Using Design Experiments to Conduct Research on Mathematics Professional Development*

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Abstract. In this paper, we propose that the emerging transformation of mathematics professional development from a practice-based to a research field would benefit from stronger connections to research on learning. In particular, we contend that design experiments represent a premier emerging methodology to study learning, and we argue that a better understanding of teacher learning through the use of design experiments in mathematics professional development can lead to improvement of mathematics professional development as both an area of practice and a field of research.
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Despite decades of work with mathematics teachers in professional development settings, the history of *research* on mathematics professional development is quite recent. Sztajn (2011) indicated that research on mathematics teacher education only began to receive significant attention in the 1990s. She noted that the first *Handbook for Research on Mathematics Teaching and Learning* (GROUWS, 1992) did not include a chapter on teacher education, signaling the lack of a significant body of research on the topic at the time. In the *Second Handbook for Research on Mathematics Teaching and Learning* (LESTER, 2007), Sowder (2007) analyzed why professional development had become a priority. She argued that the *Professional Standards for Teaching Mathematics* (NCTM, 1991) started a movement that set a new agenda for research on professional development, encouraging the field to go beyond the behavioral focus of the process-product paradigm to attend to teachers' thinking.

Examining teacher education as an emerging field of research, Grossman and McDonald (2008) noticed its recent history and claimed that research in this area had developed "in curious isolation both from mainstream research on teaching and from research on higher education and professional education more generally" (p. 185). They called research in this emerging field an

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"orphan" because of its lack of connection to other more established fields of research, and pointed to important connections between teacher education and studies of teaching, organizations, and policy implementation.

Building on the notion that teacher education needs to strengthen its connections to other fields of research, we propose that as mathematics professional development transforms from a practice-based to a research field, it would benefit from stronger connections to research on learning. In particular, we contend that design experiments represent a premier emerging methodology to study learning, and we argue that a better understanding of teacher learning through the use of design experiments in professional development can lead to improvements in mathematics professional development as an area of both practice and research.

In the learning sciences, design experiment methodology is gaining researchers' attention because the methodology allows for the development and testing of learning theories in the context of practice. Design experiments combine "the open character of design together with the self-imposed constraints of research" (KELLY et al., 2008, p.3), drawing on a central tenet of the learning sciences that people develop deeper knowledge when they engage in authentic, domain-specific tasks (SAWYER, 2006). Design experiments promote researchers' examinations of both the processes of learning and the means for supporting them.

Although design experiments have been mostly used to examine K-12 student learning, we contend that the methodology can be adapted and productively used to study teacher learning in the context of professional development. In what follows, we first briefly examine existing research on mathematics teacher learning. Next, we present the design experiment methodology and review emerging studies that have used this methodology to examine teacher learning in professional development. Then, we present an example from our current work using a design experiment to study teacher learning in the context of a professional development program focused on recent advances in the empirical development of learning trajectories for students (CLEMENTS & SARAMA, 2004; CONFREY et al., 2009). We conclude with a discussion of major differences in conducting design experiments with students versus teachers.

Studying Mathematics Teacher Learning

In the United States, emerging studies of teacher learning have taken place in the context of mathematics professional development. Sowder (2007) organized a decade of research on mathematics professional development around topics such as: the goals and design principles for professional development; teachers' knowledge; teacher change; and factors that affect professional development. Using the framework proposed by Cochran-Smith and Lytle (1999) of knowledge-for-practice, knowledge-in-practice, and knowledge-of-practice, Sowder organized studies on teacher learning around what teachers need to learn to teach mathematics (knowledge-for-practice), what teachers learn from examining teaching with their peers (knowledge-in-practice), and what teachers learn from investigating their own teaching (knowledge-of-practice). The first set of studies discussed the purposeful use of student work, pedagogical cases, and curricular materials as support for teacher learning. The second set of studies highlighted the difficulties of establishing and maintaining learning communities. The third set established differences between academic and practice-based inquiry. Most of the studies that Sowder reviewed addressed whether or not teachers learned knowledge "for", "in", and "of" practice in the context of professional development. However, the actual *process* of teacher learning in these various contexts was not addressed.

Goldsmith et al. (under review) conducted a synthesis of research focused on mathematics teacher learning, as opposed to mathematics professional development. They claimed that the question of how practicing mathematics teachers continue to learn over time was key for improving instruction, and that this question required a review that had a more specific focus on teacher learning. Their synthesis built upon Clarke and Hollingsworth's (2002) model of teacher growth, which considered the interconnections of the external domain, the personal domain, the domain of practice, and the domain of consequences in relation to teacher growth. Adding to these domains, the authors considered that teachers learn through engagement with certain catalysts such as professional development, changes in textbooks, and/or collaborative work with their peers. These catalysts, they claimed, had the potential to promote changes in teachers' knowledge, beliefs, dispositions, and classroom practices.

