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Assessment of the Knowledge of the Decimal Number System Exhibited by Students with Down Syndrome

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Assessment of the Knowledge of the Decimal Number System Exhibited by Students with Down Syndrome

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Abstract

This paper presents an assessment of the understanding of the decimal numeral system in students with Down Syndrome (DS). We followed a methodology based on a descriptive case study involving six students with DS. We used a framework of four constructs (counting, grouping, partitioning and numerical relationships) and five levels of thinking for each one. The results of this study indicate the variability of the six students in the five levels and in their mastery of the constructs. The grouping construct, which is essential to a proper development of the others, proved complex for the students. In general, we found that these students have a better procedural than conceptual understanding. However, the skills displayed by two of the students in the study group are encouraging with a view to advancing the number knowledge of these individuals.

Keywords: Down Syndrome, decimal numeral system, levels

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Evaluación del Conocimiento del Sistema de Numeración Decimal en Estudiantes con Síndrome de Down

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Resumen

Este artículo presenta una evaluación del conocimiento del sistema de numeración decimal en estudiantes con Síndrome de Down (DS). Seguimos una metodología basada en un estudio de caso descriptivo en el que participaron seis estudiantes con DS. Se utilizó un marco de cuatro constructos (conteo, agrupación, partición y relaciones numéricas) y cinco niveles de pensamiento para cada uno. Los resultados de este estudio indican la variabilidad de los seis estudiantes en los cinco niveles y en su dominio de los constructos. El constructo de agrupamiento, que es esencial para el adecuado desarrollo de los demás, resultó complejo para los estudiantes. En general, encontramos que estos estudiantes tienen una mejor comprensión procedimental que conceptual. Sin embargo, las habilidades mostradas por dos de los estudiantes de este grupo de estudio son alentadoras con vistas al avance en el conocimiento numérico de estas personas.

Palabras clave: Síndrome de Down, sistema de numeración decimal, niveles

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An understanding of the decimal number system can help people with Down Syndrome (DS) to develop more advanced mathematical skills (Lemons, Powell, King, & Davidson, 2015). Basic number skills are essential in their daily lives to finding a job and to achieving self-reliance and a good quality of life (Bird & Buckley, 2001).

There has been considerable research into how people without disabilities construct ideas involving the decimal number system. Some of this research points to the hardships present in the flexible use of multi-digit numbers and in learning place values (Baroody, 1990; Thomas, 2004). Predominant in the research on mathematical education in children with DS is that which analyses aspects on acquiring the concept of a number, such as cardinality and counting (Abdelhameed & Porter, 2006; Buckley, 2007), while there is little research on their understanding of the number system (Gaunt, Moni, & Jobling, 2012). Arithmetic and number skills are areas of particular difficulty for individuals with DS, but education has a positive influence on achievement levels in arithmetic (Nye, Buckley, & Bird, 2005). Some studies have analysed how high they can count and how effective they are when doing operations, but we have yet to find a study that considers the extent to which they understand multi-digit numbers. For example, whether they know and understand the notion of tens or hundreds, whether they group and ungroup tens in a way that lets them solve problems or if they can explain why one number is smaller than another.

In previous research on students with DS, we found that the mistakes made by students with DS when performing additions and subtractions reflected the little understanding they had of the place value of numbers, which impeded their ability to overcome these obstacles (Noda et al., 2011). The work we present herein evaluates the knowledge that six people with DS have of the decimal number system. This evaluation will help us to determine the weaknesses and strengths of this population in an effort to develop educational curricula that are suited to their cognitive characteristics and thus improve their learning of numbers.

Learning and Down Syndrome

People with DS have difficulty learning due to the changes that their trisomy causes to their brain structure and function. Not all people with DS exhibit the same cognitive ability since the brain impairment varies between individuals. They do, however, share cognitive deficiencies that affect their learning, such as attention span, perception of stimuli, memory (short-and long-term) and language. A knowledge of these different characteristics associated with their behaviour phenotype is necessary to develop effective educational programmes (Lemons et al., 2015).

In general, they have the ability to learn but are inconsistent in how they acquire knowledge, a process that is slower than in children without disabilities (Fidler & Nadel, 2007).

People with DS have attention deficits, as manifested by their inconsistency when performing tasks, their inability to retain answers, a tendency to distraction, or performing movements that have no clear purpose (Lemons et al., 2015). This requires establishing specific interventions to improve these areas. Systematic, constant work targeted at this goal must be part of the educational curriculum for these students (Fidler & Nadel, 2007). Attention span is closely related to short-term, or working, memory, which enables the use of information for brief periods of time. People with DS have problems retaining and storing information for short periods of time and giving an immediate response to a mental or motor operation. As for stimulus perception, some studies have also shown that people with DS are better able to process visual instead of auditory information (Hodapp & Freeman, 2003).

People with DS also exhibit problems when transferring information from short to long-term memory, which stores life experiences, our knowledge of the world, images, the meanings of concepts and words and the relationships between these meanings, strategies for action, and so on. This requires a systematic and organised review of topics learned. Conners, Rosenquist, Arnett, Moore and Hume (2008) found that a home-based memory intervention instructed by parents (focusing on verbal rehearsal) leads to small but significant improvements in remembering numbers among children with DS.

