Pressure Groups and Public Goods Provision: Might society efficiently decide it has enough?

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Abstract

Since Samuelson's 1954 decision rule, public goods models focus on achieving a unique efficient equilibrium that satisfies Samuelson's condition. This paper presents a new perspective on this issue. We propose that individual preferences for a public good are reflected by different pressure groups to which people belong. Through a common agency problem using a two-stage menu auction game, we show how pressure groups can aid in deciding public goods provision. Depending on the disparities in the magnitude of the influences of the pressure groups, different efficient equilibria are possible (as in a Pareto contract curve in the Edgeworth's exchange model).

I. Introduction

The model in Samuelson (1954) provides a condition to determine an efficient allocation of public goods. Despite the theoretical qualities of Samuelson's model, it has restricted empirical applications. The condition rests on the assumption of a perfect and completely informed social planner about individual preferences for the public goods. However, in practice the provision of public goods lies in a world of imperfect (even incomplete) information, which prevents an efficient allocation.

Hence, the literature on public goods has dedicated a good deal of attention to designing mechanisms that reduce inefficiencies associated to the provision of public goods. The shared characteristic of traditional models is that satisfying Samuelson's rule implies a fixed efficient provision, which is derived from individual preferences (given by the individual marginal rate of substitution) and financed by individual contributions, either voluntary or compulsory.

Consequently, the literature in this tradition starts at the individual preferences level, without considering the power of assembly. In practice, however, individuals do search for support in different types of representative groups and associations. These groups tend to reduced free-rider problems and have relatively low pressure costs. Also, the number of groups is usually lower than the number of individuals. More importantly, the groups' claims provide a clear picture of their members' preferences. For this reason, decision makers usually listen to groups rather than to individuals.

This paper sketches a very simple mechanism to reveal preferences for public goods. This mechanism is based on the assumption that individuals, concerned about how much public good should be provided, will try to influence the decision maker by resorting to specialized pressure groups. Supported by a two-stage menu auction game developed by Bernheim and Whinston (1986) and Grossman and Helpman (1994), this paper presents how pressure groups' interactions affect the available quantity of a public good. In this model, individual preferences can be inferred through the groups' public good perceptions. Conceptually, the difference between preferences and perceptions is that the latter are not necessarily private knowledge. When pressure groups battle for their interests they reveal their perceptions about public goods.

A consequence of assuming that the social planner always takes the pressure groups into consideration is that the resulting "political equilibrium" allocation will be Pareto optimal. Although different equilibria are possible depending of specific social arrays, all equilibria are efficient. Thus, in this paper, we show how changes in the relative influences of pressure groups result in distinct efficient provisions of public goods. Moreover, a "political equilibrium" satisfying Samuelson's condition exists under a few, not very restrictive, assumptions.

This paper considers the decision for the provision of public goods to be made in a socially concerted structure. First, it is socially recognized that public goods increase welfare, but there is no agreement on how much. Second, there is no consensus about the quantity of public goods to be provided because of conflicting opinions. Third, it is socially accepted that individual contributions finance public goods. Fourth, the decision maker will determine the contributions, which are the same for all. Fifth, individuals can attempt to influence the decision maker through pressure groups.

To facilitate the exposition, the paper is divided in four sections. The first section provides an introduction. The second section presents the theoretical framework. The third section presents the assumptions, results and analytical implications of the model. The final section closes the analysis with the conclusions.

II. Theoretical Framework

Since the pioneering work of Samuelson (1954) a significant number of papers and books have been written about public goods. Most of them deal with the Samuelson's optimality condition, which claims that the Pareto optimal provision of a public good occurs when the sum of the individual Marginal Rates of Substitution between a public and a private good equals the Marginal Rate of Transformation (Atkinson and Stern (1974)). In practice, the main hindrance of Samuelson's approach is that it requires an inordinate amount of information in order to ascertain these marginal rates.

The traditional literature on public goods attempts to alleviate the informational requirements of Samuelson's equation using the individual preferences to contribute for a public good. There are two approaches to this: compulsory and voluntary contributions. The former requires levying taxes, the latter entails either varying amounts paid by different individuals, depending on their willingness to pay for the public good, or a unique, general contribution.

The tax-based literature advocates non-distorting taxation as a necessary condition for the Pareto optimal provision of a public good. However, lump-sum financing is regressive and in practice poses many hurdles. Consequently, compulsory contribution is commonly associated with an inefficient provision of public goods. Stiglitz and Dasgupta (1971 and 1972), Diamond and Mirlees (1971 and 1971b) and Atkinson and Stern (1974) provide examples of the literature in this line of discourse.

With respect to voluntary contributions, Laffont (1988) shows that the first order condition is different to Samuelson's Rule, implying that consumers contribute less than the desirable amount for Pareto optimality. The lower-than-optimal provision of a public good through voluntary contributions could be reverted if every individual assumes a price for the public good. Thus, every individual pays an amount equal to her associated price for every unit of public good she consumes. The resulting Pareto optimal equilibrium is called the Lindahl Equilibrium (p. 42).

The problem with the Lindahl equilibrium is that its competitive market structure cannot be sustained because of the free rider problem. As the number of agents increase, an individual has incentives to hide her true demand for the public good and instead minimizes her revealed desire for the public good in order to reduce her contribution (Laffont (1988)).

A first line of models in the standard literature on voluntary contributions shows income neutrality in contribution for public goods. Warr (1983), Bergstrom *et al* (1986) and Bernheim (1986) are the most relevant works in this area.[1] While these models embed the idea of pure altruism in the provision of public goods,[2] Andreoni (1990) introduces impure altruism or warm-glow preferences as a generalization.[3] His model shows that contributions are not independent of income distribution because private provision increases when impure altruists obtain more income.

Following these initial papers there is an abundance of studies concerning voluntary contributions with either pure or impure altruism. Here, there are two lines of research with regard to perfect and imperfect information. Examples of the former are Fraser (1996), Itaya *et al* (2000), Diamond (2006), Kotchen and Moore (2007), Kessing (2007), Cornes and Itaya (2010), Nizar (2010), Pecorino (2010) Neslihan (2011), Furusawa and Konishi (2011), Della Vigna *et al* (2012).[4]As for the latter, Clarke (1971), Groves (1973), and Groves and Ledyard (1977) represent the pioneering analyses. Afterwards, Bohm (1984), Güth and Hellwig (1986), Twight (1993), Falkinger *et al* (2000), Marx and Matthews (2000), Menezes et al (2001), Barbieri and Malueg (2008), Lu and Quah (2009), Hon-Snir *et al* (2010) and Martimort and Moreira (2010) provide valuable insight into mechanism design with imperfect information.

Regardless of their theoretical approach, all models consider a fixed allocation such that Samuelson's rule is satisfied. In these models any equilibrium away from the Conventional Rule

[1] See Bernheim (1986) for a detailed survey of earlier work in voluntary contributions.
[2] These preferences go in opposition to the selfish (or self-interested) preferences that characterize standard utility maximization problems with private goods.
[3] Warm-glow preferences explain donations, charity contributions and any other giving where a perceptual reward is expected.
[4] A more detailed review of the literature can be found in Florenzano (2010).

is considered inefficient. In every standard model the policymaker is tasked with finding a unique efficient allocation. However, society is represented as a sum of individual tastes that are constrained by a technological limitation. This is a very restricted view about social behavior. Society does not necessarily respond to such type of fixed rules. Different allocations should be possible depending on specific social structures. Therefore, distinct individual tastes should provide different Pareto optimal equilibria, just by changing how society is defined and with independence of the Conventional Rule. From this viewpoint, the focus of the empirical problem is no longer about whether an allocation satisfies Samuelson's condition; but rather the problem focuses on which social structure guarantees a desired provision (even one satisfying the Conventional Rule).

Remarkably, the literature on public goods is not very extensive on the theory of interest groups to analyze the problem of financing public goods via contributions. Grossman and Helpman (2001) present some examples with public goods as a way of illustrating their theoretical developments in Special Interest Groups (SIG) theory. Bernheim and Whinston (1986) do the same in a very complex model of menu auctions. More recently, Martimort and Stole (2011) present a generalization of Bernheim and Whinston's model and introduce an illuminating example where interest groups propose a menu of contributions to a common agent in an effort to influence its decision.

