Reconditioning the conditional

[Recondicionando o condicional]

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This paper was written in May 2014 for a planned *Festschrift* for Patrick Suppes. Sadly, he died on 17 November. The volume was eventually published in October 2015 by College Publications (London) as the 28th volume of the series *Tributes*, under the title *Conceptual Clarifications. Tributes to Patrick Suppes (1922–2014)*. The paper, omitting the opening dedication to Suppes, is reproduced here by arrangement with the editors, J.-Y. Béziau, D. Krause, & J. R. B. Arenhart. The main idea of the paper (that some indicative conditionals are better understood in terms of deductive dependence than in terms of probability) was mentioned during my presentation 'On Deductive Dependence' at the meeting Uncertainty: Reasoning about Probability and Vagueness held at the Academy of Sciences of the Czech Republic in September 2006. The details were worked out during a visit to the University of Sassari in the spring of 2013. Warm thanks are due to my Sardinian audience, and also to Alan Hájek and Richard Bradley, who commented on an earlier version of the paper.

Resumo: Muitos autores tentaram compreender as construções condicionais indicativas da linguagem cotidiana por meio do que é geralmente chamado de probabilidade condicional. Outros autores procuraram o sentido das probabilidades condicionais em termos das probabilidades absolutas de sentenças condicionais. Apesar de todas essas tentativas terem sido frustradas pelos teoremas de trivialidade, de Lewis (1976), têm havido copiosas tentativas subsequentes tanto para resgatar a CCCP (*a interpretação condicional da probabilidade condicional*) quanto para alargar e intensificar os argumentos contra ela. Neste artigo será mostrado que a trivialidade é evitável se a função de probabilidade for substituída por uma generalização alternativa da relação de dedutibilidade, a saber, a medida de *dependência dedutiva* de Miller & Popper (1986). Se sugerirá, ainda, que este modo alternativo de orquestrar as construções condicionais está em harmonia com o teste proposto em Ramsey (1929), e também com a ideia de que a questão principal não é o valor veritativo de uma sentença condicional, mas sim a sua asseverabilidade ou aceitabilidade.

Palavras-chave: Condicionais indicativos, Probabilidade, Trivialidade, Dependência dedutiva, Adams, Lewis, Ramsey, Stalnaker.

Abstract: Many authors have hoped to understand the indicative conditional construction in everyday language by means of what are usually called conditional probabilities. Other authors have hoped to make sense of conditional probabilities in terms of the absolute probabilities of conditional statements. Although all such hopes were disappointed by the triviality theorems of Lewis (1976), there have been copious subsequent attempts both to rescue CCCP (*the conditional construal of conditional probability*) and to extend and to intensify the arguments against it. In this paper it will be shown that triviality is avoidable if the probability function is replaced by an alternative generalization of the deducibility relation, the measure of *deductive dependence* of Miller & Popper (1986). It will be suggested further that this alternative way of orchestrating conditionals is nicely in harmony with the test proposed in Ramsey (1929), and also with the idea that it is not the truth value of a conditional statement that is of primary concern but its assertability or acceptability.

Keywords: Indicative conditionals, Probability, Triviality, Deductive dependence, Adams, Lewis, Ramsey, Stalnaker.

1 Degrees of Deducibility

Since the time of Bolzano (1837), if not earlier, it has been appreciated that, when $\mathfrak{p}$ is a probability measure, the identity $\mathfrak{p}(\mathbf{c}\,|\,\mathfrak{a}) = 1$ is a necessary, but generally insufficient, condition for the deducibility in classical logic of the conclusion $\mathbf{c}$ from the assumption(s) $\mathfrak{a}$. What has been less often recognized is that there are other legitimate ways in which *degrees of deducibility* may be measured. In particular, since $\mathbf{c}$ is deducible from $\mathfrak{a}$ if and only if $\mathfrak{a}'$ is deducible from $\mathbf{c}'$ (here the prime stands for negation), the identity $\mathfrak{p}(\mathfrak{a}'\,|\,\mathbf{c}') = 1$, which is not equivalent to $\mathfrak{p}(\mathbf{c}\,|\,\mathfrak{a}) = 1$, also gives a necessary condition for the deducibility of $\mathbf{c}$ from $\mathfrak{a}$. There are a number of other interesting possibilities, which I shall elaborate on elsewhere, but they are not the concern of this paper.