Lastly, a trait of people with DS involves their difficulties with communication and language. The ones that are most evident in the language of people with DS are delay in acquiring vocabulary, a better level of receptive than expressive language, a reduced vocabulary, the use of shorter and less complex sentences, and difficulty organising their speech. These problems are bound to have a severe impact on their communications and learning skills. And yet, despite the speech and language problems they exhibit, most people with DS are enthusiastically interactive in social settings, they make good use of their non-verbal skills, such as visual contact and facial expressions, and use gestures to make themselves understood when words fail them (Roizen, 2001).

Down Syndrome and Mathematics

Research into what mathematical concepts people with DS learn and how they learn them is scarce, especially in comparison with other disciplines, such as language, an area of learning in which researchers have made greater headway. Most of the research involving DS and mathematics has focused on basic number concepts, probably as these are essential to any subsequent knowledge of mathematics.

In the area of number knowledge, the results indicate that most youngsters and adults with DS do not achieve a basic level of competence. It has also been noted that they tend to lose their number skills faster over time than their language skills (Shepperdson, 1994).

Much of the research done on groups with DS is based on analysing counting principles; that is, abstraction, stable order, irrelevance in the order, one-to-one correspondence and cardinality. There is also research on how students with DS perform addition and subtraction operations that indicates they can be successful with addition problems by using specific counting techniques. Activities involving the counting of objects are therefore key to developing more advanced abilities in these students (Abdelhameed & Porter, 2006).

There are two alternative explanations of how the cardinal meanings of the first few number words are acquired. One states that these number words are learned through counting, while other studies highlight the role of subitization (Benoit, Lehalle, & Jouen, 2004). The role of subitization has not been adequately studied in populations with DS (Nye, Fluck, & Buckley, 2001). Thus, Sella, Lanfranchi and Zorzi (2013) found that children with DS showed a specific deficit in the discrimination of small numerosities (within the subitizing range) with respect to both mental and chronological age matched typically developing children. And Belacchi et al. (2014) found that students with DS were worse at estimating collections of points than students of the same mental age without disabilities.

People with DS exhibit problems handling abstractions, generalising procedures and applying the lessons learned in one situation to another (Bird & Buckley, 2001). This makes learning mathematics particularly complex. And yet research on the number abilities of people with DS has shown that many of the shortcomings detected are indicative of improper teaching methods (Porter, 1999). Some studies have demonstrated that the mathematical knowledge of this population can be advanced by using suitable methods. These methods mainly involve individually tailored learning sequences, extensive practice with a variety of tasks and support activities that rely on specific materials or computer-based learning (Gaunt et al., 2012; Ortega-Tudela & Gómez-Ariza, 2006). Other researchers have reported advances in the overall knowledge level of students with DS, and in their knowledge of mathematics in particular, when they are taught from an early age or they are integrated into ordinary schools (Turner, Alborz, & Gayle, 2008).

The Decimal Number System

The expression *number sense* appears in curricular documents, associated with the fact that number learning has to be an activity that "makes sense" (NCTM, 2000). Promoting it has become one of the goals of mathematical learning in various countries. Sowder (192) defined number sense as a well-organised conceptual network that allows relating numbers and operations, their properties and solving problems creatively and flexibly. An analysis of number sense has also appeared in research with students who have problems learning mathematics (Berch, 2005; Brigstocke et al., 2008; Gersten & Chard, 1999) from different perspectives, though normally it involves research on how numbers are first learned. There is a consensus in this research in noting how the development of number sense allows, on the one hand, for the early detection of potential problems with learning mathematics and, on the other, for designing approaches to teach mathematics to these students. We are involved in teaching/learning methods that present *number sense* to all students, whether or not they have learning difficulties.

The decimal number system is mastered slowly over the course of one's schooling through a carefully designed educational approach that takes into account the various principles that dictate its operation. One such principle that must be understood is that of place value. Steffe, Cobb and Von Glasersfeld (1988) noted that an understanding of place value requires learning conceptual structures that lead to viewing 10 as one unit. These structures allow children to regard a set of 10 objects as one unit while maintaining its numerosity; that is, to view it as a *numerical composite unit*. The idea of an *abstract composite unit* is acquired later and is used to coordinate tens and ones.

Researchers have used different frameworks to develop an understanding of the decimal number system, suggesting that students must go through different hierarchical levels (Battista, 2012; Jones et al., 1996).

Jones et al. (1996) offer an approach for teaching and evaluating the decimal number system that involves five levels of thought. Each level addresses the four aforementioned components (*counting*, *grouping*, *partitioning* and *establishing number relationships*). Since there is no existing framework adapted for persons with DS, nor are we aware of any research that has delved into the decimal number system in this population, we opted to use a framework validated for people without disabilities, as has been done in early research into other areas of mathematics. In particular, the framework proposes a sequence of levels and is organised into four components that facilitate an evaluation of many mathematical aspects and allow for subsequent adaptation in small steps.