Although many of the papers explored in the theoretical framework section use strategic behavior when determining the contribution for a public good, this behavior is applied to isolated individuals and not to groups. In the real world, however, battling alone could be a less frequent event than a collaborative fight. It is expected for individuals to more easily reach individual objectives through groups that represent specific interests. Labor unions, chambers of commerce, entrepreneur guilds or businessmen associations are everyday examples of interest groups representing specialized interests.

The model developed in this paper does not assume the government has perfect knowledge of individual preferences. Consequently, its approach resembles a mechanism design model. Nonetheless, this paper only sketches a mechanism derived from a very specific social array, defined by the influences of pressure groups when they look after their members' interests. This paper is in correspondence with Olson´s (1965) approach and is analytically grounded on the theoretical models of SIG developed by Bernheim and Whinston (1986), Grossman and Helpman (1994), Dixit (1996), Dixit *et al* (1997) and Grossman and Helpman (2001).

As a consequence of the interaction of pressure groups, in this paper the policymaker maximizes both a social welfare function and the groups' utility functions.[5] Since groups represent their members' preferences, the model jointly maximizes the government's and individual objective

[5] See the appendix for a tentative approach to this assessment. Alternatively, see Dixit *et al* (1997) for a rigorous demonstration.

functions. This implies that the resulting contribution is sufficient to finance a Pareto optimal amount of a public good.

The strong analytical possibilities of this procedure appears to be supported by indirect empirical evidence. Alesina *et al* (1999) present an empirical paper where the groups' perceptions take the form of ethnic groups' preferences. The authors show how the differences in groups' preferences determine the quantity and the type of public goods in a city. They find that cities with high ethnic fragmentation have lower taxes to finance education and roads than cities with homogenous populations. Their result supports the suggestion that fragmented societies have selfish perceptions about public goods. In our model egoistic groups could have a low willingness to contribute, thus, our model does not exclude the presence of many egoistic groups as a consequence of ethnic fragmentation.

In our model, specific efficient provision of the public good is subject to groups' perceptions and these perceptions could vary as a function of the degree of ethnic fragmentation. Therefore, if a homogenous city has higher perceptions for a public good compared to a heterogeneous city, our model suggests that the homogeneous city will have higher contributions than the heterogeneous one. Both contributions produce optimal and Pareto efficient quantities, given the cities' perceptions.

Schwabish *et al* (2003) present empirical evidence at a national level. The authors attempt to link inequality and trust with social expenditures. They find that inequality between the middle classes and the poor has a small, positive effect in social expenditures. In contrast, inequality between the rich and the middle classes has a large and negative impact on social spending. Additionally, they find that trust implies large and positive spending. This means that more interrelated, trusting societies are more willing to share economic resources with others. Finally, they find the rich can easily substitute public goods in the private market.

Schwabish *et al* (2003) support our model's results. In this case middle classes resemble a group with a very high perception of the public good. If the middle class is the influencer, higher social spending could be expected. The opposite is true if the rich are the influencers with a lower perception of the public good and there exists a club good with a high degree of substitutability. In this case, our model suggests an efficient lower social spending.

Although these empirical efforts do not take into account the particularities of our model, their results provide empirical support to the analytical propositions of the model. Even though these empirical studies differ in their geographical scopes (cities in the case of Alesina *et al* (1999) and countries in Schwabish *et al* (2003)), in the dependent variables (public good spending in the former and social spending in the latter), and in the groups' composition (ethnic groups and social classes, respectively), their results are adjusted to the analytical propositions of our model. This is suggestive and provides abundant possibilities for an eventual direct empirical validation of our model.

III. The Model

Let there be an economy where the policymaker is considering providing a public amenity (A). Assume this amenity is financed by a general individual contribution (τ) per unit of amenity, but the policymaker is not sure about the contribution amount. Suppose that there is a similar good (x) that could substitute the amenity, and there is a market price p_x per unit of this good. Assume that only those who pay p_x can enjoy this good. Think of x as a club good financed by individual contributions p_x and the quantity of x that is consumed depends (among other things) on this market price. Assume there are no congestion problems in the consumption of A and x.

Consider that people in this economy can be assigned to three pressure groups: Left (L), Center (C) and Right (R).[6] Suppose each group represents effectively the true interests of its members. Assume members of groups do not falsify or hide their true preferences, all members in a group share similar preferences and groups do not have free riders problems.

Suppose every group uses an additively separable utility function to solve the following problem

$$\max_{x^i,y^i} U^i = u^i\left(y^i\right) + \sigma^i x^i + \theta^i A - \rho^i P^i(\tau)$$
$$s.t.\ \ M^i - c^i P^i(\tau) = p_x x^i + p_y y^i + \tau n^i A \tag{1}$$

$\forall i = L, C, R$. Where x^i is the quantity of the club good consumed by the i^{th} group; σ^i is the i^{th} group's perception of the degree of substitutability between the amenity and the club good, where $\sigma^i = 0$ indicates complete non-substitutability and $\sigma^i = 1$ implies perfect substitution, with $\sigma^i \in (0, 1]$; y^i is a standard private good consumed by the i^{th} group, with $\frac{du^i}{dy^i} > 0$ and $\frac{d^2u^i}{dy^{i^2}} < 0$; p_x and p_y are exogenous market prices;[7] M^i is the i^{th} group's endowment.[8] A is the quantity of a public amenity, which satisfies the conditions of a pure public good; n^i is the number of people in i^{th} group, with $n^L + n^C + n^R = N$; and θ^i is the i^{th} group's perception of the amenity, with $\theta^i \in \mathbb{R}^+$ and $\theta^L \neq \theta^C \neq \theta^R$.[9] Note that x^i and y^i are divisible goods such that $x^i = \sum_{j=1}^{n^i} x_j^i$ and $y^i = \sum_{j=1}^{n^i} y^i$.

$P^i(\tau)$ is the i^{th} group's pressure schedule contingent to τ. Given a specific contribution every group decides how and how much pressure. The negative effect of the pressure on utility implies $\frac{\partial U^i}{\partial P^i} < 0$.[10] Also, pressure implies a reduction on group's endowment. Thus, pressure has a double

[6] This paper assumes pressure groups already exist. See Damania and Fredrikkson (2003) for more on group formation.
[7] This implies the assumption that each pressure group is small enough to not influence the level of prices in this economy.
[8] This group's endowment could be the sum of the endowments of the group's members.
[9] This parameter is analyzed in detail later in the paper.
[10] To pressure can affect the necessary concavity condition of the utility function. For this reason, we assume concavity is retained despite of the negative effect of pressure on group's utility.

impact on groups: first, implies a reduction on utility and, second, a cost on group's budget. The ρ^i and c^i pressure coefficients are the i^{th} group's loss of utility and cost per unit of pressure, respectively. These unitary costs exist each time a group chooses pressure.[11]

For the sake of clarity, assume that the amenity is a continuous indivisible public good, for example a $300m^2$ urban park, a $200m$ long sidewalk, a $1500m^3$ municipal swimming pool, etc. Also, assume that $A = A(\tau)$, such that $\frac{dA}{d\tau} > 0$ and $\frac{d^2A}{d\tau^2} < 0$.[12] This means that A is the stock or the quantity of amenity available in the economy. A person in this economy contributes τ to help finance a quantity A of the public amenity, but this individual does not determine neither τ nor A, although they affect the individual's budget constraint and utility level. Consequently, A and τ are exogenous to the individuals' decisions. Only the interaction of all individuals determines the level of contribution and the quantity of the amenity. This assumption for an individual is also valid at the group level. For this reason, suppose that each group does not choose A and τ separately.

If an individual desires a particular level of contribution and amenity she could seek to obtain them through a group that represents people with similar desires. Though a pressure group cannot decide the level of τ and A, it can influence the social planner who is, after all, responsible of establishing the quantity of amenity and the amount of contribution needed to finance it. The combination of the groups' influences will determine the social planner's final decision. In this line of reasoning, it is convenient to think of A and τ as pseudo-exogenous election variables because the resulting quantities will depend on pressure groups' interaction and not on the specific decision of a group.

