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On $[\gamma, \gamma']$–semiopen sets

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In this work we introduce and study the concept of $[\gamma, \gamma']$–semiopen sets and their related functions in topological spaces.

Keywords: topological spaces, semiopen sets, $[\gamma, \gamma']$–open sets, $[\gamma, \gamma']$–semiopen sets.

En este trabajo se introduce el concepto de conjuntos semiabiertos $[\gamma, \gamma']$ y sus funciones relacionadas en espacios topológicos.

Palabras claves: espacios topológicos, conjuntos semiabiertos conjuntos abiertos $[\gamma, \gamma']$, conjuntos semiabiertos $[\gamma, \gamma']$.

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1 Introduction

Generalized open sets play a very important role in general topology and they are now the research topic of many topologists worldwide. Indeed, an important subject in general topology and real analysis concerns the various modified forms of continuity, separation axioms, etc., with the use of generalized open sets. Kasahara [1] defined the concept of an operation γ on topological spaces. Maki and Noiri [3] introduced the notion of $\tau_{[\gamma,\gamma']}$, which is the collection of all $[\gamma, \gamma']$–open sets in a topological space (X, τ). In this work we introduce and study the notion of $[\gamma, \gamma']$–$SO(X, \tau)$ which is the collection of all $[\gamma, \gamma']$–semiopen which use operations γ and γ' on a topological space (X, τ). We also introduce $[\gamma, \gamma']$–semicontinuous functions and investigate some of their properties.

2 Preliminaries

The closure and the interior of a subset A of (X, τ) are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

Definition 2.1. *[1] Let (X, τ) be a topological space. An operation γ on the topology τ is a function from τ into the power set $\mathcal{P}(X)$ of X such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.*

Definition 2.2. *A subset A of a topological space (X, τ) is said to be a $[\gamma, \gamma']$–open set [3] if for each $x \in A$ there exist open neighborhoods U and V of x such that $U^\gamma \cap V^{\gamma'} \subset A$. The complement of a $[\gamma, \gamma']$–open set is called a $[\gamma, \gamma']$–closed set. Also $\tau_{[\gamma,\gamma']}$ denotes the set of all $[\gamma, \gamma']$–open sets in (X, τ).*

Definition 2.3. *[3] For a subset A of (X, τ), $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)$ denotes the intersection of all $[\gamma, \gamma']$–closed sets containing A, that is, $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) = \bigcap\{F : A \subset F, X\backslash F \in \tau_{[\gamma,\gamma']}\}$.*

Definition 2.4. *Let A be any subset of X. The $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A)$ is defined as $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A) = \cup\{U : U$ is a $[\gamma, \gamma']$–open set and $U \subset A\}$.*

Definition 2.5. *A subset A of a topological space (X, τ) is said to be:*

i. *$[\gamma, \gamma']$–α–open if $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A)))$.*

ii. *$[\gamma, \gamma']$–preopen if $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))$.*

iii. $[\gamma,\gamma']$–*b–open [2] if* $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)) \cup \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$.

iv. $[\gamma,\gamma']$–*semipreopen if* $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))$.

v. $[\gamma,\gamma']$–δ–*open if* $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$.

Definition 2.6. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be:*

i. $([\gamma,\gamma'],[\beta,\beta'])$–*precontinuous if the inverse image of every* $[\beta,\beta']$–*open set of* Y *is* $[\gamma,\gamma']$–*preopen in* X.

ii. $([\gamma,\gamma'],[\beta,\beta'])$–$\alpha$–*continuous if the inverse image of every* $[\beta,\beta']$–*open set of* Y *is* $[\gamma,\gamma']$–α–*open in* X.

iii. $([\gamma,\gamma'],[\beta,\beta'])$–*b–continuous [2] if the inverse image of every* $[\beta,\beta']$–*open set of* Y *is* $[\gamma,\gamma']$–*b–open in* X.

iv. $([\gamma,\gamma'],[\beta,\beta'])$–*semiprecontinuous if the inverse image of every* $[\beta,\beta']$–*open set of* Y *is* $[\gamma,\gamma']$–β–*open in* X.

v. $([\gamma,\gamma'],[\beta,\beta'])$–$\delta$–*continuous if the inverse image of every* $[\beta,\beta']$–*open set of* Y *is* $[\gamma,\gamma']$–δ–*open in* X.