Counting is essential to learning the decimal number system. Students must progress with tasks, starting with counting by ones, and continuing on to counting on by ones, counting by tens and ones and counting on by tens and ones. Later it involves counting on or counting back by hundreds, tens and ones (Battista, 2012; Jones, et al. 1996).

An understanding of the decimal number system also requires mastering *numeric partition* (decomposing a number into the sum of smaller numbers) and *grouping* structures, which are inverse and dependent constructs. Bednarz and Janvier (1988) viewed grouping as the basis for recognising and constructing multi-digit numbers. This construct also includes numerical estimation.

Partitioning requires having stable and flexible grouping structures. Resnick (1983) distinguishes between a *unique partition* and a *multiple partition* of multi-digit numbers (*Unique partition*: 50+6; *multiple partition*: 56=30+26=16+40…).

When learning the decimal number system, it is important to establish *number relationships*, recognise the size of numbers and order them. A mastery of these concepts is indicative of a good understanding of the place value of digits. Understanding grouping structures and being able to partition can be used to order numbers using suitable strategies.

Below is a brief description of the framework in Jones et al. (1996).

Level 1. Pre-place value. *Use of individual units*. Use count all and/or count on strategies. Count informally by tens. Make numbers below 10 in different ways. Estimate amounts using groups as benchmarks. Count by fives and tens. Group to make counting easier and faster. Determine numbers greater/less than another that are no greater than 20. Order numbers less than 20.

Level 2. Initial place value (<10). *Initial understanding of the place value of figures, transition from individual use of ones to the use of tens as a unit*. Count groups of tens as if they were independent items. Form and count groups of tens and ones. Make multi-digit numbers in different ways, especially tens and ones. Estimate the number of objects in a group using the appropriate unit. Group by tens to make it quick and easy to check. Order two-digit numbers after exchanging the order of the ones. Order two-digit numbers close to and between tens.

Level 3. Developing place value (<100). *The use of two-digit numbers is extended to mental addition and subtraction*. Count on or back by tens, adding and subtracting. Count on or back by tens and ones, adding and subtracting. Make multi-digit numbers in different ways. Find the missing part of a number. Determine if the sum of two digits is within a given tens range. Order two-digit numbers. Order two-digit numbers after adding and exchanging the order of the ones.

Level 4. Extended place value (<1000). *Knowledge is expanded to three-digit numbers*. Count by hundreds, tens and ones, adding and subtracting. Make multi-digit numbers in different ways. Find the missing part of a number. Given a specific amount of tens and ones expressed verbally, determine the number without using materials. Determine if the sum of two three-digit numbers is more or less than a given number. Order two-digit numbers close to and between hundreds. Order multi-digit numbers exchanging the order of the hundreds.

Level 5 Essential place value. *Includes mental addition and subtraction problems with numbers up to 1000*. Count by hundreds, tens and ones to add and subtract mentally. Make multi-digit numbers in different ways. Find the missing part of a number. Determine if the sum or subtraction of two two- or three-digit numbers is more or less than a given number. Given a specific number of hundreds, tens and ones expressed verbally, determine the number without using materials. Order multi-digit numbers, determining which of the two is closer to a third.

The study by Jones et al. (1996) evaluated each student's consistency individually by using components and the differences between children who were at different stages of learning. The findings show that the levels are hierarchical and that the four components are equally important when evaluating one's knowledge of multi-digit numbers. This framework, validated in students without disabilities, is used in this paper to evaluate the knowledge of students with DS. In Gaunt et al. (2012), a teaching program was implemented and used to improve the number skills of children with DS, and especially their understanding of the place value of numbers. This program was based on direct instruction and repeated practice and relied on specific materials and a selection of games. Notable among the findings is the importance of instruction tailored to the individual. They also highlight the need to conduct more research on this topic so that comparisons can be made among groups with DS.

Objectives and Methodology

The objectives of our research are two-fold:

- To evaluate the knowledge that students with DS have of the decimal number system and to rank it based on the five levels of the framework described by Jones et al. (1996).
- To analyse the skills and problems involved in the four components: counting, grouping, partitioning and numeric relationships in students with DS.

The type of data required by this evaluation involves presenting students with tasks that allow us to observe how they solve them and what their rationale is. The research was conducted using a descriptive case study model with students with DS who were interviewed individually. Six individuals with DS were selected, three adolescents and three adults. The three adolescents attend regular public secondary schools where they follow inclusive education programmes with help outside the classroom at specific times.

The three adults are employed in work centres sponsored by the Tenerife Trisomy 21 Association (ATT21). The six students receive daily tutoring through this Association in different subjects, including mathematics (see Table 1).

Table 1
Characteristics of the six students with DS interviewed

	S1	S2	S3	S4	S5	S6
Age	16	16	17	25	26	36
Sex	F	M	F	F	F	F
Academic activity	Secondary School	Secondary School	Secondary School	Work Centre	Work Centre	Work Centre

This Association helps 50 individuals with DS, ranging in age from several months to 45 years, and with highly varied cognitive traits and academic knowledge. As a result, the selection of students was conditioned by the limited population and by three basic requirements for this research: that they have a minimum knowledge of two-digit numbers, an adequate ability to express mathematical ideas, and an understanding of oral and/or written language so as to respond to the tasks. The Association's teachers selected a small group of students of various ages, and the researchers opted to create two groups with the same number of students (teens and adults). The students selected, however, exhibited common cognitive traits: they were able to pay attention for 30-45 minutes, after which they tended to grow distracted, their answers relying more on intuition than understanding. The students also had no short-term memorization strategies and were better able to handle tasks requiring visual recall and auditory recognition.