Even though the decision to pressure is endogenous to every group, it is contingent to the contribution. Because the contribution is exogenous in equation 1, we can consider, momentarily, pressure in a similar manner as the amenity. For now, let us consider pressure in equation 1 as the outcome of a group's separate problem with another group's decision. Later we will pay attention on how a group chooses pressure when considering such decision within a two stages game.

The problem in equation 1 can be rewritten as

$$\max_{y^i} U^i(y^i) + \frac{\sigma^i}{p_x}\left(M^i - p_y y^i\right) + \left(\theta^i - \tau n^i \frac{\sigma^i}{p_x}\right) A(\tau) - \gamma^i P^i(\tau) \tag{2}$$

With $\gamma^i = \rho^i + \frac{\sigma^i}{p_x} c^i$ defining the aggregate marginal losses of pressure of every group.[13]

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[11] Through this section we will explain pressures in more detail.

[12] This assumption implies that the utility function is concave in τ. A negative second derivative reflects the opportunity costs of public goods. Financing the amenity reduces the wealth available to consume other goods.

[13] The second term on the right is the unit cost of pressure weighted by the club good's real degree of substitution. We return later to a discussion of this term.

As U^i is a quasi-linear utility function, a corner solution cannot be averted. For this reason, assume that there are only inner solutions. From this assumption the first order conditions of equation 2 is

$$\frac{\partial U^i}{\partial y^i} = \frac{du^i(y^i)}{dy^i} - \frac{p_y}{p_x}\sigma^i = 0 \tag{3}$$

Solving equation 3 produce the demand function $y^{i^*} = y^i(p_y, p_x, \sigma^i; \tau)$. Thus, the indirect utility function will be

$$W^i = V^i - \gamma^i P^i(\tau) \tag{4}$$

$\forall i$. Where $V^i = \frac{\sigma^i}{p_x} M^i + \left(\theta^i - \tau n^i \frac{\sigma^i}{p_x}\right) A(\tau) + s^{i^*}(p_y, p_x, \sigma^i; \tau)$ and $s^{i^*} = u^i(y^{i^*}) - \sigma^i \frac{p_y}{p_x} y^{i^*}$.

The social planner (G) should set the contribution τ per unit of amenity necessary to finance a quantity $A(\tau)$ of this amenity. Itaya et al (2000) establish that the value of τ will depend on individuals' preferences. However, suppose that G does not know these preferences, but G can obtain some signals about them through a two-stage game. In the first stage, every pressure group offers G a menu of pressures contingent on different values of τ, taking the other group's menus as given. Assume this menu is scheduled for a derivable function $P^i(\tau)$. Assume that, given a value of τ, every group actually exerts a positive pressure $(P^i(\tau) > 0)$ [14] such that a fixed utility level $V^i(\tau)$ is preserved.[15] Thereafter, on the second stage, G considers the different menus of pressures offered by the groups, selects a particular τ, and waits for the pressures associated to this value of τ. Such schedule of pressures contingent on a value of τ is akin to a menu of bids from where G makes its decision analogous to an auction process. Thus, G, considering social welfare and the groups' pressures, will set τ such that it solves the following

$$\max_{\tau} G = \alpha \sum_{i=L,C,R} V^i(\tau) + \sum_{i=L,C,R} I^i P^i(\tau) \tag{5}$$

Where $\alpha \in \mathbb{R}^+$ is the weight that G attaches to social welfare; I^i is an index which equals one if the i^{th} pressure group is powerful enough to influence the social planner's decision and zero if it does not. Assume that, if $I^i = 1$ for all pressure groups, they influence G symmetrically. Similarly, if $I^i = 0$ for all pressure groups, then they do not influence G at all.

An alternative social planner's objective function could be

$$\max_{\tau} \tilde{G} = \delta \sum_{i=L,C,R} V^i(\tau) + \beta \sum_{i=L,C,R} I^i P^i(\tau) \tag{6}$$

[14] Later we talk about positive pressures.
[15] Following Grossman and Helpman (2001) last assumption means a compensated pressure schedule.

Maximizing $\tilde{G}$ in equation 6 is equivalent to maximizing G in equation 5, with $\alpha = \delta/\beta$. If $\beta = \delta$ (implying that $\alpha = 1$), the social planner equally values the pressures associated to her decision and the implications on social welfare of this decision. If $\beta < \delta$ ($\alpha > 1$) social welfare is given more weight. Furthermore, if $\beta > \delta$ ($\alpha < 1$) pressures trump over welfare. In this paper, there are no special assumptions about the value of α, besides $\alpha > 0$. Given the equivalence between equations 5 and 6, to simplify the exposition in the rest of the paper we will use the first one.

Bernheim and Whinston (1986) develop this two stages game, known as a menu auction game, as a way of dealing with common agency problems (agency problems with one agent and many principals). They show there exists a subgame-perfect Nash equilibrium if $P^i(\tau)$ is feasible $\forall i = L, C, R$ and τ^* maximizes specific objective functions. Moreover, Bernheim and Whinston (1986) prove that under these conditions pressures associated to τ are truthful (i.e. pressures reflect the true preferences of the groups' members),[16] and groups do not bear a significant cost from playing truthful strategies. Bernheim and Whinston (1986) show that equilibria (identified as Truthful Nash Equilibria-TNE) in the menu auction game are coalition-proof; that is, they remain equilibria even if groups engage in non-binding communication before the beginning of the game.

Grossman and Helpman (1994) and Dixit (1996) establish that always pressures are positive and derivable the TNE is locally stable to little changes in pressures and it is Pareto optimal jointly for influencer groups and the social planner, although, non-necessary socially efficient.[17] Dixit *et al* (1997), and Grossman and Helpman (2001) extend common-agency models showing that pressures are always compensated (that is, they are positive and maintain a defined utility level) the TNE is globally stable. Also, they prove that if social planner's objective function includes all groups (influencers and non-influencers) the Globally Truthful Nash Equilibrium is socially efficient. Using this extension we can rewrite equation 5 as[18]

$$\max_{\tau} G = \alpha \sum_{i=L,C,R} V^i(\tau) + \sum_{i=L,C,R} \frac{I^i}{\gamma^i} V^i(\tau) \tag{7}$$

Or

$$\max_{\tau} G \sum_{i=L,C,R} \left(\alpha + \frac{I^i}{\gamma^i}\right) V^i(\tau) \tag{8}$$

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[16] Specifically Bernheim and Whinston use campaign contributions. However, this is a way to pressure. Here we use pressures as a generalized way to influence social planner's decision. In part 5 we bring additional explanation.

[17] Formally, they refer to organized or non-organized groups rather that influencer or non-influencer groups. Nevertheless, in their models they assume that only organized groups bring campaign contributions. Thus, only organized groups can aim influence the social planner's choice. Here we assume that all groups are organized and pressure, but the government could have different concerns about groups. This means that groups could influence the government asymmetrically. This describes a more general relationship between pressure groups and the social planner. In part 5 we return on this topic.

[18] See appendix.

The social planner chooses τ such that it maximizes a weighted sum of the pressure groups' indirect utility functions.

