meaningful discussions. The students in general found challenging the type of teaching that they were receiving in these classes because they were held higher standards of learning and they were asked to provide justification for their answers. However, they also found it enjoyable due to different types of activities, as well as handouts and materials that are not common in public schools. In a questionnaire that aimed to understand students' view of teaching in the mathematics lessons compared to teaching in previous years, they answered in the following ways:

> Our previous teacher agreed with our correct answers. (Aydın et al., 2016; Tunç-Pekkan et al., 2016). teacher always asks 'why?' "Math class is more challenging compared to last year but also more enjoyable."

Teaching Fractions

Four teacher-researchers (TRs) planned and taught lessons based on the Fraction Scheme Theory (Steffe & Olive, 2010) following the 5th-grade mathematics curriculum (Ministry of National Education, 2013) by addressing unit, simple, compound (improper), and equivalent fractions; location and ordering fractions on the number line; and addition and subtraction of fractions. Specifically, instruction focused on understanding how to operate on fractions as measurement units. The research team designed and taught learning activities using virtual manipulatives (Java Bars) and Cuisenaire rods, both of which have a potential to help students understand measurement interpretation of fractions (Kieran, 1976) in more effective ways.

During the project, the students had opportunities of using different types of manipulatives to understand fractions on the number line (Figures 1 and 2). The idea of equipartitioning of a whole unit by using Java Bars (Figure 1) was introduced in the beginning of the fractions unit. Later, instructors facilitated concrete meaning of the number line by using the Cuisenaire rods (Figure 2). The students showed different fractions by using the manipulatives and named different colored rods by answering questions: "if yellow rod is called 1 unit, what would we call the white rods?" The students worked on drawing their own number lines using the Cuisenaire rods.

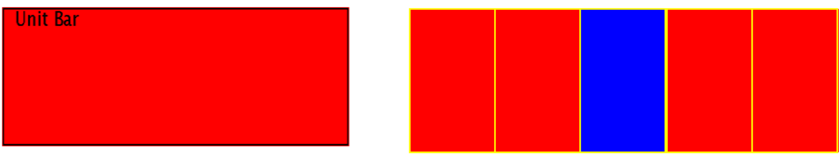

Figure 1. Java Bars.

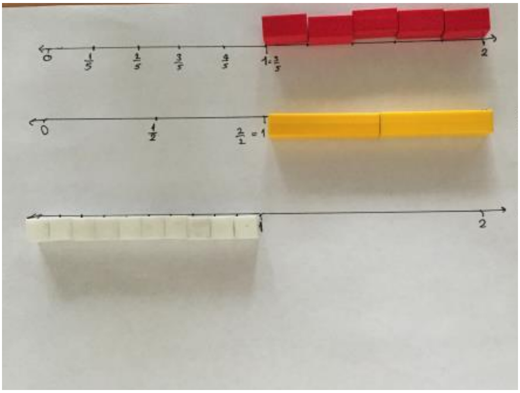

Figure 2. Cuisenaire rods.

The focus of analysis in this article are four video-taped lessons on the number line taught by a TR. In these lessons, the students worked on several number line tasks without using the manipulatives: locating different fractions including compound fractions on the number line on squared papers, and understanding the unit and whole fraction meanings as well as equivalent fractions. Because the TR did not have prior experience as a teacher, her knowledge of students and instructional strategies in teaching number line improved in the moment of her teaching. The challenging nature of teaching and learning fractions on the number line together with the instructor's intention of focusing on providing justifications of mathematical arguments in class created an interesting series of episodes, which proved worthwhile to investigate.

Self-Study of the TR

This study utilized self-study methods. A large number of teacher educators have investigated their own practices by using the self-study method which allows them to interpret their work, have a record of their own professional development and advance the field by taking into account experiences of individuals (Loughran, Mulhall, & Berry, 2004). Self-study also allows one to consider consistencies between one's beliefs and practices. In this way, self-studies facilitate a better alignment between theory and practice (Loughran, 2007).