The mathematical curriculum followed by the students involving number learning focused on natural numbers and their operations, especially addition and subtraction. The students were selected based on this number knowledge, the sole mathematical requirement being that they had worked with two-digit numbers. To complement this information we asked them questions on reading and writing numbers, finding the number immediately before and after a given number, and addition and subtraction with and without regrouping. We started with single-digit exercises and increased the number of digits as more correct answers were offered. In the case of the additions and subtractions, the maximum number of digits was four. Table 2 shows the size of the numbers up to which correct answers were given for each set of problems.

Table 2

Number knowledge of the six students

Task	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**
Writing numbers	3 digits	2 digits	2 digits	6 digits	6 digits	3 digits
Reading numbers	4 digits	3 digits	2 digits	6 digits	6 digits	3 digits
State nº. before and after	4 digits	2 digits	2 digits	2 digits	3 digits	2 digits
Add without regrouping	4 digits	4 digits	2 digits	4 digits	4 digits	4 digits
Subtract without regrouping	4 digits	1 digit	2 digits	4 digits	4 digits	4 digits
Add with regrouping	4 digits	No	No	4 digits	4 digits	4 digits
Subtract with regrouping	4 digits	No	No	No	4 digits	4 digits

The results show that their number skills are independent of age and exhibit differences between one another in terms of the size of the numbers. The strategy for carrying out operations was algorithmic with vertical operations. They were not accustomed to doing mental calculations. They were slow to perform the operations and had to use their fingers to aid them, except for S5, who had memorised basic number facts. This information helps to confirm that the six students could accomplish tasks from the various levels in the framework.

With the first questions, these students demonstrated an ability to read numbers with several digits and to operate with them. Previous research by the authors of this paper on students with DS analysed difficulties and mistakes involving addition, subtraction and problem solving (Noda et al., 2011; González et al., 2015). In that research we found that the mistakes made by students with DS when adding and subtracting stemmed from a lack of conceptual understanding. That is why we initially thought that success in the first procedural tasks would not imply that they were at the corresponding level of the framework defined by Jones et al. (1996) with regard to the size of the numbers employed. Bear in mind that in this framework, the tasks require a conceptual understanding of the decimal number system. In other words, they may know numbers with up to four digits without necessarily understanding the conceptual meanings of tens or hundreds.

The interviews were conducted at the ATT21 Association and were video recorded. Each student had between three to five sessions, depending on the progress and the level of concentration demonstrated. The interviews were semi-structured, meaning a basic protocol was followed that allowed for additional questions if those asked were insufficient to evaluate the objective. We also repeated some of the questions to check whether a faulty answer was due to a problem with attention span, fatigue or understanding the problem statement. We did this by explaining the task once more but using different numbers.

The structure of the interviews took into consideration each student's different characteristics, mentioned in Section 2, in terms of attention span, perception of stimuli, short- and long-term memory and language. The tasks were given in written form and read out loud. Occasionally, the tasks had to be solved using specific materials (candies, pencils and coins), structured materials (interlocking blocks, Herbinière-Lebert material) and numbered playing cards. The tasks included oral problem statements that were explained using drawings. Session durations were adapted to the level of concentration exhibited by each student. Tasks were broken down into small steps with short and direct problem statements featuring simple words. The explanation of a task could be repeated, and the task itself repeated with other numbers.

The interview protocol designed to evaluate levels of thought included activities from all five levels for the four constructs. The activities were arranged by level, with a minimum of three activities for each construct per level. If a student answered most of a level's tasks correctly, the interview moved on to the tasks corresponding to the next level.

To validate the data we used the researcher triangulation method and an external audit conducted by colleagues. The data were analysed independently by the two researchers. The results were later contrasted by both researchers by watching the videos and the analysis protocol to combine the results. As for the external audit by colleagues, these results have been partially presented and discussed in seminars at the researchers' university and in various conferences (González et al., 2015).

When analyzing the data we took into account the previous theoretical framework that we had used as a reference to guide our research.

Results

We present the results arranged by level, noting the features that characterise the answers for each level, the progress made and the problems faced by the students and the reasons that led us to assign the students to each level. For each construct in the various levels, we show some of the tasks (see Appendix A. Example of task).

Level 1. Pre-Place Value

Students S3, S4 and S6 were able to count objects using numbers in sequential order, one by one, and exhibited a preference for the *count all* rather than the *count on* strategy. For example, when faced with task C1.1 (see Appendix A), student S6 counted all the blocks again, while students S3 and S4 counted on from a given amount.

They did not show a tendency to make groups nor did they use them to count when they were already made and not use partitioning strategies, meaning they did not decompose numbers less than 10, either using materials or symbolically. For task P1.1, the process used by these three students was to take pieces at random (10 and 3; 8 and 6; 7 and 4) and place them atop the 8, without noticing that they were greater. They showed no signs of looking for numbers less than 8.

