Investigación

Una aplicación de las matemáticas al estudio de una máquina electrónica de Bingo

An application of the mathematics to the study of a video lottery machine

Raquel Caro Carretero y Javier Rodrigo Hitos

Revista de Investigación

G.I.E Pensamient Matemátic MAIC

Volumen V, Número 1, pp. 083–092, ISSN 2174-0410
Recepción: 21 Mar'14; Aceptación: 2 Ene'15

1 de abril de 2015

Resumen

En las máquinas "tragaperras" tipo Bingo, en las que el jugador apuesta dinero buscando obtener más dinero, hay dos aspectos que deben ser controlados para satisfacer tanto al jugador como al propietario de la máquina: 1) Tener al jugador en la máquina una cantidad razonable de tiempo para que las partidas no sean ni demasiado cortas ni demasiado largas, desanimando al jugador y 2) Tener un porcentaje de retorno en premios en torno al 90-95% para que la máquina dé beneficios. Es relativamente fácil mantener estos dos conceptos controlados en el ciclo estadístico, es decir, para tiempos grandes, pero es más difícil tenerlos en los números deseados para intervalos cortos de tiempo. El objetivo de este trabajo es examinar cómo el porcentaje de retorno en premios en intervalos cortos de tiempo, para una máquina de juego tipo Bingo, se puede controlar perturbando el precio de las bolas extra que la máquina ofrece.

Palabras Clave: Control estadístico, probabilidad, máquinas de bingo, pseudo aleatoriedad

Abstract

In the slot machines type Bingo, in which the player bets some money trying to obtain more, there are two problems that must be controlled to satisfy both the trader and player: 1) Having the player on the machine a reasonable amount of time so that the games are neither too short nor too long, discouraging the player and 2) Having a percentage of return around 90-95% for the machine to give benefits. It is desirable to maintain these two items in reasonable figures in the statistical pay cycle, i.e. for a long time interval. But it is more difficult to have these two items under control for short time intervals. The aim of this work is to examine how the percentage of return of the money that a Bingo type machine gives to the players can be controlled by perturbing the price of the extra balls the machine offers.

Keywords: Statistical control, probability, slot machines, pseudo-randomness

1. Introduction

Slot machines and other electronic gambling machines (EGMs) are gambling devices that offer a variety of games. They are inexpensive to run, compared to roulette or blackjack games, which makes it possible to offer low-stakes betting to a large number of players. As a result, they have become the most profitable form of gambling [7].

Slot machines and other EGMs also seem to attract a lot of myths. This is partly because of a lack of accurate information on how the machines work and partly due to the design of the machines [6].

The purpose of this paper is to examine how the percentage of return of the money that a Bingo type machine gives to the players can be controlled by perturbing the price of the extra balls the machine offers. We can get a percentage of return or played percentage for a player around 95.

We will use the following notation and expressions along the paper:

%J stands for the percentage of return for the gambler.

SJ stands for the played out (money in prizes that the machine has given the player)

EJ stands for the played entry (total money the player has played, including the money of the prizes that he has replayed).

It is satisfied that $\%J = \frac{SJ}{EJ}100$. This way, $\%J$ can be seen as the percentage of the played money that is returned to the player.

If *P* games are played, the cost of the game *i* is C_i, there were m_{ij} prizes in the absence of a number in the game *i* at the time of giving the extra ball *j*, $PB_{i,j}$ is the price of the *j*th extra ball drawn in the game *i* and the number of extra balls that the machine gave in the game *i* is n_i, we have that:

(1) $EJ = \sum_{i=1}^{P} C_i + \sum_{i=1}^{P}\left(\sum_{j=1}^{n_i} PB_{i,j}\right)$

The expression for the played out is (2) $SJ = \sum_{i=1}^{P}\sum_{j=1}^{n_i}\left(\sum_{z=1}^{m_{ij}} \Pr_z V_z\right)$, being $\Pr_z$ the probability of extraction of the ball that gives the prize *z*, V_z the value of the prize z (in this expression we assume that the expected winning in prizes obtained before acquiring extra balls is negligible, so we do not include the corresponding term. It will be included in the examples).