The TR worked as a teacher educator at a private university in a large city in Turkey. As part of the project "University within School", she taught mathematics in a 5th-grade class during the year 2014-2015 for ten months. Although she held a teaching certification and completed student teaching during undergraduate studies, she continued with graduate studies abroad instead of pursuing a career as a teacher. She was involved in research studies where she conducted and analyzed a large number of clinical interviews with children as well as longitudinal classroom observations and teacher interviews. She also had experience in being a teaching assistant for teacher education classes during her doctoral studies. Although she was immersed in reading and conducting research in teacher education, specifically teachers' skills of noticing student thinking, teacher knowledge, and professional development, she had limited experience as a teacher and considered herself as a novice teacher and teacher educator at the time of the study.

Data Sources

The analysis provided in this article focused on a sequence of four videotaped lessons on the topic of fractions that took place towards the end of the second semester and teacher reflections on selected episodes.

As part of the larger research project, at least one class each week during fractions instruction was recorded on video such that instructor practices could be shared and discussed among the research team. Towards the end of teaching fractions, the TR videotaped four lessons in a sequence in order to gain a deeper understanding of nature of mathematical discussions that took place in teaching number lines. The rationale for focusing on these four lessons was: 1) The concept of fractions on the number line was at the heart of the intervention, 2) A series of four lessons helped to assess both student understanding and teaching in a continuous way, 3) These lessons were not discussed in depth in weekly meetings with the research team (that is, transformation of PCK was not a result of reflection with members of the research team but only through teaching and the TR's own reflection on teaching), 4) PCK transformations during the classroom activities was evident across the four lessons. The video observations were transcribed verbatim in Turkish and translated to English by the first author. The episodes which revealed transformation of PCK were selected for in-depth analysis. Both authors agreed on the selection of significant episodes.

In addition to video observations and transcriptions, it was important to provide insights about how the TR made instructional decisions during teaching and learning. The reflections provided an account of the significant moment identified in the video observation and contributed to the transformation in PCK of the TR. The reflection focused on the TR's intention for the lesson, the big mathematical ideas, anticipation about lesson flow and student understanding and what happened during teaching. Such reflections are written in the first person singular ("I") because they stemmed from the first author's personal experience while the rest of the article is written in plural form ("we" or "authors") since it was the joint work of both authors. Like every study, there is a limitation involved in the process of data collection and analysis that the reflections may involve the TR's post-hoc rationalizations about teaching episodes. In order to address this limitation, the reflections are triangulated with observations and analyzed by also the second author.

Data Analysis

Teaching segments in which there was an evidence of PCK were selected through transcriptions of video observations. Although PCK was identified mostly through observation, teacher reflections on the selected segments and her pedagogical reasoning was also taken into account as supplementary data sources. Observations and reflections were used to ensure trustworthiness of the study. In order to ensure external validation, another researcher (the second author) observed and identified segments where transformation/ refinement of PCK was evident. After agreeing upon the initial identification of the segments, the authors conducted an in depth analysis of each segment based on the framework by Chan and Yung (2015) and utilizing definitions of KS and KISR from Lannin et al. (2013) and Park & Oliver (2008). The results focused on segments that are rich in terms of portraying relationship between transformation of PCK and interactions with students.

Results

A long vignette divided in two parts provides a compelling case of transformation of KS and KISR as a result of interaction with students.

Part 1

Vignette

The TR asked the students to draw a number line where 0 and 1/5 are separated by 4 squares and locate 1 by paying attention to precision (counting the number of squares on their notebooks) (Fig. 3). Although about one third of the class understood how fractions may be represented on number lines, the rest struggled with understanding how number lines may be used to represent equipartitioning of a whole, iterating unit fraction to identifying one whole and equal intervals on a number line. The TR walked around classroom and saw that most students had problems in understanding the task. She asked a high achieving student in class, Nadia to explain her reasoning in front of the board.

Figure 3. Representation related to the task "Locate 1 (one whole) on the number line by paying attention to precision."

