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DYNAMIC RESPONSE OF A THREE-LAYERED SANDWICH BEAM UNDER OSCILLATORY MOVING SPRUNG MASS

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Resumen: Hoy en día, el uso de nuevos materiales con una alta relación resistencia-peso, como materiales compuestos y materiales en sandwich, está aumentando en la industria del transporte. Este problema es más comúnmente usado en puentes especiales para pasar vehículos y trenes. Por lo tanto, el análisis de respuesta dinámica de la estructura bajo carga móvil es de gran importancia. Por lo tanto, una gran cantidad de investigación se ha llevado a cabo en este campo que ayuda a resolver estos problemas. El propósito de esta investigación es analizar la respuesta dinámica de un haz de sándwich en masa oscilatoria móvil. El propósito de esta investigación es analizar la respuesta dinámica de un haz de sándwich en masa oscilatoria móvil. Muchos parámetros están implicados en la respuesta dinámica del haz de emparedado, que se puede observar como sigue. 2. Relación entre la rigidez del muelle de muelle y la rigidez de flexión del haz de emparedado y 3. La relación de la frecuencia de vibración de la masa de muelle móvil a la frecuencia de vibración de El haz de sándwich. En el presente estudio, se investiga el efecto de diversos parámetros tales como las relaciones de masa, rigidez, frecuencia, etc. sobre la respuesta dinámica del haz en sandwich bajo carga móvil oscilatoria. La respuesta dinámica de un haz de sándwich de tres capas bajo carga oscilatoria de muelle móvil es mediante el uso de la teoría de tres capas componente se considera como una de las innovaciones de este trabajo. Se investigarán los parámetros de rendimiento del haz, tales como las velocidades críticas y la frecuencia crítica del haz en sandwich. Esta revisión se llevará a cabo analíticamente y por la teoría convencional en problemas de rayos Sandwich. Los resultados serán verificados comparando con los resultados en la literatura de la asignatura o realizando las simulaciones numéricas necesarias.

Palabras clave: haz de emparedado, masa oscilatoria móvil, teoría de tres capas

Abstract: Today, the use of new materials with a high strength-to-weight ratio, such as composite materials and sandwich materials is increasing in the transportation industry. This issue is most commonly used in special bridges to passing vehicles and trains. Therefore, the dynamic response analysis of the structure under moving loading is of great importance. Hence, a lot of research has been carried out on this field that helps in solving such problems. The purpose of this research is to analyze the dynamic response of a sandwich beam under oscillatory moving sprung mass.

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The purpose of this research is to analyze the dynamic response of a sandwich beam under oscillatory moving sprung mass. Many parameters are involved in the dynamic response of the sandwich beam, which can be noted as follows. 1. The mass ratio of the moving sprung mass to the sandwich beam mass, 2. The ratio of the sprung spring stiffness to the sandwich beam bending stiffness and 3. The ratio of the vibrational frequency of the moving sprung mass to the vibrational frequency of the sandwich beam. In the present study, the effect of various parameters such as mass ratios, rigidity, frequency etc. on dynamical response of the sandwich beam under oscillatory moving load is investigated. The dynamic response of a three-layered sandwich beam under oscillatory moving sprung load is by using three-layer theory component is considered as one of this work's innovations. The performance parameters of the beam, such as critical velocities and critical frequency of the sandwich beam will be investigated. This review will be carried out analytically and by the conventional theory in Sandwich beams problems. The results will be verified by comparing with the results in the subject literature or performing necessary numerical simulations.

Keywords: sandwich beam, oscillatory moving sprung mass, three-layered theory

1. INTRODUCTION

Today, the use of structures made of composite materials has been growing. Since 1891, the use of composite multi-layer coatings has become commonplace in sandwich structures. This will increase the performance and reduce the weight of the structure. The reason for the abundant use of these structures is their high bending strength caused by the charge carrier plates that are separated by the core and far from each other. Sandwich structures are made up of two composite or metal top and bottom layers that can be different, with a high weight and stiffness, usually thin, connected to a weak core with low weight, low modulus and high thickness, which is usually composed of foam, mineral wool and honeycomb. This structure increases the bending rigidity. Layers are made of solid and rigid materials such as aluminum, steel, fiber and wood composite. They must withstand a lot of plate tensile and compression forces. A sandwich structure is very light and has a much higher resistance than its components.

It also has a relatively low cost and can be used quickly and easily in construction.

Eftekhar Azam, Mofid, Afghani Khorasani, in an article (Azam, 2013) entitled "dynamic response of Timoshenko beam under moving sprung mass", examined the dynamic response of Timoshenko beam under a moving mass and a moving sprung mass using the theory of Hamilton and the principle of superposition. In a research (Meher, 2012) entitled "Dynamic Response of a Beam Structure to a Moving Mass Using Green's Function" Sudhanso Meher studied Dynamic response of two types of beams experimentally and numerically. In this study, the motion equations in the matrix form are formulated for an Euler beam under a moving load using the Green function. Arturo Cifuentes in a paper entitled (CIFUENTES, 1989) "dynamic response of a beam excited by a moving mass", has used two combined methods of finite element and finite difference to obtain an Euler-Bernoulli beam response to the motion of a moving mass.

2. PROBLEM FORMULATION

The three-layered sandwich beam is a beam consisting of two lower and upper layers and a core. The figure below shows the appearance of the problem.

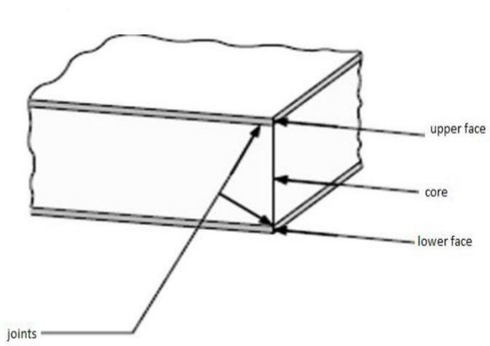

Figure 1. A Three-layered Sandwich Beam

In this problem, the three-layered sandwich beam in which the upper and lower layers are composite, are under a sprung mass that moves along the beam.

The theory used in this study is three-layer theory in which classic theory is used for layers and the first-order theory is used for the core. The displacement equations in the upper and lower layers are as follows.

$$\begin{cases} u^t(x,z,t)=u_0^t(x,t)-\left(z-\dfrac{c+d_t}{2}\right)\dfrac{\partial w_0^t(x,t)}{\partial x} \\ w^t(x,z,t)=w_0^t(x,t) \end{cases} \quad (1)$$

$$\begin{cases} u^b(x,z,t)=u_0^b(x,t)-\left(z+\dfrac{c+d_t}{2}\right)\dfrac{\partial w_0^b(x,t)}{\partial x} \\ w^b(x,z,t)=w_0^b(x,t) \end{cases}$$

And the displacement equations for the core will be as follows.

$$\begin{cases} u^c(x,z,t)=u_0^c(x,t)+zu_1^c(x,t) \\ w^c(x,z,t)=w_0^c(x,t)+zw_1^c(x,t) \end{cases} \quad (2)$$

In this theory, the terms of correlation will be written as follows.

$$\begin{cases} u^c\big|_{-\frac{c}{2}}=u^b\big|_{-\frac{c}{2}} \\ u^c\big|_{+\frac{c}{2}}=u^t\big|_{+\frac{c}{2}} \\ w^c\big|_{-\frac{c}{2}}=w^b\big|_{-\frac{c}{2}} \\ w^c\big|_{+\frac{c}{2}}=w^t\big|_{+\frac{c}{2}} \end{cases} \quad (3)$$

$$\begin{cases} u^c\big|_{-\frac{c}{2}}=u^b\big|_{-\frac{c}{2}} \\ u^c\big|_{+\frac{c}{2}}=u^t\big|_{+\frac{c}{2}} \end{cases} \rightarrow \begin{cases} u_0^c(x,t)-\dfrac{c}{2}u_1^c(x,t)=u_0^b(x,t)-\dfrac{1}{2}d_b\left(\dfrac{\partial w_0^b(x,t)}{\partial x}\right) \\ u_0^c(x,t)+\dfrac{c}{2}u_1^c(x,t)=u_0^t(x,t)+\dfrac{1}{2}d_t\left(\dfrac{\partial w_0^t(x,t)}{\partial x}\right) \end{cases} \quad (4)$$