A few historical remarks about the function $\mathfrak{q}(\mathbf{c} \mid \mathfrak{a}) = \mathfrak{p}(\mathfrak{a}' \mid \mathbf{c}')$ are offered in § 8 below. Following Miller & Popper (1986), § 1, we shall call $\mathfrak{q}(\mathbf{c} \mid \mathfrak{a})$ the *(degree of) deductive dependence* of the statement $\mathbf{c}$ on the statement $\mathfrak{a}$, where $\mathbf{c}$ is typically the conclusion of an inference from the assumption(s) or premise(s) $\mathfrak{a}$. Although, as just noted, $\mathfrak{q}(\mathbf{c}\,|\,\mathfrak{a})$, like $\mathfrak{p}(\mathbf{c}\,|\,\mathfrak{a})$, equals 1 when $\mathbf{c}$ is deducible from $\mathfrak{a}$, the two functions take the value 0 in different circumstances. Whereas $\mathfrak{p}(\mathbf{c} \mid \mathfrak{a}) = 0$ when $\mathbf{c}'$ is deducible from $\mathfrak{a}$ (provided that $\mathfrak{a}$ is consistent), that is, when $\mathfrak{a}$ and $\mathbf{c}$ are mutual *contraries*, $\mathfrak{q}(\mathbf{c} \mid \mathfrak{a}) = 0$ when $\mathbf{c}$ is deducible from $\mathfrak{a}'$ (provided that $\mathfrak{a}'$ is consistent), that is, when $\mathfrak{a}$ and $\mathbf{c}$ are mutual *subcontraries*. In other words, $\mathfrak{q}(\mathbf{c}\,|\,\mathfrak{a})$ assumes the value 1 when $\mathbf{c}$ is deductively wholly dependent on $\mathfrak{a}$, in the sense of being deducible from $\mathfrak{a}$, and the value 0 when $\mathbf{c}$ is deductively wholly independent of $\mathfrak{a}$, in the sense of having only tautological consequences in common with $\mathfrak{a}$. (This relation of deductive independence is closely related to *maximal independence*, as defined by Sheffer, 1926.) The interpretation of the function $\mathfrak{q}$ as a measure of deductive dependence is encouraged by the fact that, if the familiar function $1 - \mathfrak{p}(\mathfrak{b}) = \mathfrak{p}(\mathfrak{b}')$ is adopted as a measure of the (informative) *content* $\mathfrak{ct}(\mathfrak{b})$ of the statement $\mathfrak{b}$, and if $\mathfrak{ct}(\mathbf{c}) \neq 0$, then

$\mathfrak{q}(\mathfrak{c} \,|\, \mathfrak{a})$ is equal to $\mathfrak{ct}(\mathfrak{c} \vee \mathfrak{a})/\mathfrak{ct}(\mathfrak{c})$, the 'proportion' of the content of $\mathfrak{c}$ that resides within the content of $\mathfrak{a}$ (Hilpinen, 1970, p. 110; Miller & Popper, 1986; Miller, 1994, Chapter $10.4c$).

Although the deductive dependence function $\mathfrak{q}$ has been defined above in terms of the probability function $\mathfrak{p}$, this is not supposed to attribute to $\mathfrak{p}$ any conceptual priority. A more correct treatment would begin with an abstract measure $\mathfrak{m}$, and define each of $\mathfrak{p}$ and $\mathfrak{q}$ from $\mathfrak{m}$. But we forgo such niceties here.

2 Formalities

The function $\mathfrak{p}$ is required to satisfy the axiom system of Popper (1959), appendix $*$v, which is based on the operations of negation $'$ and conjunction (inconspicuously represented by concatenation). A dual axiomatic system for the function $\mathfrak{q}$, based on the operations $'$ and $\vee$, is presented in Miller & Popper (1986), § 2. In these systems the terms $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a})$ and $\mathfrak{q}(\mathfrak{c} \,|\, \mathfrak{a})$ are well defined for every $\mathfrak{a}, \mathfrak{c}$, including the contradiction $\bot$ and the tauto-logy $\top$. Indeed, $\mathfrak{p}(\mathfrak{c} \,|\, \bot) = 1 = \mathfrak{q}(\top \,|\, \mathfrak{a})$ for every $\mathfrak{a}$ and $\mathfrak{c}$. The usual addition or complementation law of probability therefore fails in general, since $\mathfrak{p}(\mathfrak{c} \,|\, \bot) + \mathfrak{p}(\mathfrak{c}' \,|\, \bot) = 2$. But it holds when the second argument of $\mathfrak{p}$ is not the contradiction $\bot$. Other theorems of the systems will be cited, without much proof, when they are needed. In interpreting Popper's system it is safe to restrict attention to functions $\mathfrak{p}$ for which $\forall \mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{b}) \geq \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})$ if and only if $\mathfrak{c}$ is deducible from $\mathfrak{a}$. (Since $\mathfrak{c}$ is deducible from $\mathfrak{a}$ if and only if $\mathfrak{a}'$ is deducible from $\mathfrak{c}'$, the deducibility of $\mathfrak{c}$ from $\mathfrak{a}$ can evidently be characterized also by $\forall \mathfrak{b}\, \mathfrak{q}(\mathfrak{b} \,|\, \mathfrak{c}) \leq \mathfrak{q}(\mathfrak{b} \,|\, \mathfrak{a})$.) It follows that $\mathfrak{a}$ and $\mathfrak{c}$ are interdeducible if and only if they are *probabilistically indistinguishable*: that is, $\forall \mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{b}) = \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})$. It should be recorded also that, although $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}) = 1$ is in general insufficient for $\mathfrak{c}$ to be deducible from $\mathfrak{a}$, the formula $\forall \mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}) = 1$ (whose equivalence to the formula $\forall \mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{b}) \geq \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})$ is easily demonstrated within Popper's system[1]) is both necessary and sufficient for deducibility, as is the formula $\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{a}' \,|\, \mathfrak{c}'\mathfrak{b}) = 1$. In other words, $\mathfrak{c}$ is deducible from $\mathfrak{a}$ if and only if $\forall\mathfrak{b}\, \mathfrak{q}(\mathfrak{b} \to \mathfrak{c} \,|\, \mathfrak{a}) = 1$, where the arrow $\to$ represents the material conditional.