The outline of the rest of the paper is the following: In section 2 we make an analysis of the mathematical structure of the slots machines. Section 3 presents a technical description of how the video lottery machine DStadium works and section 4 gives a statistical control of the return in prizes for the DStadium machine. This control can be considered as the main objective of the paper.

2. The pseudo-randomness

The payout of the slot machines is determined by the mathematical structure of the game, not by how recently the machine has paid out. Game structures are very complex and, as a result, the odds against winning on most EGMs are hidden from the player [4, 7]. The basis of all random-like events is a combination of complex or nonlinear relationships and initial uncertainty. Technically, a machine cannot be random. Slot machines in fact are "pseudo"-random [2]. Slot machines use a random number generator (henceforth RNG) to create an erratic sequence of numbers. If the right values are selected for the RNG, the sequence will be virtually unpredictable [7]. The RNG in slots uses Lehmer's congruential iteration [1, 4]. This makes the outcome of an EGM completely unpredictable [7].

3. The bingo type machine DStadium. Technical details

3.1 Overview

We try to explain how a particular video lottery machine (DStadium) works. DStadium is a machine based on the game of Bingo, which is built on a touch platform so that the user will interact with the machine by touching on the screen, while also has mechanical buttons. Either of these two options can be used interchangeably, according to the comfort of each player. For the development of the game 60 balls are used, of which 30 are extracted. As the balls are drawn, the corresponding numbers are automatically labeled in the card and the system will display for the player the prizes that he/she has won.

3.2 How to play

The game is similar to the Bingo game. By introducing the money in the slot, the user creates a "Game Session", which ends when he wants to leave the machine. Before the user starts playing, he has to set the parameters of the game by a previous screen configuration regarding credit value, speed of play and betting board. These parameters can also be changed thereafter at any time of the "Game Session." Once the initial setup is established, the machine goes to the main screen and bets are made, the user can play up to four cards at the same time and at least one.

Each card must have a bet of at least one credit and all the cards must have the same bet (the total bet must be an integer multiple of the cards that are in play at the time). There cannot be two cards that have some common number, so that the four cards have 60 numbers from 1 to 90, ordering each card numbers from lowest to highest from top to bottom and left to right.

In the case of obtaining a prize, the machine puts the numbers corresponding to the prize in red and increases the player's credit automatically.

Once the 30 balls that make up the draw are extracted, the machine eventually raffles ten extra balls, chosen at random from among the 30 not extracted numbers. These extra balls have an additional cost (they may also be free) and the user is the one who decides whether to buy or not. When the user decides not to buy more extra balls, the current game will end.

3.3 Description Award Chart

To get a prize, the player must match the drawn numbers marked on the card and form any of the figures presented in the pay table. Prizes are shown in Figure 1 and detailed below.

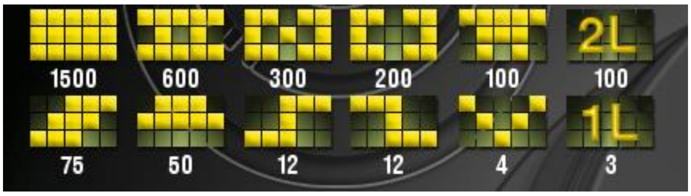

Figure 1. Pay Table

BINGO (P_1): Match all numbers on the card. The prize you get is 1500 times the bet of the awarded card.

ANVIL (P_2): Match all numbers except the ends and center of the centerline. The prize you get is 600 multiplied by the bet of the awarded card.

FLOWER (P_3): Match all numbers except those that make up the central cross, the center of the cross included. The prize you get is 300 multiplied by the bet of the awarded card.

CROSS (P_4): Hit all numbers except those which are in the central cross. The prize you get is 200 multiplied by the bet of the awarded card.

TABLE (P_5): Match all numbers except the ends of the center line and the ends and center of the bottom line. The prize you get is 100 multiplied by the bet of the awarded card.

DOUBLE LINE (P_6 , P_7 and P_8): Match the numbers that make up 2 lines. The prize you get is 100 multiplied by the bet of the awarded card.

LADDER (P_9): Match all numbers except the 3 in the upper left corner and the 3 of the bottom right corner. The prize you get is 75 times the bet of the awarded card.

HAT (P_{10}): Match the numbers that make the center line and three central numbers in the top row. The prize you get is 50 times the bet of the awarded card.

