Teacher awareness of problematic facets of meaningful metaphors of energy

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(Received February 3, 2017, accepted March 10, 2017)

Abstract

How teachers respond to students depends, in part, on what they see in their students' thinking. In a teacher professional development setting, we asked teachers to provide possible incorrect responses and explanations that students might give when discussing the gravitational potential energy of identical hikers walking to the summit of a mountain along different paths, from the same starting point. Teachers were aware of the common difficulties that students might have, including (1) energy is "used up" because of travel time, travel distance, or the effort exerted during travel (2) double-counting work and energy, and (3) energy being an intrinsic property of the hiker. Several of these difficulties use the metaphor of energy as a substance-like quantity, but teachers never made explicit that they were aware of the value of this metaphor in thinking about energy. We discuss the need for teachers to respond to multiple grain sizes of student thinking, including the metaphors they use and the different and at times problematic facets of each.

Keywords: Teacher training, Alternative conceptions, Gravity.

Resumen

La manera en que los maestros responden a los alumnos depende, en parte, de lo que ven en el pensamiento de los estudiantes. En un curso de capacitación, le pedimos a maestros que proporcionaran la posible respuesta incorrecta y la explicación de qué explicaciones podrían dar al analizar la energía gravitacional potencial de unos excursionistas idénticos caminando hacia la cumbre de una montaña por diferentes veredas, iniciando desde el mismo punto. Los maestros reconocían las dificultades comunes que los estudiantes podrían tener, incluyendo (1) la energía es "usada" en el tiempo viajado, distancia recorrida, o el esfuerzo requerido durante el viaje, (2) contar doblemente el trabajo y la energía, y (3) considerar la energía como una propiedad intrínseca del excursionista. Muchas de esas dificultades utilizan la metáfora de la energía como una cantidad del tipo sustancia, pero los maestros nunca hicieron explícito que ellos estaban al tanto del valor de dicha metáfora la pensar en energía. Discutimos la necesidad de los maestros a responder a las múltiples maneras de pensar de los estudiantes, incluyendo metáforas que usan así como las facetas que pueden ser problemáticas en ocasiones..

Palabras clave: Capacitación de maestros, Concepciones alternativas, Gravedad.

PACS: 01.40.Fk, 01.40.gb, 01.40.J- **ISSN 1870-9095**

I. INTRODUCTION

One of the roles of teachers is to notice and respond to the ideas their students into the classroom. To listen well and understand what students are saying requires more than a knowledge of the content alone. It requires understanding how students are thinking and making sense of their ideas before responding. In this paper, we discuss an investigation of teacher knowledge of student ideas and how it reveals some of the tensions in understanding the knowledge that teachers need for teaching.

This work takes place within the context of a reform movement in the United States. New standards for student learning, the Next Generation Science Standards (NGSS) [1], have been created to guide instruction. The NGSS and its parent document, *A K-12 Framework for Science Education: Practices, Crosscutting Concepts, and Core Ideas* [2], describe the scientific practices to be learned, the crosscutting concepts of science education, and the disciplinary core ideas which all students should know. Energy is one of four disciplinary core ideas in the physical sciences, showing its importance to learning in physics.

Further, the conservation and flow of energy is one of the crosscutting concepts of the NGSS, showing its importance across all disciplines.

For teachers to quickly assess what students know about a given topic, like energy, it is often easy to ask a multiple choice question that allows one to quickly see the distribution of student ideas in the classroom. This technique is common in the use of classroom response systems, such as clickers [3,4]. What an instructor learns from such a question depends both on how good the question is and whether the teacher is able to interpret the results meaningfully.

In this paper, we describe how teachers responded to one multiple-choice question about gravitational potential energy. We discuss their predictions of what students might say, in particular their awareness of the common difficulties that students have with energy in this context. We also discuss that a highly useful metaphor for understanding energy was not explicitly stated by the teachers, even as the teachers described some of the problematic applications of this idea to the context of gravitational potential energy. This raises the concern that teachers would teach in a way that fundamentally damages student use of the metaphor, rather than clarify which applications are appropriate and which not.