3 Conditionals

The appearance here of the material conditional $\mathfrak{b} \to \mathfrak{c}$ in the first argument of $\mathfrak{q}$ may quicken the hope that the substitution of the function $\mathfrak{q}$ for the probability function $\mathfrak{p}$ can in some way shed light on the *problem of indicative conditionals*, one of the most tenaciously unsolved problems of modern philosophical logic, and especially on the hypothesis of the *conditional construal of conditional probability* (facetiously dubbed CCCP by Hájek & Hall, 1994). It is the objective of this paper substantially to consummate this hope. But it should be said at once that the matter is not entirely straightforward. Pretty well the simplest form of the CCCP hypothesis worth attending to may be written as the universal identity $\forall\mathfrak{a}\forall\mathfrak{c}\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{a} \rightsquigarrow \mathfrak{c} \,|\, \mathfrak{b}) = \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b})$, according to which the absolute probability of the indicative conditional *if* $\mathfrak{a}$ *then* $\mathfrak{c}$ in ordinary language, here shortened to $\mathfrak{a} \rightsquigarrow \mathfrak{c}$, is equal to the conditional probability of $\mathfrak{c}$ given $\mathfrak{a}$, not only under the measure $\mathfrak{p}$ but under any measure obtained from $\mathfrak{p}$ by *conditionalization* on the statement $\mathfrak{b}$. We shall see below that this form of the CCCP hypothesis can hold only for the material conditional $\to$, and that when it does hold, the function $\mathfrak{p}$ is necessarily two-valued, and no more than a distribution of truth values (Leblanc & Roeper, 1990). But the identity $\forall\mathfrak{a}\forall\mathfrak{c}\forall\mathfrak{b}\, \mathfrak{q}(\mathfrak{a} \to \mathfrak{c} \,|\, \mathfrak{b}) = \mathfrak{q}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b})$, its analogue in terms of deductive dependence, may be shown to be equivalent to the CCCP hypothe-

[1] If $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{b}) \geq \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})$ for every $\mathfrak{b}$, then $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}) \geq \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{a}\mathfrak{b})$. The latter term equals 1, which is the upper bound of the function $\mathfrak{p}$. It follows that $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}) = 1$. For the converse we may note that, if $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}) = 1$ for every $\mathfrak{b}$, then, by the monotony law for the first argument of $\mathfrak{p}$ and the general multiplication law, $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{b}) \geq \mathfrak{p}(\mathfrak{c}\mathfrak{a} \,|\, \mathfrak{b}) = \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b})\mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b}) = \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})$ for every $\mathfrak{b}$.

sis, and so to force $\mathfrak{q}$ to be two-valued too.[2] Moving from $\mathfrak{p}$ to $\mathfrak{q}$ in this way does little to avoid triviality.

This result notwithstanding, it is the material conditional $\mathfrak{a} \to \mathfrak{c}$ that will be rehabilitated, in § 6 below, in terms of the deductive dependence function $\mathfrak{q}$.

A great deal has been written on various versions of the CCCP hypothesis and, in particular, on the crucial results of Lewis (1976) that show that, in the usual Kolmogorov axiomatizations of probability, the hypothesis is condemned in one way or another to triviality. In § 4 below it will be shown that, within Popper's axiom system, the triviality of the CCCP hypothesis follows from a result in Popper (1963) that is closely related to the theorems of Popper & Miller (1983). I shall not discuss directly the implosion of the CCCP hypothesis in Kolmogorov's systems. Nor shall I attempt to summarize the many extensions to Lewis's results and the many responses that have been made to them. For a useful (if dated) discussion, the reader may consult Hájek & Hall (1994), and other papers in the same volume (Eells & Skyrms, 1994), including Suppes (1994); and for surveys of the principal philosophical and technical problems posed by conditionals, Edgington (2014), Arló-Costa (2014), and the works cited therein. Mention should be made also of Mura (2011), which deepens and corrects the theory of tri-events propounded in de Finetti (1936).

[2] By the definition of $\mathfrak{q}$, the identities $\mathfrak{q}(\mathfrak{a} \to \mathfrak{c} \,|\, \mathfrak{b}) = \mathfrak{q}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b})$ and $\mathfrak{p}(\mathfrak{b}' \,|\, \mathfrak{a}\mathfrak{c}') = \mathfrak{p}(\mathfrak{a}' \vee \mathfrak{b}' \,|\, \mathfrak{c}')$ are equivalent. The hypothesis in question therefore holds if and only if $\forall\mathfrak{a}\forall\mathfrak{c}\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{b}' \,|\, \mathfrak{a}\mathfrak{c}') = \mathfrak{p}(\mathfrak{a}' \vee \mathfrak{b}' \,|\, \mathfrak{c}')$. By simultaneously replacing in this expression $\mathfrak{a}$ by $\mathfrak{b}$, $\mathfrak{b}$ by $\mathfrak{c}'$, and $\mathfrak{c}$ by $\mathfrak{a}'$, suppressing the double negations that materialize, and massaging the quantifiers, we obtain $\forall\mathfrak{a}\forall\mathfrak{c}\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{b}\mathfrak{a}) = \mathfrak{p}(\mathfrak{b}' \vee \mathfrak{c} \,|\, \mathfrak{a})$. By interchanging $\mathfrak{a}$ and $\mathfrak{b}$, and writing $\mathfrak{a} \to \mathfrak{c}$ for $\mathfrak{a}' \vee \mathfrak{c}$, we reach $\forall\mathfrak{a}\forall\mathfrak{c}\forall\mathfrak{b}\, \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}) = \mathfrak{p}(\mathfrak{a} \to \mathfrak{c} \,|\, \mathfrak{b})$, and finally the CCCP hypothesis for $\to$, as announced.