$$\begin{cases} u_0^c(x,t)=\dfrac{u_0^t(x,t)}{2}+\dfrac{u_0^b(x,t)}{2}+\dfrac{d_t}{4}\dfrac{\partial w_0^t(x,t)}{\partial x}-\dfrac{d_b}{4}\dfrac{\partial w_0^b(x,t)}{\partial x} \\ u_1^c(x,t)=\dfrac{1}{c}u_0^t(x,t)-\dfrac{1}{c}u_0^b(x,t)+\dfrac{d_t}{2c}\dfrac{\partial w_0^t(x,t)}{\partial x}+\dfrac{d_b}{2c}\dfrac{\partial w_0^b(x,t)}{\partial x} \end{cases}$$

$$u^c(x,z,t)=\frac{1}{2}u_0^t(x,t)+\frac{1}{2}u_0^b(x,t)+\frac{1}{4}d_t\left(\frac{\partial w_0^t(x,t)}{\partial x}\right)-\frac{1}{4}d_b\left(\frac{\partial w_0^b(x,t)}{\partial x}\right)+ z\left[\frac{1}{c}u_0^t(x,t)-\frac{1}{c}u_0^b(x,t)+\frac{1}{2c}d_b\frac{\partial w_0^b(x,t)}{\partial x}+\frac{1}{2c}d_t\frac{\partial w_0^t(x,t)}{\partial x}\right] \quad (5)$$

$$w^c(x,z,t)=w_0^c(x,t)+zw_1^c(x,t)$$

$$\begin{cases} w_0^c=\dfrac{1}{2}w_0^t(x,t)+\dfrac{1}{2}w_0^b(x,t) \\ w_1^c=\dfrac{1}{c}w_0^t(x,t)-\dfrac{1}{c}w_0^b(x,t) \end{cases} \quad (6)$$

$$w^c(x,z,t)=\frac{1}{2}w_0^t(x,t)+\frac{1}{2}w_0^b(x,t)+z\left[\frac{1}{c}w_0^t(x,t)-\frac{1}{c}w_0^b(x,t)\right]$$

In this study, we have five unknowns, which are the

$$u_0^t,\ u_0^b,\ w_0^t,\ w_0^b,\ w_0^m$$

The field of strain will be calculated from the following equation

$$\begin{cases} \varepsilon_x=\dfrac{\partial u(x,z,t)}{\partial x}=\dfrac{\partial u_0(x,t)}{\partial x}-z\dfrac{\partial^2 w_0(x,t)}{\partial x^2} \\ \varepsilon_z=\dfrac{\partial w(x,z,t)}{\partial z} \\ \gamma_{xz}=\dfrac{\partial u(x,z,t)}{\partial z}+\dfrac{\partial w(x,z,t)}{\partial x} \end{cases} \quad (7)$$

So, to calculate the fiel of strain in the upper and lower layers, Equations 8 and 9 can be used

$$\begin{cases} \varepsilon_x^t=\dfrac{\partial u^t(x,z,t)}{\partial x}=\dfrac{\partial u_0^t(x,t)}{\partial x}-\bar{a}\left(\dfrac{\partial^2 w_0^t}{\partial x^2}\right) \\ \varepsilon_z^t=\gamma_{xz}^t=0 \end{cases} , \quad \begin{cases} \varepsilon_x^b=\dfrac{\partial u^b(x,z,t)}{\partial x}=\dfrac{\partial u_0^b(x,t)}{\partial x}-\bar{b}\left(\dfrac{\partial^2 w_0^b}{\partial x^2}\right) \\ \varepsilon_z^b=\gamma_{xz}^b=0 \end{cases} \quad (8)$$

and core's field of strain can be written as Equation 9

$$\begin{cases} \varepsilon_x^c=\dfrac{1}{2}\dfrac{\partial u_0^t}{\partial x}+\dfrac{1}{2}\dfrac{\partial u_0^b}{\partial x}+\dfrac{1}{4}d_t\left(\dfrac{\partial^2 w_0^t}{\partial x^2}\right)-\dfrac{1}{4}d_b\left(\dfrac{\partial^2 w_0^b}{\partial x^2}\right)+z\left[\dfrac{1}{c}\dfrac{\partial u_0^t}{\partial x}-\dfrac{1}{c}\dfrac{\partial u_0^b}{\partial x}+\dfrac{1}{2c}d_t\dfrac{\partial^2 w_0^t}{\partial x^2}+\dfrac{1}{2c}d_b\dfrac{\partial^2 w_0^b}{\partial x^2}\right] \\ \varepsilon_z^c=\dfrac{1}{c}w_0^t-\dfrac{1}{c}w_0^b \\ \gamma_{xz}^c=\dfrac{1}{c}u_0^t-\dfrac{1}{c}u_0^b+\dfrac{1}{2c}d_t\dfrac{\partial w_0^t}{\partial x}+\dfrac{1}{2c}d_b\dfrac{\partial w_0^b}{\partial x}+\dfrac{1}{2}\dfrac{\partial w_0^t}{\partial x}+\dfrac{1}{2}\dfrac{\partial w_0^b}{\partial x}+z\left[\dfrac{1}{c}\dfrac{\partial w_0^t}{\partial x}-\dfrac{1}{c}\dfrac{\partial w_0^b}{\partial x}\right] \end{cases} \quad (9)$$

With using Hamilton's principles, motion equations can be written as follows

$$\int_{t_1}^{t_2}\delta(-T+U+V)\,dt=0$$

(10)

in which T, U, and V are kinetic energy, strain energy, and potential energy. The work of external force on the body is preserved in the form of potential energy in the conservative systems. location vector for mass M can be written as follows.

$$\vec{r}_M(x,t)=X(t)\hat{i}+Z(x,t)j \quad \rightarrow \quad \vec{r}_M(x,t)=\dot{X}\hat{i}+\left(\frac{\partial Z}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial Z}{\partial t}\right)j \quad (11)$$

Now according to the kinetic energy

$$T=\int_v \frac{1}{2}\rho(x,t)\left[\dot{u}^2+\dot{w}^2\right]dV+\frac{1}{2}MV_M^2 \quad (12)$$

$$\int_{t_1}^{t_2}\delta T dt=\int_{t_1}^{t_2}\int_{\forall}\rho(x,t)[\dot{u}\delta\dot{u}+\dot{w}\delta\dot{w}]d\forall dt+\int_{t_1}^{t_2}M\left(VZ'+\dot{Z}\right)\left(V\delta Z'+\delta\dot{Z}\right)dt=T_{beam}+T_{mass} \quad (13)$$

$$\int_{t_1}^{t_2}\delta T dt=-\int_{t_1}^{t_2}\int_{\forall}\rho(x,t)[\ddot{u}\delta u+\ddot{w}\delta w]d\forall dt-\int_{t_1}^{t_2}\int_0^L M\left[\left(V^2w_m''+2V\dot{w}_m'+\ddot{w}_m\right)\delta w_m\right]\delta(x-Vt)\,dxdt \quad (14)$$

The strain energy will also be written as follows.

$$\delta U=\int_{\forall}\left(\sigma_x^t\delta\varepsilon_x^t\right)d\forall+\int_{\forall}\left(\sigma_x^b\delta\varepsilon_x^b\right)d\forall+\int_{\forall}\left(\sigma_x^c\delta\varepsilon_x^c+\tau_{xz}^c\delta\gamma_{xz}^c\right)d\forall \tag{15}$$

The work of foreign forces will be written as follows.

$$\begin{aligned}
&W=\frac{1}{2}k\left(w_m-w_t\right)^2-mgw_m\ ,\\
&\int_{t_1}^{t_2}\delta W dt=\delta\int_{t_1}^{t_2}\left[\frac{1}{2}k\left(w_m-w_t\right)^2-mgw_m\right]dt=\int_{t_1}^{t_2}\left[k\left(w_m-w_t\right)\left(\delta w_m-\delta w_t\right)-mg\delta w_m\right]dt
\end{aligned} \tag{16}$$

$$\int_{t_1}^{t_2}\delta W dt=\int_{t_1}^{t_2}\int_0^L\left[k\left(w_m-w_t\right)\left(\delta w_m-\delta w_t\right)-mg\delta w_m\right]\delta\left(x-Vt\right)dxdt$$

(17)

By substituting the above equations in Hamilton's principle.

