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# PARAMETRIC WELL–POSEDNESS FOR HEMIVARIATIONAL–LIKE INEQUALITIES

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**Abstract.** In this paper, the concept of well-posedness for hemivariational– like inequalities is generalized, one metric characterization of the well–posed hemivariational–like inequalities is established and some conditions under which the parametric well–posedness for the family hemivariational–like inequalities is equivalent to the existence and uniqueness of solution are obtained.

**Key words***:* Hemivariational–like inequality, Clarke's generalized gradient, Well– posedness, Invariant monotonicity.

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## 1. Introduction

Let $X$ be a topological space and $V$ be a reflexive Banach space with dual $V^*$. We denote the duality between $V$ and $V^*$ by $<\cdot,\cdot>$, the norm of $X$ by $\|.\|_X$ and the norm of Banach space $V$ by $\|.\|$. In this paper, we suppose that $A : X \times V \to V^*$ is a single-valued operator from $X \times V$ to $V^*$, $J^o(\cdot,\cdot)$ is the Clarke's directional derivative of the locally Lipschitz functional $J : V \to \mathscr{R}$, $\eta : V \times V \to V$ and $f \in V^*$ is some given element in $V^*$. Let $K$ is a nonempty closed subset of $V$. Consider the following parametric hemivariational–like inequality associated $(A, f, J)$:

$HVLI(A, f, J)_X$**:** find $u \in K$ such that

$$<A(x,u) - f, \eta(v,u)> + J^o(u, \eta(v,u)) \geq 0,$$

$$\forall v \in K.$$

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Here, $x \in X$ and $J^o(u, \eta(v, u))$ denotes the generalized directional derivative [2] of the function $J(\cdot)$ at $u$ in the direction $\eta(v, u)$. When the operator $A$ does not depend on the parameter $x$ and $\eta(v, u) = v - u$, $HVLI(A, f, J)_X$ reduces to $HVI(A, f, J)$ in [21]. Recently, Xiao et al. [22] investigated conditions of well–posedness for hemivariational inequality $HVI(A, f, J)$ in reflexive Banach spaces.

Well–posedness is an important notion which plays a crucial role in the theory of optimization problems. The classical concept of well–posedness for a global minimization problem was first introduced by Tykhonov [17]. The Tykhonov well–posedness requires the existence and uniqueness of solution and the convergence of every minimizing sequence towards the unique solution. Since then, various kinds of well-posedness for optimization problems were introduced and studied by many researchers. (see, for example, [3, 4, 5, 6, 9, 10, 12, 14, 24]). Recently, the concept of well–posedness has been generalized to other contexts such as variational inequality problems, fixed point problems and inclusion problems.(see, for example, [11, 21]).

It is well known that variational inequalities are very closely related to an optimization problems and provide general mathematical models for a wide range of problems. The variational inequality theory was presented by Stam- pacchia [16]. Hartman and Stampacchia [8], by using variational inequali- ties, studied differential equations with applications in mechanics. In recent years, many researchers focus on the introduction of various kind of well– posedness for variational inequalities, the necessary and sufficient conditions of well-posedness and metric characterizations of well–posedness for variational inequalities and optimization problems with constraints defined by variational inequalities. (see, for example, [5, 11, 22]).

The concept of hemivariational inequality is an useful and important gener- alization of variational inequality which was first introduced by Panagiotopou- los [15]. He investigated hemivariational inequalities using the generalized gradient of Clarke for nondifferentiable and nonconvex functions. Unfortu- nately, compared with variational inequalities, the study of hemivariational inequalities



is very limited. For the hemivariational inequality theory and its applications, one can refer to [1, 7, 13, 18, 19, 20].

Inspired by the work of Lignola and Morgan [11] and Xiao et al. [22], in this paper, we investigate well–posedness of a family of hemivriational–like inequalities. The paper is organized as follows. In Section 2, some useful def- initions and results are recalled. In Section 3, we first obtain an equivalence result between The hemivariational–like inequality $HVLI(A, f, J)_X$ and an in- clusion problem. We also generalize the concept of well–posedness for a family of hemivriational–like inequalities and give one metric characterization of the well–posed hemivariational–like inequalities. Finally, We obtain conditions under which the parametric well–posedness for the family hemivariational–like inequalities is equivalent to the existence and uniqueness of solution.