Goldsmith et al. (under review) coded 106 articles published between 1985 and 2008 on practicing teachers' learning on the job with categories such as: teachers' identity, beliefs, and dispositions; mathematics content knowledge; attention to student thinking; teachers' practice; and teachers' collaboration and communities. Their results indicated that mathematics teacher learning in professional development related to their beliefs about mathematics teaching and learning; that is, teachers respond differently to mathematics professional development based on their identity and beliefs. They also showed that close attention to students and student work impacted teachers' beliefs about the mathematical competence of their students. For the studies that associated learning with changes in professional practice, Goldsmith and colleagues reported changes in four different aspects of instruction that resulted from mathematics professional development: selection of more meaningful mathematical tasks, increased attention to student thinking, structure of classroom discourse, and promotion of students autonomy.

An interesting outcome of Goldsmith et al.'s synthesis was their conclusion that:

> Most research we reviewed was designed to study the effectiveness of particular programs, curricula, or professional learning approaches. Typically, teachers' learning was treated as an indicator of the effectiveness of the program rather than as the primary object of inquiry. Studies that reported on professional development interventions did not, for the most part, focus on the processes or mechanisms of teachers' learning (GOLDSMITH et al., under review).

Thus, although teacher learning was the original focus of their review, the studies they examined considered learning as an outcome or as a pre-post measure of the effectiveness of the intervention; they did not consider teacher learning as a process.

In considering the relation of research on teacher learning and the learning sciences, Fishman and Davis (2006) noted that learning scientists often conduct professional development as an "incidental" step (although perhaps necessary) in the development of an innovation. Therefore, learning scientists typically document what teachers "should know" or focus on pre-post changes to what teachers know. Fishman and Davis (2006) claimed that learning scientists have rarely attended to professional development as the object of their research and called for investigations of "the cognitive and conceptual change that occurs *during* teacher learning" (p. 546, emphasis added). These authors noted that the learning sciences have become "strongly associated with the use of design experiments" (p. 537) because of how well this methodology is suited to research in context and in practice; they suggested that the study of teacher learning could benefit from a stronger emphasis on design experiments.

In line with Fishman and Davis, we contend that attending to the processes of teacher learning in the context of mathematics professional development is not just necessary but essential to advance the field of research in mathematics teacher education. Further, we assert that design experiment methodology is the premier tool for researchers to make such advances. The use of design experiments to investigate mathematics professional development allows researchers to work both on the design of the professional development and on generating theory about teacher learning in context. In the next section, we discuss design experiment methodology and present emerging studies that use this methodology to study mathematics teacher learning in the context of professional development.

Design Experiments and Teacher Learning

To date, the work of design experiments has mostly focused on student learning (FISHMAN & DAVIS, 2006), with some researchers engaging teachers as design partners in such work (LEHRER & SCHAUBLE, 2000; MCCLAIN & COBB, 2001). Whereas partnering with researchers to conduct design experiments in classrooms can provide powerful professional development experiences for teachers—in line with what Bannan-Ritland (2008) called teacher design research—our focus in this paper is on using professional development as the setting for conducting research on teacher learning through the use of design experiments—which is more in line with what Simon (2000) called a teacher development experiment.

Sawyer (2006) suggested that studying learning requires the scientific study of design, and stated:

> experimental and quasi-experimental designs can provide educators and policy makers with important information about the relative merits of different approaches. But they can't tell us very much about why or how a teaching method is working – the minute-by-minute structure of the classroom activity that leads to student learning... Learning scientists combine a range of methodologies to better understand learning processes…. [including] a new hybrid methodology known as design research (p. 13).

Confrey (2006) suggested that design experiments have historic roots in traditions of both the radical constructivist teaching experiment and the Vygotskian attention to development within sociocultural settings. They "entail both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them" (COBB et al., 2003, p. 9). Characteristically, their primary goal is to design learning opportunities and produce theories of learning related to these opportunities through iterative cycles of design, implementation, analysis, and redesign (DESIGN-BASED RESEARCH COLLECTIVE, 2003). Design experiments acknowledge both the complexities of authentic environments for teaching and learning, and that the theories produced are necessarily related to those environments (COBB et al., 2003), enabling the creation and careful study of the conditions that learning theories suggest are productive (DESIGN-BASED RESEARCH COLLECTIVE, 2003).