Students S3, S4 and S6 understood the meaning of "bigger than" and "smaller than" when asked to give a number of objects, and distinguished between numbers larger or smaller than another number when asked graphically or in writing, but not orally (task R1.1). They had problems ordering numbers, however (task R1.2). Furthermore, they did not use tens as a counting unit. As a result, their interviews did not continue on to level 2 tasks.

Student S2 exhibited a greater understanding of the tasks presented for the various constructs, and he continued on to level 2 tasks. When counting he used the *count all* and *count on* strategies, and resorted to counting by tens for some tasks, though on occasion he did not distinguish between tens and ones. For example, like when solving of task G1.2, he did not use pre-prepared groupings, but rather counted them out correctly by ones. And yet, when asked how much is 10 plus 10, he answered “20”. When asked again to count the 23 blocks, he said “ten, twenty, thirty” (indicating the group of 3 as if it were another group of 10). He seemed to have memorised how to count by tens, but he applied it mechanically, letting himself be carried away by his impulsiveness. The student successfully completed the partitioning tasks, using partitioning strategies quickly, both material and symbolic. He relied on the strategy of always looking for numbers smaller than the one to be decomposed; he gave different ways of decomposing one number. He distinguished between numbers larger or smaller than another number when asked graphically or in writing, but not orally (task R1.1).

Students S1 and S5 successfully completed the tasks presented to evaluate level 1, and so were given the tasks for the next level. They counted informally by tens, used a coordinated tens-one approach and they demonstrated certain grouping structures. For example, student S5 when solving of task G1.2, said “ten and three makes thirteen”, and afterward counted the other group of 10 by ones. When asked how she could count them faster, she repeated the process but instead counted the last 10 objects by threes: “ten plus three is thirteen, and three more sixteen, and three more nineteen, and three more twenty-two, plus one is twenty-three”. Were able to make different partitions for a number smaller than 10, made comparisons using proper strategies and ordered numbers.

Level 2. Initial Place Value

Of the three students who underwent the level 2 evaluation (S1, S2 and S5) students S1 and S5 went on to the level 3 evaluation.

We placed student S2 somewhere between levels 1 and 2, since some of his answers did not surpass the level 1 indicators. Even though he showed some level 2 knowledge, he sometimes confused tens and ones in the counting and grouping tasks. For part a) of task C2.2, he correctly counted by tens and ones using the *count all* strategy until he had to add the last ten (32 units), at which point he confused the ones with the tens: “Ten, twenty, thirty, forty and fifty”. After being corrected, in part b) the student was asked “how much is there now?” after adding 3 units to 32, to which he replied “thirty-three, thirty-four, thirty-five”, meaning he was back to regarding ones correctly. Student S2 had problems partitioning the larger numbers corresponding to this level. Although he showed signs of being able to do it, he did not exhibit good strategies and could not use the tens correctly as a unit for partitioning. When he picked the 45 card, S2 formed it with four cards of 10 and one of 5, meaning he decomposed it perfectly (task P2.1). However, when counting to check the result, he confused the tens and ones, saying “ten, twenty, thirty, forty and fifty”. The number 24 appeared next, which he formed by adding cards of numbers under 10 and counting with his fingers until reaching 24. Student S2 ordered two-digit numbers when shown to him in writing, though he had problems inverting the ones place and ordering (task R2.1).

Students S1 counted by tens depending on the type of activity. In task C2.1, when shown the number 45 using the Herbinière-Lebert plates and asked to state the number represented, she recognised the 10 plates and said “ten, twenty, thirty, forty”, from which point they counted the five remaining units. For task C2.2, when shown two units and adding ten, S1 *counted on* from 2, one by one, up to 12. When another ten was added, she once more *counted on* from 12, one by one, up to 22. She repeated the process when a third ten was added (up to 32). Student S1 was able to partition numbers below 100, although she did not adhere to the unique partition (into tens and ones); rather, she chose numbers with no apparent strategy and added them until the desired number was reached. For example, to partition the number 34 with the number cards (task P2.1), she used the partition 10+3+7+1+1+2+2+1+1+6, making the partition very lengthy. Students S1 ordered numbers below 100 and clearly distinguished between smaller and bigger. In task R2.1, student S1, when shown the numbers 61 and 67, inverted the units mentally and stated, “seventy-six is bigger than sixteen because seven is bigger than one”.

Student S5 was flexible in her counting and partitioned numbers under 100 very confidently and used basic number facts in every activity involving this component. She manifested the use of the tens as a composite unit. For example, with the number cards (task P2.1), she created the number 16 with 10+6; 24 with 10+10+4; and 58 with 10+10+10+10+10+8, and stated “five 10s and one 8”. We judged the knowledge level of student S5 to be above the level 2 indicators since she successfully completed all the tasks, except for estimating amounts, which we ascribe to the fact that this is not a usual task for her. In task G2.1, she counted the 53 objects visually and said “there are thirty-five”. To check her estimate, she used various groupings (by twos, fives, threes and tens), saying “two and four is six, and three makes nine…”. She then continued by ones and by twos. When instructed to make groups to count faster, she started making groups of 10 (5 groups) and then counted by tens up to 53.

