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# **OPTIMAL DESIGN OF REINFORCED CONCRETE SPECIAL SHEAR WALLS USING VIBRATING PARTICLES SYSTEM (VPS) ALGORITHM**

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**Abstract.**In this study optimal design of reinforced concrete special shear walls is performed under seismic loading utilizing the vibrating particles system (VPS) method. First the principles of this new algorithm are presented. In the following, special shear walls and boundary elements are addressed. Then design tenets of these sort of walls are explained extendedly. In the next step, by using the VPS method, seismic design optimization of special shear walls considering performance and design constraints is investigated via examples. The objective function is to minimize the total cost of the shear wall which comprises of the cost of concrete, steel bars and formwork. In order to have a precise evaluation, the outcomes of this algorithm are compared with the results of the other algorithms. Ultimate answers and the convergence history diagrams illustrate the performance of the VPS method.

**Key Words:** Special shear walls, vibrating particles system algorithm (VPS), cost minimization, design optimization, performance and seismic constraints.

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# **1. INTRODUCTION**

Nowadays optimization science plays a pivotal role in engineering. Demand for cost reduction has persuaded engineers to use concepts of optimization for dealing with their problems. Seismic design of structures often makes to build expensive structures. Therefore it is reasonable to economize structural design by optimization algorithms. These algorithms are utilized to minimize or maximize an objective function under certain specific limitations. Clearly in this process, objective function must be selected in a way that leads to minimum structural cost.

Walls that primarily withstand lateral loads due to the wind or earthquake acting on the building are called structural walls or shear walls. These walls often provide lateral bracings for the remaining part of the structure. They endure gravity loads transferred to the wall by the components of the structure tributary to the wall, besides of lateral shear loads and moments about the strong axis of the wall.

Many researchers have addressed the optimum design of seismic structures, but a few works exist on optimal design of shear walls incorporating new seismic codes' considerations. Ganzerli et al. (Ganzerli, Pantelides & Reaveley, 2000) presented a performance-based design using structural optimization. The optimum design of active seismic structures was studied by Cheng and Pantelides (Cheng & Pantelides, 1988). Fragiadakis et al (Fragiadakis & Papadrakakis, 2008) carried out performance-based optimum seismic design of structural reinforced concrete structures. Saka (Saka, 1991) offered optimum design of multistory structures with shear walls. Wallace (Wallace, 1995), (Wallace, 1995) proposed new code format for seismic design of reinforced concrete structural walls. Sasani (Sasani, 1998) proposed a performance design methodology relative to concrete structural walls. The performance level targeted in the design was life safety.

In this study the vibrating particles system (VPS) developed by Kaveh and Ilchi Ghazaan (Kaveh & Ghazaan, 2017) are utilized to determine optimum design of reinforced concrete special shear walls. The objective function considered in this paper is the cost of the structure. This function is minimized subjected to performance and design constraints. At the end two numerical examples are presented in order to illustrate the performance of the VPS method.

# **2. VIBRATING PARTICLES SYSTEM OPTIMIZATION ALGORITHM**

The VPS is a population-based algorithm that simulates a free vibration of single degree of freedom systems with viscous damping (Kaveh & Ghazaan, 2017). The VPS has a number of particles consisting of the variables of the problem. The solution candidates gradually approach to their equilibrium positions which are achieved from current population and historically best position in order to have a proper balance between diversification and intensification. In VPS, the initial locations of particles are created randomly in an *n*dimensional search space as:

$$
x_i^j = x_{\min} + \text{rand} \cdot (x_{\max} - x_{\min}), \quad (1)
$$

Where  $x_i^j$  is the *j* th variable of the particle *i*.  $x_{min}$  and  $x_{max}$  are the minimum and the maximum allowable variables vectors; *rand* is a random number uniformly distributed in the range of [0, 1].

For each particle, three equilibrium positions with different weights are defined, and during each generation, the particle position is updated by learning from them: (i) the historically best position of the entire population (HB), (ii) a good particle (GP), and (iii) a bad particle (BP). In order to select the GP and BP for each candidate solution, the current population is sorted according to their objective function values in an ascending order, and then GP and BP are chosen randomly from the first and second half, respectively.

A descending function based on the number of iterations is proposed in VPS to model the effect of the damping level in the vibration.

$$
D = \left(\frac{iter}{iter_{\text{max}}}\right)^{-\alpha} \quad \text{(2)}
$$

Where *iter* is the current iteration number and  $iter_{\text{max}}$  is the total number of iterations for the optimization process. *α* is a constant.