Central to design experiments are the articulation of a set of learning conjectures (COBB et al., 2003; CONFREY & LACHANCE, 2000) and design principles (COLLINS et al., 2004), which drive the design of the learning environment. The learning conjectures are taken as proto-theories and evidence is collected that supports or challenges them through ongoing analysis during implementation and retrospective analysis after implementation (COBB, 2000). Documentation of changes to these conjectures directly linked to the designed learning context serves as systematic variation for the methodology (COBB et al., 2003). Through progressive refinements (COLLINS et al., 2004), the products of these design experiments are both the interventions and the theories produced.

Currently, design experiment methodology is in the beginning stages of applications to the field of teacher education. Although its use is growing in mathematics professional development, few studies have published results of studying teacher learning through the use of this methodology. For example, Visnovska et al. (2006) discussed their 5-year professional development design experiment with middle-school mathematics teachers in a large urban district in the United States. The researchers developed two types of professional development activities with the intention of furthering teachers' learning. The first set of activities and conjectures focused on the analysis of student work and helping teachers learn to attend to student reasoning. The researchers found that their initial design conjectures for these activities were unviable because teachers' designed instruction based on objectives created to cover the content, not student reasoning. Teachers' examination of student work did not attend to student reasoning as anticipated but rather followed an unanticipated evaluative perspective. The researchers concluded that their "failure to support the teachers in focusing on students' reasoning resided in [their] failure to understand which interpretations of the designed PD task might be a viable extension of teachers' practices" (VISNOVSKA et al., 2006,p. 642).

The second set of professional development activities focused on helping teachers address the challenges that teachers themselves perceived in their classrooms – motivating students. The researchers conjectured that teachers could change from perceiving motivation as inherent to students to perceiving it as situational and based on whether students saw relevance in instruction. Further, they considered that when teachers perceived motivation as situational, they would attend to student reasoning. Through designing professional development activities to support teacher learning in this conjectured direction, they found that "it was possible to support the teachers in changing significant aspects of their instructional practices in envisioned ways" (p. 643).

Working with pre-service teachers, Stylianides and Stylianides (2009) conducted five research cycles over four years of design and refinement of an instructional sequence to support teachers' transition from using empirical arguments as their main method for validating mathematical generalizations. Their design principles included claims that: proof is an integral element of sense-making in mathematics; cognitive conflicts are an important mechanism to support learning; and that instructors play a role in supporting students' resolution of cognitive conflicts in the classroom. The three tasks in their instructional sequence were designed to move students from holding naïve empirical arguments, to questioning empirical arguments, to accepting the need for mathematical proofs. Analyzing data from the fifth cycle of their experiment, they concluded that the close match between their learning conjectures and participants' actual learning paths as they worked through the tasks was an outcome of their long-term, systematic approach to learning. They suggested that the designed task, after cycles of improvement and refinements, came to support learning as intended and helped students learn the limitations of empirical arguments as a method for generalizing mathematical arguments.

In science, Stolk et al. (2011) used a design experiment with six high school chemistry teachers. The researchers developed a framework for context-based science instruction and sought to apply the framework to a professional development composed of five three-hour meetings. Their overarching research question, "To what extent does the elaboration of the framework for professional development empower chemistry teachers for context-based designing?" (p. 375) was divided into 10 sub-questions directly related to the professional development activities. The activities were designed based on the researchers' proposed learning outcomes and hypothetical actions that the teachers would perform in their classrooms. Stolk and colleagues concluded that the actual learning outcomes for each activity deviated from their expectations. They explained, “Teachers were only partly empowered for designing new context-based units…the framework was not sufficiently elaborated to completely empower chemistry teachers for context-based designing” (p. 385).

Working with 33 practicing mathematics elementary teachers, Wilson (2009) used a design experiment to investigate the ways that teachers’ learning of a mathematics learning trajectory might support them in attending to student thinking and in their instructional practice. He conjectured that a learning trajectory would serve as a lens through which teachers would filter students’ activity to highlight evidence of their mathematical thinking. Further, the trajectory would support teachers in adapting their curricular materials, inform their analysis of students’ written work, and serve as a guide for making instructional decisions when teaching. After 20 hours of professional development on the learning trajectory, he found that the trajectory did support teachers in more precisely understanding their students’ mathematical thinking, suggesting how they might respond to students’ written work, and beginning to support teachers their instructional decisions when teaching. However, his initial conjectures failed to anticipate the significant interactions of teachers’ existing knowledge of their mathematics curriculum and instructional materials with the trajectory. He concluded the study with a set of revised learning conjectures and suggested, “evaluation of the learning activities, followed by revisions or reconceptualizations based on the results of this study should be conducted prior to another cycle of implementation” (p. 204).