$$\begin{aligned}
&\int_{t_1}^{t_2}\left(-\delta T+\delta U-\delta W\right)dt=\int_{t_1}^{t_2}\int_{\forall}\rho^t\left(x,t\right)\left(\ddot{u}^t\delta u^t+\ddot{w}^t\delta w^t\right)d\forall dt+\int_{t_1}^{t_2}\int_{\forall}\rho^b\left(x,t\right)\left(\ddot{u}^b\delta u^b+\ddot{w}^b\delta w^b\right)d\forall dt+\\
&\int_{t_1}^{t_2}\int_{\forall}\rho^c\left(x,t\right)\left(\ddot{u}^c\delta u^c+\ddot{w}^c\delta w^c\right)d\forall dt+\int_{t_1}^{t_2}\int_0^L M\left[\left(V^2w_m''+2V\dot{w}_m'+\ddot{w}_m\right)\delta w_m\right]\delta\left(x-Vt\right)dxdt+\\
&\int_{\forall}\left(\sigma_x^t\delta\varepsilon_x^t+\sigma_x^b\delta\varepsilon_x^b+\sigma_z^c\delta\varepsilon_z^c+\tau_{xz}^c\delta\gamma_{xz}^c\right)d\forall+\int_{t_1}^{t_2}\int_0^L\left[k\left(w_m-w_t\right)\left(\delta w_m-\delta w_t\right)-Mg\delta w_m\right]\delta\left(x-Vt\right)dxdt=0
\end{aligned} \tag{18}$$

After solving the equations of the Hamilton's principle, the unknowns of problem can be found as follows.

$\delta u_0^t:$

$$A_1\frac{\partial^2 u_0^t}{\partial t^2}+A_2\frac{\partial^3 w_0^t}{\partial x\partial t^2}+A_3\frac{\partial^3 w_0^b}{\partial x\partial t^2}+A_4\frac{\partial^2 u_0^b}{\partial t^2}+A_5\frac{\partial w_0^b}{\partial x}+A_6\frac{\partial w_0^t}{\partial x}+A_7u_0^b+A_8\frac{\partial^2 u_0^t}{\partial x^2}+A_9\frac{\partial^3 w_0^t}{\partial x^3}+A_{10}u_0^t=0 \tag{19}$$

Where A1 to A10 are defined as follows

$$\begin{aligned}
&A_1=\left(\frac{I_1^c}{4}+I_1^t+\frac{I_3^c}{c^2}\right),\ A_2=\left(\frac{I_1^cd_t}{8}-I_2^t+\frac{I_3^cd_t}{2c^2}\right),\ A_3=\left(\frac{I_3^cd_b}{2c^2}-\frac{I_1^cd_b}{8}\right),\ A_4=\left(\frac{I_1^c}{4}-\frac{I_3^c}{c^2}\right),\ A_5=\left(\frac{b\kappa G_{xz}^cd_b}{2c}+\frac{b\kappa G_{xz}^c}{2}\right)\\
&A_6=\left(\frac{b\kappa G_{xz}^cd_t}{2c}+\frac{b\kappa G_{xz}^c}{2}\right),\ A_7=\left(-\frac{b\kappa G_{xz}^c}{c}\right),\ A_8=-bQ_{11}^tH_1^t\ ,\ A_9=bQ_{11}^tH_2^t\ ,\ A_{10}=\frac{b\kappa G_{xz}^c}{c}
\end{aligned} \tag{20}$$

And

$\delta w_0^t:$

$$\begin{aligned}
&B_1\frac{\partial^2 w_0^t}{\partial x^2}+B_2\frac{\partial^2 w_0^b}{\partial x^2}+B_3\frac{\partial^4 w_0^b}{\partial x^2\partial t^2}+B_4\frac{\partial^3 u_0^b}{\partial x\partial t^2}+B_5\frac{\partial^3 u_0^t}{\partial x^3}+B_6\frac{\partial u_0^b}{\partial x}+B_7\frac{\partial u_0^t}{\partial x}+B_8\frac{\partial^4 w_0^t}{\partial x^2\partial t^2}+B_9\frac{\partial^3 u_0^t}{\partial x\partial t^2}+\\
&B_{10}\frac{\partial^2 w_0^b}{\partial t^2}+B_{11}\frac{\partial^2 w_0^t}{\partial t^2}+B_{12}\frac{\partial^4 w_0^t}{\partial x^4}+B_{13}w_0^t+B_{14}w_0^b-K\delta\left(x-Vt\right)w_0^m+K\delta\left(x-Vt\right)w_0^t=0
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
&B_1=\left(-\frac{b\kappa G_{xz}^cc}{12}-\frac{b\kappa G_{xz}^c\left(\frac{d_t+c}{2}\right)}{2}-\frac{b\kappa G_{xz}^c\left(\frac{d_t+c}{2}\right)d_t}{2c}\right),\ B_2=\left(\frac{b\kappa G_{xz}^cc}{12}-\frac{b\kappa G_{xz}^c\left(\frac{d_b+c}{2}\right)}{2}-\frac{b\kappa G_{xz}^c\left(\frac{d_b+c}{2}\right)d_t}{2c}\right)\\
&B_3=\left(\frac{I_1^cd_bd_t}{16}-\frac{I_3^cd_bd_t}{4c^2}\right)\ ,\quad B_4=\left(\frac{I_3^cd_t}{2c^2}-\frac{I_1^cd_t}{8}\right)\ ,\quad B_5=-bQ_{11}^tH_2^t\ ,\quad B_6=\left(\frac{b\kappa G_{xz}^c}{2}+\frac{b\kappa G_{xz}^cd_t}{2c}\right)\\
&B_7=\left(-\frac{b\kappa G_{xz}^c}{2}-\frac{b\kappa G_{xz}^cd_t}{2c}\right)\ ,\quad B_8=\left(-\frac{I_3^cd_t^2}{4c^2}-\frac{I_1^cd_t^2}{16}-I_3^t\right)\ ,\quad B_9=\left(I_2^t-\frac{I_1^cd_t}{8}-\frac{I_3^cd_t}{2c^2}\right)\ ,\quad B_{10}=\left(\frac{I_1^c}{4}-\frac{I_3^c}{c^2}\right)\\
&B_{11}=\left(I_1^t+\frac{I_1^c}{4}+\frac{I_3^c}{c^2}\right)\ ,\quad B_{12}=bQ_{11}^tH_3^t\ ,\quad B_{13}=\frac{bE_{33}^c}{c}\ ,\quad B_{14}=-\frac{bE_{33}^c}{c}
\end{aligned} \tag{22}$$

And

$\delta u_0^b:$

$$C_1\frac{\partial^3 w_0^t}{\partial x\partial t^2}+C_2\frac{\partial^2 u_0^b}{\partial t^2}+C_3\frac{\partial^2 u_0^b}{\partial x^2}+C_4\frac{\partial^2 u_0^t}{\partial t^2}+C_5\frac{\partial^3 w_0^b}{\partial x\partial t^2}+C_6\frac{\partial w_0^b}{\partial x}+C_7u_0^b+C_8u_0^t+C_9\frac{\partial w_0^t}{\partial x}+C_{10}\frac{\partial^3 w_0^b}{\partial x^3}=0 \tag{23}$$

where

$$\begin{aligned}
&C_1=\left(\frac{I_1^cd_t}{8}-\frac{I_3^cd_t}{2c^2}\right),\ C_2=\left(I_1^b+\frac{I_3^c}{c^2}+\frac{I_1^c}{4}\right),\ C_3=-bQ_{11}^bH_1^b\ ,\ C_4=\left(\frac{I_1^c}{4}-\frac{I_3^c}{c^2}\right),\ C_5=\left(-\frac{I_3^cd_b}{2c^2}-I_2^b-\frac{I_1^cd_b}{8}\right)\\
&C_6=\left(-\frac{b\kappa G_{xz}^c\left(\frac{d_b+c}{2}\right)}{c}\right),\ C_7=\frac{b\kappa G_{xz}^c}{c}\ ,\ C_8=-\frac{b\kappa G_{xz}^c}{c}\ ,\ C_9=\left(-\frac{b\kappa G_{xz}^c\left(\frac{d_t+c}{2}\right)}{c}\right),\ C_{10}=bQ_{11}^bH_2^b
\end{aligned} \tag{24}$$

