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ADAPTIVE CONTROL OF THE WHEELED MOBILE ROBOTS' DYNAMIC MODEL WITH REGARD TO THE LIMITATION OF INPUT TORQUES

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Resumen: En la presente investigación se ha llevado a cabo el control de robots móviles de ruedas no holonómicos en el seguimiento de una trayectoria deseada, cuando están expuestos a perturbaciones, dinámicas no moduladas e incertidumbres. En la sección 1, dada la existencia de un término bajo el título de "perturbación y dinámica no modificada en el modelo de robot", un controlador está diseñado para ser resistente a la perturbación en los modelos de robot. Entonces, suponiendo la falta de información sobre las matrices dinámicas del modelo de robot, se diseña un controlador de crucero adaptativo basado en control de modo deslizante de tal manera que lleva al robot a realizar un seguimiento de una trayectoria deseada predeterminada sin utilizar los valores de las matrices del sistema . Combinar dos controladores robustos y adaptables y crear un controlador adaptativo resistente a las perturbaciones en el sistema fue el siguiente logro del artículo. Finalmente, con el fin de considerar la condición de saturación del operador, la ley de control adaptativo está diseñada de tal manera que dada la incertidumbre en la dinámica del robot y también la limitación en el par de entrada, el robot móvil continúa rastreando una trayectoria deseada predeterminada. Los resultados de la simulación numérica apoyan la validez de los métodos de control propuestos.

Palabras clave: Robots Móviles, Seguimiento de una trayectoria deseada, Control robusto, Control adaptativo, Perturbación, Incertidumbre.

Abstract: In the present research, the control of non-holonomic wheeled mobile robots in tracking a desired trajectory, when they are exposed to disturbances, unmodeled dynamics and uncertainties has been carried out. In section 1, given the existence of a term under the title of" Disturbance and unmodeled dynamics in robot model", a controller is designed to be resistant against the disturbance in robot models. Then by supposing the lack of information about the dynamic model matrices of robot, a sliding-mode—control-based adaptive cruise controller is designed in such a way that leads the robot to track a predetermined desired trajectory without using the values of the system matrices. Combining two robust and adaptive controllers and creating an adaptive controller resistant to disturbances in the system was the next achievement of the article. Finally, in order to consider the condition of operator saturation, the adaptive control law is designed such that given the uncertainty in robot dynamics and also the limitation in the input torque, the mobile robot continues to track a predetermined desired trajectory. The results of the numerical simulation support the validity of the proposed control methods.

Keywords: Mobile Robots, Tracking a desired trajectory, Robust control, Adaptive control, Disturbance, uncertainty.

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1. INTRODUCTION

Control of the wheeled mobile robots in the form of stabilization and tracking a desired trajectory considering their various applications since the late seventies and with the approach used in transportation in the factories has attracted many attentions from the researchers (Antonini, 2006) (Krstic,1995). It included different methods for the output feedback linearization to post-step control or sliding-mode controller and finally adaptive control for the modes in which the system model is exposed to output disturbances and uncertainty. Since the controller design assuming the accuracy of dynamic model was practically a wrong assumption and relying on the control law that is designed according to this assumption leads to inaccuracy and unstable state in the system, different methods have been used to deal with this type of uncertainties including adaptive control, fuzzy control, robust control and neural networks.

In Reference (jiang,1997), the time-varying -mode feedback control method has been used to design and control the trajectory of a non-holonomic mobile robot. On the other hand, in order to enhance the mobile robots' performance and efficiency, the design of their optimal trajectory is of particular importance. In different article, the optimal trajectory design has been carried out considering the objective functions (Chettibi,2008). In (Jiang, 2010), the nonlinear feedback control law, one of the most famous control methods for mobile robots, is proposed to track a desired trajectory which is finally proved that the stable system was asymptotic and the errors tend to be zero. No uncertainty is considered in the system and the model considers the system accurate.