These examples show the promise of design experiments to study teacher learning. Design experiments allow for the refinement of professional development learning tasks in combination with the empirical testing of associated learning conjectures. They provide evidence for teacher learning in the moment and data on how teachers proceed toward more sophisticated understandings. For the purpose of illustrating the work involved in the process of conducting a design experiment, we present an example from our use of design experiment as a methodology in our current research project. The example illustrates three aspects of the work: how a learning conjecture was refined as we began to implement our intervention, the changes in design made in the first cycle of implementation as a result of the revised conjecture, and the role of retrospective analysis in the subsequent adjustments of the professional learning tasks during the second implementation.

The Learning Trajectory Based Instruction Project

The Learning Trajectory Based Instruction project (LTBI) uses design experiment research to investigate teacher learning of students' learning trajectories (LTs) and of an instructional model where LTs provide guidance for teachers' instructional decisions. LTs are defined as "a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time" (CONFREY et al., 2009, p. 347). The LTBI model of instruction (SZTAJN et al., 2012) emphasizes the importance of open instructional tasks to elicit and build upon students' mathematical thinking and a set of pedagogical practices that centralize this thinking.

To date, the LTBI project has completed one design cycle and is working through its second implementation and analysis of the professional development. Each LTBI professional development cycle had a different partner elementary school from the same mid-sized suburban school district in the Southeastern United States. The project offered teachers approximately 55 hours of face-to-face professional development, with the total number of hours varying slightly from the first to the second cycle due to local school scheduling. All teachers from grades K-5 in the partner schools were invited to join the project and received a stipend for participation. Project participation was optional, and teachers worked with the researchers in finding the best times to fit the professional development into their schedules. Principals at both schools supported the work and provided space at the school for the project to meet.

The LTBI professional development was designed for a 12-month period beginning with a 30-hour summer institute. Following the institute, teachers met regularly throughout the school year with project leaders after school hours to continue to build their knowledge and discuss their classroom implementations of tasks that incorporated LT. Although teachers learned about LT and LTBI throughout the mathematics professional development, many summer professional learning tasks were designed to support teacher learning of the LT, whereas during the year the professional development focused more specifically on LTBI. The initial focus on the LTs emerged in part from the complexity of the trajectories and in part from the fact that during the summer teachers did not have opportunities to try out ideas with their own students.

The first and the second cycles of the LTBI mathematics professional development were based on different LTs, and there were differences in the amount of time spent in the summer discussing LTs versus LTBI. Despite these differences, both cycles followed similar design principles in the development of professional learning tasks: (a) attend mostly to pedagogical content knowledge, (b) embed opportunities for teachers to examine all facets of their own knowledge for teaching, (c) employ instructional sequences that started with practice-based activities that challenged elementary teachers' views of students' mathematics and mathematics learning, and (d) use artifacts similar to the ones researchers used in developing the LT to highlight the logic of the learner (WILSON et al., 2012). Further, the tasks followed a similar sequence of activities: engage, explore, formalize, and apply.

In each cycle of design, the research team collected video recordings of all professional development meetings, audio recordings of small group discussions during these meetings, pre- and post- content assessment data, and classroom observations. Data from the first implementation were used in both ongoing and retrospective analysis to examine how teachers learned about the LT and LTBI and to redesign the intervention for the second implementation. One important change made to the summer institute between cycles regarded the attention given to discussions about attending to students and their mathematics. In particular, the project team was more purposeful in the second implementation in focusing and guiding teachers' discourse and participation around issues of student learning.

Although the LTBI project followed Sfard's (2003) recommendation that both an acquisition and a participation perspective of learning should be considered as theoretically underpinning studies of learning, the changes in design we discuss here emerged from an analysis of teacher participation. Thus, we use a socio-cultural perspective (LAVE &WENGER, 1991) to discuss teacher learning in mathematics professional development and describe how the research team came to understand changing teachers' discourse about students.

Ongoing Analysis of the First Cycle of the LTBI Project

One of the original research questions that guided the first design cycle was: in what ways are teachers' learning of the LT demonstrated through changes in their participation in the professional development learning community and in their voice and positioning within the discourse of the community? The initial conjecture proposed was that as teachers learned about the LT, they would gain specialized language to discuss student thinking, which would bring their participation closer to the center of their professional community and strengthen their voice and positioning in the discourse of the group (HERBEL-EISENMANN, 2007; WAGNER & HERBEL-EISENMANN, 2008). In this regard, the professional learning tasks were designed with a strong initial focus on teacher learning of the LT language, and the research team worked to examine the ways in which such language was incorporated into teachers' discourse during the professional development.