The student, Nadia, was able to correctly draw the number line and explained her thinking, but the teacher was not happy with her explanation:

> Teacher: Where can I locate 1 precisely? Who can tell me? When I say 1, I mean one whole?
> A lot of students raised hands. Nadia came to the board.
> Teacher: Please pay attention to drawing equal intervals like you have in your notebook. Like I did imitating notebook squares. Nadia is doing it on the board and I am gonna ask you guys if she did it correctly. Nadia please explain. I am gonna ask you guys to see if her answer makes sense.
> Nadia: Teacher you told us that we go 4 by 4.
> Teacher: Why are you going 4 by 4? I don't remember saying something like that. But okay why do you go 4 by 4?
> Nadia: … (silent).

Teacher: Why do we go like that? (Other students raising their hands). I am going to let Nadia explain and then I am going to ask you guys.
Nadia: This is a 20 pieces something. We will go 5 times.
Teacher: Why 20? Where did 20 come from?
Nadia: Let me do it on the board and then I will tell you.
Teacher: Okay.
Nadia counted 4 squares and located 2/5, 3/5, 4/5 and 5/5 correctly. She knew 5/5 was equal to one whole.
Nadia: Now we have 1 whole. It is 24 squares sorry 20 squares.
Nadia: Where did 20 squares come from? I have 4 squares between 0 and 1/5.
Nadia: How is it wrong?
Teacher: I did not say it was wrong. I did not say it is either right or wrong.
Nadia: Teacher you went 4 by 4.
Teacher: I did not go 4 by 4. How did we understand to go like that?
Nadia: You said so.
Teacher: Will you jump out of the window if I tell you to do? (Laughing)
Nadia: But teacher…
Teacher: Okay very well but why? You have to explain why. Okay let's discuss with class. Thank you Nadia. I do understand what you want to say but who can articulate it clearly to me and give reasoning? Who are participating?

Pedagogical reasoning by TR

Although as a researcher I knew the importance of selecting student ideas, sequencing them and valuing different types of student thinking, as a novice teacher it was challenging for me to apply that in the classroom. Because many students demonstrated fragile understanding and were prone to developing misconceptions, I had the intuitive idea of focusing on the correct student thinking at this time even though I knew as a researcher that incorrect student ideas could offer learning opportunities for all. As I walked around the classroom I decided to choose Nadia, who not only had a correct answer but also made sense of the answer.

I was concerned that in this class many students focused on providing the right answers and not reasons. It appeared to me that they only wanted to gain recognition of the teacher by giving the right answers but they were not worried about mathematical thinking and reasoning. Therefore, I insisted on asking "why?" Although I valued the correct answer, I wanted to emphasize the conceptual understanding that comes with it.

During the discussion, I thought Nadia was not able to explain reasoning behind her answer in front of the board although she was able to articulate her answer in one to one interactions with her colleague at her desk. I thought challenging students to provide reasoning was going to help them in understanding. I did not want to provide reasoning as the teacher myself because I did not want to be the one to "tell" students but help them make sense of the concepts. When I hinted that I may say inadequate things and it was the responsibility of students to interpret them in the best ways ("Will you jump out of the window if I tell you to do?"), it was probably not appropriate in this context for this group of students who struggled to assume responsibility in their own learning.

Reflecting back on this episode, I criticize the way I posed the same questions repeatedly. I had a specific answer in my mind and when the student did not provide it, I was disappointed. My questioning was not effective. Many other students in the class probably became more confused with my questioning. Nadia interpreted my questions as her solution was wrong, which made her quite confused and intimidated. Her thinking was perfectly valid: "Since 1/5 has 4 squares, we go 4 by 4 five times to get the unit. 5 times 4 equals 20. The unit is 20 pieces." I suspect now that most students would probably accept this explanation. What was required in this case was just to paraphrase the explanation and make it clearer to all students. Instead, I wanted to hear from Nadia to emphasize that each 1/5 represented 4 squares and that each interval should be equal. I refrained from providing the conceptual details myself because I thought it would be telling students how to think and I wanted the students to come to this understanding on their own. If I were given another opportunity, I would approach this situation differently as a teacher, rephrasing Nadia's answer because her explanation, in fact, was correct. It was only something different than I was expecting to hear. Correct student thinking may appear in different forms and I should have been able to recognize and build on it as a teacher.