3 $[\gamma,\gamma']$–semiopen sets

Definition 3.1. *Let* (X,τ) *be a topological space and* γ, γ' *be operations on* τ. *A subset* A *of* X *is said to be* $[\gamma,\gamma']$–*semiopen if and only if* $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. *The family of all* $[\gamma,\gamma']$–*semiopen sets of* (X,τ) *is denoted by* $\tau_{[\gamma,\gamma']}$–$SO(X)$. *Also, the family of all* $[\gamma,\gamma']$–*semiopen sets of* $(X,\tau_1,\tau_2,)$ *containing* x *is denoted by* $[\gamma,\gamma']$–$SO(X,x)$.

Example 3.2. *Let* $X = \{a,b,c\}$ *and* $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$. *Let* $\gamma : \tau \to \mathcal{P}(X)$ *and* $\gamma'' : \tau \to \mathcal{P}(X)$ *be the closure operation and* $\gamma' : \tau \to \mathcal{P}(X)$ *be the interior–closure operation. It is clear that,* $\tau_{[\gamma,\gamma']} = \tau$ *and* $\tau_{[\gamma,\gamma'']} = \{\emptyset, X\}$. *Then,* $\tau_{[\gamma,\gamma']}$–$SO(X) = \mathcal{P}(X)\backslash\{\{c\}\}$ *and* $\tau_{[\gamma,\gamma'']}$–$SO(X) = \tau_{[\gamma,\gamma'']}$.

Proposition 3.3.

i. *Every* $[\gamma,\gamma']$–α–*open set is* $[\gamma,\gamma']$–*semiopen.*

ii. *Every* $[\gamma,\gamma']$–*semiopen set is* $[\gamma,\gamma']$–*b–open.*

Proof. The proof follows from the definitions. □

Corollary 3.4.

- **i.** *Every $[\gamma,\gamma']$–semiopen set is $[\gamma,\gamma']$–δ–open.*
- **ii.** *Every $[\gamma,\gamma']$–semiopen set is $[\gamma,\gamma']$–semipreopen.*

The following examples show that the converses of Proposition 3.3 are not true in general.

Remark 3.5. *The converse of the above Theorem need not be true in general. From the Example 3.2 we have that $\{b,c\}$ is a $[\gamma,\gamma']$–semiopen set but not a $[\gamma,\gamma']$–α–open neither a $[\gamma,\gamma']$–preopen set. Also $\{a\}$ is both a $[\gamma,\gamma'\prime]$–preopen and a $[\gamma,\gamma']$–b–open set, but not a $[\gamma,\gamma'']$–semiopen set.*

Remark 3.6. *It is clear that $[\gamma,\gamma']$–semiopenness and $[\gamma,\gamma']$–preopenness are independent notions.*

Example 3.7. *Let $X=\{a,\,b,\,c\}$, and $\tau=\{\emptyset,\,X,\,\{a\},\,\{b,c\}\}$. Let $\gamma:\tau\to P(X)$ and $\gamma':\tau\to P(X)$ be operators defined as follows: for every $A\in\tau$,*

$$\gamma(A) = \begin{cases} \mathrm{Cl}(A) & \text{if } A=\{a\}, \\ A & \text{if } A\neq\{a\}, \end{cases}$$
$$\gamma'(A) = \begin{cases} \mathrm{Cl}(A) & \text{if } A=\{b\}, \\ A & \text{if } A\neq\{b\}. \end{cases}$$

Then, $\{b\}$ is a (γ,γ')–semipreopen set but not a (γ,γ')–semiopen set.

Theorem 3.8. *Let γ and γ' be operations on τ and $\{A_\alpha\}_{\alpha\in\Delta}$ be the collection of $[\gamma,\gamma']$–semiopen sets of (X,τ). Then, $\underset{\alpha\in\Delta}{\cup} A_\alpha$ is also a $[\gamma,\gamma']$–semiopen set.*

Proof. Since each A_α is $[\gamma,\gamma']$–semiopen, $A_\alpha\subset \underset{\alpha\in\Delta}{\cup} A_\alpha$, this implies $\underset{\alpha\in\Delta}{\cup} A_\alpha \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\underset{\alpha\in\Delta}{\cup} A_\alpha))$. Hence, $\underset{\alpha\in\Delta}{\cup} A_\alpha$ is also a $[\gamma,\gamma']$–semiopen set in (X,τ). □