II. MODELS OF TEACHER KNOWLEDGE

To begin our discussion, we look at models of teacher knowledge and a discussion of the results of the physics education research community concerning understanding of energy.

There is an extensive literature on science teachers' professional knowledge. We use a Pedagogical Content Knowledge (PCK) framework [5–7] to model teacher knowledge. To simplify this very broad topic to our specific area of interest, we attend to two aspects of teacher knowledge, using language provided by Ball and collaborators [8,9]. The first is Subject Matter Knowledge (SMK), which includes the basic knowledge of content being taught. The education research community has provided thousands of examples of research into understanding specific content areas, as summarized by Duit [10], with ample evidence showing that teachers and students often have the same difficulties with the material.

The second is Knowledge of Content and Students (KCS), one aspect of PCK that supports teachers in recognizing both common student difficulties and the productive resources that students bring to their learning or develop as they progress through instruction [8,11–14]. Work by others has investigated ideas similar to KCS. Sadler and collaborators demonstrated a statistical correlation between teacher Knowledge of Student Misconceptions and student performance on questions containing these misconceptions [15]. Wittmann, Thompson, and Christensen analyzed pre-service teachers' Knowledge of Student Ideas in the context of learning and using the results of discipline-based education research [16,17].

Knowledge of the material and how students interact with it are not sufficient for explaining the tasks given to teachers, though. Teachers must also respond to the ideas their students bring into the classroom and make pedagogical choices that affect the choice of curriculum, methods of instruction, and kinds of assessment. We mention only a few examples around this topic. Sherin and others have studied teacher knowledge and how it affects what teachers notice in their students' ideas [18]. Hammer and collaborators have extended this work to investigate how teachers respond to their students, and what this tells us about teachers' own goals and knowledge [19]. Coffey and collaborators have looked at assessment, calling for greater attention to the disciplinary substance of a field rather than a simple use of terms that may or may not be well understood [20]. Researchers involved in Cognitively Guided Instruction have used professional development activities to improve teacher knowledge of student difficulties with topics in mathematics, leading to substantial improvements in student learning [21–24].

There is a long history of studying student and teacher knowledge of topics in energy. We take a constructivist perspective in which ideas are context-dependent [25], experts use many different (and at times contradictory) models and metaphors of energy in their own work [26,27], and students' intuitive ideas are the building blocks for constructivist activities in the classroom [25,28–33]. We apply a knowledge-in-pieces model of reasoning [31], though other, similar descriptions of knowledge exist, including facets of knowledge [34,35], mathematical forms [36], intuitive rules [37], and reasoning resources [32]. In this paper, we will be discussing the difference facets of a particular metaphor of energy, for example.

There is an extensive literature on student thinking about energy. Commonly reported difficulties in energy instruction include failure to distinguish work from energy [38,39] and heat from thermal energy [40–43]. Much progress on the teaching and learning of energy was made in the 80s [44,45] and, consistent with its era, focused on alternative frameworks and misconceptions [46,47]. Subsequent curriculum development has, for example, emphasized learning progressions [48–52] for teaching energy. As is clear from our discussion above, we disagree with certain elements of this work.

There has been extensive discussion around the metaphor of energy as a substance-like quantity, a concept that plays a central role in this paper. The idea has been criticized and deemed inappropriate [53–56] but has also been found to be historically useful [27] and used by experts in their everyday activity [26] while being common [57] and productive [58] in learning. Further, the energy-as-a-substance metaphor is consistent with the idea of energy as conserved, localized, transferring among objects, and transforming among forms [59,60]. These features constitute a powerful conceptual model of energy that may be used to explain and predict energy phenomena [45,58,61,62]. But, when explaining and predicting student reasoning about energy phenomena, we are aware that there may be different facets [34] of this metaphor.

