

Back-propagation artificial neural networks in stock market forecasting. An application to the Warsaw Stock Exchange WIG20

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Abstract

Stock market forecasting plays a key role in investment practice and theory, especially given the progress made in developing automated trading systems for use in capital markets. Traditional techniques such as statistical analysis, fundamental analysis and technical analysis are no longer considered the best options in this field and artificial neural networks have gained in popularity, especially for use in stock market forecasting. The back-propagation algorithm is one of the most popular neural network training algorithms in financial time series prediction. One of the biggest problems regarding the use of neural networks trained with the back-propagation algorithm is the determination of both the number of hidden layers and the number of neurons included in each hidden layer. In this article, we present three competing architectures of a feed-forward network, all with only one hidden layer but differing in the number of neurons included in that layer. The three models will be trained with the back-propagation algorithm, in order to determine which one provides the best forecasting performance. Data on the WIG20 (Capitalization-Weighted Stock Market Index of the 20 largest companies on the Warsaw Stock Exchange) are used to evaluate the performance of the competing architectures. The results obtained show that the architecture consisting of an input layer with N neurons, one hidden layer with $3/2(N+1)$ neurons and an output layer with one neuron outperforms the other two models.

Keywords: Artificial neural network, Back propagation algorithm, Stock market forecasting. **JEL classification:** C45.

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Redes neuronales artificiales retropropagación para la predicción en mercados de valores. Una Aplicación al índice de la Bolsa de Varsovia (WIG20)

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Resumen

La predicción en los mercados de valores juega un papel crucial en la teoría y práctica de la inversión, sobre todo tras los progresos que han tenido lugar en el desarrollo de sistemas de negociación automatizados en los mercados de capitales. Las técnicas tradicionales, como el análisis estadístico, el análisis fundamental y el análisis técnico, ya no son consideradas como las mejores opciones en el terreno de la predicción financiera y las redes neuronales artificiales han ganado popularidad, sobre todo en ámbito de la predicción bursátil. El algoritmo de retropropagación es uno de los sistemas de entrenamiento de redes neuronales más populares cuando de predicción financiera se trata. Uno de los mayores problemas en lo que se refiere al uso de redes neuronales entrenadas con un algoritmo de retropropagación es la determinación tanto del número de capas ocultas como del número de neuronas en cada una de dichas capas. En este artículo se presentan tres arquitecturas de una red neuronal con conexiones hacia adelante (*feed-forward*) y con una única capa oculta que difieren en el número de neuronas que contiene dicha capa oculta. Estas tres redes, entrenadas con un algoritmo de retropropagación, compiten en cuanto a su capacidad para predecir el comportamiento del Índice de la Bolsa de Varsovia (WIG20, que incluye las veinte mayores compañías cotizadas en la Bolsa de Varsovia). Los resultados obtenidos muestran que la arquitectura consistente en una capa de entrada con N neuronas, una capa oculta con $3/2(N+1)$ neuronas y una capa de salida con una neurona es la que proporciona las mejores predicciones.

Palabras clave:

Redes neuronales artificiales, algoritmo de retropropagación, predicción bursátil.

■ 1. Introduction

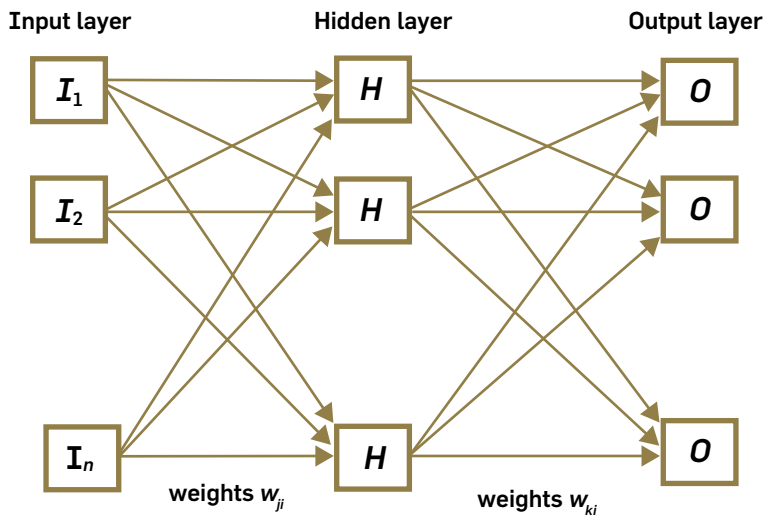
Stock market forecasting plays a key role in investment theory and practice, especially given the progress made in developing automated trading systems for use in capital markets. A capital market is a complex system that changes over time and that is influenced by many factors. Fluctuations in the capital market often indicate non-linear relationships between the response variable and the covariates that supposedly explain the changes in such a response. One of the key decision-making problems in the investment process is stock market forecasting, and so this presents one of the biggest challenges to the scientific community. Stock market prediction is a difficult task, primarily because of the uncertainties involved in the movement of the stock market (Choudhry and Garg, 2008), and so it requires continuous improvement of forecasting models.

Various conventional techniques such as statistical analysis, fundamental analysis and technical analysis have all been used for forecasting purposes in stock markets, but they are difficult to implement and none of them have provided the expected results.