Analysis

This may be an example of failing to notice students' mathematical explanation. The reflection offers an example of refinement/transformation in teacher knowledge, which are insights gained after the classroom interactions. The TR learned she could help students to improve their explanations by paraphrasing their sentences (Franke, Kazemi, & Battey, 2007), which could help with students' conceptual understanding. The TR became aware that questioning by using the same sentence structure or wording was ineffective.

The TR's knowledge was transformed in the sense that the style of challenging students with questioning may be too difficult to understand for students. If she could have elicited student thinking by adding conceptual details on her own during the discussion, it would have been different than telling students what to do procedurally and she would have contributed to their understanding (Lobato, Clarke, & Ellis, 2005). These insights can be considered as examples of refinement or transformation in the knowledge of instructional strategies and representations.

Part 2

Vignette

As the TR was concerned that many students appeared to be confused, she chose another student, Amelia, to provide reasoning for the answer on the board. The TR thought the explanation would be simple because the answer was already on the board.

> Amelia: It is 20 squares (referring to the distance between 0 and 1) because the denominator is 5 and we went 5 by 5.
> Teacher: Did we count 5 by 5s?
> (Other students object that it is not right)
> Teacher: Listen to Amelia and pay attention if she makes a mistake and you guys can correct it.
> Amelia: In order to find 20, I counted 5 by 5. All intervals are gonna be 4 (counting with fingers).
> Teacher: Okay, each interval is 4 squares.
> Students are noisy so teacher asked Amelia to repeat her answer.

Amelia: I counted 5 by 5; 5, 10, 15, 20.
Teacher challenged her: If each gap is 4 squares how come you count by 5's?
The student could not reply.
Teacher asked her to sit down: Let's think about it, let's not get too confused.
Zandra, another student that had a better understanding of fractions was nodding her head "no" as Amelia explained. The TR knew that this student was one of the best students in class in understanding fractions. She decided to choose her to help with the flow of the lesson.
Teacher: Zandra what did you say?
Zandra: My friend used a wrong strategy, instead of 5 she had to use 4. Instead of counting by 4s, she counted by 5s.
Teacher: Zandra repeat please as some people may not have heard you.
Zandra: Well it was a wrong strategy. She had to go by 4s but she counted by 5s. Even though we have same results, using 4 matches the problem situation better.
Teacher: Yes we do not even know whether 20 is our right answer. I don't care about getting 20, I want to understand what I am doing. The most important thing is how we get the answer. Zandra said we have to go by 4s, who is going explain to me more clearly? Erica?
Erica: We started by drawing the intervals in our number line. We knew 5/5 was going to be one whole.
Teacher: Yes 5/5 is one whole. But why did I go 4 by 4?
Erica: Because our number line has to be at equal intervals. That's why we divide it into equal parts.
Teacher: Yes indeed equal intervals. Okay it is a very good point. It is very important to remember the equal intervals on the number line.

Teacher wanted to make a closure on the task:

Nadia was right, because the intervals should be equal she said we would have each interval 4 squares that is the way we started the task. She was right but I wanted to get a better explanation. Did you guys get it now? Why? I went by 4s because all the intervals should be equal. Not only Amelia but most of you have this

> mistake. I think you guys are still counting the marks and that's why you think it is five. Did you count like this? (Teacher demonstrated on the board) We can't count the lines or marks, we focus on the intervals. Why? Do you know how it is like counting the intervals? That interval is like a slice of the cake. Or it is like sharing a piece of a string equally. You can think like that okay.
> At this point the teacher asked directly Amelia: What did you notice Amelia? Did you notice your mistake or something? Is that why you are raising your hand?
> Amelia: If I did each part by 4s, I would get 5 parts and get 20.
> Teacher: Okay I think you got it.