To summarize, there are multiple models of knowledge. Some describe the many ways in which teacher knowledge plays out in the classroom, including teacher SMK and KCS. Some are more general and look at how people make sense of the world, including knowledge-in-pieces or metaphors. In this paper, we will study the context-dependence of reasoning by looking at different facets of useful metaphors for understanding energy. For a teacher, this attention to the cognitive models plays a role in both SMK and KCS. For teachers to think about the content, they must understand the power of the productive metaphors of energy. For teachers to think about their students, they must understand the facets of those metaphors, and how they might be misapplied in a given setting. In this paper, we discuss an assessment question that contained useful metaphors like the energy-as-a-substance metaphor that could be applied in multiple and sometimes problematic ways.

III. DATA COLLECTION

This work took place with taking place within the Maine Physical Sciences Partnership (MainePSP), a coalition of the University of Maine, nearly 30 school districts and nearly 50 schools, the Maine State Department of Education, and several non-profit organizations (first and foremost, the Schoodic Education and Research Center) that helps teachers and students in grades 6 to 9 in the State of Maine improve the teaching and learning of the physical sciences. Since 2010, the MainePSP has affected the science learning of over 7,000 students in Maine.

As part of a larger investigation of student and teacher understanding of energy, we developed a survey on energy [63]. The multiple-choice questions on the survey were taken from the AAAS website [64]. During a process of survey revision, we added a question about three hikers of the same weight and height walking three different paths from the same starting point to the top of a mountain [65] (see Figure 1). The path taken by Hiker 1 was the shortest, going up the steepest path, while Hikers 2 and 3 wandered a bit up and down hills and through valleys before also reaching the top (see Figure 1). In multiple-choice format, students were asked, "which hiker will have the greatest amount of gravitational potential energy?" - Hiker 1, Hiker 2, Hiker 3, or "The gravitational potential energy is the same for all of the hikers" (the correct response). A correct answer assumed that the energy of the hiker was a stand-in for the potential energy in the hiker-Earth system. Among the teachers, there was no discussion of the nuance of how potential energy can exist without considering the system of Earth and hiker.

As part of a professional development activity with in-service teachers, we asked this question to have a brief conversation about student difficulties with energy. Other events from a different activity that took place that evening have been described previously [66].

We asked teachers why a student might not choose the correct answer to the question (that the hikers have the same gravitational potential energy at the top) and instead might choose hikers 1, 2, or 3, in particular asking, "What might be their reasoning for choosing one of those answers?" We recorded the conversation that followed, using audio and video. A total of 25 teachers participated in this discussion. It was often hard to identify speakers, due to crosstalk, preventing us from understanding teacher talk while they were in smaller groups. As a result, we were only able to use the elements of discussion that involved the large group setting. Further, the nature of our analysis does not require knowing individual teacher names. As a result, we do not name the teachers, and instead refer to them simple as "a teacher."

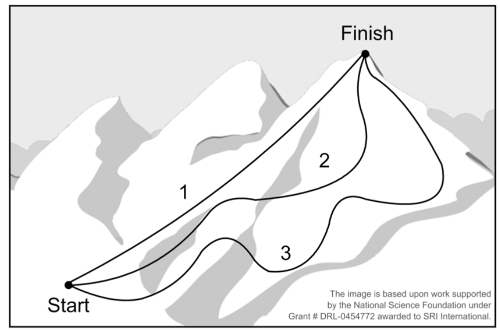

FIGURE 1. Graphic provided for the Hiker Question [63].

IV. TEACHERS PROVIDE MULTIPLE EXPLANATIONS FOR INCORRECT ANSWERS

Teachers provided several explanations for why Hiker 1, 2, or 3 might be chosen as having the highest GPE. Of these, six included reasoning about the physics. One, that a "lazy student" unwilling to read very far would pick Hiker 1 because it was listed first, will not be discussed further. Similarly, we will not discuss the suggested answer that "Hiker 1 would get there first," because it does not concern specific thinking about energy. The energy version of this response is discussed below. We summarize their responses and our interpretations in Table I.

For each teacher response, we analyze what they say to see how they describe student reasoning. This includes paying attention to their metaphors of energy and the actions of the hiker.