Remark 3.9. *If A and B are two $[\gamma,\gamma']$–semiopen sets in (X,τ), then $A\cap B$ need not be $[\gamma,\gamma']$–semiopen in (X,τ). From the Example 3.2, $\{a,c\}$ and $\{b,c\}$ are $[\gamma,\gamma']$–semiopen sets in (X,τ) but $A\cap B=\{c\}$ is not a $[\gamma,\gamma']$–semiopen set.*

Proposition 3.10. *Let (X, τ) be a topological space and γ, γ' be operations on τ. A subset A of X is $[\gamma, \gamma']$–semiopen if and only if $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) = \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$.*

Proof. Let $A \in \tau_{[\gamma,\gamma']}$–$SO(X)$. Then we have $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. Then $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$ and hence $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) = \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. The converse is obvious. □

Corollary 3.11. *If A is a nonempty $[\gamma, \gamma']$–semiopen set, then $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A) \neq \emptyset$.*

Proof. Since A is $[\gamma, \gamma']$–semiopen, by Proposition 3.10, we have $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) = \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. Suppose $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A) = \emptyset$. Then we have $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) = \emptyset$ and hence $A = \emptyset$. This contradicts the hypothesis. Therefore, $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A) \neq \emptyset$. □

Proposition 3.12. *Let (X, τ) be a topological space and γ, γ' be operations on τ. Then A is $[\gamma, \gamma']$–semiopen if and only if there exists $U \in \tau_{[\gamma,\gamma']}$ such that $U \subset A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U)$.*

Proof. Let $A \in \tau_{[\gamma,\gamma']}$–$SO(X)$. Then we have $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. Take $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A) = U$. Then $U \subset A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U)$. Conversely, let U be a $[\gamma, \gamma']$–open set such that $U \subset A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U)$. Since $U \subset A$, $U \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A)$, we have $\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. Thus, we obtain $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. □

Proposition 3.13. *Let (X, τ) be a topological space and γ, γ' be operations on τ. If A is a $[\gamma, \gamma']$–semiopen set in a topological space (X, τ) and $A \subset B \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)$, then B is a $[\gamma, \gamma']$–semiopen set in (X, τ).*

Proof. Since A is $[\gamma, \gamma']$–semiopen, there exists a $\tau_{[\gamma,\gamma']}$–open set U such that $U \subset A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U)$. Then we have $U \subset A \subset B \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U)) = \tau_{(\gamma,\gamma')}$–$\mathrm{Cl}(U)$ and hence $U \subset B \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(U)$. By Proposition 3.12, we obtain $B \in [\gamma, \gamma']$–$SO(X)$. □

Theorem 3.14. *Let (X, τ) be a topological space and γ, γ' be operations on τ. Then a subset A of X is $[\gamma, \gamma']$–semiopen if and only if it is both $[\gamma, \gamma']$–δ–open and $[\gamma, \gamma']$–semipreopen.*

Proof. Let A be a $[\gamma, \gamma']$–semiopen set. Then, we have $A \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A)) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))$. This shows that A is $[\gamma, \gamma']$–semipreopen. Moreover, $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A) \subset \tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(A))$. Therefore, A is $[\gamma, \gamma']$–δ–open. Conversely, let A be a $[\gamma, \gamma']$–δ–open and a $[\gamma, \gamma']$–semipreopen set. Then, we have $\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A)) $\subset$ $\tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int(A)). Thus we obtain that $\tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A))) $\subset$ $\tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int(A)). Since A is $[\gamma,\gamma']$–semipreopen, we have $A \subset \tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl($A$))) $\subset$ $\tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int(A)) and $A \subset \tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int($A$)). Hence, A is a $[\gamma,\gamma']$–semiopen set. □

Definition 3.15. *Let A be a subset of a topological space (X,τ) and γ, γ' be operations on τ. Then a subset A of X is said to be $[\gamma,\gamma']$–semiclosed if and only if $X\backslash A$ is $[\gamma,\gamma']$–semiopen.*

Remark 3.16. *The set of all $[\gamma,\gamma']$–semiclosed sets of a topological space (X,τ) is denoted by $\tau_{[\gamma,\gamma']}$–$SC(X)$.*