4 Triviality of the CCCP hypothesis

In order visibly not to prejudge the question of whether the connective $\rightsquigarrow$ introduced above is or is not worthy of the title of an indicative conditional, in this section we shall state the CCCP hypothesis in the ostensibly weaker form

$$\mathrm{CCCP}_0 \qquad \forall \mathfrak{a} \forall \mathfrak{c} \exists \mathfrak{y} \forall \mathfrak{b}\ \mathfrak{p}(\mathfrak{y} \,|\, \mathfrak{b}) = \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}).$$

We shall show that within Popper's axiomatic system this universal hypothesis implies that for each $\mathfrak{a}$,$\mathfrak{c}$, the object $\mathfrak{y}$ can only be the material conditional $\mathfrak{a} \to \mathfrak{c}$ and, furthermore, that the value of the function $\mathfrak{p}$ can only be 0 and 1.

We assume that $\mathfrak{b}$ is not the contradiction $\perp$. Using a version of the addition law, then the multiplication law, and finally CCCP_0 twice, we may then derive

$$\begin{aligned}
\mathfrak{p}(\mathfrak{y}\mathfrak{a}' \,|\, \mathfrak{b}) &= \mathfrak{p}(\mathfrak{y} \,|\, \mathfrak{b}) - \mathfrak{p}(\mathfrak{y}\mathfrak{a} \,|\, \mathfrak{b}) \\
&= \mathfrak{p}(\mathfrak{y} \,|\, \mathfrak{b}) - \mathfrak{p}(\mathfrak{y} \,|\, \mathfrak{a}\mathfrak{b})\mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b}) \\
&= \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}) - \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}(\mathfrak{a}\mathfrak{b}))\mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b}) \\
&= \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b})(1 - \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})).
\end{aligned}$$

Using the multiplication law, CCCP_0, and the law $\mathfrak{p}(\mathfrak{c} \mid \perp) = 1$ we may derive

$$\begin{aligned}
\mathfrak{p}(\mathfrak{y}\mathfrak{a}' \,|\, \mathfrak{b}) &= \mathfrak{p}(\mathfrak{y} \,|\, \mathfrak{a}'\mathfrak{b})\mathfrak{p}(\mathfrak{a}' \,|\, \mathfrak{b}) \\
&= \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}(\mathfrak{a}'\mathfrak{b}))\mathfrak{p}(\mathfrak{a}' \,|\, \mathfrak{b}) \\
&= 1 - \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b}),
\end{aligned}$$

by a second use of the addition law (which is valid here since $\mathfrak{b}$ is not $\perp$). It follows that if $\mathfrak{b} \not\equiv \perp$ then $\mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b})(1 - \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})) = 1 - \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})$ for all $\mathfrak{a}$,$\mathfrak{c}$, and hence that $(1 - \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}\mathfrak{b}))(1 - \mathfrak{p}(\mathfrak{a} \,|\, \mathfrak{b})) = 0$ for all $\mathfrak{a}$, $\mathfrak{c}$. Now formula (22) in Addendum 3 of Popper (1963) states without proof (and in different notation) that $(1 - \mathfrak{p}(\mathfrak{c} \,|\, \mathfrak{a}))(1 - \mathfrak{p}(\mathfrak{a}))$ is equal to the value of the arithmetical difference between the probability

$\mathfrak{p}(\mathfrak{a} \to \mathfrak{c})$ and the probability $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$. It may be shown more generally that $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{b}) - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}) = (1 - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}))(1 - \mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}))$ when $\mathfrak{ab} \not\equiv \bot$,[3] which implies that $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{b}) - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}) = 0$ when $\mathfrak{ab} \not\equiv \bot$. But $\mathfrak{ab} \equiv \bot$ implies the deducibility of $\mathfrak{a} \to \mathfrak{c}$ from $\mathfrak{b}$, and hence that $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{b}) = 1 = \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab})$. We conclude that $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{b}) - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}) = 0$ for every $\mathfrak{a},\mathfrak{b},\mathfrak{c}$.

It follows from CCCP_0 above that for all $\mathfrak{a},\mathfrak{c}$, there exists a statement $\mathfrak{y}$ such that $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{b}) - \mathfrak{p}(\mathfrak{y}\,|\,\mathfrak{b}) = 0$ holds for all $\mathfrak{b}$. What this means is that the statement $\mathfrak{y}$ is probabilistically indistinguishable from the material conditional $\mathfrak{a} \to \mathfrak{c}$, in the sense of § 2 above, and thus interdeducible with it. The equation $\mathfrak{p}(\mathfrak{y}\,|\,\mathfrak{b}) = \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab})$ can hold for every $\mathfrak{b}$ if and only if $\mathfrak{y}$ is the statement $\mathfrak{a} \to \mathfrak{c}$.

To show that the function $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$ can take only the values 0 and 1, we may set aside the case of inconsistent $\mathfrak{a}$ (since $\mathfrak{p}(\mathfrak{c}\,|\,\bot)$ always equals 1). We have proved above that if $\mathfrak{b} \not\equiv \bot$ then $(1 - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}))(1 - \mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b})) = 0$, from which it follows that if $\mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}) \neq 1$ then $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}) = 1$ for every $\mathfrak{c}$. In particular, $\mathfrak{p}(\mathfrak{a}'\,|\,\mathfrak{ab}) = 1$. But $\mathfrak{p}(\mathfrak{a}'\mathfrak{a}\,|\,\mathfrak{b}) = 0$ if $\mathfrak{b} \not\equiv \bot$, and so by the multiplication law, $\mathfrak{p}(\mathfrak{a}'\,|\,\mathfrak{ab})\mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}) = 0$. It may be concluded that if $\mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}) \neq 1$ then $\mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}) = 0$.