A. Double-counting work and energy

In explaining why a student would choose Hiker 1, one teacher stated, "Because [Hiker 1] worked the hardest to get up the mountain." For this teacher, a student might think in terms of the amount of effort exerted by Hiker 1. In colloquial language, it is hard work to climb a steep path. In a separate, commonly used definition of energy, energy is the ability to do work. A student could easily reverse the logic of that sentence: energy is created by doing work. Because this hiker is working the hardest to get up the mountain, more energy is created, and the student would have double-counted the work done and the gravitational potential energy one has at the top of the mountain.

The idea that more energy is created by working harder is contrary to the models of energy being used up, described below, but it is consistent with past work by Lindsey, Heron, and Shaffer [38,39]. In contrast, in Lancor [57,67,68] and Brewe's [58] research, we do not see this metaphor in student thinking. Scherr et al. [59] have described the metaphor of energy-as-a-stimulus, but in this case, we do not believe the ideas are connected.

B. Students believe energy is an inherent trait of the hiker

To explain why a student might say Hiker 1 had the greatest GPE at the top, a different teacher stated, "Because they chose the harder path, they're someone who is in better shape."

For this teacher, a possible explanation comes from thinking about energy as a trait of the hiker. The person who could take the harder path would have to be in better shape, so they must intrinsically have more energy than another hiker, who takes the easier path (like, say, Hiker 3).

This answer is perhaps related to the "energy is a life force" explanation described in [57]. In the teacher's statement of a possible student idea, it is unclear whether the energy changes as the hiker moves up the mountain. Instead, the response suggests that energy is an intrinsic property of the hiker, independent of the situation the hiker is in. Again, this is unlike the stimulus ontology described by Scherr et al. [59], because the energy is not a stimulus to action, but is an inherent trait of the hiker instead.

C. Students believe energy is used up

Given the prevalence of the "Used up" reasoning in the literature (we cite [44,45,57,58,67,68], but the broader literature on student ideas contains many examples of this explanation), we expected to find two different ways in which Hiker 1 might use up energy on the way to the top, related to distance traveled and time traveled. Teachers arrived at a third that included a place-based explanation, as well. In each of the three cases, the proposed model clearly makes use of the energy-as-a-substance metaphor [27,57,58,67,68]. In each of the three cases, the hiker has a quantity of energy, and some is left over at the end of the hike. The differences between the suggested student answers lies in how the energy was used up. In section IV.D, we discuss an extension of this thinking, where energy could be replenished, as well. In section V, we discuss in more detail why these particular responses are a problem for teachers.

1. Energy is used up over time

A teacher stated, "Maybe [Hiker 1] got there first, most direct path, they got there first, so they had more energy left over."

For this teacher, the model a student might use includes the idea that there is a finite amount of energy in a person at the start of a trip, and that energy gets used up while walking. Because the path is the most direct of paths and Hiker 1 gets to the top first (while perhaps assuming that the 3 hikers move at the same speed), less energy would get used up along the way. The hiker would be left with more than the others.

2. Energy is used up over a distance

Another teacher said, "they would just see shorter path, not steeper path, so they would say 'oh, it took less energy to get there'."

Like in the previous explanation, there seems to be an assumption of a fixed amount of energy at the start of the trip. When going on the shorter path (while explicitly disregarding the steepness of the path), the hiker would use up less energy to get there. We note that we are interpreting "took less" to mean "used up less," as opposed to assuming that "took" means "gathered from someplace else." Because of the short path (in terms of distance, not in terms of time traveled), the hiker would be left with more than the others.

3. Energy is used up on steep paths

Where the previous teacher had explicitly discounted steepness, a third teacher said, "[the] guy used his all up, going up that steep path."

For this teacher, it seems that a student might think that it takes more energy to go up a steep path. As opposed to saying that this person is in better shape (and has more energy), as stated in IV.B, or that the steepness would lead to harder work (and higher energy at the end, as in IV.A), going up the steep path would "use up" all the energy. For this explanation, Hiker 3 would have the greatest energy at the top, since that path is, on average, the least steep.