Theorem 3.17. *A subset A of X is $[\gamma,\gamma']$–semiclosed if and only if $\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A)) $\subset A$.*

Proof. Since $A \in \tau_{[\gamma,\gamma']}$–$SC(X)$, we have$X\backslash A \in \tau_{[\gamma,\gamma']}$–$SO(X)$. Hence, $X\backslash A \subset \tau_{[\gamma,\gamma']}$–Cl($\tau_{[\gamma,\gamma']}$–Int($X\backslash A$)) $=$ $X\backslash(\tau_{[\gamma,\gamma']}$ – Int($\tau_{[\gamma,\gamma']}$ – Cl(A))). Therefore, we obtain $\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A)) $\subset A$. The converse is clear. □

Theorem 3.18. *A subset A of X is $[\gamma,\gamma']$–semiclosed if and only if there exists a $[\gamma,\gamma']$–closed set F such that $\tau_{[\gamma,\gamma']}$–Int(F) $\subset A \subset F$.*

Proof. Suppose that A is $[\gamma,\gamma']$–semiclosed. Then by Theorem 3.17, $\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A)) $\subset A$. Let $F = \tau_{[\gamma,\gamma']}$–Cl($A$), then F is a $[\gamma,\gamma']$–closed set such that $\tau_{[\gamma,\gamma']}$–Int(F) $\subset A \subset F$. Conversely, let F be a $[\gamma,\gamma']$–closed set such that $\tau_{[\gamma,\gamma']}$–Int(F) $\subset A \subset F$. But $F \supset \tau_{[\gamma,\gamma']}$–Cl($A$), so $\tau_{[\gamma,\gamma']}$–Int(F) $\supset$ $\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A)). Hence, $\tau_{[\gamma,\gamma']}$–Int($\tau_{[\gamma,\gamma']}$–Cl(A)) $\subset A$. Therefore, A is $[\gamma,\gamma']$–semiclosed. □

Proposition 3.19. *Let (X,τ) be a topological space and γ, γ' be operations on τ. If a subset A of X is $[\gamma,\gamma']$–semipreclosed and $[\gamma,\gamma']$–δ–open, then it is $[\gamma,\gamma']$–semiclosed.*

Proof. The proof follows from the definitions. □

Theorem 3.20. *Let (X,τ) be a topological space and γ, γ' be operations on τ. Arbitrary intersections of $[\gamma,\gamma']$–semiclosed sets are always $[\gamma,\gamma']$–semiclosed.*

Proof. Follows from Theorems 3.8 and 3.17. □

Definition 3.21. *Let A be a subset of a topological space (X,τ) and γ, γ' be operations on τ. Then*

i. *the* $\tau_{[\gamma,\gamma']}$*–semiclosure of* A *is defined as the intersection of all* γ*–semiclosed sets containing* A*. That is,* $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A) = \bigcap\{F : F$ *is* $[\gamma,\gamma']$*–semiclosed and* $A \subset F\}$.

ii. *the* $\tau_{[\gamma,\gamma']}$*–semiinterior of* A *is defined as the union of all* $[\gamma,\gamma']$*–semiopen sets contained in* A*. That is,* $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A) = \bigcup\{U : U$ *is* $[\gamma,\gamma']$*–semiopen and* $U \subset A\}$.

The proof of the following theorem is obvious and therefore is omitted.

Theorem 3.22. *Let* A *be a subset of a topological space* (X,τ) *and* γ, γ' *be operations on* τ*. Then*

i. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A)$ *is the largest* $[\gamma,\gamma']$*–semiopen subset of* X *contained in* A.

ii. A *is* $[\gamma,\gamma']$*–semiopen if and only if* $A = \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A)$.

iii. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A)) = \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A)$.

iv. *If* $A \subset B$*, then* $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(B)$.

v. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A\cap B) = \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A) \cap \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(B)$.

vi. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A\cup B) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A) \cup \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(B)$.

vii. $x \in \tau_{[\gamma,\gamma']}\,\mathrm{Cl}(A)$ *if and only if* $U \cap A \neq \emptyset$ *for every* $U \in [\gamma,\gamma']$*–*$SO(X,x)$.

viii. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A)$ *is the smallest* $[\gamma,\gamma']$*–semiclosed subset of* X *containing* A.

ix. A *is* $[\gamma,\gamma']$*–semiclosed if and only if* $A = \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A)$.

x. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A) = \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A)$.

xi. *If* $A \subset B$*, then* $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(B)$.

xii. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A\cup B) = \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A) \cup \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(B)$.

xiii. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A\cap B) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A) \cap \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(B)$.

xiv. $(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(X\backslash A) = X\backslash\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(A)$;

xv. $\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Cl}(X\backslash A) = X\backslash\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(A)$.