What is so damaging about these results is not that the only conditional conforming to the CCCP hypothesis is the familiar material conditional, for several authors have held that indicative conditionals are, in their semantics, material conditionals, but that all probabilities have to be either 0 or 1. There is nothing but disappointment for the hope that since 'the abstract calculus [of probability] is a relatively well defined and well established mathematical theory . . . [and i]n contrast, there is little agreement about the logic of conditional sentences . . . [p]robability theory could be a source of insight into [their] formal structure' (Stalnaker, 1970, p. 64). Indeed,

[3] The right-hand side of the equation, $(1 - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}))(1 - \mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}))$, can be expanded, and by the multiplication law shown equal to $1 - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}) - \mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{b}) + \mathfrak{p}(\mathfrak{ac}\,|\,\mathfrak{b})$. By two applications of the addition law, this can be shown equal to $1 - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab}) - \mathfrak{p}(\mathfrak{ac}'\,|\,\mathfrak{b}) = \mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\mathfrak{b}) - \mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab})$.

the recourse to probability is otiose, since a two-valued probability function is no more than an assignment of truth values: we may define $\mathfrak{b}$ to be *true* if $\mathfrak{p}(\mathfrak{b}\,|\,\top) = 1$, and *false* if $\mathfrak{p}(\mathfrak{b}\,|\,\top) = 0$. Matters are actually worse than this, for all true statements turn out to be probabilistically indistinguishable from $\top$, and all false statements probabilistically indistinguishable from $\perp$. This belies the assumption of § 2 that probabilistic indistinguishability ought to coincide with interdeducibility.[4]

The first proof that, in Popper's system, CCCP_0 implies the two-valuedness of $\mathfrak{p}$ was given by Leblanc & Roeper (1990). The present proof dates from about 1992. The Basic Triviality Result of Milne (2003, p. 301f), which is derivable in Kolmogorov's less general (finite) system, is related but less general.

5 Updating and Relativization

One of the factors that has made the CCCP hypothesis attractive is surely the multiple use of the word *conditional* and its cognates. As Hájek & Hall (1994) put it, the hypothesis 'sounds right' (p. 80). What is not always realized, however, is that, aside from the word *conditional* in logic, here endorsed, there are two distinct uses of the words in probability theory. There is the process of (Bayesian) *conditionalization*, the generally agreed way in which a probability distribution is updated on the receipt of new information or new knowledge. There is also the result of applying the probability functor $\mathfrak{p}$ not to a single argument (in the present paper, a statement) but to two arguments, or to one statement relative to another, yielding a binary measure $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$ that is standardly called *conditional prob-*

[4] The two-valuedness of $\mathfrak{p}$ settles the truth table for negation. The other tables need also the addition and monotony laws. For example, by the general addition law, $\mathfrak{p}(\mathfrak{a} \to \mathfrak{c}\,|\,\top) = 0$ if and only if $\mathfrak{p}(\mathfrak{a}\,|\,\top) = 1 - \mathfrak{p}(\mathfrak{ac}\,|\,\top)$. By monotony and two-valuedness, this holds if and only if $\mathfrak{p}(\mathfrak{a}\,|\,\top) = 1$ and $\mathfrak{p}(\mathfrak{ac}\,|\,\top) = 0$. In short, $\mathfrak{a} \to \mathfrak{c}$ is false if and only if $\mathfrak{a}$ is true and $\mathfrak{c}$ is false. The CCCP hypothesis implies that in addition $\mathfrak{a} \to \mathfrak{c}$ is false if and only if $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a}) = 0$. But if $\mathfrak{c}$ is true, $\mathfrak{a} \to \mathfrak{c}$ is true for every $\mathfrak{a}$, and accordingly $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a}) = 1 = \mathfrak{p}(\top\,|\,\mathfrak{a})$ for every $\mathfrak{a}$.

ability. These processes of *updating* and *relativization*, as they will hereafter be called, happen to have the same mathematical effect: the result of updating the singulary measure $\mathfrak{p}$ with the information $\mathfrak{b}$ is the same as relativizing it to $\mathfrak{b}$. It follows that updating $\mathfrak{p}(\mathfrak{c})$ with $\mathfrak{b}$, and then relativizing it to $\mathfrak{a}$, is the same as relativizing $\mathfrak{p}(\mathfrak{c})$ to $\mathfrak{a}$, and then updating it with $\mathfrak{b}$. Since conjunction in the second argument of $\mathfrak{p}$ is commutative, the outcomes $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ba})$ and $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab})$ are identical. Although relativization and updating are therefore formally dead ringers for each other, they deserve to be understood as distinct undertakings. In particular, if $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a}) = r$ is a declaration of relative probability there is no presumption that the statement $\mathfrak{a}$ is known to be true, or even supposed to be true (van Fraassen, 1995, § 2), any more than this is the case in the metalogical declaration $\mathfrak{a} \vdash \mathfrak{c}$. (But the interpretation of $\mathfrak{a}$ as a statement of evidence, and of $\mathfrak{c}$ as a hypothesis, is not excluded.) This is not idle pedantry. With the function $\mathfrak{q}$, the distinction between updating and relativization emerges as a distinction with a difference.