In Reference (Aguiar, 2000), an adaptive controller lacking the information from the robot dynamic parameters and using the step-back method has been implemented. Unlike many articles, the paper has been described the robot model in the polar coordinates. In (Shojaei, 2010), the adaptive control method for non-holonomic mobile robots has been proposed despite the uncertainties in modeling. In Reference (Martins, 2010), an adaptive controller has been designed for tracking the mobile robot considering the kinematics and dynamics of robot. One of the advantages of the control law is inserting the saturation constant while the proof of its stability is also simple. In (DeVon, 2017), kinematic and dynamic control of one-wheel robot has been proposed based on the discrete time-invariant control law. The method caused the system's asymptotic stability. The uncertainties and environmental disturbances have not been considered in this model. In (Chih-Yang, 2017), a sliding-mode robust control for tracking the desired trajectory in a mobile robot has been used despite the uncertainty and disturbance in dynamic model of robot. First a kinematic controller is presented for mobile robot. Then the adaptive robust control is added to it, if the high-bandwidth mode of the uncertainty in system is unknown. Finally the problem stability is proved by the Lyapunov theory. Tracking the mobile robots trajectory in the presence of parametric uncertainties has been proposed using the adaptive output feedback controller in Reference (Park, 2011).

In Reference (Wang, 2003), in order to solve the problem of the non-holonomic dynamic systems' stability in the presence of the disturbances and uncertainties, a new controller has been designed. In this paper, an adaptive robust controller is used to compensate parametric uncertainty and a sliding model controller is applied to eliminate the disturbances.

The papers (You-Wei, 2014) (Gwo-Ruey, 2014) are recent works performed in the field of adaptive control of the mobile robots. In (You-Wei, 2014) tracking a moving target has been performed by combining both fuzzy control method and the sliding-mode controller. In the present study, the system uncertainty has been considered in modeling. In (Canigur, 2013), the direct model reference adaptive control algorithm in tracking an optimal trajectory has been used in mobile robots. The main problem of the recent papers is to ignore the input limitations that are not considered in the controller design. In (Gwo-Ruey, 2014), the fuzzy control method has been proposed to track an optimal trajectory considering the limitation of input saturation. The main problem in this method is assuming the accuracy of the system model.

In tracking an optimal trajectory in mobile robots, the main goal is that the mobile robot tracks a predetermined optimal trajectory. In general, it is likely that the model intended for the robot is not accurate and the system is exposed to limited disturbances, unmodeled dynamics or noise. All these cases have prompted us to use the robust or adaptive controllers in solving the optimal trajectory tracking problem.

The design of these controllers provides a condition to control the mobile robots when they are not accurate in dynamic model or they are exposed to the disturbance and types of the unmodeled dynamics. This is an important suggestion for the mobile robots control because many of the control algorithms are available assuming the accuracy of robot dynamic model, so they are not practically implementable. The combination of two kinds of controllers, robust and adaptive, is a new issue that less attention has been paid to it but we have tried to consider it in this paper by designing the optimal controllers and proving their stabilities. In addition, the saturation condition in the input signals of the robot dynamic model has been neglected in many articles and many designed control methods are not practically implementable due to neglecting the condition in the controllers' design.

The paper has tried to design a controller for tracking an optimal trajectory more realistic than the previous methods using a dynamic model of mobile robot exposed to the disturbance and uncertainty and considering the operator saturation condition and it is a big achievement that will discuss later in this paper.

The paper is organized as follow:

In section 2, we will discuss the modeling of mobile robot model in the presence of non-holonomic limitations. In section 3, we will deal with the design of the robust, adaptive and robust-adaptive controllers in order to track an optimal trajectory in the presence of uncertainty, disturbance and also adaptive control with the input limitations to deal with the operator saturation condition. The results of the simulation in section 4 will show the validity of the control laws and the conclusions will be presented in section 5.

2. ROBOT MODELS EXPOSED TO THE NON-HOLONOMIC LIMITATIONS

The kinematic and dynamic model of a mobile robot is considered as follow:

$$\dot{q} = S(q)v,$$
$$M_1\dot{v}(t) + C_1(q,\dot{q}) + D_m v + G_1(q) + \tau_{d1}(t,q,\dot{q}) = B_1(q)\tau_a \quad (1)$$

where, $q = [x_0, y_0, \theta]^T$ are the system matrices including M_1 and $C_1(q,\dot{q})$ and D_m, $G_1(q)$, τ_a is the input torque and $\tau_{d1}(t,q,\dot{q})$ is the external disturbance. Consider the following mobile robot:

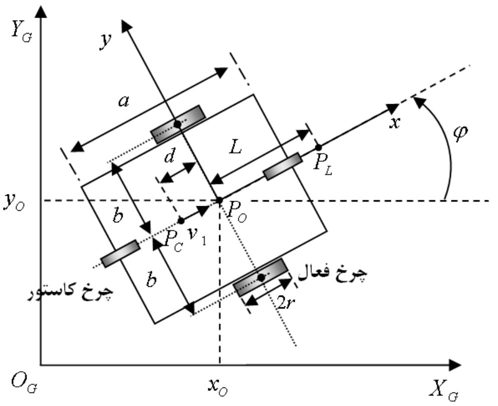

Figure 1: a sample of a wheeled mobile robot

This robot moves in a horizontal plane and has two fixed wheels that are placed on the same axis. It has also two inactive castor wheels that are intended to keep the balance of robot. The robot's center of mass places in the point of $P_C = (x_C, y_C)$. The point $P_0 = (x_0, y_0)$ is the origin of the local coordinates system that is placed on the body of robot with a distance d from the point P_c. The point $P_L = (x_L, y_L)$ is a virtual reference point on the x axis of the local coordinates with a distance L from P_0. If we choose the robot coordinates in a form of $q = [x_0, y_0, \theta]^T$ the speed limitation will be seen as $\dot{y}_0\cos(\theta) - \dot{x}_0\sin(\theta) = 0$. So the quasi-speeds vector of a non-holonomic mobile robot system is defined as $v(t) = [v_1(t), v_2(t)]^T$ where, v_1 and v_2 are shown respectively the linear speeds and the angular speeds of robot. In this case, the kinematic and dynamic models of robot are yielded as follow (You-Wei, 2014).

(2)

$$S(q) = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix}, M_1(q) = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}$$
$$C_1(q,\dot{q}) = \begin{bmatrix} 0 & m_c d\dot{\theta} \\ -m_c d\dot{\theta} & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1/r & 1/r \\ b/r & -b/r \end{bmatrix}$$
$$A(q) = [-\sin(\theta) \ \cos(\theta) \ 0]$$

We have:

(3)

$$m = m_c + 2m_w$$
$$I = I_c + 2I_m + m_c d^2 + 2m_w b^2$$

What is usually considered in control of mobile robots in two-dimensional space is to steer the front point of robot in which the detection sensors are placed. For this reason, the system output (1) is considered as a point in front of it.

The output can be considered in front of robot according to control of P_l point and is expressed as follow:

(4)

$$y = [h_1(q), h_2(q)]^T = [x_0 + L\cos(\theta), y_0 + L\sin(\theta)]^T$$

Considering equation (4), we can write the equation (1) as follow:

(5)

$$M(q)\ddot{Y} + C(q,\dot{Y})\dot{Y} + D(q)\dot{Y} + G(q) + \tau_d = B_2(q)\tau_a$$

In this case, the system matrices for the reduced model will be as follow:

$$M = \begin{bmatrix} m\cos^2\theta + \frac{I}{L^2(1-\cos^2\theta)} & 0.5\left(m - \frac{I}{L^2}\right)\sin(2\theta) \\ 0.5\left(m - \frac{I}{L^2}\right)\sin(2\theta) & \frac{I}{L^2\cos^2\theta} + m(1-\cos^2\theta) \end{bmatrix};$$
$$D = \begin{bmatrix} d_{m1}\cos^2(\theta) + d_{m2}\sin^2\theta/L^2 & (d_{m1} - d_{m2}/L^2)\sin\theta\cos\theta \\ (d_{m1} - d_{m2}/L^2)\sin\theta\cos\theta & d_{m1}\sin^2(\theta) + d_{m2}\cos^2\theta/L^2 \end{bmatrix}$$
$$C = \begin{bmatrix} -0.5\dot{\theta}\sin(2\theta)\left(m - \frac{I}{L^2}\right) & m\dot{\theta}\cos^2\theta + \left(\frac{I}{L^2}\right)\dot{\theta}\sin^2\theta + \frac{m_c d\dot{\theta}}{L} \\ -m\dot{\theta}\sin^2\theta - \left(\frac{I}{L^2}\right)\dot{\theta}\cos^2\theta - \frac{m_c d\dot{\theta}}{L} & 0.5\dot{\theta}\sin(2\theta)\left(m - \frac{I}{L^2}\right) \end{bmatrix}$$

In model (5), τ_a is the input torque applied on the left and right wheels of robot. And the aim of designing the controller is to find the torques applied on robot's wheels so that it can track an optimal trajectory in the plane.