Definition 3.23. *Let (X,τ) be a topological space. An operation γ is said to be regular[1] if for every open neighborhood U and V of each $x\in X$ there exists an open neighborhood W of x such that $W^\gamma\subset U^\gamma\cap V^\gamma$.*

Theorem 3.24. *Let (X,τ) be a topological space, γ, γ' regular operations on τ and A a subset of X. Then the following holds:*

i. $A=\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A)$.

ii. $\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A))\subset A$.

iii. $(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))\backslash(X\backslash(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A))))$ $\supset(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\backslash A)$.

Proof.

(**i**) $\Rightarrow$ (**ii**). If $A=\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A)$, then $\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A))=\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(A)\subset A$.

(**ii**) $\Rightarrow$ (**iii**). Suppose $\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A))\subset A$. Now $\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A)$ is a $[\gamma,\gamma']$–semiclosed set. So, by Proposition 3.18, there is a $[\gamma,\gamma']$–closed set F such that $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F)\subset\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A)\subset F$. Since $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F)$ is $[\gamma,\gamma']$–semiopen, $\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F))=\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F)$. Therefore, $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F)=\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}((\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F))\subset\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A))$ and hence $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F)\subset A$. But $A\subset\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A)\subset F$. Thus, $\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(F)\subset A\subset F$, where F is $[\gamma,\gamma']$–closed.

(**iii**) $\Leftrightarrow$ (**i**). We have $(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))\backslash(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))))\supset(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\backslash A)\Leftrightarrow\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))\backslash(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))))\subset A\Leftrightarrow\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\cap(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))\backslash(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))))\subset A\Leftrightarrow\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\cap(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))\cap(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)))))\subset A\Leftrightarrow\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\cap((X\backslash(\tau_{[\gamma,\gamma']}$– $\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}-\mathrm{Cl}(A))))\cup(X\backslash(\tau_{[\gamma,\gamma']}-\mathrm{Cl}(A)))))\subset A\Leftrightarrow\tau_{[\gamma,\gamma']}-\mathrm{Cl}(A)\cap((X\backslash(\tau_{[\gamma,\gamma']}-\mathrm{Cl}(X\backslash(\tau_{[\gamma,\gamma']}-\mathrm{Cl}(A))))\cup(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\cap(X\backslash(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))))))\subset A\Leftrightarrow\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A)\cap\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))\subset A\Leftrightarrow\tau_{[\gamma,\gamma']}$–$\mathrm{Int}(\tau_{[\gamma,\gamma']}$–$\mathrm{Cl}(A))\subset A$. A is $[\gamma,\gamma']$–semiclosed $\Leftrightarrow A=\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(A)$.

□

Definition 3.25. *Let (X,τ) be a topological space and γ, γ' be operations on τ. A subset B_x of X is said to be a $[\gamma,\gamma']$–semineighborhood of a point $x\in X$ if there exists a $[\gamma,\gamma']$–semiopen set U such that $x\in U\subset B_x$.*

Theorem 3.26. *Let (X, τ) be a topological space and γ, γ' be operations on τ. A subset of (X, τ) is $[\gamma, \gamma']$–semiopen if and only if it is a $[\gamma, \gamma']$–semineighborhood of each of its points.*

Proof. Let G be a $[\gamma, \gamma']$–semiopen set of X. Then, by definition, it is clear that G is a $[\gamma, \gamma']$–semineighborhood of each of its points, since for every $x \in G$, $x \in G \subset G$ and G is $[\gamma, \gamma']$–semiopen. Conversely, suppose that G is a $[\gamma, \gamma']$–semineighborhood of each of its points. Then for each $x \in G$, there exists $S_x \in [\gamma, \gamma']\text{–}SO(X)$ such that $S_x \subset G$. Then, $G = \bigcup\{S_x : x \in G\}$. Since each S_x is $[\gamma, \gamma']$–semiopen, G is $[\gamma, \gamma']$–semiopen in (X, τ). $\square$