Pedagogical reasoning by TR

When I invited Amelia to explain her thinking, I thought the answer was already on the board and her reasoning could not be misleading. But I was wrong. Amelia provided an explanation that did not represent the problem situation. As I appeared to reject Nadia's explanation, which was based on "4 by 4", Amelia tried to use the same strategy in a different way. Although the reasoning of Amelia was also correct, the wording was not, because the students did not learn yet to use the mathematical language for these notions in a proper way. Her answer of "5 by 5" did not fit the problem situation as each unit fraction represented 4 squares.

I wanted to explore student thinking but I was getting worried that the other students were becoming more and more confused. Reflecting back on the moment, if I had probed the students again I would know more about their thinking but it would also raise the level of confusion and discomfort. I also wondered whether, in addition to difficulties in using mathematical language in a shared way in the classroom, the mistake resulted from the students' struggle in number sense in constructing multiplication sentences.

In this part I intended to make a closure knowing that some students were still struggling. I was aware that I struggled in knowing how to help students. In this challenging moment, I thought of an analogy that I did not use before: that the number line was almost like a string that had to be divided equally, like sharing a piece of cake. Additionally, although I initially believed that Amelia was saying 5 by 5 randomly because she saw 1/5 in the question, later I realized that this student could have another misconception in the task that I had not considered before: that maybe she was counting marks on the number line rather than intervals. It was something I knew from common mistakes and it was what I observed in the beginning of teaching number lines but I was not aware that this misconception could be persistent.

Analysis

Although there could be better ways of making this closure to help the students, this could be regarded as another example of expansion in the knowledge of instructional representations in the moment of teaching. The students had experience of cutting strings of paper equally to demonstrate half and quarter of the fractions in previous classes but the TR understood that most students were not at the desired level of understanding the deeper concepts related to number lines. As this moment was an example of expansion in knowledge of instructional strategies, it was also an example of lack thereof because the teacher still did not know how to approach this difficulty in future teaching. Additionally, the TR initially believed that Amelia gave the answer "5 by 5" randomly because she saw 1/5 in the question. Later she realized that this student could have another misconception in the task that she had not considered before, counting marks on the number line rather than intervals. This moment was an example of expansion and reorganization of knowledge of students. The student seemed to shift her thinking in an improved way as a result of the interaction.

Summary

In summary, the TR gained insights about her teaching and her own PCK by way of reflections. The analysis of refinement in the TR's KISR and KS can be described in the following way:

1) Awareness of the need to use revoicing to help students learn the proper use of mathematical language (as addressed in Franke, Kazemy, & Battey, 2007). The TR seemed to stick to a notion of right or wrong explanation, instead of figuring out the reasoning that is behind an incomplete explanation.
2) Awareness of the need to not increase the level of insecurity of the students by suggesting that their incomplete answers are wrong.
3) Awareness of the need to decide when the classroom discourse becomes too confusing for most students and how to intervene to

> bring all students home to understand the task and a proper way of dealing with it. This is related to an awareness of the dilemma between telling and not telling (Lobato, Clarke, & Ellis, 2005).

Viewed altogether, the TR gained important insights about how to manage a discussion of a task such as this with similar groups of students.

Discussion

Scrutinizing the work of a TR in order to illuminate aspects of transformation of PCK is a way of documenting teacher learning in the complex environment of teaching. The TR had room for development in terms of PCK. Admittedly, an expert teacher could offer different explanations, representations, analogies, etc. in order to help students understand the number line representation. Although there is no single right approach to instructional strategies and knowledge of students, it is important to expand a repertoire and become flexible in adapting the right tools by time.

We argue that being mindful of classroom events during teaching and reflecting on them afterwards as recommended by previous literature, the TR gained insights, which could be considered as development, construction or refinement of KISR. Even when the TR was aware that she could have done better, she made a note to herself about the situation, which had potential to inform future instruction. In line with previous literature, the TR developed understanding of learners' difficulties during teaching. Although her KS was not advanced, it was simply being constructed during teaching and afterwards as she reflected on teaching. Through reflections and observations new insights about student understandings were evident.