The first order condition of equation 8 is

$$\sum_{i=L,C,R}\left(\alpha+\frac{I^i}{\gamma^i}\right)\frac{\partial V^i(\tau)}{\partial\tau}=0 \tag{9}$$

From equation 4

$$\frac{\partial V^i(\tau)}{\partial\tau}=\left(\theta^i-\tau n^i\frac{\sigma^i}{p_x}\right)\frac{dA(\tau)}{d\tau}-\frac{n^i\sigma^i}{p_x}A(\tau)+\frac{\partial s^{i^*}\left(p_y,p_x,\sigma^i;\tau\right)}{\partial\tau}$$

Where

$$\frac{\partial s^{i^*}\left(p_y,p_x,\sigma^i;\tau\right)}{\partial\tau}=\frac{\partial u^i\left(y^{i^*}\right)}{\partial y^{i^*}}\frac{\partial y^{i^*}}{\partial\tau}-\sigma^i\frac{p_y}{p_x}\frac{\partial y^{i^*}}{\partial\tau}$$

Reordering it

$$\frac{\partial s^{i^*}}{\partial\tau}=\left(\frac{\partial u^i\left(y^{i^*}\right)}{\partial y^{i^*}}-\sigma^i\frac{p_y}{p_x}\right)\frac{\partial y^{i^*}}{\partial\tau}$$

First Order Conditions 3 imply that $\frac{\partial s^{i^*}}{\partial\tau}=0$.[19] Then

$$\frac{\partial V^i(\tau)}{\partial\tau}=\left(\theta^i-\tau n^i\frac{\sigma^i}{p_x}\right)\frac{dA(\tau)}{d\tau}-\frac{n^i\sigma^i}{p_x}A(\tau) \tag{10}$$

$\forall\, L,C,R.$

From equation 10 in equation 9

$$\sum_{i=L,C,R}\left(\alpha+\frac{I^i}{\gamma^i}\right)\left[\left(\theta^i-\tau n^i\frac{\sigma^i}{p_x}\right)\frac{dA(\tau)}{d\tau}-\frac{n^i\sigma^i}{p_x}A(\tau)\right]=0 \tag{11}$$

And solving for τ

[19] This result is expected because for every value of τ demand functions y^{i^*} maximize utility function and, consequently, they have to satisfice First Order Condition 3.

$$\tau = p_x \frac{\varepsilon}{1+\varepsilon} \frac{\sum_{i=L.C,R}\left(\alpha + \frac{I^i}{\gamma^i}\right)\theta^i}{\sum_{i=L.C,R}\left(\alpha + \frac{I^i}{\gamma^i}\right)\sigma^i n^i} \tag{12}$$

Where $\varepsilon = \frac{dA(\tau)}{d\tau}\frac{\tau}{A}$ is the contribution-amenity elasticity. Note that $\frac{\partial\tau}{\partial\theta^i} > 0$, $\frac{\partial\tau}{\partial\varepsilon} > 0$ and $\frac{\partial\tau}{\partial n^i} < 0$. Also, from $\gamma^i = \rho^i + \frac{\sigma^i}{p_x}c^i$ we have $\frac{\partial\tau}{\partial p_x} > 0$, $\frac{\partial\tau}{\partial\sigma^i} < 0$ and $\frac{\partial\tau}{\partial\rho^i} < 0$.

The positive relationship between τ and the groups' perceptions is expected.[20] The positive relationship with ε is decreasing because of $\frac{\partial^2\tau}{\partial\varepsilon^2} < 0$. This implies that τ is concave in ε. For initial values of τ, a higher elasticity more than compensates the contribution's opportunity costs and this could induce to higher increments in τ. However, when values of τ are high enough, the correspondingly higher opportunity costs could be less than compensated for the higher elasticity consequently reducing the increases in τ.

The signal of $\frac{\partial\tau}{\partial\rho^i}$ and $\frac{\partial\tau}{\partial c^i}$ is ambiguous depending of parameters' values. For example, de sing of $\frac{\partial\tau}{\partial c^L}$ depends of the sing of this expression$(\sigma^L n^L \theta^C - \sigma^C n^C \theta^L)\Gamma^C + (\sigma^L n^L \theta^R - \sigma^R n^R \theta^L)\Gamma^R$. Where Γ^C and Γ^R are positives. Interestingly, if leftist group's perception is higher relative to other groups, and there are no significant differences in all $\sigma^i n^i$terms the contribution decreases when the unitary nominal cost to pressure c^L increases. A contrary case arises if Center and Right groups have greater perceptions for the public good. A more intuitive result occur when $\sigma^C n^C$ and $\sigma^R n^R$ are greater than $\sigma^L n^L$, and $\theta^C \sim \theta^L \sim \theta^R$. Here a small Left group could not support increasing nominal costs for pressure, or desires for the club good on Center and Right groups could reduce interest for public good if Leftist faces higher costs. There are many explanations (someone more intuitive than others) that could explain a negative or positive relationship between contributions and unitary costs of pressure.

1. Symmetry

Because pressure are compensated and the policymaker includes all groups in her objective function political equilibrium in equation 12 is socially efficient. For public good contribution this means that each balance of influences between pressure groups and the government produce a Pareto optimal equilibrium. This is, a social structure defines the contribution that is the socially best given this social structure. Although, contributions can differ between structures each political equilibrium provides the Pareto optimal contribution corresponding to a specific structure.

Think of an Edgeworth's box in a pure exchange economy. There the Pareto set is the set optimal equilibria. Every equilibrium in the box depends of the endowments distribution. A specific

[20]We return to this relationship later in the paper.

allocation correspond to and given wealth initial distribution. Consequently depending on distribution some equilibria are more egalitarian than others. But all equilibria in the Edgeworth's box are Pareto optimal. This is the same case in this model. Here, before game there is an initial influence distribution. Depending of this distribution (that defines a specific social structure) we have their corresponding optimal equilibrium. Depending of the influences distribution some equilibria produces results closer to those derived from Samuelson's rule than others. However, every equilibrium is socially efficient, and the Samuelson's allocation is one of these optimal equilibria.[21]

In this model, the social structures are determined by the influencing degree of every pressure group. Thus a social structure is one where pressure groups are symmetrically influencers ($I^L = I^C = I^R = 1$) or non-influencers ($I^L = I^C = I^R = 0$). This equilibrium, denoted the symmetrical equilibrium, is given by

$$\tau^* = p_x \xi \begin{cases} \dfrac{\sum_{i=L.C,R}\left(\alpha + \dfrac{1}{\gamma^i}\right)\theta^i}{\sum_{i=L.C,R}\left(\alpha + \dfrac{I}{\gamma^i}\right)\sigma^i n^i}, & if\ I^i = 1 \\ \dfrac{\theta^L + \theta^C + \theta^R}{\sigma^L n^L + \sigma^C n^C + \sigma^R n^R}, & if\ I^i = 0 \end{cases} \tag{13}$$

$\forall i = L, C, R$. With $\xi = \frac{\varepsilon}{1+\varepsilon}$.

A special case of this symmetrical equilibrium occurs when $\gamma^i = \kappa, \forall i = L, C, R$ and κ is a positive constant.[22] Here the equilibrium is the same regardless groups are all influencers or non-influencers

$$\tau^*|_{\gamma^i=\kappa} = p_x \xi \frac{\theta^L + \theta^C + \theta^R}{\sigma^L n^L + \sigma^C n^C + \sigma^R n^R} \tag{14}$$

When the sum of marginal losses in utilities plus unitary costs for pressure, times considerations about public good substitutions adjusted by the club's price are very similar for all groups, every influencer group compensates other influencer groups producing a final effect equivalent to that when government is only interested in social welfare. In this case, pressure implies losses and costs for groups without changes in social planner's choice. This special equilibrium is equivalent to the standard optimal equilibrium derived from the Samuelson's rule.[23]

[21] On part 6 we prove this statement.

[22] A constant γ^i requires that any difference in groups' marginal losses of pressure be compensated when they are aggregated and the real inclination for the club is considered. For example, assume that ρ^i is the same for all groups while σ^i and c^i are different among groups. Hence, if a group incurs in higher (lower) unit cost, c^i, it would have a lower (higher) liking for the club good. Both effects will be compensated across groups and, consequently, γ^i will be constant for all groups.

[23] We prove this assessment in part 6.