## 2.Notations and Preliminaries

In this section, we recall some useful concepts and results in nonlinear analysis and nonsmooth analysis (see, for example [2]). Let $J : V \to \mathrm{R}$ be a locally Lipschitz functional on Banach space $V$.

The Clarke's generalized directional derivative of $J$ at $u \in V$ in the direction of a given $v \in V$, denote by $J^o(u,v)$ is defined by

$$J^o(u,v) := \limsup_{\substack{w \to u \\ \lambda \downarrow 0}} \frac{J(w+\lambda v) - J(w)}{\lambda}.$$

The Clarke's generalized gradient of $J$ at $u$, denoted by $\partial_C J(u)$, is defined by

$$\partial_C J(u) := \{\omega \in V^* : <\omega, v> \leq J^o(u,v), \ \forall v \in V\}.$$

The following proposition provides some properties for the Clarke's generalized gradient and the Clarke's generalized directional derivative.

**Proposition 2.1.** [2] *Let $V$ be a Banach space, $u,v \in V$ and $J : V \to \mathcal{R}$ a locally Lipschitz functional defined on $V$. Then*

(1) *the function $v \mapsto J^o(u,v)$ is finite, positively homogeneous, subadditive*



*and then convex on $V$,*

(2) *$J^o(u,v)$ is upper semicontinuous on $V \times V$ as a function of $(u,v)$, i.e., for all $u,v \in V$, $\{u_n\} \subset V$, $\{v_n\} \subset V$ such that $u_n \to u$ and $v_n \to v$ in $V$, we have that*

$$\limsup J^o(u_n,v_n) \leq J^o(u,v),$$

(3) $J^o(u,-v) = (-J)^o(u,v)$,

(4) *for all $u \in V$, $\partial_C J(u)$ is a nonempty, convex, bounded and weak$^*$- compact subset of $V^*$,*

(5) *for every $v \in V$, one has*

$J^o(u,v) = \max\{(\xi,v) : \xi \in \partial_C J(u)\}$,

(6) *the graph of the Clarke's generalized gradient $\partial_C J(u)$ is closed in $V \times (w^* - V^*)$ topology, where $(w^* - V^*)$ denotes the space $V^*$ equipped with weak$^*$ topology, i.e., if $\{u_n\} \subset V$ and $\{u_n{}^*\} \subset V^*$ are sequences such that $u_n{}^* \in \partial_C J(u_n)$, $u_n \to u$ in $V$ and $u_n{}^* \to u^*$ weakly$^*$ in $V^*$, then*

$u^* \in \partial_C J(u)$.

**Definition 2.2.** Let $\eta : V \times V \to V$. A subset $K$ of $V$ is said to be invex with respect to $\eta$ if, for any $u,v \in K$ and $\lambda \in [0,1]$, $u + \lambda\eta(v,u) \in K$.

**Condition C:** Let $\eta : V \times V \to V$. Then, for any $u,v \in V$ and $\lambda \in [0,1]$

$\eta(v, v+\lambda\eta(u,v)) = -\lambda\eta(u,v)$,



$$\eta(u, v+\lambda\eta(u,v)) = (1-\lambda)\eta(u,v).$$

**Definition 2.4.** Let $A(x,.) : V \to V^*$ be a set-valued mapping, for all $x \in X$. $A(x,\cdot)$ is said to be invariant monotone with respect to $\eta$, if for any $u,v \in V$, $\xi \in A(x,u)$ and $\gamma \in A(x,v)$, one has

$$<\xi, \eta(v,u)> + <\gamma, \eta(u,v)> \leq 0.$$

**Definition 2.5.** Let $V$ be a Banach space with its dual $V^*$, $x$ be an arbitrary element in $X$ and $A(x,\cdot) : V \to V^*$ an operator from $V$ to $V^*$. $A(x,\cdot)$ is said to be hemicontinuous if, for any $u,v \in V$, the function

$$\lambda \mapsto <A(x, u+\lambda v), v>$$

from $[0,1]$ into $(-\infty,\infty)$ is continuous at $0+$.

