A Note on University's Choices: the Role of Skewed Abilities and Financial Constraints

Bralind Kiri*

October 30, 2016

Abstract

We theoretically analyze university's choices under skewed abilities and financial constraints. In order to model skewed abilities, we introduce a triangular distribution. On the other hand, we consider financial constraints as a crucial factor influencing individuals' schooling decisions. Moreover, we embrace views from contemporary discussions about the aim of a modern university by assuming that independently of ownership, a university plays rational by simply seeking human capital maximization. A strategic interaction between a university and potential students takes place under two different financial conditions, i.e. perfect capital markets (PCM), and borrowing constraints (BC). Assuming a quadratic cost function for the university, the effect of borrowing constraints on optimal choices is derived. Ultimately, we conduct a comparative analysis in terms of social welfare (SW), and equilibrium vectors composed of three components, namely quality, ability threshold, and tuition fee. Our results suggest that the mode of the distribution will be an intrinsic part of subgame perfect equilibria, and that a human capital maximizing university will make additional efforts in terms of pricing and non-pricing strategies in order to alleviate the inconvenience brought by skewed abilities and financial constraints.

*UNYT, Albania and University of Extremadura, Spain. Email: bralindkiri@unyt.edu.al

1 Introduction

In education markets, unobservable features do not only appear in the demand side (e.g. students' ability), but also in the supply side (e.g. institutional quality). Any negligence of such features would consequently lead to inefficient allocations (Fernández, 2008).

As a matter of fact, universities' choices reflect much of what it has been achieved, and what it can further be achieved with respect to efficiency and equity aspects. In reality, such choices can be indirectly influenced by the regulator itself through several instruments. The latter may take the form of competition or (and) funding policies (e.g. subsidies). Therefore, to the aim of accomplishing certain objectives (e.g. income redistribution, social cohesion, equity, etc.), regulators can either seek to influence the demand or the supply side, or even both sides at the same time. However, all of such interventions will be insufficient unless rigid and time-depedent features of education markets are taken into consideration. That is, in short-run, characteristics of the demand side (e.g. individuals' income, ability, etc.), may result more difficult to handle than those of the supply side (e.g. universities' aim, funding, legal framework, etc.).

A major concern lies on abilities. Given that students' abilities are inputs into the production of human capital as it has been elegantly modeled in Rothschild and White (1995), the distribution of these abilities takes a significant importance. At the same time, borrowing constraints are significant obstacles related to economic environment, which indirectly affect individuals' schooling decisions (Romero and Del Rey 2004; Romero, 2005; Fernández, 2008). It is precisely the combination of each of the aforementioned aspects, which will finally determine the genuine demand for higher education. Consequently, it will be the interaction of the former demand with the HE's supply, the one which will characterize outcomes (e.g. personal, societal) for the whole higher education system.

This paper investigates on how decisions of a human capital maximizing university are influenced by variations on the ability distribution of potential students. Additionally, we calculate the variations on decisions across two possible financial scenarios, perfect capital markets (PCM), and borrowing constraints (BC). Ultimately, welfare implications across scenarios are analyzed. The latter are also compared to the results of earlier studies, which rely on the assumption of uniformity in ability distribution.

Our results indicate that when quadratic costs for the monopolist (i.e. human capital maximizing university) are presumed, the skewness' parameter turns out to be an intrinsic part of subgame perfect equilibria, suggesting that the university will accommodate its choices (i.e. quality, tuition fee, admission standards) in accordance with exogenous characteristics of the environment. Moreover, the university will attempt to internalize negative externalities deriving from borrowing constraints and skewed abilities. In other words, the presence of non-uniformity in ability distributions (i.e. skewed abilities), and financial constraints, leads to higher efforts from the university in order to internalize negative externalities.

Our approach builds upon the contributions of Romero and Del Rey (2004) and Romero (2005). However, we focus on a particular case of monopolistic competition, where a human capital maximizing university solely operates in the higher education market. In realistic terms, our approach captures situations, where university system is highly regulated and centralized. We believe that as long as most of universities in a country, operate according to the same set of rules, and aim the same (or similar) objectives, one can, for the sake of simplicity, consider the whole system as monopolistic. The assumption about human capital maximization, is quite in line with recent trends of university behavior in places, where decisions on issues such as universities' subsidies are based on welfare criteria, and not narrowly on university's ownership criteria, as traditionally used to be. Our approach is also aligned with Dill's viewpoint about recent trends in universities behavior. Dill (2005) points out that what nowadays really matters, is the behavior of universities in the marketplace, which in fact is converging towards a type of behavior aiming to balance the preferences of a vast number of agents. Furthermore, according to him, distinctions such as "public vs private", and "non-profit vs. for profit", for the US higher education market, are blurring.