The task involved in the illustrative vignette was challenging, as it involved the reconstruction of the unit, the number line representation, and a second "hidden" unit (the "squares") in which 1/5 was divided. The students had to coordinate several pieces of information and find suitable representations to solve the problem. It was not surprising that they had trouble in solving the task and in explaining their reasoning. After all, number line fraction tasks are considered as involving the most difficult representations for students. In retrospect, it is possible that the teacher's naming the number line intervals as "squares" may also have led to misconceptions because the representation squares did not match what students saw on the blackboard and choosing students who struggled for explaining their reasoning might have hindered other students' learning as well.

To reconstruct the unit involved iterating the picture with four "squares" five times as some students indicated. So the response 5 times 4 made sense and was related to the representation in Figure 4.

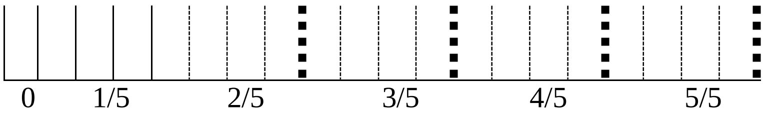

Figure 4. Students' representation of the question in Figure 3.

It was likely that some of the students thought in this way. However, the TR was asking for an explanation in another representation (such as the one in Fig. 3) and the students had trouble in relating their representations to that intended by the teacher. Although the TR's goal was that students would consider the intervals as equal lengths, they may have already considered them as equal on their squared notebooks. The TR kept posing the question "why?" during teaching but it appeared as though she expected to hear some ideas that were important to her. Although her goal was to teach by building on student thinking (student-centered), ultimately instruction appeared to follow a teacher-centered approach. For instance, in the first episode student Nadia was giving the right answer by saying "we go by 4 squares" but TR wanted her to explain for each unit fraction there was an equal interval. The answer she looked for did not come until late in discussion and it disrupted the flow of lesson. It was interesting that another student perceived it as going 5 by 5. In both cases the result would be 20 squares after 0 (or 20 equal length intervals) but understanding the question required them to have 5 intervals of length 4 squares and not 4 intervals of length 5 squares. Some of the students had such difficulties in number sense that maybe it was also a problem with their multiplication skills.

When students did not offer the ideas the TR was looking for (either because they really did not understand or made different kinds of explanations), she felt uncomfortable and was unsure of what to do next in her interaction with students. Being familiar with research made the TR become aware of her weaknesses and challenges as a teacher. One of the key contributions of this article is that different roles of being a teacher, teacher educator, and a researcher afforded opportunities to gain insights on how to develop/improve complex knowledge required during the moment of teaching, and analyze it such that similar learning experiences can be organized in order to facilitate future teachers' learning from teaching.

Implications

This study has implications for both teacher educators' and teachers' learning. The analysis of in-the-moment teaching provided opportunities of getting inside the head of the teacher. The analysis suggested that when the teacher reflects on her teaching, even when she was not aware of something during teaching, there is a chance that she may become aware of that by reflecting on teaching afterwards.

There seemed to be both opportunities and constraints of choosing the subject as a TR. The TR was able to provide connections between research and her own teaching. On the other hand, she was more likely to act based on theoretical knowledge instead of what she experienced in class on during the moment. This might be because she was a novice teacher and she viewed herself as a researcher instead of a teacher first. The transformation in her knowledge of students and instructional strategies suggests that teacher educators may benefit from renewing their teaching experience.

Investigating one's own practices with the help of an experienced researcher allowed providing insights on the link between theory and practice in teacher learning. Scrutinizing teaching practices, identifying strengths and weaknesses served as a professional development experience, which can help to improve future practices both as teacher and teacher educator. When teacher educators know about the context and type of interactions that help transform teachers' PCK during teaching, there is a better potential for designing similar experiences to help with teachers' learning and gaining new insights about their own teaching. Although PCK is prevalent in mathematics education research, more studies are needed on how PCK is enacted and translated during the moment of teaching as a result of interactions in different contexts. Future research may find it helpful to analyze how teachers at different levels in their professional career demonstrate this complex knowledge and what one can learn from especially expert teachers.