4 $([\gamma, \gamma'], [\beta, \beta'])$–semicontinuous functions

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $([\gamma, \gamma'], [\beta, \beta'])$–semicontinuous if for each $x \in X$ and each $[\beta, \beta']$–open set V containing $f(x)$ there exists a $[\gamma, \gamma']$–semiopen set U such that $x \in U$ and $f(U) \subset V$.*

Proposition 4.2.

i. *Every $([\gamma, \gamma'], [\beta, \beta'])$–$\alpha$–continuous function is $([\gamma, \gamma'], [\beta, \beta'])$–semicontinuous but not conversely.*

ii. *Every $([\gamma, \gamma'], [\beta, \beta'])$–semicontinuous function is $([\gamma, \gamma'], [\beta, \beta'])$–$b$–continuous but not conversely.*

iii. *$([\gamma, \gamma'], [\beta, \beta'])$–semicontinuity and $([\gamma, \gamma'], [\beta, \beta'])$–precontinuity are independent.*

Proof. The proof follows from Proposition 3.3, Examples 3.2 and 3.7. $\square$

Corollary 4.3.

i. *Every $([\gamma, \gamma'], [\beta, \beta'])$–semicontinuous function is $([\gamma, \gamma'], [\beta, \beta'])$–$\delta$–continuous but not conversely.*

ii. *Every $([\gamma, \gamma'], [\beta, \beta'])$–semicontinuous function is $([\gamma, \gamma'], [\beta, \beta'])$–semiprecontinuous but not conversely.*

Theorem 4.4. *For a function $f : (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:*

i. *f is $([\gamma,\gamma'],[\beta,\beta'])$–semicontinuous;*

ii. *The inverse image of each $[\beta,\beta']$–closed set in Y is $[\gamma,\gamma']$–semiclosed in X;*

iii. *For each subset A of X, $f(\tau_{[\gamma,\gamma']}\text{-}s\,\mathrm{Cl}(A)) \subset \sigma_{[\beta,\beta']}\text{-}\mathrm{Cl}(f(A))$;*

iv. *For each subset B of Y, $\tau_{[\gamma,\gamma']}\text{-}s\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_{[\beta,\beta']}\text{-}\mathrm{Cl}(B))$;*

v. *For each subset C of Y, $f^{-1}(\sigma_{[\beta,\beta']}\text{-}\mathrm{Int}(C)) \subset \tau_{[\gamma,\gamma']}\text{-}s\,\mathrm{Int}(f^{-1}(C))$.*

Proof. The proof is obvious. □

Theorem 4.5. *Let $f:(X,\tau)\to(Y,\sigma)$ be a $([\gamma,\gamma'],[\beta,\beta'])$–semicontinuous function. Then, for each subset V of Y, $f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(V)) \subset \tau_{[\gamma,\gamma']}\text{–}\mathrm{Cl}(\tau_{[\gamma,\gamma']}\text{–}\mathrm{Int}(f^{-1}(V)))$.*

Proof. Let V be any subset of Y. Then $\sigma_{[\beta,\beta']}$–$\mathrm{Int}(V)$ is $[\beta,\beta']$–open in Y and so $f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(V))$ is $[\gamma,\gamma']$–semiopen in X. Hence, $f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(V)) \subset \tau_{[\gamma,\gamma']}\text{–}\mathrm{Cl}(\tau_{[\gamma,\gamma']}\text{–}\mathrm{Int}(f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(V)))) \subset \tau_{[\gamma,\gamma']}\text{–}\mathrm{Cl}(\tau_{[\gamma,\gamma']}\text{–}\mathrm{Int}(f^{-1}(V)))$. □

Corollary 4.6. *Let $f:(X,\tau)\to(Y,\sigma)$ be a $([\gamma,\gamma'],[\beta,\beta'])$–semicontinuous function. Then, for each subset V of Y, $\tau_{[\gamma,\gamma']}\text{–}\mathrm{Int}(\tau_{[\gamma,\gamma']}\text{–}\mathrm{Cl}(f^{-1}(V))) \subset f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Cl}(V))$.*

Theorem 4.7. *Let $f:(X,\tau)\to(Y,\sigma)$ be a bijective function. Then, f is $([\gamma,\gamma'],[\beta,\beta'])$–semicontinuous if and only if $\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(f(U)) \subset f(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(U))$ for each subset U of X.*