...the federal government offers competitive contracts with public universities, private universities, and for that matter with profit-making institutions to conduct the research and scholarship that is believed to be in the broader public interest (Dill, 2005: 3).

Of course, we do not pretend that Dill's view, along with our assumption of human capital striving, will capture much of what goes on everywhere. Big exceptions are the operation of universities, especially those of private ownership, in some particular parts of Eastern-Europe, Asia, Africa, etc., where government policies concerning many aspects of HE's regulation are still fragile, unbalanced, and sometimes intrinsically driven by the ownership criteria (Lutran, 2007). However, we believe that our theoretical setup can provide some insights into well-established systems, in nations where universities have already acquired the ability to self-regulate and strive for benevolent objectives (e.g. human capital, inclusion, equity, etc.), satisfaction of a wide spectrum of economic agents, and support nations' prosperity.

The model presented here is very much in the same spirit with that of Rothschild and White (1995), in which it is assumed that students are at the same time input and output into human capital production. Our approach is also in line with most recent and sophisticated modeling of university competition, such as that of Epple, et. al. (2006), where colleges strive for quality maximization. However, in our simple analysis, quality is not narrowly an end rather than a mean to achieve the maximization of university's human capital.

The paper is organized as it follows. In the next section we present the basic model with the central assumptions, players' preferences, and the setup of interactions. Section 3 reports the equilibria of the higher education game, comparative statics, and welfare implications. Section 4 concludes by summarizing the main findings of the paper. All proofs are in the Appendix.

2 The model

2.1 The higher education game

We assume to have only two main players in the higher education market: a university, and individuals. The university acts as a monopoly aiming to better pursue its interest given the scarcity of resources (e.g. funds, classrooms, labs, professors, etc.). As stated in the introduction, we avoid the classical distinction between public and private universities by assuming that a modern university fully exerts its rationality through pursuing human capital maximization. In the other side, individuals interact with the university in order to acquire their education in the most beneficiary way, i.e. with the highest quality and the lowest feasible cost. Consequently, the university (individuals) will pay attention and it (they) will adjust its (their) behavior in response to the choices and the behavior of individuals (university), given the payoffs' structure. The interaction will be considered as totally strategic.

2.2 The potential students

To the final aim of accounting for particularities, we adopt the basic framework provided by Romero and Del Rey (2004), and Romero (2005) in defining individuals' (i.e. potential students) measure, and preferences for higher education. There is a continuum of potential students of measure one. The utility obtained from matriculating in the university for a potential student i, is

$$u_i = w_i + h_i - f \tag{1}$$

where w_i is the initial endowment, h_i is the human capital which will be embodied in the individual i from matriculating at the university, and f is the tuition fee paid by the individual to the university.

Concurrently, we follow the track of earlier contributions, which point out complementarities between individuals' intrinsic capabilities (i.e. ability a_i), and institutional value (i.e. quality q). Thus, we define human capital embodied in an individual as

$$h_i = a_i q. \tag{2}$$

For a potential student, university's quality and tuition fee will be exogenous since she will have no power to directly affect these. Therefore, it will be her endowment and intrinsic ability, which will mainly characterize her utility obtained from matriculating in the university. A key assumption here is that endowment is uniformly distributed across individuals. We support the assumption by twofold arguments: first, young individuals from developed countries have to independently (to a great extent) make their way to higher education; second, in places with a sound tradition in human capital investment, substantial income inequalities might not be so substantial.

In the other side, differently from earlier contributions, we assume a non-uniform distribution of individuals' abilities. A comprehensive summary of empirical studies about intelligence in Gottfredson (1994), suggests that non-uniformity of human skills and intelligence is widely confirmed.

Therefore, for the sake of simplicity, and believing that it will yet be possible to get sound insights, we employ a triangular distribution of abilities. In order to account for a wide-range of theoretical scenarios, we allow for skewness through variations on the distribution's mode.