According to the above concepts, the update rules in the VPS are given by:



Where  $x_i^j$  is the *j* th variable of the particle *i*. *w*1, *w*2 , and *w*3 are three parameters to measure the relative importance of HB, GP and BP, respectively. *rand1* , *rand2* , and *rand3* are random numbers uniformly distributed in the range of [0, 1].

In order to have a fast convergence in the VPS, the effect of BP is sometimes considered in updating the position formula. Therefore, for each particle, a parameter like  $p$  within  $(0,1)$  is defined, and it is compared with rand (a random number uniformly distributed in the range of  $[0,1]$ ) and if  $p <$  rand, then *w*3 = 0 and *w*2 = 1 − *w*1.

# **3. SPECIAL SHEAR WALLS**



*Figure 1. Boundary elements in special shear walls*

Shear walls are often used to resist lateral loads imposing on the building. If these loads are due to the earthquake, they will have a dynamic nature. Because of the great importance of designing RC members in the regions with medium or high seismic risk level, codes determine special reinforcement which makes the whole structure more ductile and increases energy absorbing dramatically in order to have a favorable performance of building against earthquake. One of these cases is boundary elements. Regions comprising of concentrated and tied reinforcement are known as boundary elements, irrespective of whether or not they are thicker than the rest of the wall. A wall containing boundary elements is called special shear wall. Figure 1 shows two types of boundary elements in special shear walls (Wight & Macgregor, 2012).

### **4. BOUNDARY ELEMENTS FORMULATION**

First boundary elements parameters are defined as follows:

 $200 \lt t$ w $\lt 400$  mm is assumed as the limitation for thickness of the web.

 $600 < t < 1200$  mm is assumed as the limitation for length of the flange.

200<br/>bf<1200 mm is assumed as the limitation for width of the flange.

 $300 \leq Ssh \leq 450$  mm is the distance of the vertical and horizontal shear bars.

Asf min= $0.01*$ tf\*bf is the minimum

reinforcement area of one flange.

Asf max= $0.04*$ tf\*bf is the maximum reinforcement area of one flange.

 $\Phi_{be}$  is the diameter of each bar of the flange (boundary element) that is selected as 32 or 36 mm.

Other parameters are as follows: *as* is the area of selected reinforcement bars of the boundary elements. *db* is the flexural reinforcing bar diameter; *dt* is the diameter of tie bar; *sc* is spacing between longitudinal bars in the boundary element; *tc* is the cover thickness.

# $\triangleright$  **Caution:** *Int(x)* rounds to integer part of the x.

Reference (Kaveh & Zakian, 2012) has proposed following formulations for boundary elements reinforcement arrangement:

$$
mv_1 = 2Int\left(\frac{b_r - 2(t_s + d_s) + s_r}{d_b + s_r} - 0.5\right)
$$
 (6)

$$
v_2 = 2Int\frac{b_f - 2(t_e + d_r) + s_e}{d_b + s_e} + \frac{t_f - 2(t_e + d_r) + s_e}{d_b + s_e} - 0.5)
$$
 (7)

m

**Size** 

$$
V_{\text{max}} = \max(\text{Int}(\frac{r_f}{r_f + b_f} \times \frac{r_g \text{ max}}{a_i} + 0.5).4)
$$
\n
$$
\tag{8}
$$

$$
m_{{\rm max}} = \min(Int(\frac{b_f}{t_f + b_f} \times \frac{A_{\rm d' max}}{a_s} - 0.5), m v_i)
$$
\n(9)

$$
u d_{\text{max}} = \max(Int(\frac{A_{q\text{-max}}}{a_s} + 0.5), 4) - m l_{\text{min}} \tag{10}
$$

$$
nud_{\text{max}} = \min(int(\frac{A_{\text{tf}} - 0.5), mv_2) - mrl_{\text{max}} \qquad (11)
$$

Unlike steel structures in which beam and column sections have limited sizes, in RC structures by setting different bars disparate sections are built. For simplicity in the recent researches for RC frames optimization distinct beam and column databases have been utilized. Therefore we apply the created shear wall section database in (Kaveh & Zakian, 2012).

The considered database is provided in Table 1 containing 7568 wall sections which have been generated for discrete optimization.