And

$\delta w_0^b:$

$$\begin{aligned}
&D_1\frac{\partial^4 w_0^t}{\partial x^2\partial t^2}+D_2\frac{\partial^2 w_0^b}{\partial x^2}+D_3\frac{\partial^3 u_0^b}{\partial x^3}+D_4\frac{\partial^2 w_0^t}{\partial x^2}+D_5\frac{\partial u_0^b}{\partial x}+D_6\frac{\partial u_0^t}{\partial x}+D_7\frac{\partial^2 w_0^t}{\partial t^2}+\\
&D_8\frac{\partial^4 w_0^b}{\partial x^2\partial t^2}+D_9\frac{\partial^3 u_0^t}{\partial x\partial t^2}+D_{10}\frac{\partial^3 u_0^b}{\partial x\partial t^2}+D_{11}\frac{\partial^2 w_0^b}{\partial t^2}+D_{12}w_0^t+D_{13}w_0^b+D_{14}\frac{\partial^4 w_0^b}{\partial x^4}=0
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
&D_1=\left(\frac{I_1^cd_bd_t}{16}-\frac{I_3^cd_bd_t}{4c^2}\right),\ D_2=\left(-\frac{b\kappa G_{xz}^c\left(\frac{d_b+c}{2}\right)}{2}-\frac{b\kappa G_{xz}^cd_b\left(\frac{d_b+c}{2}\right)}{2c}-\frac{b\kappa G_{xz}^cc}{12}\right),\ D_3=-bQ_{11}^bH_2^b\\
&D_4=\left(\frac{b\kappa G_{xz}^cc}{12}-\frac{b\kappa G_{xz}^c\left(\frac{d_t+c}{2}\right)}{2}-\frac{b\kappa G_{xz}^cd_b\left(\frac{d_t+c}{2}\right)}{2c}\right),\ D_5=\left(\frac{b\kappa G_{xz}^c}{2}+\frac{b\kappa G_{xz}^cd_b}{2c}\right),\ D_6=\left(-\frac{b\kappa G_{xz}^c}{2}-\frac{b\kappa G_{xz}^cd_b}{2c}\right)\\
&D_7=\left(\frac{I_1^c}{4}-\frac{I_3^c}{c^2}\right),\ D_8=\left(-\frac{I_3^cd_b^2}{4c^2}-I_3^b-\frac{I_1^cd_b^2}{16}\right),\ D_9=\left(\frac{I_1^cd_b}{8}-\frac{I_3^cd_b}{2c^2}\right),\ D_{10}=\left(\frac{I_1^cd_b}{8}+I_2^b+\frac{I_3^cd_b}{2c^2}\right)\\
&D_{11}=\left(I_1^b+\frac{I_3^c}{c^2}+\frac{I_1^c}{4}\right)\ ,\quad D_{12}=-\frac{bE_{33}^c}{c}\ ,\quad D_{13}=\frac{bE_{33}^c}{c}\ ,\quad D_{14}=bQ_{11}^bH_3^b
\end{aligned} \tag{26}$$

And

$\delta w_0^m:$

$$\begin{aligned}
&MV^2\delta\left(x-Vt\right)\frac{\partial^2 w_0^m}{\partial x^2}+2MV\delta\left(x-Vt\right)\frac{\partial^2 w_0^m}{\partial x\partial t}+M\delta\left(x-Vt\right)\frac{\partial^2 w_0^m}{\partial t^2}+K\delta\left(x-Vt\right)w_0^m-K\delta\left(x-Vt\right)w_0^t=Mg\delta\left(x-Vt\right)\\
&MV^2\frac{\partial^2 w_0^m}{\partial x^2}+2MV\frac{\partial^2 w_0^m}{\partial x\partial t}+M\frac{\partial^2 w_0^m}{\partial t^2}+Kw_0^m-Kw_0^t=Mg
\end{aligned} \tag{27}$$

With using the method of modal superposition, PDE equations can be transformed into ODE equations. So, ODE equations will be written as follows.

$$\begin{aligned}
&\sum_{m=1}^3A_1\cos\left(\alpha_mx\right)\ddot{U}_m^t+\sum_{m=1}^3A_2\alpha_m\cos\left(\alpha_mx\right)\ddot{W}_m^t+\sum_{m=1}^3A_3\alpha_m\cos\left(\alpha_mx\right)\ddot{W}_m^b+\sum_{m=1}^3A_4\cos\left(\alpha_mx\right)\ddot{U}_m^b\\
&+\sum_{m=1}^3A_5\alpha_m\cos\left(\alpha_mx\right)W_m^b+\sum_{m=1}^3A_6\alpha_m\cos\left(\alpha_mx\right)W_m^t+\sum_{m=1}^3A_7\cos\left(\alpha_mx\right)U_m^b-\sum_{m=1}^3A_8\alpha_m^2\cos\left(\alpha_mx\right)U_m^t\\
&-\sum_{m=1}^3A_9\alpha_m^3\cos\left(\alpha_mx\right)W_m^t+\sum_{m=1}^3A_{10}\cos\left(\alpha_mx\right)U_m^t=0
\end{aligned} \tag{28}$$

$$\begin{aligned}
&-\sum_{m=1}^3B_1\alpha_m^2\sin\left(\alpha_mx\right)W_m^t-\sum_{m=1}^3B_2\alpha_m^2\sin\left(\alpha_mx\right)W_m^b-\sum_{m=1}^3B_3\alpha_m^2\sin\left(\alpha_mx\right)\ddot{W}_m^b-\sum_{m=1}^3B_4\alpha_m\sin\left(\alpha_mx\right)\ddot{U}_m^b+\\
&\sum_{m=1}^3B_5\alpha_m^3\sin\left(\alpha_mx\right)U_m^t-\sum_{m=1}^3B_6\alpha_m\sin\left(\alpha_mx\right)U_m^b-\sum_{m=1}^3B_7\alpha_m\sin\left(\alpha_mx\right)U_m^t-\sum_{m=1}^3B_8\alpha_m^2\sin\left(\alpha_mx\right)\ddot{W}_m^t\\
&-\sum_{m=1}^3B_9\alpha_m\sin\left(\alpha_mx\right)\ddot{U}_m^t+\sum_{m=1}^3B_{10}\sin\left(\alpha_mx\right)\ddot{W}_m^b+\sum_{m=1}^3B_{11}\sin\left(\alpha_mx\right)\ddot{W}_m^t+\sum_{m=1}^3B_{12}\alpha_m^4\sin\left(\alpha_mx\right)W_m^t+\\
&\sum_{m=1}^3B_{13}\sin\left(\alpha_mx\right)W_m^t+\sum_{m=1}^3B_{14}\sin\left(\alpha_mx\right)W_m^b-\sum_{m=1}^3K\delta\left(x-Vt\right)\sin\left(\alpha_mx\right)W_m^m+\sum_{m=1}^3K\delta\left(x-Vt\right)\sin\left(\alpha_mx\right)W_m^t=0
\end{aligned} \tag{29}$$

$$\begin{aligned}
&\sum_{m=1}^3C_1\alpha_m\cos\left(\alpha_mx\right)\ddot{W}_m^t+\sum_{m=1}^3C_2\cos\left(\alpha_mx\right)\ddot{U}_m^b-\sum_{m=1}^3C_3\alpha_m^2\cos\left(\alpha_mx\right)U_m^b+\sum_{m=1}^3C_4\cos\left(\alpha_mx\right)\ddot{U}_m^t+\\
&\sum_{m=1}^3C_5\alpha_m\cos\left(\alpha_mx\right)\ddot{W}_m^b+\sum_{m=1}^3C_6\alpha_m\cos\left(\alpha_mx\right)W_m^b+\sum_{m=1}^3C_7\cos\left(\alpha_mx\right)U_m^b+\sum_{m=1}^3C_8\cos\left(\alpha_mx\right)U_m^t+\\
&\sum_{m=1}^3C_9\alpha_m\cos\left(\alpha_mx\right)W_m^t-\sum_{m=1}^3C_{10}\alpha_m^3\cos\left(\alpha_mx\right)W_m^b=0
\end{aligned} \tag{30}$$