The axiomatic system of Popper (1963) that we adopted in § 2 above is a system of relative probability $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{a})$. It is easy to check that if the function $\mathfrak{p}$ satisfies the axioms, and if $\mathfrak{b} \not\equiv \bot$, then $\mathfrak{p}_{\mathfrak{b}}(\mathfrak{a}\,|\,\mathfrak{c}) = \mathfrak{p}(\mathfrak{a}\,|\,\mathfrak{cb})$ also satisfies them. (The function $\mathfrak{p}_{\bot}$ is identically equal to 1, and violates the axiom that requires the function $\mathfrak{p}$ to have at least two distinct values.) The subscript notation embodied in $\mathfrak{p}_{\mathfrak{b}}$ will be used whenever we wish to refer to the updating of a function with the information $\mathfrak{b}$. Since $\mathfrak{p}_{\mathfrak{b}}(\mathfrak{c}\,|\,\mathfrak{a})$ equals $\mathfrak{p}(\mathfrak{c}\,|\,\mathfrak{ab})$ for every $\mathfrak{a}$, and hence $\mathfrak{p}_{\mathfrak{b}}(\mathfrak{b}\,|\,\mathfrak{a}) = \mathfrak{p}(\mathfrak{b}\,|\,\mathfrak{ab}) = 1 = \mathfrak{p}(\top\,|\,\mathfrak{ab}) = \mathfrak{p}_{\mathfrak{b}}(\top\,|\,\mathfrak{a})$, updating with $\mathfrak{b}$ amounts to a decision to treat $\mathfrak{b}$ as probabilistically indistinguishable from $\top$.

Since $\mathfrak{q}(\mathfrak{c}\mid\mathfrak{a}) = \mathfrak{p}(\mathfrak{a}'\,|\,\mathfrak{c}')$, the updated function $\mathfrak{q}_{\mathfrak{b}}$ is defined by $\mathfrak{q}_{\mathfrak{b}}(\mathfrak{c}\,|\,\mathfrak{a}) = \mathfrak{p}_{\mathfrak{b}}(\mathfrak{a}'\,|\,\mathfrak{c}') = \mathfrak{p}(\mathfrak{a}'\,|\,\mathfrak{c}'\mathfrak{b}) = \mathfrak{q}((\mathfrak{c}'\mathfrak{b})'\,|\,\mathfrak{a})$, which equals $\mathfrak{q}(\mathfrak{b} \to \mathfrak{c}\,|\,\mathfrak{a})$. In general, this term differs from $\mathfrak{q}(\mathfrak{c}\,|\,\mathfrak{ab})$. Updating with $\mathfrak{b}$ is not the same as relativizing to $\mathfrak{b}$. The distinction is especially transparent when the second argument of the function $\mathfrak{q}$ is the tautology $\top$. For except when $\mathfrak{a} \equiv \bot$, the value of $\mathfrak{p}(\bot\,|\,\mathfrak{a})$ is

0 for every probability measure; and therefore $\mathfrak{q}(\mathbf{c}\,|\,\top) = 0$ except when $\mathbf{c} \equiv \top$. (The function $\mathfrak{q}$, unlike the function $\mathfrak{p}$, has an almost flat prior distribution.) Updating $\mathfrak{p}$ to $\mathfrak{p}_{\mathbf{b}}$ does not change matters: $\mathfrak{q}_{\mathbf{b}}(\mathbf{c}\,|\,\top)$ still equals 0 (unless $\mathbf{c} \equiv \top$). But relativization of $\mathfrak{q}(\mathbf{c})$ to $\mathbf{b}$ yields $\mathfrak{q}(\mathbf{c}\,|\,\mathbf{b})$, which may well not be 0.

6 The Reconditioned Conditional

Armed with these considerations we are at last in a position to understand how and why the replacement in the CCCP hypothesis of the probability measure $\mathfrak{p}$ by the deductive dependence measure $\mathfrak{q}$ makes such a dramatic difference. The first formula displayed below is CCCP_0, exactly as it was displayed in § 4. The formula CCCP_1 is a notational variant, obtained from CCCP_0 by writing $\mathfrak{p}_{\mathbf{b}}(\mathbf{c}\,|\,\mathbf{a})$ for $\mathfrak{p}(\mathbf{c}\,|\,\mathbf{ab})$. The formula CCCP_2 is obtained from CCCP_0 by first commuting the terms in the conjunction $\mathbf{ab}$, then interchanging the letters $\mathbf{a}$ and $\mathbf{b}$ throughout, and finally writing $\mathfrak{p}_{\mathbf{b}}(\mathbf{c}\,|\,\mathbf{a})$ for $\mathfrak{p}(\mathbf{c}\,|\,\mathbf{ab})$, as before. It is because updating and relativization are formally equivalent manoeuvres that each of CCCP_1 and CCCP_2 is equivalent to CCCP_0, though they look different.

$$\begin{array}{ll}
\mathrm{CCCP}_0 & \forall\mathbf{a}\forall\mathbf{c}\exists\mathbf{y}\forall\mathbf{b}\;\mathfrak{p}(\mathbf{y}\,|\,\mathbf{b}) = \mathfrak{p}(\mathbf{c}\,|\,\mathbf{ab}) \\
\mathrm{CCCP}_1 & \forall\mathbf{a}\forall\mathbf{c}\exists\mathbf{y}\forall\mathbf{b}\;\mathfrak{p}(\mathbf{y}\,|\,\mathbf{b}) = \mathfrak{p}_{\mathbf{b}}(\mathbf{c}\,|\,\mathbf{a}) \\
\mathrm{CCCP}_2 & \forall\mathbf{b}\forall\mathbf{c}\exists\mathbf{y}\forall\mathbf{a}\;\mathfrak{p}(\mathbf{y}\,|\,\mathbf{a}) = \mathfrak{p}_{\mathbf{b}}(\mathbf{c}\,|\,\mathbf{a}).
\end{array}$$