Very early in the ongoing data analysis for the project, teachers' discourse during the mathematics professional development indicated the prevalence of language that talked about students' successes and failures as related to students being "high or low" or to whether contextual features of mathematical tasks matched students' experiences. For example, teachers talked about students not completing a task because they were "low students," or talked about students failing to properly produce a fair share of a rectangular shape because the task proposed presented the shape as a cake and "no one shares all the cake at once," leading students to not fulfill the requirement that the entire whole had to be used. Statements such as these indicated that it was important for the research team to attend not only to the ways in which teachers positioned *themselves* in the community but also to the ways in which teachers positioned *students* in their discourse within this community. Thus, our initial conjecture about positioning was only partial, as it did not take into account teachers' positioning of students.

The professional learning tasks designed for the LTBI professional development included many opportunities for teachers to talk about students' mathematics. These opportunities originated from the team's design principles and built on previous research that reported the value of using student work to support teacher learning (LITTLE, 1999; KAZEMI & FRANKE, 2004; SOWDER, 2007). However, as the first design cycle unfolded, the research team understood that professional learning tasks using student work provided the needed space for the research team to attend to the ways in which teachers' talked about students' successes or failures when completing the mathematical work under examination.

In regular meetings throughout the first implementation of the LTBI mathematics professional development, the research team examined how teachers' positioned students in their discourse. Our ongoing analysis sought to understand and question the ways teachers talked about students, and the team began to include specific notes about the ways teachers talked about students in the field notes. We watched videos segments of teachers talking about students and discussed similarities and differences in the ways the discourse during mathematics professional development positioned students.

From a design perspective, we continued to refine the mathematics professional development to create new opportunities for teachers to examine the ways in which they talked about their students as mathematics learners. At the end of the first LTBI professional development cycle, for example, we developed a professional learning task that presented back to teachers some of their own ways of talking about students. The task consisted of a series of four teacher-dialogues, each one highlighting a different way that teachers talked about students that we had documented in the ongoing analysis. (See the Appendix for an example of one of these dialogues proposed to teachers.) These four dialogues focused on students' previous achievement, out of school contexts, the mathematical task posed to students, and teachers' own teaching. They related students' successes or failures to these four aspects and elicited teachers' perspective on each of them. Teachers were purposefully given a series of questions to discuss in small groups related to these different ways to talk about students with the goal of having teachers' examine some of the perspectives on students they had used during the mathematics professional development.

Retrospective Analysis of the First Cycle of LTBI Project

To understand the language teachers used to talk about students when participating in the discussion around the professional learning tasks, during the retrospective data analysis that followed the first implementation of the LTBI mathematics professional development we turned our attention to the concept of stereotypes within the positioning literature. We used Langnehove and Harré's (1999) definition that stereotypes are speech acts "used in order to position both speaker and the object of the stereotyping, and draws upon social representations of the stereotyped objects (cultural stereotypes) which are available in certain moral orders" (p.132). Thus, people appropriate and use stereotypes that are available in their discourse communities, and to change stereotypes, it is necessary to change the rules of conversation and the discursive conventions within these communities.

From a participation perspective on learning, our retrospective analysis examined the ways in which teachers used stereotypes about students' mathematics in their professional discourse during the LTBI mathematics professional development, as well as the implications of such stereotypes for teacher learning of students' mathematical thinking. We tested an emerging conjecture that teachers would come to use the language of the LT to talk about students' mathematics, thus substituting their initial set of stereotypes with more mathematical language to describe and assess students' mathematics. Because we realized that our initial conjecture about teacher learning was incomplete, we developed the following revised learning conjecture to work with as we conducted the retrospective analysis from the first LTBI professional development cycle: as teachers learn about the LT, they gain specialized language to discuss student thinking, which brings their participation closer to the center of their professional community and allows them to attend to *how they position students as learners of mathematics as well as how they position themselves as mathematics teachers of these students*.

The retrospective analysis used Decuir-Gunby et al.'s (2010) recommendations for the development of a codebook. We began with an iterative comparison of our field notes and theory-driven codes coming from the literature on attribution theory (SZTAJN et al., 2012). However, in line with our perspective on learning, we came to consider that teachers did not hold a particular attribution for students; rather, they appropriated and used various ways of talking about students that were available to them at their school professional community. Thus, all teachers used varied stereotypes over the course of the MPD instead of aligning with one particular attribution for student success.

To document the various stereotypes available and their usage, we created operational definitions for the following stereotypes that had been identified in teachers' discourse during the first cycle of the professional development: ability/achievement, age/grade, effort, luck/random, out of school contexts, quality of task, teaching, learning trajectory (see Table 1). Having documented the presence of each stereotype in teachers' discourse, we used our definitions to code every turn in all whole group and small group conversations from the first cycle of the LTBI mathematics professional development. Five coders were trained to identify turns and code these turns when stereotypes were present.