$$-\sum_{m=1}^{3} D_1\alpha_m^2\sin(\alpha_m x)\ddot{W}_m^t-\sum_{m=1}^{3} D_2\alpha_m^2\sin(\alpha_m x)W_m^b+\sum_{m=1}^{3} D_3\alpha_m^3\sin(\alpha_m x)U_m^b-\sum_{m=1}^{3} D_4\alpha_m^2\sin(\alpha_m x)W_m^t$$
$$-\sum_{m=1}^{3} D_5\alpha_m\sin(\alpha_m x)U_m^b-\sum_{m=1}^{3} D_6\alpha_m\sin(\alpha_m x)U_m^t+\sum_{m=1}^{3} D_7\sin(\alpha_m x)\ddot{W}_m^t-\sum_{m=1}^{3} D_8\alpha_m^2\sin(\alpha_m x)\ddot{W}_m^b$$
$$-\sum_{m=1}^{3} D_9\alpha_m\sin(\alpha_m x)\ddot{U}_m^t-\sum_{m=1}^{3} D_{10}\alpha_m\sin(\alpha_m x)\ddot{U}_m^b+\sum_{m=1}^{3} D_{11}\sin(\alpha_m x)\ddot{W}_m^b+\sum_{m=1}^{3} D_{12}\sin(\alpha_m x)W_m^t+$$
$$\sum_{m=1}^{3} D_{13}\sin(\alpha_m x)W_m^b+\sum_{m=1}^{3} D_{14}\alpha_m^4\sin(\alpha_m x)W_m^b=0 \quad (31)$$

$$-\sum_{m=1}^{3} MV^2\alpha_m^2\sin(\alpha_m x)W_m^m+\sum_{m=1}^{3} 2MV\alpha_m\cos(\alpha_m x)\dot{W}_m^m+$$
$$\sum_{m=1}^{3} M\sin(\alpha_m x)\ddot{W}_m^m+\sum_{m=1}^{3} K\sin(\alpha_m x)W_m^m-\sum_{m=1}^{3} K\sin(\alpha_m x)W_m^t=Mg \quad (32)$$

To solving ODEs all of them will be multiplied by $\cos(\alpha_n x)$ and $\sin(\alpha_n x)$.

$$\int_0^L\left(\begin{aligned}&\sum_{m=1}^{3} A_1\cos(\alpha_m x)\cos(\alpha_n x)\ddot{U}_m^t+\sum_{m=1}^{3} A_2\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)\ddot{W}_m^t+\sum_{m=1}^{3} A_3\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)\ddot{W}_m^b+\\&\sum_{m=1}^{3} A_4\cos(\alpha_m x)\cos(\alpha_n x)\ddot{U}_m^b+\sum_{m=1}^{3} A_5\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)W_m^b+\sum_{m=1}^{3} A_6\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)W_m^t+\\&\sum_{m=1}^{3} A_7\cos(\alpha_m x)\cos(\alpha_n x)U_m^b-\sum_{m=1}^{3} A_8\alpha_m^2\cos(\alpha_m x)\cos(\alpha_n x)U_m^t-\sum_{m=1}^{3} A_9\alpha_m^3\cos(\alpha_m x)\cos(\alpha_n x)W_m^t+\\&\sum_{m=1}^{3} A_{10}\cos(\alpha_m x)\cos(\alpha_n x)U_m^t=0\end{aligned}\right)dx \quad (33)$$

$$\int_0^L\left(\begin{aligned}&-\sum_{m=1}^{3} B_1\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)W_m^t-\sum_{m=1}^{3} B_2\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)W_m^b-\sum_{m=1}^{3} B_3\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^b\\&-\sum_{m=1}^{3} B_4\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)\ddot{U}_m^b+\sum_{m=1}^{3} B_5\alpha_m^3\sin(\alpha_m x)\sin(\alpha_n x)U_m^t-\sum_{m=1}^{3} B_6\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)U_m^b\\&-\sum_{m=1}^{3} B_7\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)U_m^t-\sum_{m=1}^{3} B_8\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^t-\sum_{m=1}^{3} B_9\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)\ddot{U}_m^t\\&+\sum_{m=1}^{3} B_{10}\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^b+\sum_{m=1}^{3} B_{11}\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^t+\sum_{m=1}^{3} B_{12}\alpha_m^4\sin(\alpha_m x)\sin(\alpha_n x)W_m^t\\&+\sum_{m=1}^{3} B_{13}\sin(\alpha_m x)\sin(\alpha_n x)W_m^t+\sum_{m=1}^{3} B_{14}\sin(\alpha_m x)\sin(\alpha_n x)W_m^b-\sum_{m=1}^{3} K\delta(x-Vt)\sin(\alpha_m x)\sin(\alpha_n x)W_m^m\\&+\sum_{m=1}^{3} K\delta(x-Vt)\sin(\alpha_m x)\sin(\alpha_n x)W_m^t=0\end{aligned}\right)dx \quad (34)$$

$$\int_0^L\left(\begin{aligned}&\sum_{m=1}^{3} C_1\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)\ddot{W}_m^t+\sum_{m=1}^{3} C_2\cos(\alpha_m x)\cos(\alpha_n x)\ddot{U}_m^b-\sum_{m=1}^{3} C_3\alpha_m^2\cos(\alpha_m x)\cos(\alpha_n x)U_m^b\\&+\sum_{m=1}^{3} C_4\cos(\alpha_m x)\cos(\alpha_n x)\ddot{U}_m^t+\sum_{m=1}^{3} C_5\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)\ddot{W}_m^b+\sum_{m=1}^{3} C_6\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)W_m^b\\&+\sum_{m=1}^{3} C_7\cos(\alpha_m x)\cos(\alpha_n x)U_m^b+\sum_{m=1}^{3} C_8\cos(\alpha_m x)\cos(\alpha_n x)U_m^t+\sum_{m=1}^{3} C_9\alpha_m\cos(\alpha_m x)\cos(\alpha_n x)W_m^t\\&-\sum_{m=1}^{3} C_{10}\alpha_m^3\cos(\alpha_m x)\cos(\alpha_n x)W_m^b=0\end{aligned}\right)dx \quad (35)$$

$$\int_0^L\left(\begin{aligned}&-\sum_{m=1}^{3} D_1\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^t-\sum_{m=1}^{3} D_2\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)W_m^b+\sum_{m=1}^{3} D_3\alpha_m^3\sin(\alpha_m x)\sin(\alpha_n x)U_m^b\\&-\sum_{m=1}^{3} D_4\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)W_m^t-\sum_{m=1}^{3} D_5\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)U_m^b-\sum_{m=1}^{3} D_6\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)U_m^t\\&+\sum_{m=1}^{3} D_7\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^t-\sum_{m=1}^{3} D_8\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^b-\sum_{m=1}^{3} D_9\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)\ddot{U}_m^t\\&-\sum_{m=1}^{3} D_{10}\alpha_m\sin(\alpha_m x)\sin(\alpha_n x)\ddot{U}_m^b+\sum_{m=1}^{3} D_{11}\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^b+\sum_{m=1}^{3} D_{12}\sin(\alpha_m x)\sin(\alpha_n x)W_m^t\\&+\sum_{m=1}^{3} D_{13}\sin(\alpha_m x)\sin(\alpha_n x)W_m^b+\sum_{m=1}^{3} D_{14}\alpha_m^4\sin(\alpha_m x)\sin(\alpha_n x)W_m^b=0\end{aligned}\right)dx \quad (36)$$

$$\int_0^L\left(\begin{aligned}&-\sum_{m=1}^{3} MV^2\alpha_m^2\sin(\alpha_m x)\sin(\alpha_n x)W_m^m+\sum_{m=1}^{3} 2MV\alpha_m\cos(\alpha_m x)\sin(\alpha_n x)\dot{W}_m^m+\sum_{m=1}^{3} M\sin(\alpha_m x)\sin(\alpha_n x)\ddot{W}_m^m\\&+\sum_{m=1}^{3} K\sin(\alpha_m x)\sin(\alpha_n x)W_m^m-\sum_{m=1}^{3} K\sin(\alpha_m x)\sin(\alpha_n x)W_m^t=Mg\sin(\alpha_n x)\end{aligned}\right)dx \quad (37)$$

By using Orthonormal function

$$\int_0^L\sin(\alpha_m x)\sin(\alpha_n x)dx=\begin{cases}if\quad m=n & \frac{L}{2}\\ if\quad m\neq n & 0\end{cases}\quad,\quad\int_0^L\cos(\alpha_m x)\cos(\alpha_n x)dx=\begin{cases}if\quad m=n & \frac{L}{2}\\ if\quad m\neq n & 0\end{cases}$$
$$\int_0^L\cos(\alpha_m x)\sin(\alpha_n x)dx=\frac{Ln\left[(-1)^{m+n}-1\right]}{\pi(m^2-n^2)}(1-\delta_{mn}) \quad (38)$$