2. Asymmetry

Depending on the kind of asymmetry there are various efficient political equilibria. First, consider the case when $I^L = 1$ and $I^C = I^R = 0$

$$\tau^L = p_x \xi \frac{\left(\alpha + \frac{1}{\gamma^L}\right)\theta^L + \alpha(\theta^C + \theta^R)}{\left(\alpha + \frac{1}{\gamma^L}\right)\sigma^L n^L + \alpha(\sigma^C n^C + \sigma^R n^R)} \tag{15}$$

Here influencer group receive a greater weigh compare with other groups. However, the consequence of this major concern about the influencer group depends on parameters. Comparing equation 15 with equation 13 can be shown that

$$\tau^L \begin{cases} > \tau^*, if\ \theta^L > Z \\ = \tau^*, if\ \theta^L = Z \\ < \tau^*, if\ \theta^L < Z \end{cases}$$

With

$$Z = \frac{\sigma^L n^L \left(\alpha + \frac{1}{\gamma^L}\right)\left(\frac{\theta^C}{\gamma^C} + \frac{\theta^R}{\gamma^R}\right) + \alpha(\sigma^R n^R - \sigma^C n^C)\left(\frac{1}{\gamma^C} - \frac{1}{\gamma^R}\right)}{\sigma^C n^C \left(\frac{\alpha}{\gamma^C} + \frac{1}{\gamma^L \gamma^C}\right) + \sigma^R n^R \left(\frac{\alpha}{\gamma^C} + \frac{1}{\gamma^L \gamma^R}\right)} \tag{16}$$

Equation 16 is difficult for an intuitive explanation, for this reason let us take $\gamma^i = \kappa, \forall i = L, C, R$ and to use equation 15 to compare how influencer's perception for the public good can affect the contribution. Thus, from equation 16

$$\tau^L|_{\gamma^i=\kappa} \begin{cases} > \tau^*|_{\gamma^i=\kappa}, if\ \theta^L > \frac{\sigma^L n^L}{\sigma^C n^C + \sigma^R n^R}(\theta^C + \theta^R) \\ = \tau^*|_{\gamma^i=\kappa}, if\ \theta^L = \frac{\sigma^L n^L}{\sigma^C n^C + \sigma^R n^R}(\theta^C + \theta^R) \\ < \tau^*|_{\gamma^i=\kappa}, if\ \theta^L < \frac{\sigma^L n^L}{\sigma^C n^C + \sigma^R n^R}(\theta^C + \theta^R) \end{cases} \tag{17}$$

Analogous results are obtained when only C or only R can influence G.

Equation 17 suggests that asymmetrical equilibrium could replicate symmetrical provision depending of parameters. If the influencer group's amenity perception is equivalent to a fraction $\frac{\sigma^L n^L}{\sigma^C n^C + \sigma^R n^R}$ of the sum of non- influencers' perceptions, the amount of τ set by G would be the symmetrical Pareto optimal. But, if the influencer's perception is low enough, a low level of contribution implies a lower quantity of amenity compared with that under the symmetrical equilibrium. The opposite is true if influencer's perception is high enough. Note that the fraction

$\frac{\sigma^L n^L}{\sigma^C n^C + \sigma^R n^R}$ is increasing both in the amount and the perceptions for the club good of the members of the Left, but is decreasing in the same parameters of the other groups.

The intuition behind this fraction is not evident. However, if the sum $\theta^C + \theta^R$ represents the other groups' perceptions for the public amenity, then the fraction is the weight of these perceptions. Therefore, this weight will be lower if other groups increase their degree of substitutability between the amenity and the club good. If the center and rightist groups have a near substitute for the amenity, it is realistic to reduce the importance of their perceptions for the amenity. On the other hand, if the Left group increases its own degree of substitutability, it is reasonable to increase the weight of the other groups' perceptions, because the Left has a near substitute for the public amenity and this implies a reduced perception for the amenity by the Left.

The impact of a growing population is similar because a demographic increase in other groups involves a quantitative disparity between leftists and others that should be balanced by reducing the weight of the other groups' perceptions. This is a way of preserving the balance of forces in this economy if a group increases its size relative to the others. Finally, if the Left group increases in size, the balance implies a lower weight in its perceptions and, accordingly, higher weights for other groups' perceptions. Therefore, the fraction could be considered as a weight to balance the relative differences in size and degree of substitutability between the groups.

Now consider $I^C = 0$ and $I^L = I^R = 1$. In this case

$$\tau^{LR} = p_x \xi \frac{\alpha\theta^C + \left(\alpha + \frac{1}{\gamma^L}\right)\theta^L + \left(\alpha + \frac{1}{\gamma^R}\right)\theta^R}{\alpha\sigma^C n^C + \left(\alpha + \frac{1}{\gamma^L}\right)\sigma^L n^L + \left(\alpha + \frac{1}{\gamma^R}\right)\sigma^R n^R} \tag{18}$$

Comparing equation 18 with equation 13, we have

$$\tau^{LR} \begin{cases} > \tau^*, & if\ \theta^C < T \\ = \tau^*, & if\ \theta^C = T \\ < \tau^*, & if\ \theta^C > T \end{cases}$$

With

$$T = \frac{\sigma^C n^C \left(\left(\alpha + \frac{1}{\gamma^L}\right)\theta^L + \left(\alpha + \frac{1}{\gamma^R}\right)\theta^R\right)}{\left(\alpha + \frac{1}{\gamma^L}\right)\sigma^L n^L + \left(\alpha + \frac{1}{\gamma^R}\right)\sigma^R n^R} \tag{19}$$

Similar to equation 16 let us take $\gamma^i = \kappa$, $\forall i = L, C, R$ and recurring to equation 15 to get a more intuitive explanation about possible contributions when only one group is non-influencer. Consequently equation 19 now implies

$$\tau^{LR}|_{\gamma^i=\kappa} \begin{cases} > \tau^*|_{\gamma^i=\kappa}, if\ \theta^C < \dfrac{\sigma^C n^C}{\sigma^L n^L + \sigma^R n^R}(\theta^L + \theta^R) \\ = \tau^*|_{\gamma^i=\kappa}, if\ \theta^C = \dfrac{\sigma^C n^C}{\sigma^L n^L + \sigma^R n^R}(\theta^L + \theta^R) \\ < \tau^*|_{\gamma^i=\kappa}, if\ \theta^C > \dfrac{\sigma^L n^L}{\sigma^L n^L + \sigma^R n^R}(\theta^L + \theta^R) \end{cases} \tag{20}$$

The same can be said when only L or R cannot influence G.

In this case, equation 20 shows that, if the non-influencer group has a sufficiently high perception, G could choose a smaller optimal amount of amenity compared to the symmetrical equilibrium. The opposite is true if influencers groups have high enough perceptions versus the non-influencer group. Finally, as before, if the fraction $\frac{\sigma^C n^C}{\sigma^L n^L + \sigma^R n^R}$ of the sum of influencers' perceptions equals non-influencer's perception, the quantity of amenity is the same as the symmetrical allocation. Again, note the structure of the fraction.

From expressions 17 and 20, it is possible to deduce that knowledge of influencers and non-influencers' perceptions is important to anticipate the final contribution to finance the amenity. Apparently, G is responsible of knowing about groups' perceptions. However, G is a passive element subjected to pressures. G merely selects a contribution depending of the menu of pressures offered by the groups. The pressure groups are the ones who influence G. They are the ones who decide how and how much pressure, and, as has been indicated, whenever these pressures are truthful the groups' perceptions will reflect the members' preferences. In this manner, the problem of type identification is transferred from G to the groups, who are responsible of pressuring for their own amenity perception, given others groups' amenity perceptions.

3. Pressure groups' perceptions

Even though perceptions are essential in order to define how much amenity will be supplied in the economy, so far nothing has been said about what pressure groups' perceptions are. To start, different results are Nash equilibria and, particularly, Truthful Nash Equilibria. That is, as has been mentioned before, strategies of pressure groups reflect their actual members' preferences for their preferred amount of amenity. In this line of ideas, parameter θ^i signals the i^{th} group's preferences. In a practical sense, θ^i could be understood as the willingness to contribute of the i^{th} group in accordance with its tastes, wishes, beliefs, identities or any other perceptions about the amenity.

Consider a specific public good. For instance, a public park in a town. It would not be illogical to suppose that in this town different individuals have different perceptions about the park. Further, suppose that every group in town values the amenity positively (i.e. $\theta^i > 0$).[24] First, a Leftist group

[24]This assumption is valid because, by definition, public goods increase welfare for all people, despite some groups disdain for them. Additionally, positive perceptions eliminate the existence of an "exaggerator" group. If a group

perceives the park as an improvement for the quality of life in the town, and believes it is good for the general welfare, as the park offers more possibilities for hedonic activities. Second, a Rightist group, even though it considers the park essentially good, also feels the amenity could reduce resources for more profitable projects and could motivate loitering or attract undesirable visitors. Lastly, there is a Center group that considers the park to have a low impact on quality of life and that it would only benefit the population of a part of town. This group does not believe that the park has its downsides, as the Rightist group does, but it is not as excited as the Leftist group.