Level 3: Developing Place Value

Students S2 and S1 used similar reasoning for some constructs, like ordering and partitioning, but S1 seemed more confident and offered more reasoned replies, so we decided to evaluate her knowledge of level 3 constructs.

In the counting activities resorted to using the more basic *count all* strategy to count, evidence that student S1 had not yet acquired the abstract composite unit structure for the tens. In task C3.1, she recognised the 10 plates and pointing to them, said “ten, twenty, thirty, forty”, after which she counted out the five remaining units. When asked to add 20, she needed to represent the new situation with the material and counted again from the start: “ten, twenty, thirty, forty, fifty, sixty and sixty-five”. She used the same process with the remaining questions, which shows she has mastered level 2 counting concepts, but not level 3, since she cannot operate mentally with the tens as the abstract composite unit. With the more complex tasks, where they were presented with two kinds of numeric representations, like in task C3.2 for example, not process the two representations mentally, which is the method of operation for this level of thinking; first said that the number represented by the material was “fifty-six”, and then wrote down the equation 56 + 12 = 68. Though she was able to do partitioning tasks in level 2, that was not the case in this level. For task P3.1, where they had to partition the number 37, she used a faulty rule that consisted of repeating the number in the tens place as many times as indicated in the ones place (37=30+30+30+30+30+30+30). Lastly, her answers to the ordering tasks demonstrated an incomplete knowledge of the meaning of tens. This student's answers reveal the influence of the tasks and of the words used in those tasks. In task R3.1, when adding 2 tens to 43 she placed the number 2 to the left of the tens place, writing 243. When asked "What happens if you add two tens to 43?", she wrote 43+2=45, but then, when asked "what if you add 2 tens to 52?", she again wrote down 252, which demonstrates her confusion with the vocabulary associated with the decimal number system.

S5 possessed good knowledge of two-digit numbers and completed the tasks for this level correctly, though in some cases she preferred to write down the operations, instead of doing them mentally. We thus proceeded to test her at level 4. Student S5 was able to operate mentally using the tens as an abstract composite unit. For example in task C3.1, counted the first 10 plate by saying "two times five is ten" (multiplicative strategy), and then continued with the remaining plates, saying "ten, twenty, thirty, forty and forty-five". When asked the number that would result from adding 20, she calculated it mentally, stating that "four and two is six, so we have 65". She used the same process with the remaining questions; that is, she operated mentally, demonstrating the use of the tens as an abstract composite unit. On the other hand, recognised and effectively used multi-digit partitions and used basic numeric facts, though she could not do so mentally in every situation. For task P3.2, S5 used the tens as the counting unit, and visually and mentally answered that there were 10 units hidden. In task P3.3, she did not see 40 as a part of 65; instead, she correctly employed the subtraction algorithm. That is to say, she recognised and used the partition but did not compute it mentally in every case.

Level 4: Extended Place Value

Student S5 gave correct answers for many level 4 tasks, though in every construct she exhibited problems, which leads us to believe that she is in the process of building the skills attributed to level 4. S5 had troubles understanding the meaning of hundreds, tens and ones and the relationships between them so she could not determine how many tens there were in a three-digit number, identifying it only by the position it occupies in the written number. In other words, she does not always ascribe 10 tens or 100 ones as the numerosity of one hundred. For example, when shown a hundred using the Herbinière-Lebert material and asked "how many units of 10 are there in this hundred?" she counted and replied "ten". When asked how many ones there were, she counted by tens and said "a hundred". However, when asked "how many tens are there in the hundreds plate", she said "zero tens". In other words, she indicated the digit occupying the tens place in the written number. Not use the hundreds or tens as the counting unit to give a mental reply, but instead resorted to writing the algorithm, she used this same method for the grouping and partitioning activities. In task G4.1 part a), when was asked to write 4 tens and 3 ones, she correctly wrote 43. For part b), she thought of 400 and replied "zero because four are hundreds". For question c) she stated 31, while for part d) she had to write out the equation, which she did correctly.

Was able to give different decompositions for numbers below 1000, she was not, however, able to see when hundreds, tens and ones can be combined mentally, having instead to write out the equation, as she did for task P4.2, where she directly wrote out the operation 462-342. Finally, the student S5 knew the number sequence for ordering, and she successfully completed those tasks requiring her to order various three-digit numbers. But her problems with the positional value of numbers kept her from correctly determining the distance between two quantities. In part a) of task R4.1, she only focused on the hundreds, ignoring the tens and ones, and answered "closer to three hundred, since in four hundred the hundreds place is bigger". For part b) she focused only on the hundreds and tens, ignored the ones and said "closer to 320 since in 330 the tens place is bigger".

Summary of the evaluation

Table 3, showing the profiles of the six students and their levels, is based on the results and on the analysis presented. The graph reflects the level of thought each one was capable of engaging in.