*Table 1. Section database of special shear walls*

No.	t,	tr	bŗ	$\mathbf{S_{sh}}$	ዋ <sub>ሥ</sub>	$\mathbf{N}_{\mathbf{d}}$	$N_{ud}$
ı	200	600	400	500	32	4	6
2	200	600	400	350	32	4	6
3	200	600	400	400	32	4	6
4	200	600	400	450	32	4	6
3290	300	700	1100	300	36	14	8
3291	300	700	1100	300	36	16	10
3292	300	700	1100	300	36	18	10
3293	300	700	1100	300	32	б	4
7565	400	1200	1200	400	36	20	20
7566	400	1200	1200	400	36	22	22
7567	400	1200	1200	400	36	24	24
7568	400	1200	1200	400	36	26	26

### **6. OPTIMAL DESIGN PROCESS**

The main goal of optimization for every structure is to minimize the construction cost. Unlike steel structures, RC structures optimization is more complicated. In RC structures at least three cost items should be considered which comprise of concrete cost, steel cost and formwork cost(Sarma & Adeli, 1998).

General form of an optimization problem is defined as follows:





*Figure 2. Reinforcement notations of the boundary element*

*nud* and *nrl* are the number of bars defined in Figure 2. Note that each value which is selected for *nrl* and *nud* must be an even number, and if any of which is an odd number, then it must be round to the nearest even number in its allowable domain.

Figure 3 shows the topology and notations of the considered special shear wall cross- section.



*Figure 3. Topology of a special shear wall section*

### **5. CREATING DATABASE**

#### **7. LIMITATION CONSTRAINTS**

Penalty approch is used for constraint handling:

$$
f_{\text{penalty}}(X) = (1 + \varepsilon_1 \cdot \mathbf{v})^{\varepsilon_2}, \ \mathbf{v} = \sum_{i=1}^{n} \max[0, v_i] \tag{18}
$$

In this paper, the parameters  $\varepsilon_1$  and  $\varepsilon_2$  for the penalty function, are chosen as 1 and 2, respectively. υ is the sum of the disapproval constraints.

Optimization constraints consist of design and performance criteria which are defined as follows:

Plastic rotation limitation is considered as a performance constraint and imposed on the first level (story) of the shear wall. Because initial plastic hinges are formed at places near the base of the wall, in order to calculate the plastic rotations this equation can be utilized:

$$
\Theta_p = (\phi_u - \phi_y)L_p \quad (19)
$$

 $\theta_p$  is the plastic rotation,  $\phi_y$  is the yield curvature,  $\phi_u$  is the ultimate curvature,  $L_p$  is the assumed plastic hinge length.

Yield curvature  $(\phi_v)$  is defined as follow:

$$
\phi_y = \frac{0.003}{l_w} \qquad (20)
$$

Plastic hinge length  $(L_n)$  value is half of the shear wall length:

$$
L_p = \frac{l_w}{2} \tag{21}
$$

 $X_i$  and  $f(X)$  are the design variables and objective function of the problem respectively.  $g_i(X)$  are optimization constraints.  $R<sup>d</sup>$  is the design domain of the variables.

to minimize 
$$
fit(X) = f(X) \times f_{\text{penalty}}(X)
$$

Where  $fit(X)$  is the fitness function, *f* penalty(X) is penalty function utilized for constraint handling.

objective function in this article is the total cost of the shear wall which is determined as follows:

$$
f = \text{Concrete cost} + \text{Steel cost} + \text{Formwork cost} \tag{14}
$$

Concrete cost = 
$$
C_c
$$
 \* $(2 * b_f * t_f + h_w * t_w - 2 * m_l * A_{sf} - m_2 * A_{sw}) * H_w$  (15)  
Stel cost =  $\gamma_{st} * C_{st} * (2 * m_l * A_{sf} + m_2 * A_{sw}) * H_w + 2 * l_w * tnt \frac{H_w}{S_{sh}}) * A_{sw}$  (16)

$$
Formwork cost = C_f * (4 * (b_f + t_f - 0.5 * t_w) * H_w + 2 * h_w * H_w)
$$
 (17)

*Hw* is the total height of the wall; *Asf is* crosssection area of each bar in the flange of the wall*; Asw*  is the cross-section area of each bar in the web of the wall, for longitudinal and transversal shear reinforcement considered as one cross-section area*;*  hw is the length of the shear wall's cross-section web;  $m1$  is the number of reinforcement bars in each flange;  $m2$  is the number of reinforcement bars in the web.

Constant values:  $C_c = 60 \frac{\epsilon}{m^3}$  is the unit cost of the concrete;  $C_s = 0.9 \frac{1}{3} m^3$  is the unit cost of the steel;  $C_t$ =18 \$/m<sup>2</sup> is the unit cost of the formwork;  $\gamma_s$  = 7850 $kg/m^3$  is the density of the steel.