REVERSED Z (P_{11}): Match the numbers that make up an inverted Z. The prize you get is 12 times the bet of the awarded card.

Z (P_{12}): Match the numbers that make up the letter Z. The prize you get is 12 times the bet of the awarded card.

V (P_{13}): Match the numbers that form the letter V. The prize you get is 4 times the bet of the awarded card.

LINE (P_{14} , P_{15} and P_{16}): Match the numbers that form a horizontal line. The prize you get is 3 times the bet of the awarded card.

4. Statistical control of the prizes in a short time frame

4.1. Statement of the Problem

In the Bingo type game machines, in which the player bets an amount of money in order to obtain more money, there are two aspects that must be controlled to satisfy both the trader and the player's claims:

- to have a percentage return in prizes around 95%, so that the machine gives benefits to the owner.

- to have the player on the machine a reasonable amount of time, so that the games are not too short or too long. Ideally, for each coin inserted the player gets five games. This concept will be called replayed. If we consider the D Stadium machine as an infinitely-reach adversary, then in order to get a suitable replayed we have to ensure that the probability of ruin of the gambler is not high [3].

For random type machines, you can have these two items in the desired figures in the statistical cycle, that is to say, for a long time interval, but it is a more difficult problem to get a rate of return close to 95% and around 5 replayed in short time intervals, which is necessary in order to provide the trader with a greater sense of control of the machine because it gives an almost cyclical aspect to the machine.

In order to achieve the desired control of the bingo machine in short periods of time, one input that the machine has will be considered (see the introduction): the price of the extra balls. The goal is to perturb this item so as to get the percentage of return and replayed in the desired numbers and that the machine passes the mandatory test of randomness based on the chi squared algorithm [5].

4.2. Control of the percentage of return by perturbing the price of extra balls

If we set the price of extra balls in the game P+1 to get a percentage of return after P+1 games (%J(P+1)) of 95%, depending on the percentage of return after we split P games (%J(P)) and the played entry in the first P games (EJ(P)), we consider the following expression derived from (1), (2):

$$0.95=\frac{\%J(P+1)}{100}=\frac{SJ(P+1)}{EJ(P+1)}=\frac{SJ(P)+\sum_{j=1}^{n_{P+1}}\sum_{z=1}^{m_{P+1j}}\mathrm{Pr}_z\,V_z}{EJ(P)+C_{P+1}+\sum_{j=1}^{n_{P+1}}PB_{P+1,j}}=\frac{\frac{\%J(P)}{100}EJ(P)+\sum_{j=1}^{n_{P+1}}\sum_{z=1}^{m_{P+1j}}\mathrm{Pr}_z\,V_z}{EJ(P)+C_{P+1}+\sum_{j=1}^{n_{P+1}}PB_{P+1,j}} \qquad (3)$$

Then, by manipulating (3):

$$\sum_{j=1}^{n_{P+1}}PB_{P+1,j}=\frac{\left(\frac{\%J(P)}{100}-0.95\right)EJ(P)+\sum_{j=1}^{n_{P+1}}\sum_{z=1}^{m_{P+1j}}\mathrm{Pr}_z\,V_z-0.95\,C_{P+1}}{0.95} \qquad (4)$$

Interpretation: If the percentage of return until game P is high (greater than 95%), the first term of the numerator of (4) is positive, which increases the price of the extra balls and refrains the player from buying extra balls in order to decrease the percentage of return. If the percentage of return until game P is low (less than 95%), the first term of the numerator of (4) is negative, which decreases the price of the extra ball and encourages the player to buy extra balls to raise the percentage of return. If the percentage is just 95%, the price of the extra ball will not depend on what happened in the previous games but only on the prizes the player can get if the right extra ball is drawn at the time of giving this extra ball.

If there are high prizes in the absence of a number, then the second term of the numerator of (4) is large and so the price of the extra balls is high (the player has an incentive to buy them despite their high price). If prizes in the absence of a number are low, then the second term of the numerator is smaller and so the price of the extra balls is low (so the player has an incentive to buy them despite the small reward in prizes). If the total price of extra balls becomes negative because the percentage of return until game P is very low with a high played entry, then the price of the extra balls is set to 0, that is to say, they would be given for free to encourage the user to continue playing.