Table 1: Stereotype Codes

Code	Description
Ability/ Achievement	A personal trait of the students, a characteristic that defines the student as a person, a fixed quality that related to teachers' views of students' aptitude in math due to either innate capacity or previous work.
Age/Grade	Developmental and curricular expectations of students' performance given teachers' normalized definitions of what the generic student should be able to do at certain points in their age/grade growth
Effort	Level of student attention and engagement with a particular task at a particular moment; not necessarily a fixed characteristic of the student or of the task, but a factor of how carefully or how speedy that particular student progressed through the work at a particular point.
Luck/ Random	What students do has no intentionality behind it; students do things without really having an explanation for what they did or knowing what they were doing. Students are just acting and getting lucky (or not) about the work.
Out of School Contexts	Experiences students bring with them from their own lives; outside school understandings and explanations that students generalize to the academic context. Out of school use of vocabulary, and words we bring into the classroom of mathematical problems.
Tasks	Represents the idea that what students can or cannot do mathematically depends on the types and quality of tasks posed to them, as well as the form in which the task is presented and the materials or support offered to them. It includes the idea that the clarity or lack of clarity of the question posed to students is what determines what they can do, as well as the idea that there is a perfect way to ask a question so that students would not make a mistake.
Teaching	The idea that what students do depends on what teachers have presented to them; student work depends on whether a teacher has already taught a particular topic to the students and therefore it should be expected that students would know a topic taught and cannot know a topic not yet taught. Or, teachers' descriptions of how they would teach in order for kids to learn.
Learning Trajectory	The idea that students' work can be attributed to students' cognitive development or mathematical experiences, or *descriptions* of students mathematical work/thinking using the language provided by the trajectory.

After coding and analyzing the data from our first partner school, our findings suggested that whereas the MPD on LTs offered teachers a new way to talk about students' mathematical work, it added to, but did not displace, other stereotypes teachers used to conceptualize students as learners of mathematics. The use of stereotypes varied over time, but also varied based on the professional learning task given to teachers, with certain tasks allowing certain stereotypes to emerge more often. This finding had implications for the redesign and implementation of the mathematics professional development with the second partner school.

Re-design of the Second Cycle of the LTBI Project

As we redesigned the LTBI mathematics professional development for its second implementation, we considered how the discourse in place in the teachers' professional community within partner schools allowed teachers to use particular language to talk about students. Teachers brought the acceptable language from their school community to the professional development and, as they engaged with the projects' professional learning tasks, they used the ways of talking about students that were familiar to them. We conjectured that the stereotypes we found in the previous school existed beyond that particular school, and we revised the mathematics professional development considering that we would find similar stereotypes in the new school. We redesigned many of the summer institute tasks to purposefully act on changing teachers' discursive patterns very early on the professional development. Our goal was to focus teachers' discourse on the mathematical features of students work and on using the LT language to describe what students could and could not do. Therefore, although many of the tasks were still designed to teach teachers about the LT, they were also designed to support teachers in more carefully attending to students' mathematics and using the LT language to talk about students' mathematical successes and failures.

For the second design cycle of the mathematics professional development, we purposefully established a set of discursive norms to define what we considered acceptable ways of talking about students in the professional development setting. During the first day of the LTBI summer institute, we proposed to participants a set of productive ways to talk about students' mathematical work, including 1) describe what students can do, 2) develop hypotheses about the mathematical reasoning for the work students do, 3) provide evidence for claims about what students know or do not know, and 4) recognize when statements are speculations or judgments. In essence, we wanted teachers to focus on who students were as mathematical thinkers and use evidence to substantiate their claims, rather than stereotypes. Throughout the summer institute, teachers were reminded of these norms and when the conversation deviated from these norms, the research team purposefully intervened to remind participants of these more acceptable ways of talking about students in the LTBI professional development setting.

The analysis for the second design cycle is still under way. Throughout the duration of the mathematics professional development with the second partner school, the research team continues to meet regularly to revisit and revise our conjectures related to teachers' use of stereotype. We have created a conjecture log (COBB et al., 2003) to document evidence of teachers' uses of stereotypes, the role of the professional learning tasks in bringing forth certain stereotypes, and the nature of the school culture that support certain discursive patterns over others.

As the analysis unfolds, we continue to contend that learning about students' LTs provides language that allows teachers to move from describing students' mathematics using stereotypes such as ability, grade level, task, or out of school experiences and focus more clearly on the mathematical meaning of students' work. Teachers who learn about the LT come to attend to the mathematics; they then become empowered to act on supporting student learning rather than considering they cannot change their "high or low" students or change students' out of school experiences. We argue that designing professional learning tasks that offer teachers a clear set of expectations for how to talk about students' mathematics, together with a framework to describe the development of students' mathematical thinking and a related model of instruction, allows teachers to attend to student learning and find ways to support students' growth.