D. Students may show complex combinations of ideas

Teachers did not just give answers that required one kind of thinking to arrive at the incorrect response. At one point, a teacher elaborated on previous ideas about steepness and path length. This teacher referred to different paths that go up a local mountain, which we will call The Mountain, which is famous throughout Maine and was known by the teachers. One path is very steep, and we will call it Steep Path. The other has areas where there are flat sections, and we will call Resting Path. The teacher stated, "They could also be thinking of their hiking experience … like, thinking of [The Mountain], … Steep Path is straight up [group laughter], [but] if I go Resting Path, I've got the plateau to rest..." The laughter of the teachers indicated, to us, a familiarity with what the teacher was saying, indicating that the teachers recognized the hike and the path.

We suggest that the reason someone taking Resting Path (akin to Hiker 2 or 3, compared to Hiker 1) would have more energy is because they had a chance to rest after having lost energy during the hike. We again see evidence of the energy-as-a-substance metaphor [27,57,58,67,68]. When energy is replenished by resting, it suggests a connection to the "trait of a hiker" explanation (IV.B). People get tired as they use up energy, but return to their baseline energy, given a chance.

E. Summary of teacher responses

In these examples of teachers suggesting answers that students would give to explain an incorrect answer, we observe that teachers are aware of the common difficulties that students might have with the material. First, they are aware of students possibly saying that energy is "used up" because of travel time, travel distance, or the effort exerted during travel. Second, they are aware that students might double-count work and energy, implying that more work leads to more energy. Finally, they are aware that students might believe that energy is an intrinsic property of the hiker. These are all reasonable difficulties for students to have and are aligned with the research literature and it was notable that the entire discussion took place in only a few minutes. Teachers were confident about their responses and not in disagreement.

TABLE I. Summary of teacher statements about possible student incorrect responses, and our interpretation of these in terms of correct and incorrect ideas about energy.

Teacher Statement of a student response	Interpretation of reasoning needed for the "student answer"	Metaphors used in response
"Because [Hiker 1] worked the hardest to get up the mountain."	Hard work leads to increased energy – a possible double-counting of work and potential energy.	Energy can be created.
"Because they chose the harder path, they're someone who is in better shape."	Energy is an intrinsic trait of the hiker, regardless of activity.	Energy is constant.
"Maybe [Hiker 1] got there first, most direct path, they got there first, so they had more energy left over."	Energy is used up, and the less time you spend hiking, the less energy is used up.	Energy is used up.
"They would just see shorter path, not steeper path, so they would say 'oh, it took less energy to get there'."	Energy is used up, and the less distance you travel, the less energy is used up.	Energy is used up.
"[The] guy used his all up, going up that steep path."	Energy is used up, and the steeper your path, the more energy is used up.	Energy is used up.
"They could also be thinking of … [The Mountain], … Steep Path is straight up, [but] if I go Resting Path, I've got the plateau to rest..."	Energy is used up, but if you have a chance to rest, you will return to your natural energy level (i.e., energy is an intrinsic trait of the hiker).	Energy is like a reservoir which refills if it is emptied.

V. DISCUSSION

In discussing our results, we look at both how we model student reasoning and how a teacher might respond to assessment data in their teaching. We argue that useful ideas can be used in problematic ways, making the task of teaching difficult. We further argue that assessments that do not help clarify student thinking make the task of listening to students harder.

A. Facets and metaphors in the classroom

The research literature (using [27,57,58,67,68] as examples) includes discussion of the energy-as-a-substance metaphor as well as the stimulus ontology, the life-force metaphor, and more. In the examples of possible student responses, teachers provided a nuanced view of how these metaphors apply to classroom practice. To interpret these views, we turn to Minstrell's facets of knowledge [34,35].

We have argued for a substance metaphor when interpreting the statement that a hiker *has* a certain amount of energy that is *used up* while hiking. We have also observed teachers presenting three different interpretations of this metaphor, one related to time traveled, one related to distance traveled (explicitly not attending to steepness), and one related to the steepness (and perhaps, therefore, the effort) of the travel. These are three different interpretations of the same idea. In Minstrell's language, they are different facets of the same metaphor.