As the right side of the equation (4) depends on n_{P+1}, the total price of the extra balls the player buys in the game P+1 depends on the number of such extra balls purchased, number not known a priori. We then can take two options to avoid this problem:

Option 1. Assuming that the player buys the 10 balls that are offered and they all are offered at the same price (so we can put PB_{P+1} instead of $PB_{P+1,j}$ and $n_{P+1}=10$) and taking into account that the probability of hitting a ball that gives a prize in the absence of one number is $\frac{1}{30}$ when the first extra ball is offered, $\frac{1}{29}$ when the second extra ball is offered and in general $\frac{1}{30-(j-1)}$ when the extra ball number j is offered, to get the price of each extra ball we consider the following:

$$\sum_{j=1}^{10} PB_{P+1,j} = 10\, PB_{P+1} = \frac{\left(\frac{\%J(P)}{100}-0.95\right)EJ(P)+\sum_{j=1}^{10}\sum_{z=1}^{m_{P+1j}}\frac{1}{31-j}V_z-0.95\,C_{P+1}}{0.95}$$

Thus the price of each extra ball would be:

$$PB_{P+1} = \frac{\left(\frac{\%J(P)}{100}-0.95\right)EJ(P)+\sum_{j=1}^{10}\frac{1}{31-j}\left(\sum_{z=1}^{m_{P+1j}}V_z\right)-0.95\,C_{P+1}}{9.5} \qquad (5)$$

Example 1. We assume that the player has introduced 1 credit with which he has played 5 games, because he has won prizes that has been replaying at the rate of 1 credit per game, so we have that $EJ(P)=5$, $\%J(P)=80$ and in the sixth game, in which the player also has bet one credit, he does not get any prizes with the first 30 balls but he is at a number of the first and the third prizes, and there are no more awards in the absence of a number as he plays the extra balls in that game. Then the price of each extra ball, taking (5) into account, would be:

$PB_{P+1} = \dfrac{(0.8-0.95)5+\sum_{j=1}^{10}\frac{1}{31-j}(1500+300)-0.95}{9.5} = 75.089$ credits. Assuming that the player has set the value of each credit at 0.05 €, this gives the following price for each extra ball in Euros: $75.089 \times 0.05 \approx 3.75$

Option 2. We assume now that the extra ball number *i* that is offered to the player in the game *P*+1 has a price $PB_{P+1,i}$ which depends on the percentage of return and played entry that the machine had at the time of giving the previous extra ball or at the time of playing the 30 first balls if it is the first extra ball ($\%J(P+1,i-1)$, $EJ(P+1,i-1)$). We note the following:

$$0.95 = \frac{\%J\ P+1,i}{100} = \frac{SJ\ P+1,i}{EJ\ P+1,i} = \frac{SJ\ P+1,i-1\ +\sum_{z=1}^{m_{P+1i}}\mathrm{Pr}_z\ V_z}{EJ\ P+1,\ i-1\ +PB_{P+1,\ i}} = $$

thus,

$$= \frac{\frac{\%J\ P+1,\ i-1}{100}\ EJ\ P+1,\ i-1\ +\sum_{z=1}^{m_{P+1i}}\mathrm{Pr}_z\ V_z}{EJ\ P+1,\ i-1\ +PB_{P+1,\ i}}$$

$$PB_{P+1,i} = \frac{\left(\frac{\%J(P+1,i-1)}{100}-0.95\right)EJ(P+1,i-1)+\sum_{z=1}^{m_{P+1i}}\frac{1}{31-i}V_z}{0.95} \qquad (6)$$

Example 2. We see an example for this case obtained by computer simulation: The user plays four cards and he makes a bet of 1 credit for each one, setting the value of the credit at €0.05. In the first two games he has not won prizes and has not bought extra balls, so $SJ(2)=0$, $\%J(2)=0$, $EJ(2)=8$. In the third game he plays with cards 3-4-20-22-30-32-33-34-49-52-53-57-60-83-85, 13-15-21-24-28-29-40-42-47-54-68-69-81-88-89, 8-9-10-17-19-26-37-38-61-65-71-72-76-79-87, 1-2-5-39-44-48-55-63-70-75-77-78-80-84-90 and the first 30 balls drawn are 75-49-87-42-79-13-8-33-37-70-32-60-61-20-63-88-17-1-68-28-5-48-80-90-89-39-55-29-40-52.