3. CONTROLLER DESIGN

In this section, considering the dynamic equation of robot and the non-holonomic limitations and the presence of the disturbance and uncertainty and also the limitation in input torque, new suitable controllers have been designed to track an optimal trajectory in mobile robots.

3.1. The design of robust controller to deal with the disturbance and unmodeled dynamics

Considering the error as $\tilde{y} = y - y_{des}$ in model (5), the integral switch board s (t) is selected as follow [22]:

$$s(t) = \dot{y} - v' \quad (6)$$
$$v' = \dot{y}_{des} - \Lambda\tilde{y}$$

where , $s(t) \in \mathbb{R}^2$ and $\Lambda > 0$ is a positive definite matrix y_{des}. shows the determined optimal trajectory and the robot needs to track it in two-dimensional plane and y is also the system output (5), that is the front position of robot in the plane x-y.

In order to design a controller that is resistant against the disturbance and unmodeled dynamics, we consider the control input τ_a in equation (5) as follow:

$$\tau_a = B_2(q)^{-1}\{M\dot{v}' + Cv' + Dv' + G - Ks + u_0\} \quad (7)$$

In this equation, s (t) and υ are defined like equation (6) and k>0 is an arbitrary positive definite matrix. The value of u_0 is a term added to the control law to cope with the disturbances and uncertainty in model (5), that is τ_d the structure of u_0 is formed by being negative of the Lyapunov's function derivative.

Proposition1. Consider the equation of non-holonomic mobile robot in (5). By applying the control law (7), if the control gain K>0 is an arbitrary definite positive matrix, wherein γ shows the high bandwidth of the uncertainty and disturbance in (5), then the closed-loop system of error is asymptotically stable and the value γ tends to exponentially reach the optimal value γ_{des}.

Proof: by inserting the designed input torque in (7), we have in (5):

$$M\dot{Y} + C\dot{Y} + D\dot{Y} + G + \tau_d = B_2(q)\left\{B_2(q)^{-1}\{M\dot{v}' + Cv' + Dv' + G - Ks + u_0\}\right\} = M\dot{v}' + Cv' + Dv' + G - Ks + u_0 \quad (8)$$

Thus we have:

$$M(\dot{Y} - \dot{v}') + C(\dot{Y} - v') + D(\dot{Y} - v') + Ks = u_0 - \tau_d$$
$$\Rightarrow \quad M\dot{s} + Cs + Ks = u_0 - \tau_d \quad (9)$$

Now we select the following candidate Lyapunov's function, so we have

$$V = \frac{1}{2}s^T M s \quad (10)$$

According to Lyapunove's stability proposition, if its time derivative becomes negative in direction of the system trajectory, it can be concluded that the system is asymptotically stable. By deriving from V in respect to time and substituting the value $M\dot{s}$ from equation (9) and after simplification, we have:

$$\dot{V} = s^T\{-Ks + u_0 - \tau_d\} = -s^T Ks + s^T u_0 - s^T \tau_d \quad (11)$$

Now by substituting the value of $u_0 = -\gamma \frac{s}{\|s\|}$ in proposition 1, we have:

$$\dot{V} = -s^T Ks - \gamma\|s\| - s^T\tau_d \quad (12)$$

Considering that $-s^T\tau_d \leq \|s\|\tau_d$, we can write (12) as follow:

$$\dot{V} = -s^T Ks + (\tau_d - \gamma)\|s\| \leq 0 \quad (13)$$

We have concluded that V becomes negative considering that $\|\tau_d\| \leq \gamma$ and that K>0.

Finally, since the Lyapunove's function was determined positive and its derivative was negative, using the Lyapunove's proposition, the closed-looped error system with controller (7) is asymptotically stable and the error value according to zero s will be tended to be zero.

3.2. The design of adaptive controller to deal with the uncertainty

Although the controller that was developed in the previous section was able to cope with the disturbance and unmodeled dynamics, its use in robot, as shown in equation (7), requires us to have the complete information about the system's matrices, M, C, D and G. Of course having the exact knowledge about these matrices is practically impossible and the matrices are always exposed to the uncertainty. In this section, an adaptive controller to cope with the uncertainty in modeling will be developed. In the controller, we will use the estimated values of the system's matrices and the system dynamic parameters will be estimated by adaptive control law.