Perceptions about the park signal the actual preferences of individuals in the town. Thus θ is a manifestation of how much interest there is for the park. It is expected for the Leftist group to have a higher θ, while the Rightist group's θ is the lowest, and the Center has a θ in between them. Interestingly, θ is not necessarily private knowledge. Think of your neighborhood. Do you not have some idea about your neighbors' general perceptions? At the level of group associations, knowledge about others' perceptions improves. Visualize, for instance, a group called "Friends of Laissez Faire" what would you say its ideals are? Can you not glean some notion about this group's interests?

Government does not require knowledge of individual preferences. Nonetheless, it can obtain signals through the specific way in which a group aims to pressure for the park.[25] Suppose the Leftist group organizes a free artistic festival in the location where the park will be built. What do you think is this group's perception about the objective of the park? A week later, the Rightist group presents figures and data showing loitering and robbery associates to the locations of public parks in different cities. How does this group perceive the park? And what about the Center group? Assume it is an association of immigrants who consider the park would be a welcome addition and they set up some meetings to inform other immigrants about the park's advantages and disadvantages. Again, like in the Left and Right groups, it is possible to infer the Center group's perceptions about the park. Suppose every group knows about the activities of the other groups. Obviously, the Leftist group understands the Rightist group does not want a big park and the Center group is somewhat skeptical or neutral. The Rightist group, meanwhile, knows the park appeals to the first group and that the Center group does not lose sleep over the park. Similarly, the Center group thinks people in the Leftist group are very enthusiastic while those in the Right are reticent.

Nevertheless, equations 16 and 19 suggest that a more precise knowledge of groups' perceptions is necessary. This paper assumes θ is a positive non-zero real number that indicates a group's perception. In practice, this perception could be measured using contingent valuation surveys, willingness to pay polls, etc. However, the model allows us to infer θ in a more intuitive and practical way.

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considers the amenity as a bad they could have incentives to exaggerate their perception in order to reduce the contribution in equations 13, 15 or 18. Later it will be shown that positive perceptions reduce this incentive.

[25]This knowledge is valid for groups too.

Suppose the government is not sure about how much park to provide, but it decides that the quantity of park depends on the total amount of individual contributions. In this case a new problem arises: what will be the contribution amount? Then, the government tries to obtain knowledge about how much people want to contribute suggesting the necessity of the park and waiting for the public to react. Suppose every group has a well-defined willingness to contribute. Assume the Leftist group decides to pressure for its willingness to contribute.

Because pressure is public knowledge, supporting this contribution suggests to the Rightist and Center groups notions about the Leftist's perception. Thus, they can anticipate (from equation 15) what the final contribution would be if the Leftist group imposes its perceptions. Consequently, the Rightist group has to decide whether it supports a different contribution or not. Suppose the Right considers the Left's willingness to contribute is too high (from equation 16) and decides to strive for a lower contribution. Analogously, the Center group anticipates the contribution if the Left and Right influence the decision of the government (from equation 18). Additionally, the Center finds that the Left and Right groups' willingness to contribute is different to its own perception (from equation 19). For this reason, it decides to pressure for a τ that better reflects its own willingness to contribute.[26]

This way, through different groups' pressures, the government now has enough information to make its decision. If pressure groups can influence the government symmetrically the optimal contribution is given by equation 13. If influences are not symmetrical, the contribution is given by the equations 15 or 18 and, depending on the relationship between groups' perceptions (according to equations 16 or 19, respectively), the asymmetrical equilibrium contribution will be equal, higher or lower than the symmetrical equilibrium contribution.

Bernheim and Whinston (1986), and Grossman and Helpman (1994) prove that in menu auctions models groups do not have incentives to distort their pressures, contingent on a value of τ. They show that under certain conditions pressures always reflect real preferences. For this reason, the Nash equilibria are truthful. An interesting consequence of the truthful Nash equilibria is derived from the following reasoning: The perceptions are understood as the willingness to contribute. Pressures are truthful in the contributions. Thus, perceptions and pressures are related through the contributions. For this reason, it is expected that perceptions reflect real preferences too.[27]

An intuitive analysis of this idea can be obtained from equations 13, 15 and 18 where the perceptions define the amount of contribution. Should there be concern about an "exaggerator" group? Could a group exaggerate or understate its actual perceptions? Equations 13, 15 and 18 show τ is positively related with groups' perceptions. In this case the Left group could exaggerate

[26] Remember this example considers that groups' perceptions reflect their willingness to contribute.
[27] Bernheim and Whinston's (1986) common agency model is developed from Clarke's (1971) and Groves's (1973) mechanisms. In these mechanisms individuals do not have incentives to lie. Thus, Bernheim and Whinston's (1986) menu auction model is built in a way that it mimics standard auctions process, where bidders do not have incentives to distort their bids. Remember that, in an auction, the bids signal preferences for a good. Here, the perceptions signal the preferences for the public amenity.

θ^L while the Right could understate θ^R. However, equation 1, and $\frac{d^2A}{d\tau^2} < 0$ indicate that the higher τ, the higher the opportunity cost. Consequently, the Left group will be careful not to reveal a θ^L that is higher than its actual perception. Meanwhile, the Right group, with $\theta^R > 0$, increases utility with the quantity of amenity. Thus, despite a lower perception than the Left, the Right understands that a lower than actual θ^R could imply a lower τ and, accordingly, a quantity of amenity lower than they would like. In practical terms this implies that the Left and the Right do not have incentives to pressure for contributions higher or lower, respectively, than their actual perceptions.

This analysis intuitively supports the existence of both a truthful pressures condition and a non-distorted perceptions condition in the model. However, in the case of perceptions, this condition it is not sufficient. Hence, it is necessary to assume that perceptions are strictly positive in order to avoid incentives for a group to be an "exaggerator". For example, if the Right considers the amenity as an evil, its perception is $\theta^R < 0$. This denotes that the Right's willingness to contribute is negative (or null) and it will pressure for a negative or a least zero contribution. In this case, it has incentives to understate (or exaggerate negatively) its perception. These incentives disappear when the Right perceives the amenity as a good.

Every pressure group understands that the final contribution does not depend on itself, but that it rather depends on the interaction with other groups' perceptions and that this might motivate them to misrepresent its perceptions. However, the magnitude of this distortion is limited by what has been described before and by the groups' information about the other groups' perceptions. It has been mentioned before that perceptions are not necessarily private knowledge, that a group could have some idea about other groups' perceptions. Grossman and Helpman (2001) describe interest groups as social institutions with a wealth of information about their concerns. Interest groups obtain easier and cheaper information compared to individuals and policymakers. Groups are permanently trying to get relevant information from different sources. For this reason, it is feasible to assume that a pressure group has information about others groups, and that every group can use this information to regulate any possible distortion from other groups.

In this model it is expected that every group has enough information about other group's perceptions, such that this information could be used to reduce, still more, any incentive that other groups could have to distort their perceptions.

4. The degree of substitutability

Conceptually, the group's degree of substitutability (GDS) between the amenity and the club good is equivalent to the group's perception for the amenity. Both are determined in accordance with the tastes, wishes, beliefs or identities of every group. The GDS, similarly to perceptions, could be measured via questions about the club good in contingent valuation surveys or in willingness to pay polls. Despite the similarities, the amenity and the club good have different interpretations. In the model, the GDS offsets or reinforces the perceptions when the GDS is high or low, respectively. The groups agree with this balance because everyone will mitigate its own perceptions if they realize that they, and other groups, consider the club good as a feasible near substitute; and this balance increases the perceptions if the amenity does not have substitutes. Every group understands that the social need of the amenity is lower than it would be if it had a far substitute.

The GDS, similarly to group's perceptions, has the same distortionary problems. Nevertheless, what has been mentioned about the existence of non-distorted perceptions can be extended to the GDS too. Additionally, if a group can obtain information about other groups' perceptions, it is reasonable to assume that it can obtain information about other groups' GDS without incurring in significant costs; and utilize this to regulate any possible distortion in the degree of substitutability of the other groups.