Based on FEMA criteria (*Federal Emergency Management Agency*, 2000) allowable plastic rotation  $(\theta_{\text{pall}})$  of shear walls that are controlled by flexure for IO, LS and CP performance levels are 0.005, 0.010 and 0.015, respectively. In this study IO level is utilized for optimization procedure, and thus it is equal to  $\theta_{pall} = 0.005$ .

Plastic rotation constraint is shown in Eq. (22):



For seismic design of special shear walls some important design constraints must be used. The ACI 318-08 (American Concrete Institute, 2008) express these restrictions for design as:

*c* is compression region length of the wall section.

$$
c \geq \frac{l_w}{600(\delta \frac{l_w}{H_w})} \quad (23)
$$

 $\delta_u$  is the design displacement. In this paper for risk category of IV it is equal to 0.0045Hw based on the ASCE 7-10 (*American Society of Civil Engineers,*  2010).

Wallace (Wallace, 1995), (Wallace, 1995) has also provided the following relationship for the calculation of *c* as:

$$
C=0.25l_w
$$
 (24)

Minimum length of each flange is:

$$
t_{f\min} = \max\{c - 0.1l_w, c/2\} \quad (25)
$$

The second constraint is a limitation for flange length:

$$
g_2 = \frac{t_f}{t_{f\min}} - 1 \ge 0 \quad (26)
$$

Ultimate shear strength of the wall is defined as:

$$
V_{u\max} = \frac{2}{3}\varphi_v \sqrt{f_c'} A_{cv} \qquad A_{cv} = t_w \times l_w \qquad (27)
$$

Shear force of the wall *Vu* must be controlled by the ultimate shear strength *Vu* max. This restriction can be expressed as:

$$
g_3 = \frac{V_u}{V_{u\text{ max}}} - 1 \le 0 \qquad (28)
$$

The proportion of horizontal shear bars area to the concrete vertical section gross area  $(\rho_t)$  should not be less than 0.0025. This constraint is defined as :

$$
g_4 = \frac{\rho_t}{0.0025} - 1 \ge 0 \qquad (29)
$$

Eq. (24) can be used for the maximum compressive strain of  $\varepsilon_{c \, max} \leq 0.005$ .

Furthermore, the proportion of vertical shear bars area to the concrete horizontal section gross area  $(\rho_l)$ should not be less than 0.0025.

$$
\rho_l = 0.0025 + 0.5(2.5 - \frac{H_w}{l_w}) \left( \rho_t - 0.0025 \right) \ge 0.0025 \quad (30)
$$

This constraint is defined as :

$$
g_5 = \frac{\rho_l}{0.0025} - 1 \ge (8.1)
$$

Considering the combination of axial force  $P_u$  and bending moment  $M_u$  demands is necessary in the design of walls. Due to tall height of the wall and neglecting the longitudinal (vertical) shear reinforcement effect, with good approximation we can convert the moment to a couple of compressive force  $C_u$  and tension force  $T_u$ . Hence, we can design flanges such as a column by these force demands.

$$
T_u = \frac{M_u}{z} \qquad (32)
$$

$$
C_u = P_u + \frac{M_u}{Z} \qquad (33)
$$

 $T_{ua}$  and  $C_{ua}$  are allowable tension and compressive forces, respectively; Z is the distance between the center of two flanges.

$$
T_{u\sigma} = \varphi_t A_s f_v \qquad (34)
$$

$$
C_{ua} = \varphi_c [0.85 f_c' (A_g - A_{st}) + A_{st} f_y] \quad (35)
$$

 $\Phi_v$ ,  $\Phi_t$  and  $\Phi_c$  are the strength reduction factors being equal to 0.75, 0.90 and 0.65 according to the ACI, respectively.  $f'_c$  is the compressive strength of concrete and  $f_v$  is the yield stress of steel.

Tension and compressive strength constraints are as follows:

$$
g_6 = \frac{T_u}{T_{ua}} - 1 \le 0 \qquad (36)
$$

$$
g_7 = \frac{C_u}{C_{ua}} - 1 \le 0 \qquad (37)
$$

### **8. TEST PROBLEMS AND OPTIMIZATION RESULTS**

In this section the performance of the VPS algorithm in optimal design of special shear walls is studied by two examples. For this purpose a computer program is written in matlab for design optimization. The design is in the form of a function which is called by the optimization program. As it was discussed seven constrains are imposed to the problem containing six seismic design constraints and one performance constraint. Plastic rotation constraint often is not violated in the optimization process for two reasons: The first is due to the implementation of six effective design constraints, and the second because of using ACI code special criteria for creating database.