Formally, the functional form of the distribution will be

$$p(a) = \begin{cases} \frac{2a}{m} & if \quad 0 \leq a \leq m \\ \frac{2(1-a)}{1-m} & if \quad m < a \leq 1 \end{cases}. \tag{3}$$

In our approach, the parameter m will be a proxy for the distribution's skewness. As long as $m < 1/2$, less able students will constitute a larger proportion of the total population, which as a matter of fact is a widespread scenario. By constrast, if $m > 1/2$, abler students will constitute a larger proportion over the total population. And finally, for $m = 1/2$, the proportion of high ability students equals that of low ability students.

2.3 The university

The university, in our case a monopoly, will aim to maximize human capital over the incurred cost to provide a certain level of quality. As in one of the scenarios proposed by Romero (2005), institution's surplus will positively depend on human capital production, while negatively on costs incurred to provide the former. More specifically, university's preferences are measured by the following

$$U = H - c(q), \tag{4}$$

where H denotes the total human capital produced by the university, and $c(q)$ is the total cost incurred by the university to provide a quality level $q \in [0,1]$.

As for university's incurred costs, we assume these to be quadratic in quality (q), which implies increasing marginal costs due to the limited capacity of the university (e.g. professors, classrooms, funds, etc.). More formally, a sharp increase in costs occurs if quality improves

$$c(q) = q^2 \ . \tag{5}$$

The bounded structure of quality ensure us that university's budget will also be bounded. Thus, no matter what university's financial performance will be, government's subsidy to the aim of keeping the system working will not ever exceed the unity measure (Equation 4). Moreover, in order to focus on quality's decision, we omit fixed costs.

Next, given that variables move in continuous and closed intervals $[0,1]$, and the university deals with the coverage of a market demand composed of individuals, who differ in endowment and ability, the university surplus accounting for triangular distribution of abilities, can be expressed as follows

$$U = \begin{cases} \int_0^1 \int_a^m (\frac{2a}{m}q - q^2) dadw + \int_0^1 \int_m^1 (\frac{2(1-a)}{(1-m)}q - q^2) dadw & if \quad 0 \leq a \leq m \\ \int_0^1 \int_a^1 (\frac{2(1-a)}{(1-m)}q - q^2) dadw & if \quad m < a \leq 1 \end{cases} . \tag{6}$$

As in earlier studies, we allow first integral to move in endowment's full extension $[0,1]$, indicating the existence of PCM, and the fact that individuals face no barriers with respect to education's financing. The latter will constitute a benchmark against other cases, such as the one when financial constraints are present.

In fact, with BC, it will not be possible anymore to sum individuals along the full extension of endowment. As individuals will bear the matriculating costs f, the integral will move in a reduced segment $[f,1]$, indicating that education's funding becomes an obstacle. Thus, we will have

$$\bar{U} = \begin{cases} \int_f^1 \int_a^m (\frac{2a}{m}q - q^2) dadw + \int_f^1 \int_m^1 (\frac{2(1-a)}{(1-m)}q - q^2) dadw & if \quad 0 \leq a \leq m \\ \int_f^1 \int_a^1 (\frac{2(1-a)}{(1-m)}q - q^2) dadw & if \quad m < a \leq 1 \end{cases} . \tag{7}$$

We preserve the university's choice sequence proposed in earlier contributions (Romero and Del Rey, 2004; Romero, 2005). The assumption that price decision follows quality decision is in line with a wide range of models on provision of consumer goods. As for admission standards, they finalize the sequence given their key role as instruments for fixing pricing deficiencies caused by demand's relevant but unobservable characteristics such as students' abilities (Fernández, 2008). The decisions' sequence is illustrated in Figure 1.

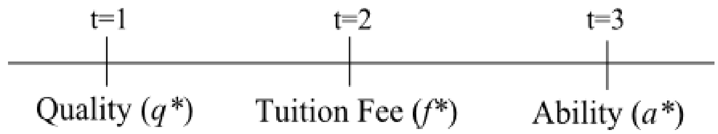

Figure 1: Decisions' sequence

3 Results

An equilibrium is a vector composed by quality, tuition fee, and ability threshold for which: (i) the university maximizes its surplus; (ii) individuals maximize their utility.

We employ the backward induction technique for solving the individuals-university game, in which university will make choices according to decisions' sequence illustrated in Figure 1. Solutions will represent subgame perfect equilibria.