5. Influences and pressures

From the model it is evident that, as groups' interaction induces to reveal true preferences for the park, social welfare equilibrium is conditioned in the capacity of groups to influence the government. The model shows that pressuring is a necessary, but not a sufficient condition, so that a group can influence the government's decision. If the government does not pay attention to the pressure from a specific group, this group cannot influence by pressure the final decision and the result will be given by equations 15 or 18, and the corresponding contribution (compared to the social welfare symmetrical equilibrium) will depend on equations 16 or 19, respectively.

Pressure could take many forms: Festivals, voting, campaign contributions, advertising, lobbying, etc. In a democracy, whether a group's members vote is important too for the group's influence. Nevertheless, it is not important how loud or bright the pressure is if the decision maker cannot or does not like to afford it special consideration. Additionally, a group's pressure could quiet down or obscure from the government's ears or eyes another group's claims. However, although pressures and influences seem to be different problems, in practice they are related. If a group feels it is a non- influencer (because many members cannot vote, for example) it would not pressure because either pressuring or not pressuring will not change the result. Consequently, in the real world, it is fundamental for the legislature to define rules that avoid discrimination, social exclusion, and lack of representativeness of certain social groups.

This model assumes that a group always pressures, regardless of whether it is an influencer or not. This assumption is important in order to assure that a group seeks to reveal its perceptions. A group should be motivated to battle for its willingness to contribute. It should not feel as if it is a waste of time. It should be confident that the result will be one if they fight and another if they do not. After all, information about the strength of influences belongs to the government. A priori, a group fights without knowing how much it will influence the final decision. In this model, groups could only obtain some information about its influences after the final decision. In practice, many people fight battles that are lost before even being fought. The relevant point here is that they have the freedom to express their convictions, wishes, frustrations and, as is required in this model, their willingness to contribute.

The stronger property of this model is that every political equilibrium is socially efficient. This result requires pressures to be compensated. This means that pressure should compensate any change in utility derive from a change in contribution. In practice this implies a group only pressure if a desired or established group's utility is preserved. We can think that groups never will pressure if that reduce group's welfare considerably. Compensation requires considering pressure contradictorily. On a size, pressure entails reductions in utilities and group's wealth. That is, pressure has negative effects on a group's welfare. Pressure requires enforcement, energy, resources, time, etc. Pressure entails opportunity costs. However, pressure has a positive effect on the group's welfare because it improves the odds of obtaining a more favorable policy to the group's interests.

By definition, compensated pressure requires pressure to be positive. In standard models of SIG, pressure takes the form of campaign contributions. Therefore, assuming positive pressure becomes very intuitive. A positive contribution means that a group is not subsidized for pressure. Then, if a group contributes, it loses money and the group knows that pressure involves incurring in costs and, consequently, choose to minimize them. This behavior supports the Truthful Nash Equilibria. If a group receives money to pressure it behaves in order to maximize this money, distorting group's preferences in the process. Therefore, positive campaign contributions are transfer of money from groups to social planner.[28]

Nevertheless, in this paper we have a general idea about pressure. The intuition here, as in the case of campaign contributions, is that groups does not perceive pressure as a good by itself. If this is the case a group pressure how a target not as a tool to fight for its interests. A positive pressure means that there is a transfer of wellbeing from the groups to the policy maker not the contrary. A pressure positive denotes that groups incur in a cost to fight, while the policy maker receive a gain from this group's fighting. In practice, a positive pressure could be a festival which is very hard to make for a pressure group, but bring knowledge to the policy maker about the preferences of this group.[29]

6. The Samuelson case

The 1954 Samuelson's condition established a decision rule that defines much of the following public good literature. The condition is about a benevolent social planner interested in social welfare. Samuelson's rule assume an isolate and completely informed decision maker. There is no chance for strategy behavior between economic agents. Also, the policymaker's decision is

[28] With a quasi-linear utility function this implies a utility transfer from groups to the police maker. Dixit *et al* (1997) employ a general utility function where any transfer is possible.

[29] In this sense a riot could be a positive pressure too. However the important thing here is that the government experiment a gain from the pressure and pressure groups do not enjoy greatly perturbing social peace.

keeping out of any social structure that could enclose different perceptions and interests about specific policies. Then social planner is influence and pressure free.

Under this perspective, it is possible consider Samuelson`s condition as a special case of the more general model developed in this paper. To show that, let us to employ the Laffont's (1988) public good provision model. There the social planner problem is:[30]

$$\max \sum_{i=L,C,R} \beta^i V^i\left(y^i, A\right)$$

$$s.t. \ \sum_i m^i - p_y \sum_i y^i - p_y \sum_i n^i \tau = 0 \tag{21}$$

$$A = A(\tau)$$

$$\beta^i \geq 0$$

Using Laffont's assumptions we assume the contribution τ has the form of private good used as a public good's input. Thus the Langrange function could be:

$$\mathcal{L} = \sum_{i=L,C,R} \beta^i V^i\left(y^i, A\right) - \lambda\left(p_y \sum_i y^i - p_y \tau \sum_i n^i - \sum_i m^i\right) - \mu\big(A - A(\tau)\big) \tag{22}$$

First Order Conditions

$$\beta^i \frac{\partial V^i}{\partial y^i} - \lambda p_y = 0 \quad \forall\, i = L, C, R$$

$$\sum_i \beta^i \frac{\partial V^i}{\partial A} - \mu = 0 \tag{23}$$

$$-\lambda p_y \sum_i n^i + \mu \frac{dA}{d\tau} = 0$$

After some no complicate algebra we have:

$$\sum_{i=L,C,R} \frac{\frac{\partial V^i}{\partial A}}{\frac{\partial V^i}{\partial y^i}} = \frac{1}{dA/d\tau} \sum_{i=L,C,R} n^i \tag{24}$$

[30] Notation has been adapted to the particular one of this paper.

In Laffont's model, contribution is considered as the total of private good used as a public good's input. For these reason here is convenient to assume $\sum_i n^i = 1$. This implies:

$$\sum_{i=L,C,R} \frac{\frac{\partial V^i}{\partial A}}{\frac{\partial V^i}{\partial y^i}} = \frac{1}{dA/_{d\tau}} \tag{25}$$

This is equivalent to the Samuelson´s (1954) optimality condition.

Now, we can use the model develop in this paper to show under which assumptions its political equilibrium becomes in a Samuelson's equilibrium.

First assumption: In equation 1 assume there is no a club good, Thus, indirect utility function 4 acquires the following specific form:

$$W^i(\tau) = V^i\left(y^i(\tau), A(\tau)\right) - \gamma^i P^i(\tau) \tag{26}$$

Therefore, the social planner's problem in equation 8 is:

$$\max_{\tau} \sum_i \left(\alpha + \frac{I^i}{\gamma^i}\right) V^i\left(y^i(\tau), A(\tau)\right) \tag{27}$$

First Order Condition

$$\sum_i \left(\alpha + \frac{I^i}{\gamma^i}\right) \frac{\partial V^i\left(y^i(\tau), A(\tau)\right)}{\partial \tau} = 0 \tag{28}$$

With

$$\frac{\partial V^i\left(y^i(\tau), A(\tau)\right)}{\partial \tau} = \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau} + \frac{\partial V^i}{\partial A}\frac{dA}{d\tau} \tag{29}$$

From equation 28

$$\sum_i \left(\alpha + \frac{I^i}{\gamma^i}\right) \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau} + \frac{dA}{d\tau}\sum_i \left(\alpha + \frac{I^i}{\gamma^i}\right) \frac{\partial V^i}{\partial A} = 0 \tag{30}$$

Reordering

$$-\frac{\sum_i \left(\alpha + \frac{I^i}{\gamma^i}\right) \frac{\partial V^i}{\partial A}}{\sum_i \left(\alpha + \frac{I^i}{\gamma^i}\right) \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau}} = \frac{1}{dA/_{d\tau}} \tag{31}$$