In order to use the adaptive control law that doesn't use the system dynamic matrices, the robot dynamic model in equation (5) is written in the following regression form: (now we suppose that τ_d=0)

(14)

$$M(q)\ddot{Y} + C(q,\dot{Y})\dot{Y} + D(q)\dot{Y} + G(q) = Y'(q,\dot{Y},\ddot{Y})\theta$$

In this equation, we have

$$\theta = [m, I, m_c d, d_{m1}, d_{m2}] \quad (15)$$

$$Y'(q,\dot{Y},\ddot{Y}) = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} \end{bmatrix} \quad (16)$$

To summarize, we waive from stating the details of the matrix elements $Y'(q,\dot{Y},\ddot{Y})$.

To propose a control algorithm that makes the system stable even in the presence of modeling uncertainties, we consider the control law as follow:

$$\tau_a = B_2(q)^{-1}\{\hat{M}\dot{v}' + \hat{C}v' + \hat{D}v' + \hat{G} - Ks\}$$
(17)

The matrices $\hat{M}$ ،$\hat{C}$ ،$\hat{D}$ و $\hat{G}$ ،are the estimates of the system's matrices. However, regarding the regression form in (14), we have:

$$\hat{M}\dot{v}' + \hat{C}v' + \hat{D}v' + \hat{G} = Y'(\dot{v}', v', Y, \dot{Y})\hat{\theta} = Y_d\hat{\theta} \quad (18)$$

where, it is assumed that $Y'(\dot{v}', v', Y, \dot{Y}) \triangleq Y_d$ Thus the estimated control law can be considered as follow:

$$\tau_a = B_2(q)^{-1}\{Y_d\hat{\theta} - Ks\} \quad (19)$$

Through the substitution of adaptive control algorithm in (19) in model (14), we have:

$$M(q)\ddot{Y} + C(q,\dot{Y})\dot{Y} + D(q)\dot{Y} + G(q) + \tau_d = Y_d\hat{\theta} - Ks \quad (20)$$

In order to obtain the closed-loop system model in terms of s and $\dot{s}$, we add and distract the expression of $\pm\{M\dot{s} + Cs + Ds + G\}$ to the first side of equation (20). Finally, the equation of closed-loop system will be obtained as follow:

$$M\dot{s} + Cs + Ds + Y_d\theta = Y_d\hat{\theta} - Ks \quad (21)$$

and finally we have

$$M\dot{s} + Cs + Ds + Ks = Y_d\tilde{\theta} \quad (22)$$

Where $\tilde{\theta} = \hat{\theta} - \theta$ and indicates the error in estimating the parameters. Indeed, if the value of parameters is accurate in the control input (19), that is, $\hat{\theta} = \theta$ the second side in equation (22) is zero and as we have stated in the previous section, the system will be stable. But the problem is that the accurate value of the system parameters is not practically available and it should be estimated.

The control algorithm (19) along with an adaptive algorithm is recommended and proved to estimate the system parameters, namely, $\hat{\theta}$ in the following proposition.

Proposition 2:

Consider the equation of the non-holonomic mobile robot in (5). By using the control law (19), if the control gain K>0 is a positive arbitrary definite matrix, the closed-loop system of error is asymptotically stable and the γ value tends to reach exponentially to optimal value of y_{des} In this law, the $\hat{\theta}$ value is obtained from the below estimation:

$$\dot{\hat{\theta}} = -\Gamma Y_d^T s \tag{23}$$

where, Γ is a positive definite matrix.

Proof: Unlike the prior propositions, we choose the candidate Lyapunov's function as follow:

$$V = \frac{1}{2} s^T M s + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{24}$$

where, Γ>0 is a constant matrix. By deriving from V with respect to time and substituting from equation (22), we have

$$\dot{V} = -s^T K s + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} + s^T Y_d \tilde{\theta} \tag{25}$$

Given, $s^T Y_d \tilde{\theta}$ is scalar , we can write $s^T Y_d \tilde{\theta} = \tilde{\theta}^T {Y_d}^T s$and thus the equation (25) will be written as follow:

$$\dot{V} = -s^T K s + \tilde{\theta}^T \{\Gamma^{-1} \dot{\tilde{\theta}} + {Y_d}^T s\} \tag{26}$$

Now, by selecting the adaptive law as (23) and whereas the value of the system parameters are constant and thus $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, the derivative value of the Lyapunov's function will be as follow and as mentioned earlier it will be negative.

$$\dot{V} = -s^T K s < 0 \tag{27}$$

Since, the Lyapunove's function is positive definite and its derivative is negative definite and using the Lyapunove's proposition, the closed-loop error system with the controller (19) is asymptotically stable and as we have shown in the previous section, the error value will tend to reach zero regarding zero s.