## 3. PARAMETRIC WELL-POSEDNESS FOR HEMIVARIATIONAL-LIKE INEQUALITIS

In this section, we present an equivalence result between the hemivariational– like inequality $HVLI(A,f,J)_X$ and an inclusion problem. We also gener- alize notion well–posedness for a family of hemivariational–like inequalities and obtain one metric characterization of well–posedness for this family of hemivariational–like inequalities. Finally, we give necessary and sufficient con- ditions for well–posedness of this family.

**Condition H:** Let $\eta : V \times V \to V$ be a function and $K$ be an invex set with

respect to $\eta$. Then for any $u \in K$, we have

$$\forall w \in V, \exists t > 0, v \in K; \quad tw = \eta(v,u).$$

The following Lemma generalized Lemma 3.1 in [22] for the hemivariational–

like inequalities. Suppose that $K$ be a nonempty, closed and invex subset of $V$.

**Lemma 3.1.** *Let $\eta$ satisfies Condition H and $x \in X$ is arbitrary element in X. Then, $u_X \in V$ is a solution to the hemivariational–like inequality $HVLI(A, f, J)_X$ if and only if $u_X$ is a solution to the following inclusion prob- lem:*

*$IP(A,f,J)_X$: find $u_X \in K$ such that $f \in A(x,u_X) + \partial_C J(u_X)$.*

*Proof.* Let $x \in X$ and $u_X \in V$ be a solution to the hemivariational–like in- equality $HVLI(A, f, J)_X$, which means

$$<A(x,u_X) - f, \eta(v,u_X)> + J^o(u_X,\eta(v,u_X)) \geq 0, \quad \forall v \in K. \qquad (3.1)$$

Since $\eta$ satisfies Condition H, for any $w \in V$ there exist $t > 0$ and $v \in K$ such

that $tw = \eta(v,u_X)$. On the other hand, $J^o(.,.)$ is positively homogeneous with respect to the second argument. Thus, it follows from (3.1) that

$$< A(x,u_X) - f, \frac{1}{t}\eta(v,u_X) > + J^o(u_X,\frac{1}{t}\eta(v, u_X)) \geq 0, \quad \forall v \in K.$$

Hence,

$$< A(x,u_X) - f, w > + J^o(u_X, w) \geq 0, \quad \forall w \in V.$$

which implies that $f - A(x,u_X) \in \partial_C J(u_X)$.

Now, let $u_X \in V$ be a solution to the inclusion problem $IP(A,f,J)_X$. Therefore, there exists $\xi \in \partial_C J(u_X)$ such that $f = A(x,u_X) + \xi$. Hence,

$$<f - A(x,u_X), \eta(v,u_X)> = <\xi, \eta(v,u_X)>$$

$$\leq J^o(u_X,\eta(v,u_X)),$$

$$\forall v \in K,$$

which implies that $u_X$ is a solution to the $HVLI(A,f,J)_X$. This completes the proof.

PARAMETRIC WELL–POSEDNESS FOR HEMIVARIATIONAL–LIKE INEQUALITIES

**Example 3.2.** Let $X = V = K = \mathscr{R}$, $\eta(u,v) = \frac{u^3 - v^3}{3}$, $A(x,u) = ux$, $f \equiv 0$, and consider the locally Lipschitz function $J$ defined by $J(u) = |u|$.It can be verified that $\partial_C J(0) = [-1, 1]$. Thus, $f \equiv 0 \in A(x,0) + \partial_C J(0)$, for all $x \in X$. It implies that $u = 0$ solves $IP(A,f,J)_X$, for all $x \in X$. Also, it can be seen that $\eta$ satisfies Condition $H$. Therefore, $u = 0$ solves $HVLI(A,f,J)_X$, for all $x \in X$.

Now, let us consider the family

$$(HVLI(A,f,J)) := \{HVLI(A,f,J)_X : x \in X\}.$$

**Definition 3.3.** Let $x \in X$ and $\{x_n\}$ be a sequence converging to $x$. A sequence $\{u_n\}$ is said to be an approximating sequence with respect to $\{x_n\}$ for the parametric hemivariational–lik inequality $HVLI(A, f, J)_X$, if $u_n \in K$ for any $n \in$ N and there exists a positive sequence $\{\varepsilon_n\}$ with $\varepsilon_n \to 0$ as $n \to \infty$ such that

$$<A(x_n,u_n) - f, \eta(v,u_n)) + J^o(u_n,\eta(v,u_n)> \geq -\varepsilon_n \|\eta(v,u_n)\|,$$

$$\forall v \in K.$$

Now, we present concept of well–posedness for the family hemivariational– like inequality $(HVLI(A,f,J))$.