3.1 Equilibria

By solving the model with PCM in place, we find that each component of the choice vector $\{q^*, f^*, a^*\}$ increases in m, indicating full capability of the university to internalize externalities. In the other side, when BC are in place, university's capability, considerably shrinks. More specifically, under BC, q^* will decrease in m, a^* will increase for pre-intermediate values of m, and it will then starts to fall until it reaches zero when $m \approx 0.62$. Results are formalized in the following corollaries.

Proposition 1 *Under PCM, the subgame perfect equilibrium vector is*

$$\left\{ q^* = \frac{1-\sqrt{(1-m)}}{m};\ f^* \leq \frac{1-\sqrt{(1-m)}}{m} - \frac{1}{2};\ a^* = \frac{1-\sqrt{(1-m)}}{2} \right\}. \tag{8}$$

Proposition 2 *Under BC, the subgame perfect equilibrium vector is*

$$\left\{ q^* = \frac{2}{3m}\left(2-\sqrt{4-3m(\frac{1-m-m^2}{1-m})}\right);\ f^*=0;\ a^* = \frac{1}{3}\left(2-\sqrt{4-3m(\frac{1-m-m^2}{1-m})}\right) \right\}. \tag{9}$$

Proofs are given in Appendix.

In Figure 2, we can clearly observe that quality under PCM is always equal (or superior with respect) to the case of BC. Moreover, the former gap becomes larger as the proportion

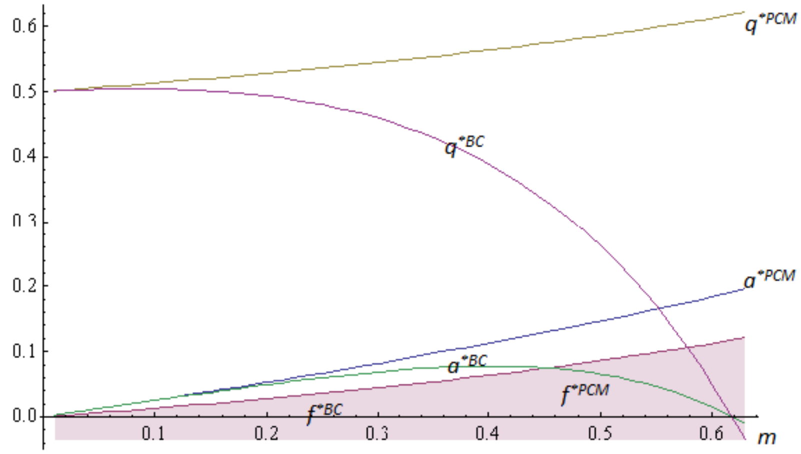

Figure 2: Possible equilibria bundles: PCV vs. BC

of abler individuals over the total population raises, i.e., $m \to 1$. In a similar fashion, ability thresholds and tuition fees are always equal (or higher compared) to those with BC. Nevertheless, with BC, ability threshold reaction is not monotonically increasing. Surprisingly, it increases for certain levels of m, and later it starts to decline. Furthermore, with BC, price reaction becomes perfectly inelastic with respect to m, indicating that even the least able individuals will be indifferent between matriculating and not matriculating at the university. Across scenarios, as for quality, tuition fee and selectivity gaps, enlarge as m raises.

3.2 Welfare

We define social welfare as a function which positively depend on those factors which contribute to equity, inclusion, cohesion, etc., while negatively on factors such as education's cost, exclusion, etc. There is evidence that raising human capital in a society, leads to more welfare, while raising costs, and imposition of other obstacles to education's access, contribute towards deepening inequalities, and so undermining welfare. More formally,

Lemma 1 *Social welfare is a function which increases in human capital, and decreases in tuition fee and selectivity standard:*

$$SW = \frac{2q^*}{m}\int_{a^*}^{m} ada + \frac{2q^*}{1-m}\int_{m}^{1}(1-a)da - (1-\frac{2}{m}\int_{0}^{a^*} ada)f^* - \frac{2}{m}\int_{0}^{a^*} ada. \qquad (10)$$

By plugging vectors from Corollaries 3 and 4, into Lemma 1, welfare performances for any exogenous change on m, and under both financial scenarios are generated. Additionally, to the aim of getting a whole picture, we plug the equilibria results from Romero (2005) into Lemma 1, so we can also show the welfare performance with uniform abilities[1]. The former are denoted by SW^{UPCM} and SW^{UBC}. The overall patterns are illustrated in Figure 3.