We now replace $\mathfrak{p}$ by $\mathfrak{q}$ in both CCCP_1 and CCCP_2, to produce the formulas

$$\begin{array}{ll}
\mathrm{CCCQ}_1 & \forall\mathbf{a}\forall\mathbf{c}\exists\mathbf{y}\forall\mathbf{b}\;\mathfrak{q}(\mathbf{y}\,|\,\mathbf{b}) = \mathfrak{q}_{\mathbf{b}}(\mathbf{c}\,|\,\mathbf{a}) \\
\mathrm{CCCQ}_2 & \forall\mathbf{b}\forall\mathbf{c}\exists\mathbf{y}\forall\mathbf{a}\;\mathfrak{q}(\mathbf{y}\,|\,\mathbf{a}) = \mathfrak{q}_{\mathbf{b}}(\mathbf{c}\,|\,\mathbf{a}).
\end{array}$$

These formulas are far from equivalent to each other: one is refutable, the other is demonstrable. CCCQ_1 is refuted by identifying $\mathbf{b}$ with $\top$. This shows that, for each $\mathbf{a}$ and $\mathbf{c}$, $\mathfrak{q}(\mathbf{c}\,|\,\mathbf{a}) = \mathfrak{q}_{\top}(\mathbf{c}\,|\,\mathbf{a})$ can take only the value 1 or the value 0; the value 1 if $\mathbf{y}$ (which may depend on $\mathfrak{a}$ and $\mathfrak{c}$) is equivalent to $\top$, and the value 0 if it is not. In contrast, CCCQ_2 is demonstrable, since $\mathfrak{y}$ may be the conditional $\mathfrak{b} \to \mathfrak{c}$. As was shown near the end of § 5 above, $\forall\mathfrak{b}\forall\mathfrak{c}\forall\mathfrak{a}\, \mathfrak{q}(\mathfrak{b} \to \mathfrak{c} \,|\, \mathfrak{a}) = \mathfrak{q}_{\mathfrak{b}}(\mathfrak{c} \,|\, \mathfrak{a})$.

7 Discussion

In the interests of amity and brevity, I shall limit my discussion of these results to three items. One concerns their relation to the well-known Ramsey test. A second concerns the tenability of the thesis that, at least with regard to conditionals, measures of deductive dependence offer an attractive alternative to measures of probability. The third matter, dealt with first, and in only a couple of sentences, is whether the unassailability of CCCQ_2 vindicates the identification of all indicative conditionals, at a semantic level, with material conditionals. This remains an open question. But I am not able here to provide solace to those who, having resolved to learn about indicative conditionals by studying their synergy with probabilities, are dismayed by what has been learnt.

Ramsey's test Much work on the connection between conditionals and probability has been guided by the words of Ramsey in (1929), p. 247: 'If two people are arguing "If p, will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge, and arguing on that basis about q; ... We can say that they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered *void*.' In Stalnaker (1968, p. 101), this description becomes a piece of advice: 'your deliberation ... should consist of a simple thought experiment: add the antecedent (hypothetically) to your stock of knowledge (or beliefs), and then consider whether or not the consequent is true. Your belief about the conditional should be the same as your hypothetical belief, under this condition, about the consequent.' Hájek & Hall (1994, p. 80), add that the agent's system of beliefs may need to be revised (but as little as possible) if it is to accommodate the antecedent consistently, a qualification that imports new problems. What lies behind the advice, if I understand it, is the idea that evaluating the probability of the consequent of a conditional, relative to its antecedent, is a way in which the agent might 'consider whether or not the consequent is true'.

I suggest that the explicit identity that we may extract from CCCQ_2, namely $\mathfrak{q}(\mathbf{b} \to \mathbf{c} \,|\, \mathbf{a}) = \mathfrak{q}_{\mathbf{b}}(\mathbf{c} \,|\, \mathbf{a})$, heeds this advice as well as does any identity derivable from the CCCP hypothesis. To be sure, there is a difference. In the case of an identity of the form $\mathfrak{p}(\mathbf{a} \rightsquigarrow \mathbf{c} \,|\, \mathbf{b}) = \mathfrak{p}(\mathbf{c} \,|\, \mathbf{ab})$, it is likely that what Stalnaker (and others) had in mind was that the antecedent of the conditional $\mathbf{a} \rightsquigarrow \mathbf{c}$ be 'added to your stock of knowledge (or beliefs)' by further relativizing $\mathfrak{p}(\mathbf{c} \,|\, \mathbf{b})$ to $\mathbf{a}$. I do not know that this strategy has ever been described (equivalently) as one of updating $\mathfrak{p}(\mathbf{c} \,|\, \mathbf{b})$ with $\mathbf{a}$. But in the identity $\mathfrak{q}(\mathbf{b} \to \mathbf{c} \,|\, \mathbf{a}) = \mathfrak{q}_{\mathbf{b}}(\mathbf{c} \,|\, \mathbf{a})$, the antecedent of the conditional $\mathbf{b} \to \mathbf{c}$ is unambiguously used to update the function $\mathfrak{q}$. This is how $\mathbf{b}$ is to be 'added to your stock of knowledge (or beliefs)'.