There are also implications for teacher education policies resulting from this case study of a teacher educator. Teacher educators should be given opportunities to gain and revisit teaching experience in a variety of contexts so that they can refresh their professional knowledge, identify gaps and commonalities between theory and practice and design future research based on their experiences. This has an important potential to help integrate theory and practice in teacher learning in the field of mathematics education.

References

Abell, S. K., Park-Rogers, M. A., Hanuscin, D. L., Lee, M. H., & Gagnon, M.J. (2009). Preparing the next generation of science teacher educators: A model for developing PCK for teaching science teachers. *Journal of Science Teacher Education*, *20,* 77–93. doi:10.1007/s10972-008-9115-6

Alonzo, A.C., Kobarg, M., & Seidel, T. (2012). Pedagogical content knowledge as reflected in teacher–student interactions: Analysis of two video cases. *Journal of Research in Science Teaching*, *49*(10), 1211–1239. doi: 10.1002/tea.21055

Association of Teacher Educators (ATE) (2008). *Standards for teacher educators*. http://www.ate1.org/pubs/Standards.cfm. (accessed June 15, 2016)

Aydın, U., Tunç-Pekkan,Z., Taylan, R. D., Birgili, B., & Özcan, M. (2016, online). Impacts of a university-school partnership on middle school students' fractional knowledge: A quasiexperimental study. *The Journal of Educational Research*. doi:10.1080/00220671.2016.1220358

Blanco, L. (2004). Problem solving and the initial practical and theoretical education of teachers in Spain. *Mathematics Teacher Education and Development*, *6*, 31-42.

Borko, H., Roberts, S., & Shavelson, R. (2008). Teachers' decision making: From Alan J. Bishop to today. In P. Clarkson & N. Presmeg (Eds.), *Critical issues in mathematics education: Major contributions of Alan Bishop* (pp. 37–67). New York, NY: Springer.

Chan, K.K.H., & Yung, B.H.W. (2015). On-site pedagogical content knowledge development. *International Journal of Science Education*, *37*(8), 1246-1278. doi:10.1080/09500693.2015.1033777

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12-25. doi: 10.1016/j.tate.2013.03.001

Demirdöğen, B., Aydın, S., & Tarkın, A. (2015). Looking at the mirror: A self-study of science teacher educators' PCK for teaching teachers. *EURASIA Journal of Mathematics, Science and Technology Education*, *11*(2), 189-205.

van Driel, J. H., de Jong, O., & Verloop, N. (2002). The development of pre-service chemistry teachers' pedagogical content knowledge. *Science Education*, *86*(4), 572–590. doi: 10.1002/sce.10010

van Driel, J.H., Verloop, N., & de Vos, W. (1998). Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching*, *35*(6), 673–695.

Faikhamta, C.C., & Clarke, A. (2013). A self-study of a Thai teacher educator developing a better understanding of PCK for teaching about teaching science. *Research in Science Education*, *43*(3), 955-979. doi:10.1007/s11165-012-9300-7

Franke, M.L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-356). Charlotte, NC: Information Age.

Grossman, P., Hammerness, K., McDonald, M. (2009). Redefining teacher: Re-imagining teacher education. *Teachers and teaching: Theory and practice*, *15(*2), 273-290.

Hashweh, M. Z. (2005). Teacher pedagogical constructions: A reconfiguration of pedagogical content knowledge. *Teachers and Teaching: Theory and Practice*, *11*(3), 273–292.

Janik, T., Najvar, P., Slaviik, J., & Trna, J. (2009). On the dynamic nature of physics teachers' pedagogical content knowledge. *Orbis Scholae*, *3*(2), 47–62.