Then with the fourth card he gets the prizes P_{11} (12 credits) and P_{14} (3 credits) and he has been in the absence of a number for the following prizes: P_{11} (12 credits) in the first card, P_{15} (3 credits) in the second card and P_7 (100 credits), P_{12} (12 credits), P_{16} (3 credits) in the fourth card. We then have,

$SJ(3,0)=15$, $EJ(3,0)=12$, $\dfrac{\%J(3,0)}{100}=\dfrac{15}{12}=\dfrac{5}{4}$

Then the price of the first extra ball, taking (6) into account, is:

$PB_{3,1} = \dfrac{\left(\frac{5}{4}-0.95\right)12+\frac{1}{30}(12+3+100+12+3)}{0.95} = 8.35$ credits, which gives the following price of the first extra ball in Euros: $8.35 \times 0.05 = 0.42$

This first extra ball is 78, which gives prizes P_7, P_{12}, P_{16} and does not leave any other prize in the absence of a number.

For the price of the second extra ball, we must then consider:

$$EJ\ 3,1\ = 12 + 8.35 = 20.35\,,\ SJ\ 3,1\ = 3 + 12 + 115 = 130\,,$$

$$\frac{\%J\ 3,1}{100} = \frac{130}{20.35} \approx 6.38821$$

So that the price of the said ball, according to (6), is:

$$PB_{3,2} = \frac{(6.38821 - 0.95)\,20.35 + \frac{1}{29}(12+3)}{0.95} \approx 117.037$$ credits, which gives the following price of the second extra ball in Euros:

$$117.037 \times 0.05 \approx 5.85$$

Note that the price of this ball has increased with respect the previous one because the percentage of return has increased since the player has won prizes with the first extra ball, which increases the first term of the numerator. The slight decrease of the second term, due to the less number of possible prizes, does not offset this.

5. Conclusions

As it can be seen from the examples, the control we have performed acts in a dynamic manner by adjusting the control parameter to the machine outputs up to the present time. So we have obtained the desired percentage of return in short time intervals, providing both the owner and the player with a greater sense of security.

Acknowledgements

The authors thank freelancer Francisco Ortiz for suggesting the work hypothesis and providing data for this paper.

References

[1] BRYSBAERT, Marc. *Algorithms for randomness in the behavioral sciences: A tutorial*, pp. 45-60, Behavior Research Methods, Instruments, & Computers, USA, 1991.

[2] GLEICK, James. *Chaos: Making a New Science*, Viking Penguin Inc., New York, USA, 1987.

[3] KATRIEL, Guy. *Gambler´s ruin probability-A general formula*, pp. 2205-2210, Statist. Probab. Lett., North Holland, 2013.

[4] ONGHENA, Patrick. *A theoretical and empirical comparison of mainframe, microcomputer, and pocket calculator pseudorandom number generators*, pp. 384-395, Behavior Research Methods & Instrumentation, USA, 1993.

[5] SHARMA, Subhash. *Applied Multivariate Techniques*, John Wiley & Sons, New York, USA, 1996.

[6] TURNER, Nigel, FRITZ, Barry and MACKENZIE, Bronwyn. *How to gamble: Information and*

misinformation in books and other media on gambling, The Electronic Journal of Gambling Issues, Ontario, Canada, 2003. Available at http://jgi.camh.net/doi/full/10.4309/jgi.2003.9.13

[7] TURNER, Nigel and HORBAY, Roger. *How do slot machines and other electronic gambling machines actually work?*, The Electronic Journal of Gambling Issues, Ontario, Canada, 2004. Available at http://jgi.camh.net/doi/full/10.4309/jgi.2004.11.21

Sobre los autores:

Nombre: Raquel Caro Carretero
Correo Electrónico: rcaro@upcomillas.es
Institución: Universidad Pontificia Comillas, España.

Nombre: Javier Rodrigo Hitos
Correo Electrónico: jrodrigo@upcomillas.es
Institución: Universidad Pontificia Comillas, España.