Second assumption: if $I = 0$ or $I = 1$, and $\gamma^i = 1 \; \forall \; i = L, C, R$. Which are the special case when influences are symmetrical we have

$$-\frac{\sum_i \frac{\partial V^i}{\partial A}}{\sum_i \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau}} = \frac{1}{dA/_{d\tau}} \tag{32}$$

Assuming $y(\tau) = \sum_{i=L,C,R} y^i(\tau)$ we have

$$\frac{\partial y(\tau)}{\partial \tau} = \sum_{i=L,C,R} \frac{\partial y^i(\tau)}{\partial \tau} \tag{33}$$

Third assumption: The government faces a budget restriction. Assuming $m = \sum_{i=L,C,R} m^i$ from budget restriction in equation 21 we have

$$m - p_y y(\tau) - p_y \sum_i n^i \tau = 0 \tag{34}$$

Taking derivatives respect to contribution

$$\frac{\partial y(\tau)}{\partial \tau} = -\sum_i n^i \tag{35}$$

This implies:

$$\sum_{i=L,C,R} \frac{\partial y^i(\tau)}{\partial \tau} = -\sum_i n^i \tag{36}$$

Fourth assumption: If $n = \sum_i n^i$, the i^{th} derivative of private good consumption respect to contribution is expected to be constant. In this manner, assume $\frac{\partial y^i(\tau)}{\partial \tau} = -\beta_i$, with $\beta_i \geq 0 \; \forall i = L, C, R$. From equation 36

$$-\sum_i \beta_i = -\sum_i n^i \tag{37}$$

Taking the denominator of left member in equation 32, and applying the fourth assumption

$$\sum_i \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau} = -\sum_i \frac{\partial V^i}{\partial y^i}\beta_i \tag{38}$$

From third assumption and equation 23

$$\beta^i \frac{\partial V^i}{\partial y^i} = \lambda p_y \;\; \forall i = L, C, R. \tag{39}$$

Applying equation 39 in equation 38

$$\sum_i \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau} = -\sum_{i=L,C,R} \lambda p_y = -3\lambda p_y = -3\beta^i \frac{\partial V^i}{\partial y^i} \tag{40}$$

Fifth assumption: If specifically we assume $\beta_i = \frac{n}{3}$, equation 38 becomes

$$\sum_i \frac{\partial V^i}{\partial y^i}\frac{\partial y^i}{\partial \tau} = -n \frac{\partial V^i}{\partial y^i} \tag{41}$$

Replacing equation 41 in equation 32

$$-\frac{\sum_i \frac{\partial V^i}{\partial A}}{-\frac{\partial V^i}{\partial y^i}} = \frac{1}{{dA}/{d\tau}} n \tag{42}$$

In equation 24 we assume that $n = \sum_i n^i = 1$, If we maintain this assumption equation 42 converts

$$\sum_{i=L,C,R} \frac{\frac{\partial V^i}{\partial A}}{\frac{\partial V^i}{\partial y^i}} = \frac{1}{{dA}/{d\tau}} \tag{43}$$

This is the same Samuelson's optimal condition as in equation 25.

In consequence, when interest groups are considered, always their influences are symmetrical and utility's weights (in the social planner's objective function) are equal,[31] it can be showed that the political equilibrium of public goods satisfies the Samuelson's rule.

IV. Conclusions

This paper exposes a simple model where the interaction of interest groups determines the amount of contribution required to finance a public good. Interestingly, the model sketches mechanism such that an optimal and socially efficient provision is acquire always groups offers compensate pressure schedule, and the social planner shows concern for the welfare of every group in the economy. This result is more general compared to standard public good literature. Additionally, some indirect empirical evidence appears to support the analytical results of the model.

[31] And a few other not very restrictive assumptions.

Nevertheless, the model could be improved by introducing discrete analysis rather than the continuous analysis that has been used. It would also be of interest to focus on developing a model in which interest groups can change the form in which the public good is provided.[32] Here the interaction of interest groups determines how much public good is financed, irrespective of how the good is produced or by whom. However, a step further would be to investigate the decision between public versus private production when groups interact. [33]

Moreover, our model considers public goods with non-exclusion in consumption. But, what happens if we assume that private production implies exclusion, as in club goods?[34] For instance, consider a city where some residents pressure for amenities financed by prices-for-use. What are the consequences in terms of social discrimination, urban segregation, social conflict or economic performance of the city? Urban planning policies could be affected due to selfish interests or exaggerated well-being concerns, neither of which take into account the externalities that can ensue.

Appendix

In equation 1 the i^{th} group chooses the quantities of private and club goods that solve its consumer problem. However, this decision does not include any group's choice about public good and pressure because of these variables are considered exogenous to group's decision. Equation 4 provides the maximum i^{th} group's welafare under these conditions. Thus, regardless pressure the i^{th} group waits get at less an utility level V^i for a given value of τ. This is valid result when group does not define individually the contribution τ

Now let us consider that i^{th} group is worry about both $V^i$and τ. It knows that contribution bounds the amenity level A and consequently its utility level V^i. In this case i^{th} group is interested in gain some control about contribution τ and understand that given other groups interests it need to defend a specific contribution. Knowing that the government establish the contribution the i^{th} group design a pressure schedule contingent to contribution. Consider that on a first stage before the social planner chooses contribution the i^{th} group should announce its pressure schedule. This schedule would be that regardless contribution a utility V^i is get it. Because of Amenity affects utility and contribution affects amenity when pressure groups aim design its schedule it is trying to solve the following problem[35]

$$\max_{\tau} W^i = V^i(p_y, p_x, \sigma^i; \tau) - \gamma^i P^i(\tau) \tag{1A}$$

Because $P^i(\tau)$ is assumed as a derivable function the First Order Condition is

[32] In a way that extends and generalizes the analysis of Padon (1999).
[33] Le Breton and Zaporozhets (2010) develop a model where a legislature has to choose between two alternatives while it faces pressures from two opposing lobbies.
[34] This is the type of analysis proposed by Demsetz (1970).
[35] We follow Dixit's *et al* (1997) Corollary 1 to Proposition 4, and Grossman and Helpman (2001) chapter 8.

$$\frac{\partial V^i}{\partial \tau} - \gamma^i \frac{\partial P^i}{\partial \tau} = 0 \qquad (2A)$$

Where

$$\frac{\partial P^i}{\partial \tau} = \frac{1}{\gamma^i}\frac{\partial V^i}{\partial \tau} \qquad (3A)$$

$\forall\ i = L, C, R.$

Thus i^{th} group offers to the social planner a pressure schedule $P^i(\tau)$ such that equation 3A is satisfied.

On the second stage, from equation 5 the government`s problem is given by

$$\max_{\tau} G = \sum_{i=L,C,R} \alpha V^i(\tau) + \sum_{i=L,C,R} I^i P^i(\tau)$$

The First Order Condition is

$$\sum_{i=L,C,R} \alpha \frac{\partial V^i(\tau)}{\partial \tau} + \sum_{i=L,C,R} I^i \frac{\partial P^i(\tau)}{\partial \tau} = 0 \qquad (4A)$$

Remember $P^i(\tau)$ is the group's pressure schedule contingent to τ. Thus, any choice of τ, such that equation 5A is satisfied, should satisfies equation 3A too. Therefore, substituting equation 3A in equation 5A we have:

$$\sum_{i=L,C,R} \alpha \frac{\partial V^i(\tau)}{\partial \tau} + \sum_{i=L,C,R} \frac{I^i}{\gamma^i} \frac{\partial V^i(\tau)}{\partial \tau} = 0 \qquad (5A)$$

Consequently, equation 4A can be rewritten as:

$$\max_{\tau} G = \sum_{i=L,C,R} \alpha V^i(\tau) + \sum_{i=L,C,R} \frac{I^i}{\gamma^i} V^i(\tau)$$

or

$$\max_{\tau} G = \sum_{i=L,C,R} \left(\alpha + \frac{I^i}{\gamma^i}\right) V^i(\tau) \qquad (6A)$$

Equation 6A is the same Equation 8.[36]

[36] See Dixit *et al* (1997), and Grossman and Helpman (2001) for a very rigorous theoretical supporting of this simple prove.

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