Table 3
Profiles of the levels of thought of the six students

	S1	**S2**	**S3**	**S4**	**S5**	**S6**
Age	N3	N2	N1	N1	N4	N1

Conclusions

In our research we used a framework consisting of five levels of thought, each one with four constructs, to evaluate the understanding that persons with DS have of the decimal number system. This framework, already validated for students without disabilities, has proven useful for evaluating the number knowledge of this population with DS. Jones et al. (1996) interviewed twelve students (six in 1st grade and six in 2nd grade) after concluding a teaching sequence that used the constructs and the different levels. At the end of the first year, one of the six students was at thinking level 5 (which is higher than that required for the curriculum of this academic year), one at level 4, one at level 3, two at level 2 and one at level 1, thus demonstrating the high variability in the levels. As for the six 2nd-grade students, the results were more homogeneous, with three at level 5, two at level 3, transitioning to 4, and one at level 2. Although the empirical data from the research by Jones et al. (1996) are not comparable to this study due to the age difference of the students and the type of study carried out, both research efforts involved a framework that is consistent in its constructs and hierarchy.

In our research, there was a variability in their understanding of multi-digit numbers that evidenced their individual differences. One possible explanation might be that their cognitive skills and behavioural characteristics lead to different ways to solve the tasks. Their numerical education could also play a role. Such is the case of student S5, who is at level 4 and who exhibited fewer deficits in attention span, perception, memory and language. This student exhibited greater concentration and motivation in the tasks, which resulted in fewer procedural mistakes. She also gave verbal explanations with certain agility and had a good grasp of basic number facts. This agrees with Lemons et al. (2014), who point to the clear need for increased of mathematics intervention research for people with DS that takes into account the behavioural phenotype.

As we had anticipated, these results show a gap between the ability to carry out numerical procedures (reading and writing numbers or writing addition and subtraction equations) and those tasks in which they have to take the initiative and determine what number knowledge is needed. For example, three of the six students used improper strategies to complete the level 1 partitioning tasks with numbers under 10 despite knowing the numbers up to 100. It is characteristic of this population to engage in repetitive procedures, which is why open-ended activities with different solutions pose such a challenge to them, as reflected by the greater uncertainty in their answers.

The framework served to demonstrate the need to expand the number task curriculum taught to these students. We noted that counting is the predominant resource used to solve every task, to the detriment of other skills like estimating, grouping, partitioning and subitization. There has been little research on the role of subitization in populations with DS (Belacchi et al., 2014), though we did observe some students, like S5 and S2, use subitization for small sets, but the strategy was not used spontaneously by every student in our study. In fact we found that in one activity, they were able to recognise ten with the Herbinière-Lebert material without counting before resorting to counting all the units in the same activity.

The answers of the six students show that the skill of estimating is not developed in their learning, since they said numbers at random with no strategy to look for the most suitable quantity. Moreover, four of the six students did not show any inclination to group during the tasks for the various constructs, and often resorted to counting objects one by one. We do, however, agree with Jones et al. (1996) in thinking that the grouping construct is key to developing the other constructs.

The methodologies used for these students need to be changed so that more emphasis is given to those conceptual aspects that will help them further their understanding of the mathematics that follow. This would also improve their cognitive development (Lemons et al., 2015).

Another important observation with regard to this population is its problem with mentally manipulating multi-digit numbers. In the case of the students who have achieved levels 3 and 4 of thinking, whose progression to the higher levels was conditioned by this fact, we noticed that they were dependent on writing out equations to deal with those tasks in which they were expected to mentally manipulate two numbers. This aspect of the framework would require adaptation if it were to be used as a basis for developing a teaching-learning sequence.

The results of students S1 and S5 show how students with DS can advance their knowledge of the various constructs. Their reasoning is encouraging as they seek to further their understanding and achieve a mastery of numbers that is not based solely on algorithms and memorisation. Our findings show that we must not put a cap on the knowledge of the decimal number system that can be attained by students with DS.

Lastly, the framework utilised in our research is not only useful for evaluating the knowledge that students with DS have of multi-digit numbers; it also provides a basis for creating teaching programs, an objective that we are currently putting into practice. As stated in Belacchi et al. (2014), individuals with DS can achieve good academic results if their strengths are recognised and developed. At the completion of the evaluation, the framework allows us to expand the number activities from the start of the learning process, emphasising the four constructs without prioritising the counting construct. The framework needs to be adapted to the estimation tasks and to those that require using numbers mentally. These aspects require a long-term learning process that teaches them to look for certain strategies not associated with routine procedures, something rarely seen in this population.

We started by working with level-1 students using tasks from the four constructs broken down into small steps using carefully worded language. We designed the tasks taking into account the strengths and weaknesses in their development profiles. We used activities with visual aids, and employed different materials (like arithmetic blocks and everyday objects), card and dice games with rules and computer games, which we then complemented with other activities involving paper cards and pencils. This was done because as the research shows, students with DS benefit from using a variety of materials familiar to them from everyday life to generalise acquired knowledge (Bird & Buckley, 2001). We worked with small groups of two or three students to encourage communication and the use of mathematical vocabulary between them and with their teachers, thus making use of their great social and imitative skills.

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Appendix A

Examples of Task

Task level 1.

Counting. C1.1.

Show a written number and say “give me this many blocks”. Add 2 more and ask “How many are there now?”