**Definition 3.4.** The family hemivariational-like inequalities $(HVLI(A, f, J))$ is said to be parametrically well–posed if

(1) there exists a unique solution $\overline{u}_X$ to $HVLI(A,f,J)_X$, for all $x \in X$.

(2) for all $x \in X$ and for all $\{x_n\}$ converging to $x$, every approximating



sequence for the problem $HVLI(A,f,J)_X$ with respect to $\{x_n\}$strongly converges to $\overline{u}_X$.

For any $x \in X$ and $\varepsilon > 0$, we define the following two sets:

$$\Omega(x,\varepsilon) := \{u \in K : <A(x,u) - f, \eta(v,u)> + J^o(u,\eta(\square u)) \geq -\varepsilon \|\eta(v,u)\|, \ \forall v \in K\}.$$

and

$$\Psi(x,\varepsilon) := \{u \in K : <A(x,v) - f, \eta(v,u)> + J^o(u,\eta(v,u)) \geq -\varepsilon \|\eta(v,u)\|, \ \forall v \in K\}.$$

**Definition 3.5.** The function $\eta : V \times V \to V$ is called skew function, if

$$\eta(u,v) + \eta(v,u) = 0,$$

for all $u, v \in V$.

**Lemma 3.6.** *Let $A(x,\cdot) : V \to V^*$ be invariant monotone with respect to $\eta$ and hemicontinuous, for all $x \in X$. Suppose that $\eta$ is skew function and satisfies Condition C. Then, $\Omega(x,\varepsilon) = \Psi(x, \varepsilon)$, for all $x \in X$ and $\varepsilon > 0$.*

*Proof.* Let $u \in \Omega(x,\varepsilon)$. Since $A(x,\cdot)$ is invariant monotone with respect to $\eta$

that $\eta$ is skew function, we obtain

$<A(x,v) - f, \eta(v,u)> + J^o(u,\eta(v,u)) \geq <A(x,u) - f, \eta(v,u)> + J^o(u,\eta(v,u))$

$\geq - \varepsilon \, \|\eta(v,u)\|,$

$\forall v \in K.$

Therefore, $u \in \Psi(x,s)$.

Now, we proof that $\Psi(x,s) \subset \Omega(x,s)$. In fact, for any $u \in \Psi(x,s)$, we have



$<A(x,v) - f, \eta(v,u)> + J^o(u,\eta(v,u)) \geq - \varepsilon \, \|\eta(v,u)\|, \ \forall v \in K.$

(3.2)

Suppose that $w$ is arbitrary element in $K$ and $\lambda \in [0,1]$. Since $K$ is invex, letting $v = u + \lambda\eta(w,u) \in K$ in (3.2) yields

$<A(x,u+\lambda\eta(w,u)) - f, \eta(u+\lambda\eta(w,u),u)> + J^o(u,\eta(u + \lambda\eta(w,u),u))$

$\geq - \varepsilon \, \|\eta(u + \lambda\eta(w,u),u)\|, \ \forall w \in K.$

By Remark 2.3 and the positive homogeneousness of $J^o(u,v)$ with respect to $v$, we obtain

$$<A(x,u+\lambda\eta(w,u)) - f, \eta(w,u)> + J^o(u,\eta(w,u)) \geq - \varepsilon \, \|\eta(w,u)\|,$$

$$\forall w \in K. \tag{3.3}$$

Taking the limit $\lambda \to 0+$ in (3.3), we get from the hemicontinuity of mapping

$A(x,\cdot)$ that

$$<A(x,u) - f, \eta(w,u)> + J^o(u,\eta(w,u)) \geq - \varepsilon \, \|\eta(w,u)\|,$$

$$\forall w \in K.$$

Therefore, $u \in \Omega(x,\varepsilon)$.