3.3. The design of adaptive-robust controller

The developed control law in section 3-1 was designed to deal with the disturbance and unmodeled dynamics. The control law obtained in section 3-2 was also developed to deal with the modeling uncertainties in dynamic model of robot. But the question is that how can we develop a control law to be able to consider simultaneously both the disturbance and unmodeled dynamics and also the parametric uncertainties in the system. By combining both robust control in equation (7) and adaptive control in equation (19), the adaptive-robust control law will be yielded as follow:

$$\tau_a = B_2(q)^{-1} \{Y_d \hat{\theta} - K s + u_0\} \tag{28}$$

This combined control law makes robotic system stable in the presence of disturbance and unmodeled dynamics and also in the presence of the uncertainty in the system. It also changes the output into the optimum output. Indeed u_0 was added to adaptive control law in the previous section to deal with the disturbance and unmodeled dynamic τ_d.

Proposition3. Consider the equation of non-holonomic mobile robot in (5). By applying the control law (28), if the control gain K>0 is an arbitrary positive definite matrix, the closed-loop system of the error is asymptotically stable and the y value will exponentially tend to reach to an optimal value of $y_{des.}$ In this algorithm, the $\hat{\theta}$ and u_0 values are obtained from the following equations:

$$\dot{\hat{\theta}} = -\Gamma Y_d^T s \tag{29}$$

$$u_0 = \frac{-\gamma^2 s}{\gamma \|s\| + \sigma(t)} \tag{30}$$

where, Γ is a positive definite matrix, γ shows the high bandwidth of the disturbance and σ (t) is an arbitrary positive function so that $\int_0^\infty \sigma(t) dt < \infty$

Proof: The proof for the proposition is simply done and achieved by combining the proofs stated in the both previous propositions and for its simplicity, we waive from its replication.

3.4. The adaptive control design by taking the operator saturation condition into consideration

In the previous section, a controller was designed to track a predetermined optimal trajectory of a robot in the presence of uncertainty in mobile robot dynamics. In this section, we will focus on the limitation in the input control due to the presence of saturation in the existing operators in robot wheels to produce the necessary torque. The proposed adaptive control law has error terms (tanh) and will be always limited. In addition, by using the adaptive law, we will not need having the knowledge about the dynamics of the system.

We will rewrite the control law in (5) as follow:

$$\tau_a = B_2(q)^{-1} \left(-K_p \tanh(e) - K_i \tanh(z) - K_d \tanh(\dot{e}) + Y \hat{\theta}\right)$$

(31)

We apply the equation (31) in equation (5):

$$M(q)\ddot{Y} + C(q, \dot{Y})\dot{Y} + D(q)\dot{Y} + G(q) = -K_p \tanh(e) - K_i \tanh(z) - K_d \tanh(\dot{e}) + Y\hat{\theta}$$

By substituting $\dot{y} = \dot{e} + \dot{y}_d$ and $\ddot{y} = \ddot{e} + \ddot{y}_d$ in equation (32), we have

$$M_2(q)(\ddot{y}_d + \ddot{e}) + C_2(q, \dot{e} + \dot{y}_d)(\dot{e} + \dot{y}_d) + D_2(q)(\dot{e} + \dot{y}_d) = M_2(q)(\ddot{e}) + C_2(q, \dot{e})(\dot{e}) + D_2(q)(\dot{e}) + 2C_2(q, \dot{y}_d)(\dot{e}) + Y\theta$$
$$= -K_p \tanh(e) - K_i \tanh(z) - K_d \tanh(\dot{e}) + Y\hat{\theta}$$

(33)

Given that $\tilde{\theta} = \hat{\theta} - \theta$ we have:

$$M_2(q)(\ddot{e}) + C_2(q, \dot{e})(\dot{e}) + D_2(q)(\dot{e}) + 2C_2(q, \dot{y}_d)(\dot{e}) = -K_p \tanh(e) - K_i \tanh(z) - K_d \tanh(\dot{e}) + Y\tilde{\theta}$$

(34)