Proof. Let U be any subset of X. Then, by Theorem 4.4, $f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(f(U))) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(f^{-1}(f(U)))$. Since f is a bijective function, $\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(f(U)) = f(f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(f(U))) \subset f(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(U))$. Conversely, let V be any subset of Y. Then, $\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(f(f^{-1}(V))) \subset f(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(f^{-1}(V)))$. Since f is a bijection, $\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(V) = \sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(f(f^{-1}(V))) \subset f(\tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(f^{-1}(V)))$. Hence, $f^{-1}(\sigma_{[\beta,\beta']}\text{–}\mathrm{Int}(V) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\mathrm{Int}(f^{-1}(V))$. Therefore, by Theorem 4.4, f is (α,β)–semicontinuous. □

Definition 4.8. *A function $f:(X,\tau)\to(Y,\sigma)$ is said to be $([\gamma,\gamma'],[\beta,\beta'])$–irresolute if the inverse image of every $[\beta,\beta']$–semiopen set of Y is $[\gamma,\gamma']$–semiopen in X.*

Proposition 4.9. *Every $([\gamma,\gamma'],[\beta,\beta'])$–irresolute function is $([\gamma,\gamma'],[\beta,\beta'])$–semicontinuous but not conversely.*

Proof. Straightforward. □

Theorem 4.10. *If* $f : (X,\tau) \to (Y,\sigma)$ *is* $([\gamma,\gamma'],[\beta,\beta'])$*–continuous and* $f^{-1}(\sigma_{[\beta,\beta']}\text{–Cl}(V)) \subset \tau_{[\gamma,\gamma']}\text{–Cl}(f^{-1}(V))$ *for each* $V \in \sigma_{[\beta,\beta']}$*, then* f *is* $([\gamma,\gamma'],[\beta,\beta'])$*–irresolute.*

Proof. Let B be any (i,j)–semiopen subset of Y. By Proposition 3.12, there exists $V \in \sigma_{[\beta,\beta']}$ such that $V \subset B \subset \tau_{[\gamma,\gamma']}\text{–Cl}(V)$. Therefore, we have $f^{-1}(V) \subset f^{-1}(B) \subset f^{-1}(\tau_{[\gamma,\gamma']}\text{–Cl}(V)) \subset \tau_{[\gamma,\gamma']}\text{–Cl}(f^{-1}(V))$. Since f is $([\gamma,\gamma'],[\beta,\beta'])$–continuous and $V \in \sigma_{[\beta,\beta']}$, $f^{-1}(V)$ is a $[\gamma,\gamma']$–semiopen set of X. Hence, $f^{-1}(B)$ is a $[\gamma,\gamma']$–semiopen set of X. This shows that f is $([\gamma,\gamma'],[\beta,\beta'])$–irresolute. □

Theorem 4.11. *For a function* $f : (X,\tau) \to (Y,\sigma)$ *the following statements are equivalent:*

- **i.** f *is* $([\gamma,\gamma'],[\beta,\beta'])$*–irresolute;*
- **ii.** *the inverse image of each* $[\beta,\beta']$*–semiclosed subset of* Y *is* $[\gamma,\gamma']$*–semiclosed in* X*;*
- **iii.** *for each* $x \in X$ *and each* $[\beta,\beta']$*–semiopen set* V *of* Y *containing* $f(x)$*, there exists a* $[\gamma,\gamma']$*–semiopen set* U *of* X *containing* x *such that* $f(U) \subset V$*.*

Proof. The proof is obvious from the fact that the arbitrary union of $[\gamma,\gamma']$–semiopen subsets is $[\gamma,\gamma']$–semiopen. □

Theorem 4.12. *Let* $f : (X,\tau) \to (Y,\sigma)$ *be a function. Then:*

- **i.** f *is* $([\gamma,\gamma'],[\beta,\beta'])$*–irresolute;*
- **ii.** $\tau_{[\gamma,\gamma']}\text{–}s\,\text{Cl}(f^{-1}(V)) \subset f^{-1}(\sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(V))$ *for each subset* V *of* Y*;*
- **iii.** $f(\tau_{[\gamma,\gamma']}\text{–}s\,\text{Cl}(U) \subset \sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(f(U))$ *for each subset* U *of* X*.*

Proof.