So, ODEs will be written as follows

$$\sum_{m=1}^{3} A_1\frac{L}{2}\delta_{mn}\ddot{U}_m^t+\sum_{m=1}^{3} A_2\alpha_m\frac{L}{2}\delta_{mn}\ddot{W}_m^t+\sum_{m=1}^{3} A_3\alpha_m\frac{L}{2}\delta_{mn}\ddot{W}_m^b+\sum_{m=1}^{3} A_4\frac{L}{2}\delta_{mn}\ddot{U}_m^b+\sum_{m=1}^{3} A_5\alpha_m\frac{L}{2}\delta_{mn}W_m^b+$$
$$\sum_{m=1}^{3} A_6\alpha_m\frac{L}{2}\delta_{mn}W_m^t+\sum_{m=1}^{3} A_7\frac{L}{2}\delta_{mn}U_m^b-\sum_{m=1}^{3} A_8\alpha_m^2\frac{L}{2}\delta_{mn}U_m^t-\sum_{m=1}^{3} A_9\alpha_m^3\frac{L}{2}\delta_{mn}W_m^t+\sum_{m=1}^{3} A_{10}\frac{L}{2}\delta_{mn}U_m^t=0$$
$$A_1\ddot{U}_n^t+A_2\alpha_n\ddot{W}_n^t+A_3\alpha_n\ddot{W}_n^b+A_4\ddot{U}_n^b+A_5\alpha_nW_n^b+A_6\alpha_nW_n^t+A_7U_n^b-A_8\alpha_n^2U_n^t-A_9\alpha_n^3W_n^t+A_{10}U_n^t=0 \quad (39)$$

$$-\sum_{m=1}^{3} B_1\alpha_m^2\frac{L}{2}\delta_{mn}W_m^t-\sum_{m=1}^{3} B_2\alpha_m^2\frac{L}{2}\delta_{mn}W_m^b-\sum_{m=1}^{3} B_3\alpha_m^2\frac{L}{2}\delta_{mn}\ddot{W}_m^b-\sum_{m=1}^{3} B_4\alpha_m\frac{L}{2}\delta_{mn}\ddot{U}_m^b+\sum_{m=1}^{3} B_5\alpha_m^3\frac{L}{2}\delta_{mn}U_m^t-$$
$$\sum_{m=1}^{3} B_6\alpha_m\frac{L}{2}\delta_{mn}U_m^b-\sum_{m=1}^{3} B_7\alpha_m\frac{L}{2}\delta_{mn}U_m^t-\sum_{m=1}^{3} B_8\alpha_m^2\frac{L}{2}\delta_{mn}\ddot{W}_m^t-\sum_{m=1}^{3} B_9\alpha_m\frac{L}{2}\delta_{mn}\ddot{U}_m^t+\sum_{m=1}^{3} B_{10}\frac{L}{2}\delta_{mn}\ddot{W}_m^b+$$
$$\sum_{m=1}^{3} B_{11}\frac{L}{2}\delta_{mn}\ddot{W}_m^t+\sum_{m=1}^{3} B_{12}\alpha_m^4\frac{L}{2}\delta_{mn}W_m^t+\sum_{m=1}^{3} B_{13}\frac{L}{2}\delta_{mn}W_m^t+\sum_{m=1}^{3} B_{14}\frac{L}{2}\delta_{mn}W_m^b-$$
$$\sum_{m=1}^{3} K\sin(\alpha_m Vt)\sin(\alpha_n Vt)W_m^m+\sum_{m=1}^{3} K\sin(\alpha_m Vt)\sin(\alpha_n Vt)W_m^t=0 \quad (40)$$

$$\sum_{m=1}^{3} C_1\alpha_m\frac{L}{2}\delta_{mn}\ddot{W}_m^t+\sum_{m=1}^{3} C_2\frac{L}{2}\delta_{mn}\ddot{U}_m^b-\sum_{m=1}^{3} C_3\alpha_m^2\frac{L}{2}\delta_{mn}U_m^b+\sum_{m=1}^{3} C_4\frac{L}{2}\delta_{mn}\ddot{U}_m^t+\sum_{m=1}^{3} C_5\alpha_m\frac{L}{2}\delta_{mn}\ddot{W}_m^b+$$
$$\sum_{m=1}^{3} C_6\alpha_m\frac{L}{2}\delta_{mn}W_m^b+\sum_{m=1}^{3} C_7\frac{L}{2}\delta_{mn}U_m^b+\sum_{m=1}^{3} C_8\frac{L}{2}\delta_{mn}U_m^t+\sum_{m=1}^{3} C_9\alpha_m\frac{L}{2}\delta_{mn}W_m^t-\sum_{m=1}^{3} C_{10}\alpha_m^3\frac{L}{2}\delta_{mn}W_m^b=0$$
$$C_1\alpha_n\ddot{W}_n^t+C_2\ddot{U}_n^b-C_3\alpha_n^2U_n^b+C_4\ddot{U}_n^t+C_5\alpha_n\ddot{W}_n^b++C_6\alpha_nW_n^b+C_7U_n^b+C_8U_n^t+C_9\alpha_nW_n^t-C_{10}\alpha_n^3W_n^b=0 \quad (41)$$

$$-\sum_{m=1}^{3} D_1\alpha_m^2\frac{L}{2}\delta_{mn}\ddot{W}_m^t-\sum_{m=1}^{3} D_2\alpha_m^2\frac{L}{2}\delta_{mn}W_m^b+\sum_{m=1}^{3} D_3\alpha_m^3\frac{L}{2}\delta_{mn}U_m^b-\sum_{m=1}^{3} D_4\alpha_m^2\frac{L}{2}\delta_{mn}W_m^t-\sum_{m=1}^{3} D_5\alpha_m\frac{L}{2}\delta_{mn}U_m^b$$
$$-\sum_{m=1}^{3} D_6\alpha_m\frac{L}{2}\delta_{mn}U_m^t+\sum_{m=1}^{3} D_7\frac{L}{2}\delta_{mn}\ddot{W}_m^t-\sum_{m=1}^{3} D_8\alpha_m^2\frac{L}{2}\delta_{mn}\ddot{W}_m^b-\sum_{m=1}^{3} D_9\alpha_m\frac{L}{2}\delta_{mn}\ddot{U}_m^t-\sum_{m=1}^{3} D_{10}\alpha_m\frac{L}{2}\delta_{mn}\ddot{U}_m^b+$$
$$\sum_{m=1}^{3} D_{11}\frac{L}{2}\delta_{mn}\ddot{W}_m^b+\sum_{m=1}^{3} D_{12}\frac{L}{2}\delta_{mn}W_m^t+\sum_{m=1}^{3} D_{13}\frac{L}{2}\delta_{mn}W_m^b+\sum_{m=1}^{3} D_{14}\alpha_m^4\frac{L}{2}\delta_{mn}W_m^b=0$$
$$-D_1\alpha_n^2\ddot{W}_n^t-D_2\alpha_n^2W_n^b+D_3\alpha_n^3U_n^b-D_4\alpha_n^2W_n^t-D_5\alpha_nU_n^b-D_6\alpha_nU_n^t+D_7\ddot{W}_n^t-D_8\alpha_n^2\ddot{W}_n^b-D_9\alpha_n\ddot{U}_n^t-D_{10}\alpha_n\ddot{U}_n^b$$
$$+D_{11}\ddot{W}_n^b+D_{12}W_n^t+D_{13}W_n^b+D_{14}\alpha_n^4W_n^b=0 \quad (42)$$

$$-MV^2\alpha_n^2W_n^m+\sum_{m=1}^{3} 2MV\alpha_m\frac{2n\left[(-1)^{m+n}-1\right]}{\pi(m^2-n^2)}(1-\delta_{mn})\dot{W}_m^m+M\ddot{W}_n^m+KW_n^m-KW_n^t=\frac{2Mg}{n\pi}\left[1-(-1)^n\right] \quad (43)$$

where

$$\alpha_m=\frac{m\pi}{L}\quad,\quad\alpha_n=\frac{n\pi}{L} \quad (44)$$

By giving numbers to m and n

$$I)A_1\ddot{U}_1^t+A_2\alpha_1\ddot{W}_1^t+A_3\alpha_1\ddot{W}_1^b+A_4\ddot{U}_1^b+A_5\alpha_1W_1^b+A_6\alpha_1W_1^t+A_7U_1^b-A_8\alpha_1^2U_1^t-A_9\alpha_1^3W_1^t+A_{10}U_1^t=0$$
$$II)A_1\ddot{U}_2^t+A_2\alpha_2\ddot{W}_2^t+A_3\alpha_2\ddot{W}_2^b+A_4\ddot{U}_2^b+A_5\alpha_2W_2^b+A_6\alpha_2W_2^t+A_7U_2^b-A_8\alpha_2^2U_2^t-A_9\alpha_2^3W_2^t+A_{10}U_2^t=0$$
$$III)A_1\ddot{U}_3^t+A_2\alpha_3\ddot{W}_3^t+A_3\alpha_3\ddot{W}_3^b+A_4\ddot{U}_3^b+A_5\alpha_3W_3^b+A_6\alpha_3W_3^t+A_7U_3^b-A_8\alpha_3^2U_3^t-A_9\alpha_3^3W_3^t+A_{10}U_3^t=0 \quad (45)$$