Now we choose the Lyapunov's function as follow:

$$V = \frac{1}{2}\dot{e}^T M \dot{e} + \frac{1}{\alpha} tanh^T(e) M \dot{e} - e^T + \sum_{i=1}^{n}\left(k_{pi} + \frac{d_i}{\alpha}\right) \ln(\cosh(e_i)) + \frac{1}{\alpha^2}\int_0^{\phi} \sigma^T K_i cosh^2(z)\, d\sigma + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

(35)

By deriving from (35), we have

$$\dot{V} = \frac{1}{2}\dot{e}^T \dot{M}_2(q)\dot{e} + \dot{e}^T M_2(q)\ddot{e} + \frac{1}{\alpha}(sech^2(e)\dot{e})^T M_2(q)\dot{e} + \frac{1}{\alpha} tanh^T \dot{M}_2(q)\dot{e} + \frac{1}{\alpha} tanh^T(e) M_2(q)\ddot{e}$$
$$+ \dot{e}^T k_p\, tanh(e) + \frac{1}{\alpha} tanh^T(e) D_2 \dot{e} + \frac{1}{\alpha^2}\varphi^T cosh^2(z) k_i \dot{\varphi} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}}$$

(36)

According to (34), the equation (36) is written as follow:

$$\dot{V} = -\dot{e}^T K_d\, tanh(\dot{e}) - \dot{e}^T D_2 \dot{e} - \frac{1}{\alpha} tanh^T(e) K_d\, tanh(\dot{e}) - \frac{1}{\alpha} tanh^T(e) K_p\, tanh(e) - 2\dot{e}^T C_2(q, \dot{y}_d)\dot{e}$$
$$- \frac{2}{\alpha} tanh^T(e) C_2(q, \dot{y}_d)\dot{e} + \frac{1}{\alpha}\left[tanh^T(e) C_2^T(q, \dot{e})\dot{e} + (sech^2(e)\dot{e})^T M_2(q)\dot{e}\right] + \dot{e}^T Y\tilde{\theta} + \frac{1}{\alpha} tanh^T(e) Y\tilde{\theta}$$
$$+ \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}$$

(37)

and finally considering the adaptive control law as follow:

$$\dot{\hat{\theta}} = -\Gamma Y^T\left(\dot{e} + \frac{1}{\alpha}\tanh(e)\right) \quad (38)$$

The derivative of the Lyapunov's function becomes negative and when it is occurred similar to the previous section, the error value reaches to zero and the system output will track an optimal output.

4. SIMULATION

The results obtained from applying the developed adaptive controllers in the previous section in tracking an optimal trajectory for a mobile robot based on dynamic model will be shown in this section by simulating in MATLAB.

4.1. First simulation

In first simulation, adaptive-robust control law (28) by assuming that the disturbance as $\tau_{d1} = [cos\left(\frac{t}{4}\right), sin\left(\frac{t}{4}\right)]^T$ will be applied on the model of robot. In order to simulate, we choose an optimal trajectory that robot needs to track as a circular path:

$$Y_d = \begin{bmatrix} x_0 + Rcos(\omega t) \\ y_0 + Rsin(\omega t) \end{bmatrix}$$

where we have:

$$\omega = 0.05 \quad , R = 3 \ , x_0 = y_0 = 10$$

We choose the controller parameters as follow:

$$K = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.8 \end{bmatrix}; \quad \Lambda = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.8 \end{bmatrix}; \ \sigma(t) = \frac{1}{1 + t^3}, \gamma = 2$$

$$\Gamma = \begin{bmatrix} 80 & 0 & 0 & 0 & 0 \\ 0 & 70 & 0 & 0 & 0 \\ 0 & 0 & 70 & 0 & 0 \\ 0 & 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 0 & 90 \end{bmatrix}$$

Figure 2 shows the position of robot. We see that by applying the adaptive-robust controller, the robot can well track the optimal circular trajectory despite the unmodeled dynamics. Note that in this case, none of the system matrices has been used.

In figure 3, the values of the input torques are shown by applying the adaptive-robust control law and as you can see both of the torques have been limited and tended toward zero when the robot reaches to the optimal trajectory.