[1]By referring to that paper, SPE with exponential costs, are the following: $SPE^{PCM} = \{c\'(q^*) = \frac{1+a^*}{2}$; $f^* \leq c(q)$; $a^* = \frac{c(q)}{2}\}$, $SPE^{BC} = \{c\'(q^*) = \frac{1+a^*}{2}$; $f^* = 0$; $a^* = \frac{c(q)}{2}\}$. But if quadratic cost are assumed, i.e.,

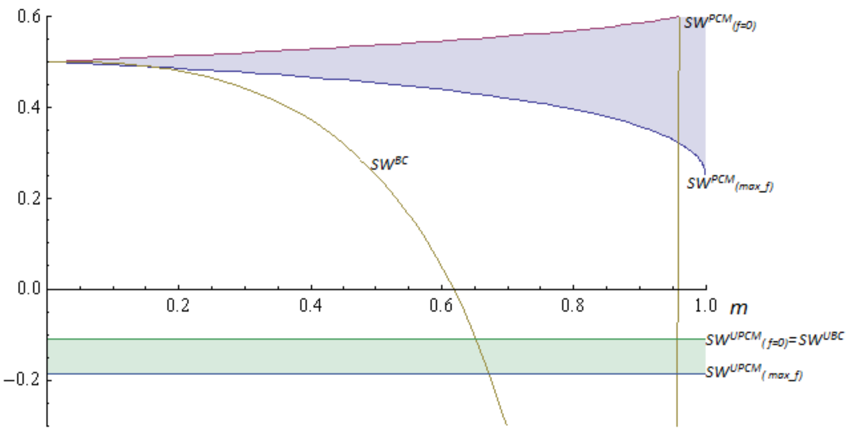

Figure 3: SW across scenarios

Proposition 3 *Under triangularly distributed abilities, social welfare is always higher than the one with uniformly distributed abilities. Moreover, the university will always contribute more to welfare under a PCM scenario than under a BC one. The contribution intensifies as abilities become more right-skewed.*

4 Conclusions

Within the proposed setup of the higher education game, we get several insights into the role of skewed abilities and borrowing constraints in university's choices. Among other points, we would like to underline the following.

First of all, a human capital striving university will make additional efforts to alleviate adverse effects emerging from the presence of non-uniformity in ability distribution (i.e. skewed abilities).

Second, as financial constraints impose a real obstacle to individuals' access to education, the university will try to internalize negative externalities deriving from the former. However, we see that such capability will significantly fade when skewed abilities are present (see Figure 3).

Third, when no borrowing constraints are faced from potential students, and abilities are triangularly distributed, university's welfare performance is superior with respect to all alternative scenarios (i.e. BC and triangularly distributed abilities, BC and uniformly distributed abilities, PCM and uniformly distributed abilities).

Fourth, the university's welfare performance with triangular ability distribution is superior to that with uniform distribution. However, under BC, such superiority is expoited up to a limited extent.

To conclude, we have proved that the overall presence of non-uniformly distributed abilities and borrowing constraints has a substantial effect on equilibria and welfare performance

---

$c(q) = q^2$, then we would have: $SPE^{PCM} = \{q^* = \frac{1}{3};\ f^* \leq \frac{1}{3};\ a^* = \frac{1}{3}\}$, $SPE^{BC} = \{q^* = \frac{1}{3};\ f^* = 0;\ a^* = \frac{1}{3}\}$. By plugging the previous results into Lemma 1, we will get: $SW^{PCM}_{(\max\_f)} = -\frac{1}{3}$, $SW^{PCM}_{(f=0)} = -\frac{1}{9}$, $SW^{BC} = -\frac{1}{9}$.

of a human capital maximizing university. Despite possible pitfalls related to the simplicity of the approach, we believe that we have shed more light on some aspects which have not been considered by the earlier contributions.

References

[1] Dill, D., D. (2005). "The Public Good, the Public Interest and Public Higher Education". University of Carolina at Chapel Hill. Working Paper.

[2] Epple, D., Romano, and R., Sieg, H. (2006). Tuition, and Financial Aid Policies in the Market for Higher Education. *Econometrica*, 74(4), 885-928.

[3] Fernández, R. (2008). "Education and Borrowing Constraints: Tests vs. Prices". NBER Working Paper.