Kieren, T. E. (1976). On the mathematical, cognitive, and instructional foundations of rational numbers. In R. Lesh (Ed.), *Number and measurement: Papers from a research workshop* (pp. 101-144). Columbus, OH: ERIC/SMEAC.

Korthagen, F., & Lunenberg, M. (2004). Links between self-study and teacher education reform. In J. Loughran, M. Hamilton, V.

LaBoskey, & T. Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (Vol. 1, pp. 421-449). Dordrecht: Kluwer.

Koster, B., Dengerink, J., Korthagen, F. & Lunenberg, M. (2008). Teacher educators working on their own professional development: Goals, activities and outcomes of a project for the professional development of teacher educators, *Teachers and Teaching*, *14*(5-6), 567-587. doi: 10.1080/13540600802571411

Lannin, J.K., Webb, M., Chval, K., Arbaugh, F., Hicks, S., Taylor, C., & Bruton, R. (2013). The development of beginning mathematics teacher pedagogical content knowledge. *Journal of Mathematics Teacher Education, 16*(6), 403-426. doi:10.1007/s10857-013-9244-5

Lobato, J., Clarke, D., & Ellis, A.B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education, 36*(2), 101-136.

Loughran, J. (2007). Researching teacher education practices: Responding to the challenges, demands, and expectations of self-study. *Journal of Teacher Education*, *58*, 12–20. doi: 10.1177/0022487106296217

Loughran, J., Berry, A., & Mulhall, P. (2012). *Understanding and developing science teachers' pedagogical content knowledge*. New York, NY: Springer.

Loughran, J., Mulhall, P., & Berry, A. (2004). In search of pedagogical content knowledge in science: Developing ways of articulating and documenting professional practice. *Journal of Research in Science Teaching*, *41*(4), 370–391. doi: 10.1002/tea.20007

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics*, *38*(1-3), 135-161. doi: 10.1023/A:1003622804002

Ministry of National Education (2013). *Ortaokul matematik dersi öğretim programı*. [Middle school mathematics teaching program]. Ankara: Talim ve Terbiye Kurulu Başkanligi.

Özcan, M. (2013). *University within school: A model to re-structure teacher education in Turkey*. İstanbul: TÜSİAD.

Park, S., & Oliver, J. (2008). Revisiting the conceptualization of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education*, *38*(3), 261–284. doi:10.1007/s11165-007-9049-6

Schön, D.A. (1996). *Educating the reflective practitioner: Towards a new design for teaching and learning in the professions.* San Francisco, CA: Jossey-Bass.

Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–22. doi: 10.17763/haer.57.1.j463w79r56455411

Steffe, L.P., & Olive, J. (2010). *Children's fractional knowledge*. New York, NY: Springer.

Tunç-Pekkan, Z., Taylan, R. D., Birgili, B., Aydın,U. & Özcan, M. (2016). Academicians as teachers: Nurturing teacher experience. Paper presented at the *International Congress on Mathematics Education*, Hamburg, Germany.

Tunc-Pekkan, Z. (2015). An analysis of elementary school children's fractional knowledge depicted with circle, rectangle, and number line representations. *Educational Studies in Mathematics, 89*, 419–441. doi:10.1007/s10649-015-9606-2

Yılmaz, K. (2007). Learner-centred instruction as a means to realise democratic education: The problems and constraints confronting learner-centred instruction in Turkey. *Studies in Learning, Evaluation, Innovation and Development*, *4*(3), 15-28.

Rukiye Didem Taylan is Assistant Professor, at the Department of Mathematics Education, in the MEF University, Turkey.

João Pedro da Ponte is Full Professor and Director of the Instituto de Educação, Universidade de Lisboa, Portugal.

Contact Address: Direct correspondence concerning this article, should be addressed to the author. Postal address: MED Üniversitesi Huzur Mah. Ayazağa Yolu Cad. No:4, Maslak, Sarıyer, İstanbul 34396 Türkiye **Email**: tayland@mef.edu.tr and jpponte@ie.ulisboa.pt