From a teacher's perspective as they interact with and listen to students, one might need to hear both the metaphor (of energy-as-a-substance) and the facets (thinking of a particular way in which substances get used up). The substance metaphor is of value to novices and experts alike [26], so a teacher might seek to sustain students' use of the idea while addressing that some facets of it may be problematic. The teacher needs to determine whether the *used up* facet is a problem in and of itself, or if a version of the idea might be useful for instruction. A teacher might consider whether GPE is *used up* as a falling ball gains kinetic energy, *e.g.*, if there are ways to modify the *used up* idea to look at forms of energy rather than total energy, for example. Similarly, the different facets (based on time, distance, and effort) might present value in some situations and be problematic in others. This is a situation in which an intuition might have a useful and an incorrect application, as discussed in other settings by Hammer and Elby [69].

How teachers respond to their students' ideas is suggestive of how they value the ideas that students bring to the classroom. Some facets of ideas have value, while others are a problem in some particular setting. But even those that are problematic may have value in later discussions [32]. In responding to a student using incorrect facets of a useful metaphor, the teacher is faced with the difficult task of recognizing the many different ways a single idea can play a role in the classroom.

B. Assessing how energy is used up

The role of a good assessment should be to guide instruction to help students learn the material better. This question was not a good assessment for such a purpose because it did not provide teachers with enough information about how to proceed.

With so many compelling arguments for Hiker 1, students might easily be distracted from correctly saying that all three hikers have the same gravitational potential energy at the top. For those who answered correctly, we can conclude that the issue of path dependence, which so clearly distinguishes Hikers 1, 2, and 3, did not distract them. We might also assume that they recognized that gravitational potential energy depends only on one's location (on the hill). We should not assume that they have thought about the equivalence of these explanations, though. In sum, answering the hiker question correctly gives relatively strong evidence that students are thinking appropriately about gravitational potential energy.

Problems arise when a teacher tries to find value in or guidance from the incorrect responses a student might give. In our data, discussed with teachers later that evening, roughly 40% of their students had answered "Hiker 1" at the beginning of the school year. What did this response mean? It might mean any of the explanations given above.

A responsive teacher giving the hiker question as a pretest would like to be able to use these data to guide decisions about instruction. When 40% of the students answer Hiker 1, there is no guidance for how to address their needs. Are they thinking hikers use up energy, and in which of the three ways? Are they thinking that Hiker 1 created energy by working harder, instead? These explanations might lead to the same answer but for very different reasons. A teacher using tools such as clickers (or other multiple-choice tools for which no explanations are readily given by students) would need to ask additional questions to gather more information about students' ideas before proceeding. This is a situation where asking students to "Explain their reasoning" (as is described in more detail in [70]) would be helpful, but would take away from quickly and easily reading student responses. For a classroom conversation, though, it is likely a highly productive question to ask.

VI. CONCLUSIONS

In a professional development setting, teachers provided multiple explanations for possible incorrect answer to a question about gravitational potential energy. Importantly, these explanations spanned a broad space of (at times contradictory) explanations. Collectively, the teachers were aware of more ideas than they might have named on their own. Their explanations made clear that teachers were aware of multiple ways of thinking about energy. Further, their explanations highlighted the ways in which similar, basic ideas (such as the substance metaphor for energy) could be used differently to answer the question. That the basic idea is so productive, but the facets of the idea so problematic, was not discussed by the teachers.

For teachers to listen well, they need an effective assessment that uncovers the details of student reasoning. This particular question did not do so. But, even with a more effective question, teachers would be left with the difficult task of understanding how to help students use their existing knowledge to develop new ideas and how to determine which ideas to support and which to confront.

ACKNOWLEDGEMENTS

We thank the teachers of the Maine Physical Sciences Partnership who participated in the activity. This work was supported in part by NSF grants MSP-0962805 and DRL-1222580. A previous version of this paper was published as [71].

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