[4] Gottfredson, S., L. (1994). Mainstream Science on Intelligence: An Editorial With 52 Signatories, History, and Bibliography. *Intelligence*, 24(1), 13-23.

[5] Lutran, J. (2007). *Private Higher Education in Western Europe*. Athens: Hellenic Colleges Association.

[6] Romero, L. (2005). On the role of borrowing constraints in public and private universities' choices. *Economics Bulletin*, 9, 1-8.

[7] Romero, L., and Del Rey, E. (2004). "Competition between Public and Private Universities: Quality, Prices and Exams". Department of Economics, Universidad Carlos III de Madrid, Economics Series 23. Working Paper.

[8] Rothschild and White (1995). The Analytics of the Pricing of Higher Education and Other Services in Which the Customers Are Inputs. *The Journal of Political Economy*, 103(3), 573-586.

A Appendices

A.1 Proof of Proposition 1

A.1.1 The extended form of the utility function

If a^* falls on the left of m: $U^l = \int_0^1 \int_a^m (\frac{2a}{m}q - q^2)dadw + \int_0^1 \int_m^1 (\frac{2(1-a)}{1-m}q - q^2)dadw = -\frac{qa_b^2}{m} + a_b q^2 + qm - q^2 + \frac{q-2qm+qm^2}{1-m}$. While if a^* falls on the right of m, then $U^r = \int_0^1 \int_a^1 \frac{2(1-a)}{(1-m)}q - c(q))dadw = \frac{q}{1-m} - q^2 - \frac{2qa}{1-m} + \frac{qa^2}{1-m} + aq^2$.

A.1.2 Optimal ability

First, let us check if the optimal a^* falls on left of m. For this to occur, we need: $\frac{\partial U^l}{\partial a} = 0$, and $\frac{\partial^2 U^l}{\partial^2 a} < 0$. $\frac{\partial U^l}{\partial a} = -\frac{2qa}{m} + q^2 = 0 \Longrightarrow a = \frac{mq}{2}$. $\frac{\partial^2 U^l}{\partial^2 a} = -\frac{2q}{m} < 0$, $\forall (q,\ m) \geq 0$. Thus, $a = \frac{mq}{2}$ is a maximum.

Let us check now if we can find an optimum a^* on the right of m. For that to happen, we need: $\frac{\partial U^r}{\partial a} = 0$, and $\frac{\partial^2 U^r}{\partial^2 a} < 0$. $\frac{\partial U^r_b}{\partial a} = -\frac{2q}{1-m} + \frac{2qa}{1-m} + q^2 = 0 \implies a^* = 1 - \frac{(1-m)q}{2}$. $\frac{\partial^2 U^r}{\partial^2 a} = \frac{2q}{1-m} > 0$, $\forall q > 0$, and $m \,\epsilon [0,1)$. Hence, $a^* = 1 - \frac{(1-m)q}{2}$ is not a maximum. We cannot find an optimal ability threshold falling on the right of m. Finally, we can say that in a strategic game individuals-university, the optimal ability will depend not only on the university's choices but also on individuals' ones. That is, $a^* = \frac{mq}{2}$ will be optimal for the university as long as it exceedes individuals' indifference level: $a^* = \frac{f}{q}$. More formally, the optimal reaction of university with respect to exams will be better represented by the discontinuous function $a^* = \begin{cases} \frac{mq}{2} \; if \;\; f \leq \frac{mq^2}{2} \\ \frac{f}{q} \; if \;\; f > \frac{mq^2}{2} \end{cases}$.