Grouping. G1.2.

a) Arrange a group of 17 blocks in front of the student, grouped by fives and two single units, and ask how many there are.
b) Arrange a group of 23 blocks in front of the student, grouped by tens and three single units, and ask how many there are.

Partitioning. P1.1.

Give the student the piece for number 8 from the Herbinière-Lebert material and ask for different ways to get it.

Relationship and ordering. R1.1.

Write a number much bigger or much smaller than a given number.

Relationship and ordering. R1.2.

Order two or three numbers less than 20.

Task level 2.

C2.1. Counting.

Show a number represented using Herbinière-Lebert (45). “What amount is shown?”

Figure 1. Task C2.1

C2.2. Counting.

a) Show 2 units using Herbinière-Lebert (Figure 2). “What amount is shown?”
Add ten. “What amount is shown now?”
Repeat the process adding tens up to 32.
b) Add 3 units to the previous number (Figure3). “What amount is shown now?”

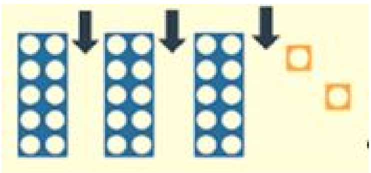

Figure 2. Task C2.2(1)

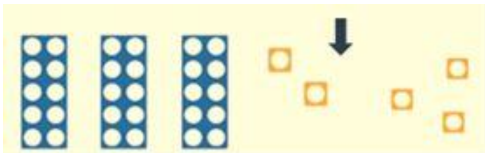

Figure 3. Task C2.2(2)

G2.1. Grouping.

a) Give student 53 blocks, “How many do you think there are?”
b) Ask student to check it. “How many are there?”
“What would you do to make it easier and faster to count them?”

P2.1. Partitioning.

Cards with the numbers from 1 to 10 are shown on the table.
Cards with two-digit numbers are placed face down in a pile: 24, 16, 45, etc. A card from the pile that is face down is chosen and its two digits revealed. The student must take cards from 1 to 10 whose sum equals the two-digit number just revealed.

R2.1. Relationship and ordering.

Show the student the written numbers 61 and 67 and ask, “Which is bigger?” Invert the order of the tens and ones. “Which is bigger now?”

Task level 3.

C3.1. Counting.

Show an Herbinière-Lebert representation of the number 45. “What amount is represented?”

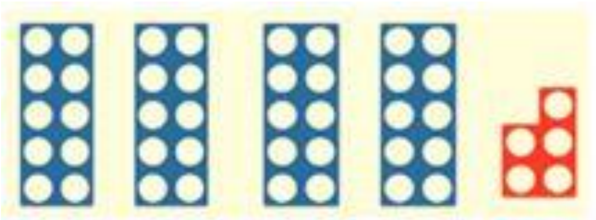

Figure 4. Task C3.1

“If you add 20, how much will you have?”
“If you now give me 10, how much will be left?”
“And if you give me 25 now, how much will be left?”

C3.2. Counting.

Show an amount in which one part is represented using Herbinière-Lebert material and another symbolically. “If 12 holes are covered up, how many are shown in total?

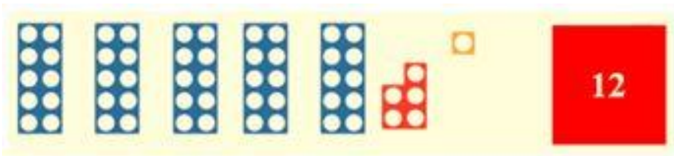

Figure 5. Task C3.2

P3.1. Partitioning.

“Decompose the number 37”.

P3.2. Partitioning.

“There are 34 units shown with the Herbinière-Lebert material. A part is hidden under some paper. How many units are hidden?”

Figure 6. Task P3.2

P3.3. Example of partitioning tasks for Level 3.

“We’re planning a birthday party. We have 40 balloons but we need 65. How many more balloons do we need?”

R3.1. Relationships and ordering.

Is the number 43 closer to 40, 50, 60 or 70?
If we add 2 tens to it, is it now closer to 40, 50, 60 or 70?

Task level 4.

G4.1. Grouping.

a) “What number is formed with 4 tens and 3 ones?”
b) “How many ones are there in 40 tens?”
c) “How many ones are there in 31 tens?”
d) “What number is formed with 31 tens and 12 ones?”

P4.2. Partitioning.

"There are 462 units in all. How many are hidden?"

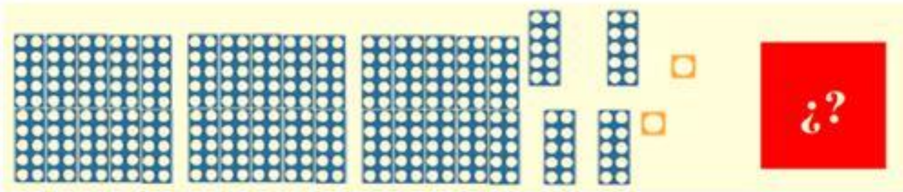

Figure 7. Task P4.2

R4.1. Relationship and ordering.

a) "Is the number 327 closer to 300 or 400?"
b) "Is it closer to 320 or 330?"