Conclusion: Using Design Experiments to Mathematics Professional Development

In this paper, we showed how the use of design experiment methodology to study teacher learning allows researchers to develop theories of learning in the context of mathematics professional development. Although design experiments have been more often used to examine K-12 student learning, our review of emerging findings from studies that used design experiments in teacher education, including our own work in analyzing and revising learning conjectures and tasks within the LTBI project, indicated the strength of the methodology for improving mathematics professional development design and researching the process of teacher learning as it unfolds during the professional development. We conclude with important issues to consider that make it different to conduct design experiments with students and with teachers. As the use of design experiment methodology in mathematics professional development increases, understanding and documenting these issues support the improvement of the design of appropriate tasks as well as the specification of requirements for using this methodological approach to study learning in the context of teacher education.

Issues to Consider in Conducting Design Experiments with Teachers

Teachers' mathematical knowledge for teaching. Design experiments begin with the learner and take into account learners' previous knowledge to develop tasks and initial learning conjectures. Therefore, similar to work with K-12 students, design experiments with teachers attend to the knowledge teachers bring into the mathematics professional development. Teacher knowledge, however, is not restricted to subject matter knowledge; it includes other domains of teachers' mathematical knowledge for teaching (BALL et al., 2008). Thus, attending to the learners' knowledge is more complex in the case of works with teachers when compared to work with K-12 students. When conducting design experiments with teachers in the context of mathematics professional development, the pedagogical content knowledge participating teachers bring with them has to be taken into account.

Teachers' identities. Beyond attending to the various domains of knowledge teachers bring with them, design experiments with teachers should consider the professional identities teachers bring into the mathematics professional development. Identities shape teachers interactions with the tasks as well as the learning that can result from working on these tasks. As such, identities also play an important role in the development of the learning conjectures about teacher learning that one examines through design experiments. Goldsmith et al. (under review) reported that teacher learning in mathematics professional development related to their beliefs about mathematics teaching and learning. Similarly, Visnovska et al. (2006) concluded that their initial conjecture was unviable because it failed to consider the interpretations teachers' brought to the MPD. These findings support the importance of considering teachers as learners from a perspective that goes beyond the various domains of knowledge to also include teachers' identities.

Discursive patterns and school culture. Adding to the attention to teachers' identity, design experiments in mathematics professional development settings are also impacted by the discourse patterns teachers bring with them and, more generally, by the values and cultural assumptions that exist at their schools. The culture of schools, along with the various ways in which teachers come to talk about and discuss students and mathematics teaching, shape teachers' participation in professional development discourse, supporting and suppressing the perspectives that become part of the work within the mathematics professional development. Cobb et al. (2009) considered that it was important to locate teachers' learning in the institutional setting of their schools and also the setting of their district. Their work emphasized the importance of taking teachers' institutional settings into account when creating or adjusting the mathematics professional development design.

Goal-oriented professionals. In addition to attending to all aspects of who participants are in a design experiment within a mathematics professional development setting, it is important to consider that teachers are goal-oriented adults who come to a mathematics professional development with their own professional agendas. Differently from K-12 students, teachers come to mathematics professional development learning opportunities with a set of professional expectations; they set goals for their own learning that often precede those of the mathematics professional development. Richardson (1992) referred to the "agenda-setting dilemma" of professional development, as both teachers and professional development designers come to the mathematics professional development with their own agendas, which then have to be negotiated. Because teachers' agendas shape their engagement and work with the professional learning tasks used in the design experiment, they need to be considered in the design of such tasks for them to be effectively carried out with goal-oriented, professional learners.

Learning conjectures beyond subject matter knowledge. Beyond taking into account the complex set of knowledge, identity, values, and goals that teachers bring to a mathematics professional development context, design experiments with teachers differ from design experiments with students in that the learning goals for the intervention – that is, the goals for the mathematics professional development – often go beyond subject matter learning goals. Many mathematics professional development projects are designed to promote teacher learning of pedagogical content knowledge or changes in their identities and values, requiring that learning conjectures in these experiments to address changes that transcend subject matter knowledge. The concept of pedagogical content knowledge is a relatively recent idea (SCHULMAN, 1986), and although work has begun to define what this knowledge encompasses, talking about teacher learning of pedagogical content knowledge and how such learning may progress and change over time to become more sophisticated is still a novel idea. Thus, there is a significant difference in developing learning conjectures about subject matter knowledge in K-12 settings and developing learning conjectures about pedagogical content knowledge or identities in mathematics professional development settings.