**Lemma 3.7.** *Suppose that $A : X \times V \to V^*$ be hemicontinuous with respect*

*to second argument and $\eta$ be continuous with respect to the second argument. Then, $\Psi(x,\varepsilon)$ is closed in $V$, for all $x \in X$ and $\varepsilon > 0$.*

*Proof.* Let $\{u_n\} \subset \Psi(x,\varepsilon)$ and $u_n \to u$ in $V$. Then

$<A(x, v) - f, \eta(v, u_n)> + J^o(u_n, \eta(v, u_n)) \geq - \varepsilon \, \|\eta(v,u_n)\|, \ \forall v \in K.$ (3.4) Since the Clarke's generalized directional derivative $J^o(u,v)$ is upper semi- continuous with respect to $(u,v)$ and $\eta$ is continuous with respect to second



argument, taking lim sup at both sides of (3.4), we obtain

$$<A(x,v) - f, \eta(v,u)> + J^o(u,\eta(v,u)) \geq - \varepsilon \, \|\eta(v,u)\|,$$

$$\forall v \in K.$$

So, $u \in \Psi(x,\varepsilon)$.

Unfortunately, differently from strongly well–posedness in [21] and [22], parametrically well–posedness is not equivalent to the $diam\Omega(x,\varepsilon) \to 0$.

**Theorem 3.8.** If the family (HVLI(A, f, J)) is parametrically well–posed, then $\Omega(x, \varepsilon) \neq \emptyset$, for every $x \in X$ and every $\varepsilon > 0$, and $diam\Omega(x_n, \varepsilon_n) \to 0$, for all $\{x_n\}$ converging to x and all $\{\varepsilon_n\}$ converging to 0.

*Proof.* The proof is similar to the Proposition 2.3 in [11].

**Remark 3.9.** By some modifications in the proof of Proposition 2.3 (bis) in [11], we can obtain the following stronger result. Let $A$ does not depend on the parameter $x$ and $A$ be hemicontinuous and invariant monotone with respect to $\eta$. Suppose that $\eta$ is continuous with respect to the second argument and satisfying Condition $C$. Then, $HVLI(A,f,J)$ is strongly well-posed(in the

sense of [21]) if and only if $\Omega(\varepsilon) \neq \emptyset$, for all $\varepsilon > 0$ and $diam\Omega(\varepsilon_n) \to 0$ as

$\varepsilon_n \to 0$.

**Condition D:** Let $A : X \times V \to V^*$ and $u$ be an arbitrary element in $K$.

Then

$<A(x,u) - A(y,u), \eta(v,u)> \leq \|x - y\|_X \, \|\eta(v,u)\|, \ \forall x, y \in X, \forall v \in K.$

For example, let $X = V = K = \mathscr{R}$, $\eta(u,v) = u - v$ and $A(x,u) = x - u$, for

all $x \in X$ and $u \in V$. Then $A$ satisfies Condition $D$.

PARAMETRIC WELL–POSEDNESS FOR HEMIVARIATIONAL–LIKE INEQUALITIES

**Theorem 3.10.** *Let $A(x,\cdot)$ be hemicontinuous and invariant monotone with respect to $\eta$, for all $x \in X$. Suppose that $\eta$ is continuous with respect to the second argument and $\eta$ satisfies Condition C. If operator $A : X \times V \to V^*$ satisfies Condition D, then the family $(HVLI(A, f, J))$ is parametrically well- posed if and only if $\Omega(x,\varepsilon) \neq \emptyset$, for every $x \in X$ and every $\varepsilon > 0$, and $diam\Omega(x_n, \varepsilon_n) \to 0$ as $x_n \to x$ and $\varepsilon_n \to 0$.*

*Proof.* Obviously, the necessity follows immediately from Theorem 3.8. It re- mains to prove the sufficiency. Let $x$ be an arbitrary element in $X$ and $\{x_n\}$ be a sequence converging to $x$. Assume that $\{u_n\}$ be an approximating sequence for $HVLI(A, f, J)_X$(w.r. to $\{x_n\}$). Then, there exists a positive sequence $\varepsilon_n \to 0$ such that

$$<A(x_n, u_n) - f, \eta(v,u_n)> + J^o(u_n; \eta(v,u_n)) \geq -\varepsilon_n \|\eta(v,u_n)\|, \quad \forall v \in K \tag{3.5}$$