$$I)-B_1\alpha_1^2W_1^t-B_2\alpha_1^2W_1^b-B_3\alpha_1^2\ddot{W}_1^b-B_4\alpha_1\ddot{U}_1^b+B_5\alpha_1^3U_1^t-B_6\alpha_1U_1^b-B_7\alpha_1U_1^t-B_8\alpha_1^2\ddot{W}_1^t-B_9\alpha_1\ddot{U}_1^t+B_{10}\ddot{W}_1^b+B_{11}\ddot{W}_1^t$$
$$+B_{12}\alpha_1^4W_1^t+B_{13}W_1^t+B_{14}W_1^b-\frac{2K}{L}\sin(\alpha_1Vt)\left(\sin(\alpha_1Vt)W_1^m+\sin(\alpha_2Vt)W_2^m+\sin(\alpha_3Vt)W_3^m\right)$$
$$+\frac{2K}{L}\sin(\alpha_1Vt)\left(\sin(\alpha_1Vt)W_1^t+\sin(\alpha_2Vt)W_2^t+\sin(\alpha_3Vt)W_3^t\right)=0$$
$$II)-B_1\alpha_2^2W_2^t-B_2\alpha_2^2W_2^b-B_3\alpha_2^2\ddot{W}_2^b-B_4\alpha_2\ddot{U}_2^b+B_5\alpha_2^3U_2^t-B_6\alpha_2U_2^b-B_7\alpha_2U_2^t-B_8\alpha_2^2\ddot{W}_2^t-B_9\alpha_2\ddot{U}_2^t+B_{10}\ddot{W}_2^b+B_{11}\ddot{W}_2^t$$
$$+B_{12}\alpha_2^4W_2^t+B_{13}W_2^t+B_{14}W_2^b-\frac{2K}{L}\sin(\alpha_2Vt)\left(\sin(\alpha_1Vt)W_1^m+\sin(\alpha_2Vt)W_2^m+\sin(\alpha_3Vt)W_3^m\right)$$
$$+\frac{2K}{L}\sin(\alpha_2Vt)\left(\sin(\alpha_1Vt)W_1^t+\sin(\alpha_2Vt)W_2^t+\sin(\alpha_3Vt)W_3^t\right)=0$$
$$III)-B_1\alpha_3^2W_3^t-B_2\alpha_3^2W_3^b-B_3\alpha_3^2\ddot{W}_3^b-B_4\alpha_3\ddot{U}_3^b+B_5\alpha_3^3U_3^t-B_6\alpha_3U_3^b-B_7\alpha_3U_3^t-B_8\alpha_3^2\ddot{W}_3^t-B_9\alpha_3\ddot{U}_3^t+B_{10}\ddot{W}_3^b+B_{11}\ddot{W}_3^t$$
$$+B_{12}\alpha_3^4W_3^t+B_{13}W_3^t+B_{14}W_3^b-\frac{2K}{L}\sin(\alpha_3Vt)\left(\sin(\alpha_1Vt)W_1^m+\sin(\alpha_2Vt)W_2^m+\sin(\alpha_3Vt)W_3^m\right)$$
$$+\frac{2K}{L}\sin(\alpha_3Vt)\left(\sin(\alpha_1Vt)W_1^t+\sin(\alpha_2Vt)W_2^t+\sin(\alpha_3Vt)W_3^t\right)=0 \quad (46)$$

$$I)C_1\alpha_1\ddot{W}_1^t+C_2\ddot{U}_1^b-C_3\alpha_1^2U_1^b+C_4\ddot{U}_1^t+C_5\alpha_1\ddot{W}_1^b+C_6\alpha_1W_1^b+C_7U_1^b+C_8U_1^t+C_9\alpha_1W_1^t-C_{10}\alpha_1^3W_1^b=0$$
$$II)C_1\alpha_2\ddot{W}_2^t+C_2\ddot{U}_2^b-C_3\alpha_2^2U_2^b+C_4\ddot{U}_2^t+C_5\alpha_2\ddot{W}_2^b+C_6\alpha_2W_2^b+C_7U_2^b+C_8U_2^t+C_9\alpha_2W_2^t-C_{10}\alpha_2^3W_2^b=0$$
$$III)C_1\alpha_3\ddot{W}_3^t+C_2\ddot{U}_3^b-C_3\alpha_3^2U_3^b+C_4\ddot{U}_3^t+C_5\alpha_3\ddot{W}_3^b+C_6\alpha_3W_3^b+C_7U_3^b+C_8U_3^t+C_9\alpha_3W_3^t-C_{10}\alpha_3^3W_3^b=0 \quad (47)$$

$$I)-D_1\alpha_1^2\ddot{W}_1^t-D_2\alpha_1^2W_1^b+D_3\alpha_1^3U_1^b-D_4\alpha_1^2W_1^t-D_5\alpha_1U_1^b-D_6\alpha_1U_1^t+D_7\ddot{W}_1^t-D_8\alpha_1^2\ddot{W}_1^b-D_9\alpha_1\ddot{U}_1^t-D_{10}\alpha_1\ddot{U}_1^b$$
$$+D_{11}\ddot{W}_1^b+D_{12}W_1^t+D_{13}W_1^b+D_{14}\alpha_1^4W_1^b=0$$
$$II)-D_1\alpha_2^2\ddot{W}_2^t-D_2\alpha_2^2W_2^b+D_3\alpha_2^3U_2^b-D_4\alpha_2^2W_2^t-D_5\alpha_2U_2^b-D_6\alpha_2U_2^t+D_7\ddot{W}_2^t-D_8\alpha_2^2\ddot{W}_2^b-D_9\alpha_2\ddot{U}_2^t-D_{10}\alpha_2\ddot{U}_2^b$$
$$+D_{11}\ddot{W}_2^b+D_{12}W_2^t+D_{13}W_2^b+D_{14}\alpha_2^4W_2^b=0$$
$$III)-D_1\alpha_3^2\ddot{W}_3^t-D_2\alpha_3^2W_3^b+D_3\alpha_3^3U_3^b-D_4\alpha_3^2W_3^t-D_5\alpha_3U_3^b-D_6\alpha_3U_3^t+D_7\ddot{W}_3^t-D_8\alpha_3^2\ddot{W}_3^b-D_9\alpha_3\ddot{U}_3^t-D_{10}\alpha_3\ddot{U}_3^b$$
$$+D_{11}\ddot{W}_3^b+D_{12}W_3^t+D_{13}W_3^b+D_{14}\alpha_3^4W_3^b=0 \quad (48)$$

$$I)-MV^2\alpha_1^2W_1^m+\left(2MV\alpha_2\frac{2\left[(-1)^{2+1}-1\right]}{\pi(2^2-1^2)}\dot{W}_2^m+2MV\alpha_3\frac{2\left[(-1)^{3+1}-1\right]}{\pi(3^2-1^2)}\dot{W}_3^m\right)+M\ddot{W}_1^m+KW_1^m-KW_1^t=\frac{2Mg}{n\pi}\left[1-(-1)^n\right]$$
$$I)\left(K-MV^2\alpha_1^2\right)W_1^m-\frac{8}{3\pi}MV\alpha_2\dot{W}_2^m+M\ddot{W}_1^m-KW_1^t=\frac{4Mg}{\pi}$$
$$II)-MV^2\alpha_2^2W_2^m+\left(2MV\alpha_1\frac{2\times2\left[(-1)^{1+2}-1\right]}{\pi(1^2-2^2)}\dot{W}_1^m+2MV\alpha_3\frac{2\times2\left[(-1)^{3+2}-1\right]}{\pi(3^2-2^2)}\dot{W}_3^m\right)+M\ddot{W}_2^m+KW_2^m-KW_2^t=0$$
$$II)\left(K-MV^2\alpha_2^2\right)W_2^m+\frac{16}{3\pi}MV\alpha_1\dot{W}_1^m-\frac{16}{5\pi}MV\alpha_3\dot{W}_3^m+M\ddot{W}_2^m-KW_2^t=0$$
$$III)-MV^2\alpha_3^2W_3^m+\left(2MV\alpha_1\frac{2\times3\left[(-1)^{1+3}-1\right]}{\pi(1^2-3^2)}\dot{W}_1^m+2MV\alpha_2\frac{2\times3\left[(-1)^{2+3}-1\right]}{\pi(2^2-3^2)}\dot{W}_2^m\right)+M\ddot{W}_3^m+KW_3^m-KW_3^t$$
$$=\frac{2Mg}{n\pi}\left[1-(-1)^n\right]$$
$$III)\left(K-MV^2\alpha_3^2\right)W_3^m+\frac{24}{5\pi}MV\alpha_2\dot{W}_2^m+M\ddot{W}_3^m-KW_3^t=\frac{4Mg}{3\pi} \quad (49)$$

With Using the method of Runge-Kutta 4th order, these equations will be solved.