(**i**) ⇒ (**ii**). Let V be any subset of Y. Then $V \subset \sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(V)$ and $f^{-1}(V) \subset f^{-1}(\sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(V))$. Since f is $([\gamma,\gamma'],[\beta,\beta'])$–irresolute, $f^{-1}(\sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(V))$ is a $[\gamma,\gamma']$–semiclosed subset of X. Hence, $\tau_{[\gamma,\gamma']}\text{–}s\,\text{Cl}(f^{-1}(V)) \subset \tau_{[\gamma,\gamma']}\text{–}s\,\text{Cl}(f^{-1}(\sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(V))) = f^{-1}(\sigma_{[\beta,\beta']}\text{–}s\,\text{Cl}(V))$.

(ii) $\Rightarrow$ **(iii)**. Let U be any subset of X. Then $f(U) \subset \sigma_{[\beta,\beta']}$–$s\,\mathrm{Cl}(f(U))$ and $\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(U) \subset \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(f^{-1}(f(U))) \subset f^{-1}(\sigma_{[\beta,\beta']}$–$s\,\mathrm{Cl}(f(U)))$. This implies $f(\tau_{[\gamma,\gamma']})$–$s\,\mathrm{Cl}(U)) \subset f(f^{-1}(\sigma_{[\beta,\beta']}$–$s\,\mathrm{Cl}(f(U)))) \subset (i,j) - s\,\mathrm{Cl}(f(U))$.

(iii) $\Rightarrow$ **(i)**. Let V be a (i,j)–semiclosed subset of Y. Then, $f(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(f^{-1}(V)) \subset \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(f^{-1}(f(V))) \subset \sigma_{[\beta,\beta']} - s\,\mathrm{Cl}(V) = V$. This implies $\tau_{[\gamma,\gamma']} - s\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(f(\tau_{[\gamma,\gamma']})) - s\,\mathrm{Cl}(f^{-1}(V)))) \subset f^{-1}(V)$. $f^{-1}(V)$ is a $[\gamma,\gamma']$–semiclosed subset of X and consequently f is a $([\gamma,\gamma'],[\beta,\beta'])$–irresolute function.

□

Theorem 4.13. *A function $f : (X,\tau) \to (Y,\sigma)$ is $([\gamma,\gamma'],[\beta,\beta'])$–irresolute if and only if $f^{-1}(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(V)) \subset \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(f^{-1}(V))$ for each subset V of Y.*

Proof. Let V be any subset of Y. Then, $\sigma_{[\beta,\beta']}$–$s\,\mathrm{Int}(V) \subset V$. Since f is $([\gamma,\gamma'],[\beta,\beta'])$–irresolute, $f^{-1}(\sigma_{[\beta,\beta']}$–$s\,\mathrm{Int}(V))$ is a $[\gamma,\gamma']$–semiopen subset of X. Hence, $f^{-1}(\sigma_{[\beta,\beta']}$–$s\,\mathrm{Int}(V)) = \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(f^{-1}(\sigma_{[\beta,\beta']}$–$s\,\mathrm{Int}(V))) \subset \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(f^{-1}(V))$. Conversely, let V be a (i,j)–semiopen subset of Y. Then, $f^{-1}(V) = f^{-1}(\sigma_{[\beta,\beta']}$–$s\,\mathrm{Int}(V)) \subset \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Int}(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is a $[\gamma,\gamma']$–semiopen subset of X and consequently f is a $([\gamma,\gamma'],[\beta,\beta'])$–irresolute function. □

Corollary 4.14. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function. Then, f is $([\gamma,\gamma'],[\beta,\beta'])$–closed and $([\gamma,\gamma'],[\beta,\beta'])$–irresolute if and only if $f(\tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(V)) = \sigma_{[\beta,\beta']}$–$s\,\mathrm{Cl}(f(V)))$ for every subset V of X.*

Corollary 4.15. *Let $f : (X,\tau) \to (Y,\sigma)$ be a function. Then f is $([\gamma,\gamma'],[\beta,\beta'])$–open and $([\gamma,\gamma'],[\beta,\beta'])$–irresolute if and only if $f^{-1}((\sigma_{[\beta,\beta']}$–$s\,\mathrm{Cl}(V))) = \tau_{[\gamma,\gamma']}$–$s\,\mathrm{Cl}(f^{-1}((V)))$ for every subset V of Y.*

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