which implies that $u_n \in \Omega(x_n, \varepsilon_n)$. It follows from $\lim_{n\to\infty} diam\Omega(x_n,\varepsilon_n) = 0$ that $\{u_n\}$ is a Cauchy sequence and so $\{u_n\}$ converges strongly to some point $u_X \in K$. Since Clarke's generalized directional derivative $J^o$ $(u, v)$ is upper semicontinuous with respect to $(u,v)$, $A$ satisfies Condition D and the mapping $A(x,\cdot)$ is invariant monotone with respect to $\eta$ that $\eta$ is continuous with respect to the second argument, the inequality (3.5) implies that

$$<A(x,v) - f, \eta(v,u_X)> + J^o\ (u_X, \eta(v,u_X))$$

$$\geq \limsup <A(x,v) - f, \eta(v,u_n)> + J^o\ (u_n, \eta(v,u_n))$$



$$\geq \limsup <A(x,u_n) - f, \eta(v,u_n)> + J^o\ (u_n, \eta(v,u_n))$$

$$\geq \limsup <A(x_n,u_n) - f, \eta(v,u_n)> + J^o(u_n, \eta(v,u_n))$$

$$- \|x_n - y\|_X \|\eta(v,u_n)\|$$

$$\geq \lim \sup(-\varepsilon_n - \|x_n - y\|_X \|\eta(v,u_n)\| = 0, \quad \forall v \in K.$$

Similar to the proof of Lemma 3.6, for any $w \in K$ and $\lambda \in [0,1]$, letting $v = u_X + \lambda\eta(w,u_X)$ in last inequality, we obtain

$$<A(x,u_X) - f, \eta(w,u_X)> + J^o(u_X, \eta(w,u_X)) \geq 0,$$

$$\forall w \in K.$$

So, $u_X$ solves $HVLI(A,f,J)_X$.

Now, assume that $HVLI(A, f, J)_X$ has two distinct solution $u_X, v_X \in V$. Then $u_X$,

$v_X \in \Omega(x,\varepsilon)$, for all $\varepsilon > 0$. Since $0 \leq \|u_X - v_X\| \leq diam\Omega(x,\varepsilon) \to 0$,

Therefore $u_X = v_X$.

**Remark 3.11.** The Theorem 3.10, improves Proposition 2.3 in [11].

In the following theorem, we obtain classes of families parametrically well- posed in the finite dimensional cases. The following theorem improves Propo- sition 2.8 in [11] for invex case.

**Theorem 3.12.** *Let $V$ be a finite dimensional space and $K$ be a bounded invex closed subset of $V$. Suppose that $A$ be an operator on $X \times V$ such that $A(x,\cdot)$ is hemicontinuous and invariant monotone with respect to $\eta$, for all $x \in X$, and $A$ satisfies Condition D. If $\eta$ be continuous with respect to the second argument and satisfying Condition C, then the family $(HVLI(A,f,J))$ is parametrically well–posed if and only if $HVLI(A,f,J)_X$ has a unique solution on $V$, for all $x \in X$.*

*Proof.* The necessity follows immediately from definition of the parametrically well–posedness for the $(HVLI(A, f, J))$. Now, assume that $HVLI(A,f,J)_x$ has a unique solution $u_x$, for all $x \in X$. If the family $(HVLI(A,f,J))$ is not parametrically well–posed, then there exist $x \in X$, a sequence $\{x_n\} \subset X$ such that $x_n \to x$ and an approximating sequence $\{u_n\}$ (w.r. to $\{x_n\}$) for $HVLI(A, f, J)_x$ which does not converge to $u_x$. Hence, there exists a positive sequence $\{\varepsilon_n\}$ such that $\varepsilon_n \to 0$ and

$$\langle A(x_n, u_n) - f, \eta(v, u_n)\rangle + J^o(u_n, \eta(v, u_n)) \geq -\varepsilon_n \|\eta(v, u_n)\|,$$

$\forall v \in K$.

On the other hand, $u_n \in K$. Therefore, $\{u_n\}$ is bounded and, for some subse- quence, $\{u_n\}$ converges to a point $\bar{u}_x$. Now, by a similar argument as that in the proof of Theorem 3.10, we can deduce that $\bar{u}_x$ solves the hemivariational–like inequality $HVLI(A,f,J)x$. Hence, $\bar{u}_x = u_x$ and this is a contradiction. □

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