Stated quite literally, what is here being proposed is this: in order to assess the deductive dependence of the material conditional $\mathbf{b} \to \mathbf{c}$ on the statement $\mathbf{a}$, the agent should (provisionally and hypothetically) update the function $\mathfrak{q}$ to $\mathfrak{q}_{\mathbf{b}}$ and then, using this updated function, assess the deductive dependence of $\mathbf{c}$ on $\mathbf{a}$. This procedure cannot properly be described as 'evaluating the dependence of the consequent of a conditional on its antecedent'. But if $\mathbf{a}$ is supposed to state truthfully some information about the world, it is surely one way in which the agent might 'consider whether or not the consequent is true'.

Assertability and Acceptability of Conditionals It has been suggested by several writers, especially Adams (1965), that conditionals cannot be true or false, and that $\mathfrak{p}(\mathbf{c} \,|\, \mathbf{a})$ measures not the probability of the truth of $\mathbf{a} \rightsquigarrow \mathbf{c}$, but its *assertability*; that is to to say, the appropriateness of its utterance. Others, including Adams himself in a later phase (Adams, 1998), have favoured the term *acceptabil-*

ity, that is to say, the reasonableness of the belief in $\mathfrak{a} \rightsquigarrow \mathfrak{c}$. Hájek (2012), § 2, has ventured the neologism *assentability*. Although this has to my ears a subjectivist ring that is absent from *acceptability* and, to a lesser extent, *assertability*, for our present purposes the differences between these ideas are less important than what they have in common, which is an origin in the justificationist doctrine that an agent is entitled fully to assert or to accept or to assent to a statement only if he knows it to be true. The word *probably*, and similar expressions such as *in my opinion* and *I think*, are often used to qualify statements that are not fully asserted. The less probable that $\mathfrak{c}$ is, given $\mathfrak{a}$, the less the agent is entitled to assert it, or the more tentatively he asserts it. In this vein, Lucas (1970), Chapter 1, called probability 'a guarded guide'.

Those of us who dismiss as not quite serious the goal of justified truth never worry that we are not entitled to assert a statement. We think that we are entitled to say what we like, whatever the epistemological authorities may enjoin. But we may worry whether a proposition asserted is true, and if we suspect that it is not, we may qualify our assertion by such expressions as *about* or *or so* or *roughly* or *more or less*. Since the quantity $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$, the deductive dependence of a non-tautological statement $\mathfrak{c}$ on a statement $\mathfrak{a}$, is a straightforward measure of how well (the content of) $\mathfrak{c}$ is approximated by (the content of) $\mathfrak{a}$, ranging from 0, when $\mathfrak{a}$ contains none of $\mathfrak{c}$, to 1 when it contains it all, it does appear that $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$ may serve also as a measure of the assertability or the acceptability of the statement $\mathfrak{c}$ in the presence of $\mathfrak{a}$. If our aim is truth, then the higher $\mathfrak{q}(\mathfrak{c} \mid \mathfrak{a})$ is, the more successful is the statement (or hypothesis $\mathfrak{c}$), given the statement (or evidence) $\mathfrak{a}$. More generally, the assertability or acceptability of the conditional $\mathfrak{b} \to \mathfrak{c}$ may be measured by $\mathfrak{q}(\mathfrak{b} \to \mathfrak{c} \mid \mathfrak{a})$, that is, by $\mathfrak{q}_{\mathfrak{b}}(\mathfrak{c} \mid \mathfrak{a})$. It is vigorously denied here that the 'highly entrenched tenet of probabilistic semantics ... [that] the assertability of conditionals goes by conditional probability' (Arló-Costa, 2001, p. 584) exhausts the senses in which a conditional statement may be assertable or acceptable, but not completely so.

8 Conclusion

The goal of this paper has been to elucidate one of the gains that can be made in epistemology by replacing probability measures (understood as degrees of belief) by measures of deductive dependence (understood as degrees of approximation). On this theme, much more needs to be said than can be said here. In the first place, it must be recognized that variants of the function $\mathfrak{q}$ of deductive dependence have been introduced before, in rather different contexts. Hempel & Oppenheim (1948), Part IV, for example, interpreted $\mathfrak{q}(\mathbf{a}\,|\,\mathbf{c})$ as a measure of the *systematic power* of the hypothesis $\mathbf{c}$ to organize the evidence $\mathbf{a}$. Reichenbach (1954), appendix, espied in the divergence between the functions $\mathfrak{p}$ and $\mathfrak{q}$ a potential solution to Hempel's paradoxes of confirmation. Hilpinen (1970), § IV, interpreted $\mathfrak{q}(\mathbf{a}\,|\,\mathbf{c})$ as a measure of the *information transmitted* by the evidence $\mathbf{a}$ about the hypothesis $\mathbf{c}$, and used it to answer Ayer's question of why those who assay hypotheses by their relative probabilities ever search for new evidence. The function $\mathfrak{q}$ has similarities also with the idea of probabilistic validity advanced in Adams (1998), and especially with the use of p-values in modern classical (non-Bayesian) statistics. All these connections will have to be explored in due course. Interested readers may glean from Miller (2014) meanwhile a glimpse of the versatility of the function $\mathfrak{q}$, and of the role that it may perform in a saner philosophy of knowledge than is fashionable at present.

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