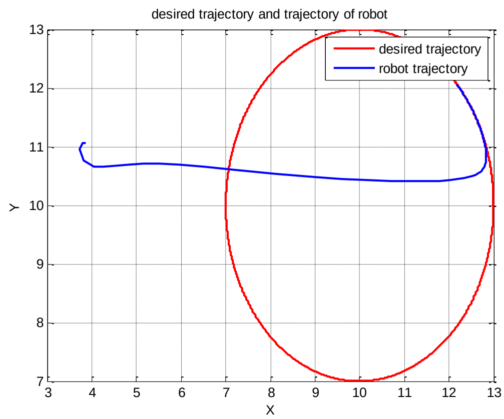

Figure 2. Trajectory of mobile robot to a desired site in simulation 1

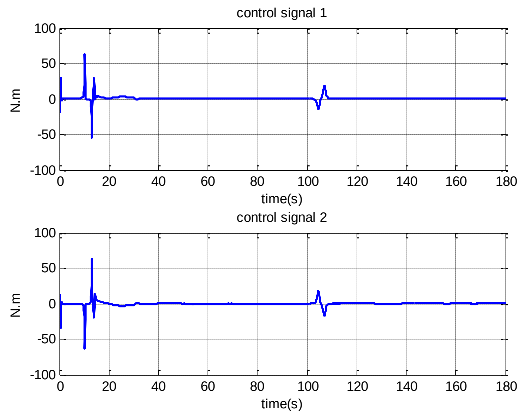

Figure 3. Input torques in simulation 1

4.2. Second simulation

In simulation 2, the adaptive- robust control law has been simulated in (31) and (38) considering the operator saturation condition. The optimal trajectory is similar to the first simulation and the controller parameters are selected as follow:

$\alpha = 100$

$$K_p = \begin{bmatrix} 30 & 0 \\ 0 & 80 \end{bmatrix}; \quad K_i = \begin{bmatrix} 45 & 0 \\ 0 & 20 \end{bmatrix}; \quad K_d = \begin{bmatrix} 32 & 0 \\ 0 & 25 \end{bmatrix};$$

$$\Gamma = diag\{20,30,40,50,50\}$$

The optimal trajectory and the robot trajectory are plotted in the plane in the following figure and as we have shown, robot can well track the circular path.

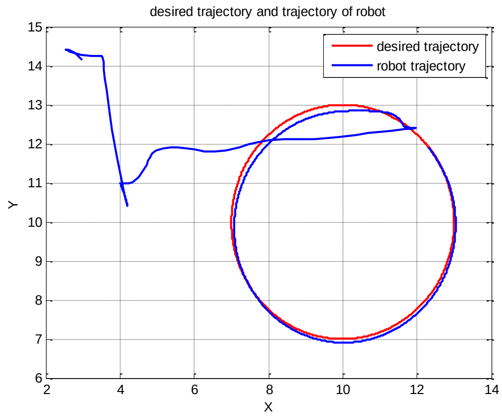

Figure 4. Robot and the optimal trajectory in simulation 2

As we expected, the control signals in figure 5 have limited values, while we spend a lot of costs for this kind of the controller, as shown in control signals, is "chattering phenomenon" that causes discontinuity in controlled inputs.

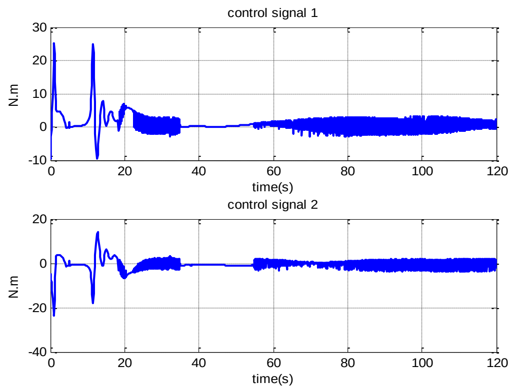

Figure 5. The limited control signals in Simulation 2

5. CONCLUSION

In the present study, the issue of tracking the optimal trajectory in the wheeled mobile robots in the presence of uncertainty, disturbance and unmodeled dynamics in robot dynamic model and also considering the operator saturation condition and the necessity for applying the limited torques on the robot's wheels has been considered. In order to cope with the disturbance and the unmodeled dynamics, the sliding-mode controller was developed in such a way that the system can be resistant and robust with respect to the disturbance in the model of robot. In the following, the parameters of some parts in dynamic models used in control law have been estimated using an adaptive law, so an adaptive controller that also considers the operator saturation condition is obtained. Applying both of the adaptive- robust control law and the adaptive control law while meeting the operator saturation condition have simulated in the form of two different examples. The results obtained from the simulations show the validity and the consistency of the developed controllers.

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