Mathematics professional development tasks designed to support teacher learning of aspects of teachers' pedagogical content knowledge such as knowledge of students, teaching and the curriculum (BALL et al., 2008) and the associated learning conjectures for such learning are beginning to emerge. Further work is needed to map how teachers come to learn such knowledge, and define what constitutes a progression of more sophisticated ways to think about students, teaching and curriculum. Discussions related to the nature of conjectures about learning pedagogical content knowledge and appropriate grain sizes for learning this domain are necessary to clarify this particular aspect of learning. Design experiment methodology in mathematics professional development can advance these discussions.

It is not unusual for the goal of mathematics professional development to go beyond the various domains of teachers' mathematical knowledge for teaching to include changes in teachers' practices. In this case, it is important to consider what it means to design tasks and propose learning conjectures that relate to teachers' classroom practices rather than solely participation in the mathematics professional development. As Cobb et al. (2009) noted, this attention to practice represents an important modification to the design experiment methodology, as the learning under investigation is to be demonstrated in a setting other than the mathematics professional development setting itself. This shift creates the challenge of "reconceptualizing the relations between teachers' activity in the PD setting and in their classrooms" (p. 165).

Duration. Finally, one other difference in using design experiments to research mathematics professional development is the duration of the interventions. Research on effective professional development suggests that to promote changes in teachers' knowledge and practices, professional development needs to be ongoing and sustained over time. Thus, similarly to LTBI, many projects last for a whole school year—making the design experiment in mathematics professional development span over many months. A 40-hour professional development project can last for a full school year, whereas a 40-hour mathematics unit in K-12 settings last perhaps for a quarter. Working for a year on one intervention supports careful on-going data analysis, but slows the process for careful retrospective data analysis prior to a new design and implementation cycle as well as the conduction of repeated design cycles. The development of appropriate tasks and parallel conjectures in mathematics professional development settings can therefore take many years before results are useful and well developed.

Despite the list of added challenges to conducting design experiments with teachers rather than students, we consider that this research methodology and its connections to the learning sciences finally allow researchers to focus on *learning* in mathematics professional development settings. Design experiments offer a new perspective for researchers, strengthening the knowledge base on teacher learning. They generate theories about teachers' learning rather than evaluative results about the overall effectiveness of mathematics professional development. Studying the phenomenon while creating it (WILSON & BERNE, 1999) continues to be a challenge for the use of design experiments in mathematics professional development. However, the positive outcomes of the use of this research methodology in mathematics professional development outnumber the problems.

Final Thoughts

In this paper, we presented a case for considering design experiment methodology as the premier tool for studying teacher learning in the context of mathematics professional development. Further, we argued that the use of design experiment could strengthen mathematics professional development as a field of research because it connects this emerging field to the learning sciences. Design experiments change the study of mathematics professional development from an evaluative focus on the effectiveness of interventions to an examination of the learning processes that takes place in context. For all these reasons, we conclude that focusing on the design of domain-specific professional learning tasks and on the learning that takes place as teachers engage with such tasks supports research on teacher learning in the context of mathematics professional development and leads to the development of local learning theories in context.

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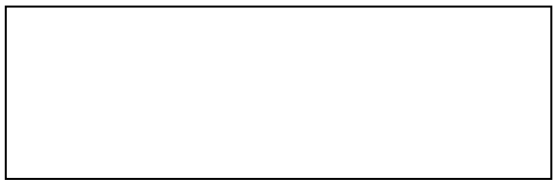
Appendix
Teacher Dialogues Example

Marcus is going to be absent from school for a few days, and Lin has agreed to teach his math lessons for him while he is away. The class will be starting its study of fractions, and Marcus has chosen the task below to open the unit.

> Eighteen students want to make a class flag by creating an equal-sized part for each student to decorate. Below is a picture of the sheet. Help the students create equal-sized parts to decorate. What mathematical name would you give to one person's part?
>
> 
>
> Jewell and Maria decided to combine their parts. What mathematical name would you call their combined share?

Below is a conversation they had as Marcus was sharing his plans with Lin:

Marcus: Thanks for helping, Lin. We are starting our unit on fractions, and I do not want the students to get behind. Here is the task for the first day.

Lin: Marcus, you have the low class. This task is way too hard for most third graders, especially the low-level kids. There is no way your students can work on this.

Marcus: I don't know, Lin – every time I give them task that I think they cannot do, they surprise me with what they come up with.

Lin: I just don't want them to not experience any success on their first day with me.

Discussion Questions:

1. What does each teacher think is the role of student thinking in mathematics instruction?
2. What do you see as the pros and cons of each teacher's perspective on math teaching?
3. In what ways can an understanding of learning trajectories support these teachers?

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