#### **8.1. PROBLEM 1**

Schematic view of a special shear wall and the loading details are illustrated in Fig 4. The total height and length of the wall are 42 meter and 6.7 meter respectively. Height of each story is equal to 3.5 meter. This example already had been optimized by CSS (Kaveh & Talatahari, 2010) algorithm in (Kaveh & Zakian, 2012). Therefore the outcomes of the VPS method are compared with the results derived from CSS algorithm. ( $f'_y = 400MPa$  &  $f'_c = 25MPa$ 



*Figure 4. The special shear wall loading*

After optimization process the results are presented in Table 2. As it can be seen the optimal solutions of the VPS are much less than the CSS's. Convergence history diagrams of the two methods are also depicted in Fig 5. This figure shows that the VPS finds better fitness in comparison with CSS for design of special shear wall and the rapid convergence of the VPS is considerable.

Table 2. Optimum section properties and cost of the special shear wall

	$t_w$ (r	$t_f$ ( m)	$b_f($ m)	$\boldsymbol{S}_{\mathcal{S}\boldsymbol{h}}$ (m)	$\Phi_b$	$N_{rl}$	$N_{ud}$	Co st( $\boldsymbol{\$})$
$\mathbf{CS}$ S (K <sup>2</sup> ) ave $\boldsymbol{h}$ $\&$ Za kia n, 20 12)	0.3	1.2	0.8	0.4 5	3 $\overline{2}$	1 6	$\overline{2}$ $\overline{4}$	418 31

	0.2	0.6	0.5	0.4	3		4	$\frac{390}{24}$
				5	$\overline{2}$	4		
VP )S Pre								
sen								
t								
wo (rk								

*Table 2. Optimum results*



*Figure 5. Convergence history of the CSS and VPS algorithms*

### **7.2. Problem 2**

A special shear wall with length of 6 meters has been placed between two columns of a construction. The construction has 5 stories which the height of the first story is 5 meters and the other stories height are 3 meters. The wall is under the 4500 KN shear strength and 60000 KN bending moment at the basis.  $(f'_y = 400MPa \& f'_c = 30MPa)$ 

We optimize this example via three divergent algorithms to get a precise evaluation. VPS, CBO(Kaveh & Mahdavi, 2014) and ECBO (Kaveh & Ghazaan, 2014) algorithms are utilized for this problem. After optimization process the results are presented in Table 3. The table illustrates that the VPS method has more optimized result. Convergence history diagrams of the three methods are also depicted in Fig 6. This figure shows that the VPS finds better fitness in comparison with CBO and ECBO for design of the concerned shear wall.

	$t_w(n)$	$t_f($ m)	$b_f($ m)	$S_{Sh}$ (m) $\mathcal{E}$	$\Phi_b$	$N_{rl}$	$N_{uc}$	Co st( $\boldsymbol{\$})$
$\overline{\mathbf{C}}\mathbf{B}$ $\overline{\mathbf{0}}$ Pre sent wor $\left( k\right)$	$\overline{0.2}$	0.6	$\overline{0.5}$	0.4 5	$\mathfrak{Z}$ $\overline{2}$	4	6	971 9
EC <b>BO</b> $\mathcal{E}$ Pre sent wor $\left( k\right)$	0.2	0.6	0.5	0.4	3 $\overline{2}$	$\overline{4}$	$\overline{4}$	957 8
<b>VP</b> $\mathbf{S}$ Pre sent wor $\left( k\right)$	0.2	0.6	0.4	0.4	$\mathfrak{Z}$ $\overline{2}$	$\overline{4}$	$\overline{4}$	948 $\overline{2}$

*Table 3. Optimum section properties and cost of the special shear wall*



*Figure 6. Convergence history of the CBO, ECBO and VPS algorithms*

#### **9. CONCLUDING REMARKS**

In this article VPS algorithm was utilized to optimize special shear walls under performance and design criteria. VPS has a number of individuals consisting of the variables of the problem. The solution

candidates gradually approach to their equilibrium positions that are achieved from current population and historically best position in order to have a proper balance between diversification and intensification. The VPS was applied to reduce the expenses. The aim was minimizing the total cost of the shear wall as objective function which comprised of the cost of concrete, steel bars and formwork. After processing optimal solutions of VPS and other algorithms were compared. Outcomes illustrated the VPS generally has better performance than the others in terms of accuracy and speed of convergence.

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