3. VALIDATION

In this section, by comparing the results of this study with the results of previous works, the results of this study will be confirmed. For this purpose, the natural frequencies of the present study are obtained and compared with the results of Khalili's research [4]. In the table below, the properties of material are presented.

Table1 _Material Property Of The Beam

ρ_c	$\rho_f\,(kg/m^3)$	G_c	E_c	$E_f\,(N/m^2)$	b	h_c	h_f	$L(mm)$
52.06	4400	2.00E+07	5.00E+07	3.60E+10	20	20	0.5	300

To obtain natural frequencies, the eig function is used in MATLAB software. In the table below, the value of five natural frequencies of the present study is shown by the value of the natural frequencies of the Khalili [4] research

Table 2 _ First Natural Frequencies of The Beam

	First freq	Second freq	Third freq	Fourth freq	Fifth freq
	(Rad/s)	(Rad/s)	(Rad/s)	(Rad/s)	(Rad/s)
This study	2027.27	5110.004	8124.995	11081.41	14011
Khalili's study [4]	2048.41	5189.672	8250.199	11225.27	14139
Difference percentage	1%	1%	1%	1%	1%

As can be seen, the results are consistent with the results of Khalili's research (Damanpack, 2012) and the first phase of verification has been well done.
The second validation relates to the vibrational equations of a Bernoulli Euler beam. First, the vibration equations of a single-layer Bernoulli beam Euler is written and then its figures are compared with the figures of this paper.

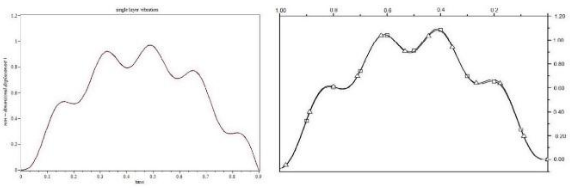

Figure 2. comparision between this study and Khalili's study

After assuring the correctness of the single-layer beam vibrational equations, the main equations of the present research are written for the three-layer beam. In the third validation, mathematical equations are written for an Euler Bernoulli single layer beam. Then the dynamics deflection of the beam is obtained based on the dynamic deflection of the moving mass

$$w_0^t = w_0^m + \frac{M}{K}V^2\frac{\partial^2 w_0^m}{\partial x^2} + 2\frac{M}{K}V\frac{\partial^2 w_0^m}{\partial x \partial t} + \frac{M}{K}\frac{\partial^2 w_0^m}{\partial t^2} - \frac{M}{K}g$$

It can be seen that if the spring stiffness coefficient goes to infinity, the dynamics deflection of the beam and the moving mass will be exactly equal. This indicates the correctness of the equations. So the validation shows the correctness of the mathematical equations. Now, the results of the research will be obtained.

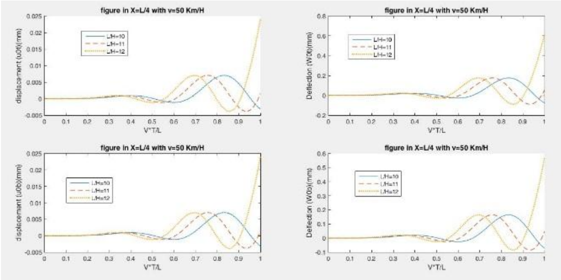

Figure 3. comparision between different amount of L/H

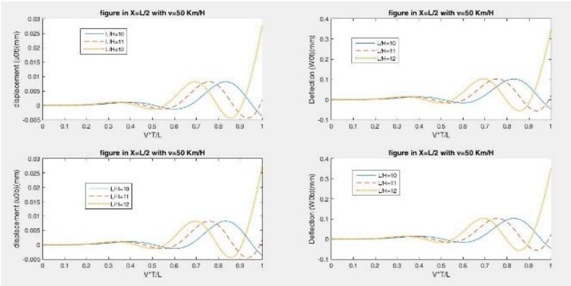

Figure 4. comparision between different amount of L/H

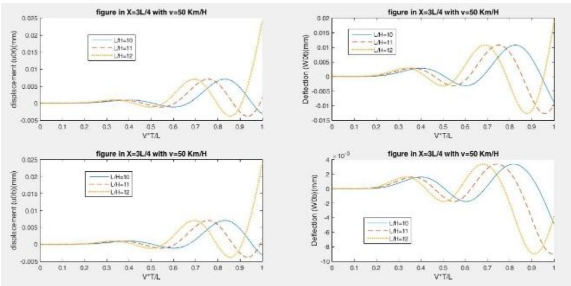

Figure 5. comparision between different amount of L/H

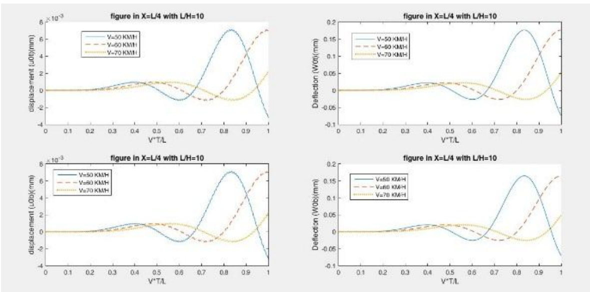

Figure 6. comparision between different amount of V

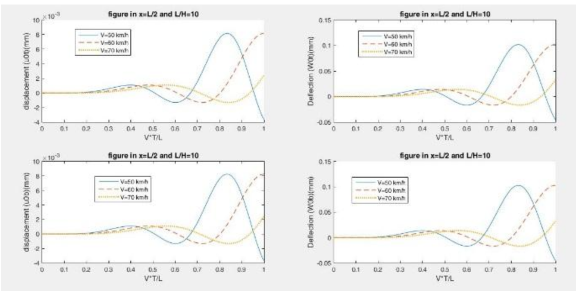

Figure 7. comparision between different amount of V

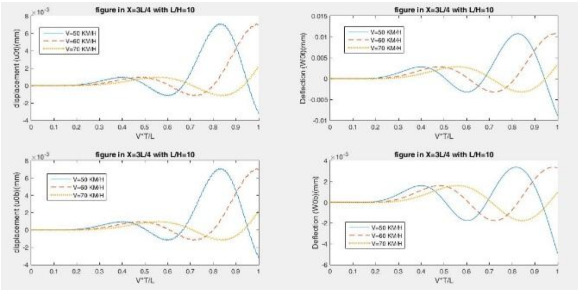

Figure 8. comparision between different amount of V

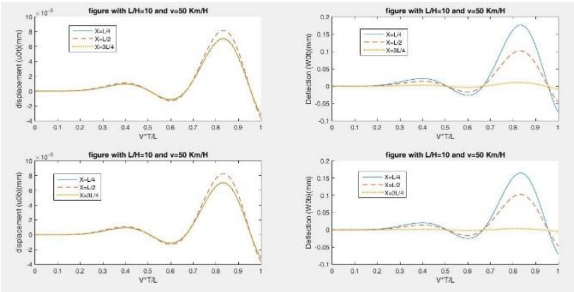

Figure 9. comparison between different places

4. CONCLUSION

Dynamic response of a three-layered sandwich beam under a moving sprung mass is obtained. It is understood that with increasing speed of mass motion, the vibration of the beam significantly decreases. As the length of the beam increases, the vibration of the layers and the transverse displacement of the mass will increase. The increase in the ratio of the moving mass to the beam mass leads to an increase in vibrations.

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