A.1.3 Optimal fee

We plug two possible threshold levels a^* in the extended utility function A.1.1 to the aim of obtaining an optimal fee. We first calculate the reaped utility for $a^* = \frac{mq}{2}$, i.e., $U(a^* = \frac{mq}{2}) = \frac{mq^3}{4} + qm - q^2 + \frac{q-2qm+qm^2}{1-m}$. Since, f does not appear in $U(a^* = \frac{mq}{2})$, we cannot find an optimal f through FOC. However, we can calculate $U(a^* = \frac{f}{q})$, and compare this with $U(a^* = \frac{mq}{2})$. Thus, $U(a^* = \frac{f}{q}) = -\frac{f^2}{qm} + qf + qm - q^2 + \frac{q-2qm+qm^2}{1-m}$. As f appears in the utility function, we can get the first order condition with respect to f. That is, $\frac{\partial U(a^* = \frac{f}{q})}{\partial f} = -\frac{2f}{qm} + q = 0 \implies f^* = \frac{mq^2}{2}$. Moreover, $\frac{\partial^2 U(a^* = \frac{f}{q})}{\partial^2 f} = -\frac{2}{qm} < 0, \forall (q,m) > 0$, indicating that $f^* = \frac{mq^2}{2}$ is a maximum. Does it lead to a higher utility than $U(a^* = \frac{mq}{2})$? To check that we calculate $U(f = \frac{mq^2}{2})$ by using the outcome of $U(a^* = \frac{f}{q})$. Therefore, we have: $U(f = \frac{mq^2}{2}) = \frac{mq^3}{4} + qm - q^2 + \frac{q-2qm+qm^2}{1-m}$. Thus, $U(f^* = \frac{mq^2}{2}) = U(a^* = \frac{mq}{2})$. But $f^* = \frac{mq^2}{2}$ cannot be applied for $a^* = \frac{f}{q}$ due to the violation of condition for optimality of a^*, i.e. $f^* > \frac{mq^2}{2}$. Therefore, $U(a^* = \frac{f}{q})$ will not exceed $U(a^* = \frac{mq}{2})$, $\forall \; f^* > \frac{mq^2}{2}$. Finally, we can definitely say that $f^* \leq \frac{mq^2}{2}$ is the optimal tuition, and it occurs for $a^* = \frac{mq}{2}$.

A.1.4 Optimal quality

Since f does not appear in the extended U^l of appendix A.1.1, $U(a^* = \frac{mq}{2})$ may serve as the function for which we can get the optimal quality.

$$\frac{\partial U(a^* = \frac{mq}{2}, \; f^* = \frac{mq^2}{2})}{\partial q} = mq^2 - 2q + 1 = 0$$

The solution of the quadratic equation will be: ${}_1q_2 = \frac{1 \pm \sqrt{(1-m)}}{m}$. Given that $q \in (0,1)$, we only consider $q^* = \frac{1-\sqrt{(1-m)}}{m}$, $\forall m \in (0,1)$.

As $\frac{\partial U^2(a^* = \frac{mq}{2}, \; f^* = \frac{mq^2}{2})}{\partial^2 q} = 2mq - 2 < 0, \forall m, q \in (0,1)$, we can be sure that q^* is a maximum. Finally, by plugging q^* in SPE^{PCM}, the values for a^*, f^* are directly obtained.

A.2 Proof of Proposition 2

A.2.1 The extended form of the utility function

The extended $\bar{U}$ if a^* falls on the left of m is: $\bar{U}^l = \int_f^1 \int_a^m (\frac{2a}{m}q - q^2)dadw + \int_f^1 \int_m^1 (\frac{2(1-a)}{(1-m)}q - q^2)dadw = \left(qm - \frac{q}{m}a^2 + q^2a + \frac{q}{1-m} - q^2 - \frac{2q}{1-m}m + \frac{q}{1-m}m^2\right)(1-f)$, while if a^* falls on the right of m: $\bar{U}^r = \int_f^1 \int_a^1 \frac{2(1-a)}{(1-m)}q - q^2)dadw = \left[\frac{q}{1-m} - q^2 - \frac{2qa}{1-m} + \frac{qa^2}{1-m} + q^2a\right](1-f)$.

A.2.2 Optimal ability

We can calculate FOC with respect to a $\frac{\partial \bar{U}^l}{\partial a} = \left[-\frac{2qa}{m} + q^2\right](1-f) = 0 \Longrightarrow \frac{2qa}{m} = q^2 \Longrightarrow a^* = \frac{mq}{2}$. We check if the former point is a maximum or minimum by taking SOC: $\frac{\partial^2 \bar{U}^l}{\partial^2 a} = -\frac{2q}{m}(1-f) \leq 0,\ \forall (q,\ m,\ f) \in (0,1)$. Thus, we can ensure that $a^* = \frac{mq}{2}$ is a maximum.

Let us check now if we have another maximum for a^* falling on the right of m. We know from A.2.1 that $\bar{U}^r = \left[\frac{q}{1-m} - q^2 - \frac{2qa}{1-m} + \frac{qa^2}{1-m} + q^2a\right](1-f)$. Thus, $\frac{\partial U_b^r}{\partial a} = \frac{2qa-2q}{1-m} - \frac{2qa-2q}{1-m}f + q^2 - q^2 f = 0 \Longrightarrow a^* = \frac{2q-(1-m)q^2}{2q} = 1 - \frac{(1-m)q}{2}$. SOC can help us to know whether the former point is a max or min, thus $\frac{\partial^2 \bar{U}^r}{\partial^2 a} = \frac{2q}{1-m}(1-f) \geq 0,\ \forall (q,\ m,\ f) \in (0,1)$. We can ensure that $a^* = 1 - \frac{(1-m)q}{2}$ is not a maximum, and so it cannot be taken into account as a solution. Hence, as with PCM, the optimal reaction of university with respect to exams will be represented by the discontinuous function $a^* = \begin{cases} \frac{mq}{2}\ if\ \ f \leq \frac{mq^2}{2} \\ \frac{f}{q}\ if\ \ f > \frac{mq^2}{2} \end{cases}$.

A.2.3 Optimal fee

As the maximization solution turns out to be met on the left of m, then we use the following utility function: $\bar{U}^l = \left(qm - \frac{q}{m}a^2 + q^2a - q^2 + \frac{q-2qm+m^2}{1-m}\right)(1-f)$. By using the backward induction technique, we plug the optimal a^* into the former utility function, i.e., $\bar{U}^l(a^* = \frac{mq}{2}) = \left(qm + \frac{mq^3}{4} - q^2 + \frac{q-2mq+m^2}{1-m}\right)(1-f)$. We now take FOC with respect to f in order to check for a possible maximum: $\frac{\partial \bar{U}^l}{\partial f} = -mq - \frac{mq^3}{4} + q^2 - \frac{q-2mq+m^2}{1-m} = 0$. No optimal f is obtained from the first order condition, although the corner solution: $f^* = 0$ can be a feasible maximum. However, to check the latter, we need to rule out the possibility that $\bar{U}^l(a^* = \frac{f}{q})$ is superior.

$\bar{U}^l(a^* = \frac{f}{q}) = \left(mq - \frac{f^2}{mq} + qf - q^2 + \frac{q-2mq+m^2}{1-m}\right)(1-f)$. In order to compare the former with $\bar{U}^l(a^* = \frac{mq}{2})$, we should analyze the distance between the only two different terms, i.e. $-\frac{f^2}{mq} + qf$, and $\frac{mq^3}{4}$. We know that $f > \frac{mq^2}{2}$ for $a^* = \frac{f}{q}$, therefore the highest weight will be focused on the negative term $-\frac{f^2}{mq}$. That is $-\frac{f^2}{mq} + qf \leq \frac{mq^3}{4}, \forall (q,\ m,\ f) \in (0,1)$ $\Longrightarrow \bar{U}^l(a^* = \frac{f}{q}) < \bar{U}^l(a^* = \frac{mq}{2}),\ \forall (q,\ m,\ f) \in (0,1)$. Finally, we can conclude that optimal tuition fee under borrowing constraints is set to zero, i.e. $f^* = 0$.

A.2.4 Optimal quality

By plugging a^*, and f^* into $\bar{U}$, we will have: $\bar{U}(a^*,\ f^*) = \left(mq + \frac{mq^3}{4} - q^2 + \frac{q-2mq+m^2}{1-m}\right)$.
FOC: $\bar{U}(a^*,\ f^*) = \left(mq + \frac{mq^3}{4} - q^2 + \frac{q-2mq+m^2}{1-m}\right)$. Therefore, FOC: $\frac{\partial \bar{U}(a^*,\ f^*)}{\partial q} = \left(\frac{3mq^2}{4} - 2q + \frac{(1-m-m^2)}{1-m}\right) =$

$0, \forall f < 1 \Longrightarrow {}_2q_1^* = \frac{2}{3m}\left(2 \pm \sqrt{4 - 3m(\frac{1-m-m^2}{1-m})}\right)$. Given that $m,\ q \in (0,1)$, we only consider $q^* = \frac{2}{3m}\left(2 - \sqrt{4 - 3m(\frac{1-m-m^2}{1-m})}\right)$. It is a maximum since $\frac{\partial^2 U(a^*,\ f^*)}{\partial^2 q} = \frac{3mq}{2} - 2 < 0$, $\forall m,\ q \in (0,1)$. By plugging q^* in SPE^{BC}, the values for $a^*,\ f^*$ are obtained straightforward.