An artificial neural network (ANN) is a large-scale, nonlinear dynamic system (Han, 2002), which is capable of performing highly nonlinear operations, self-learning, and self-organizing (Liu and Wang, 2011). ANN is regarded as more suitable for stock market forecasting than other techniques, since it is able to detect and learn patterns or relationships from the data itself (Lam, 2004; Lee and Chiu, 2002; Lee and Chen, 2002; Sun *et al.*, 2008; Leigh *et al.*, 2002). In practice, feed-forward neural networks are usually used in forecasting (see Figure 1). In feed-forward ANNs, the information moves in only one direction — forward — from the input nodes, through the hidden nodes (if any) and to the output nodes. There are no cycles or loops in the network. The most popular neural network training algorithm for financial forecasting is the backward propagation of errors or back-propagation (BP) algorithm, which demonstrates powerful problem-solving ability (Werbos, 1988; Rumelhart, 1995). As is well known, back-propagation is used in conjunction with an optimization method (usually the gradient descent method). The algorithm repeats a two-phase cycle: propagation and updating the weights of the neurons connections. When an input vector containing observed information is presented to the network, it is propagated forward through the network, layer by layer, until it reaches the output layer. The output of the network is then compared to the desired output (also observed), using a loss function, and the error value is calculated for each of the neurons in the output layer. These error values are then propagated backwards, starting from the output, until each neuron has an associated error value which roughly represents its contribution to the original output. Back-propagation uses these error values to calculate the gradient of the loss function with respect to the weights in the network. In the second phase, this gradient is fed to the optimization

method, which in turn uses it to update the weights, in an attempt to minimize the loss function. The three-layer feed-forward neural network is considered appropriate for prediction in the financial field due to both its simplicity and capability to approximate any complex continuous function. However, two problems have to be faced when using ANN: (i) selecting the number of hidden layers, and (ii) determining how many neurons should be included in each hidden layer.

■ **Figure 1. Feed-forward ANN with one hidden layer**



SOURCE: OWN ELABORATION BASED ON CILIMKOVIC (2011).

In practice, there is no reason to use more than one hidden layer in financial time series forecasting (Kaastra and Boyd, 1996). In addition, using too few neurons in the hidden layer will result in underfitting problems. On the contrary, using too many neurons in the hidden layer can result in several problems including overfitting and an increase in the time it takes to train the network, among others. Basically, the determination of the number of neurons in the hidden layer will come down to trial and error, although three main rules should be borne in mind when choosing the proper number of neurons in the hidden layer. They are as follows (Heaton, 2008):

- The number of neurons in the hidden layer should be between the number of neurons in the input layer and the number of neurons in the output layer.
- The number of neurons in the hidden layer should be $2/3$ the number of neurons in the input layer plus the number of neurons in the output layer.
- The number of neurons in the hidden layer should be less than twice the number of neurons in the input layer.

There have been many studies on stock market forecasting using ANN over the past three decades (White, 1989; Kimoto *et al.*, 1990; Trippi and DeSieno, 1992; Duke, 1993; Nikolopoulos and Fellrath, 1994; Azoff, 1994; Hiemstra, 1995; Hall, 1994; Kohara *et al.*, 1997; Aiken and Bsath, 1999; Garliauskas, 1999; Romahi and Shen, 2000; Chan *et al.*, 2000; Hwang, 2001; Thawornwong, 2005; Badaway *et al.*, 2006; Atsalakis *et al.*, 2009; Boyacioglu and Avci, 2010; Guresen *et al.*, 2011; Asadi, 2012; Adebisi *et al.*, 2012; Ticknor, 2013; Mantri *et al.*, 2014).

This article compares the forecasting performance of three competing architectures of a feedforward ANN with only one hidden layer, which meet the criteria set out by Heaton (2008). A BP mechanism was implemented for training the corresponding ANN. In order to evaluate their forecasting performance, data on WIG20 were used as an illustrative example. The empirical results show that an architecture consisting of an input layer with N neurons, one hidden layer with $2/3(N+1)$ neurons and an output layer with one neuron provide more accurate predictions than the other two competing models.

The article is organized as follows. After this introductory section, section 2 is devoted to methodological questions, section 3 contains the prediction results provided by the competing ANNs for the WIG20 returns, and section 4 concludes.

■ 2. Research methodology

BP is one of the most popular ANN training algorithms for financial time series forecasting. A back-propagation artificial neural network (BP-ANN) is characterized by the back-propagation of errors (Kao *et al.*, 2012; Lee, 2009; Lu, 2010). BP uses the gradient descent method for minimizing the error function in the weights space. The error function has to be continuous and differentiable because this training method requires the computation of the gradient of the error function in each iteration (Benvenuto and Piazza, 1992). The combination of weights which minimizes the error function is considered a solution of the learning problem.

The training of a BP-ANN is as follows (Liu and Wang, 2011):

Let $Inp(n)$ and $Out(n)$ denote the input and output corresponding to the node n respectively, where:

$$Inp_n = \sum_m w_{mm} Out_m \tag{1}$$

$$Out_n = f(Inp_n + \beta_n) , \tag{2}$$

with w_{mm} being the weight of the connection from the m th node in the previous layer to the node n , and

$$f(\text{net}) = (2/1 - e^{-2\text{net}}) - 1 \quad (3)$$

being the activation function of the nodes ('net' represents the neuron's net input), which introduces non-linearity into the output of the neuron (one of the most popular activation functions in BP-ANNs is the sigmoid function), while β_n is the bias input to the node, that is, a connection weight from a special unit with a constant, nonzero activation value.

The error E in the output is calculated as follows:

$$E = \frac{1}{2N} \sum_N \sum_o (P_{No} - Out_{No})^2, \quad (4)$$

where N and o denote the number of elements in the training set and the number of neurons in the output layer, respectively, while P_{No} and Out_{No} represent the target and the output values, respectively. The training ends when the error E falls below the threshold or tolerance level. The error e_o in the output layer and the error e_n in the hidden layer are calculated as follows:

$$e_o = \lambda(P_o - Out_o) f'(Out_o) \quad (5)$$

$$e_n = \lambda \sum_m e_o w_{nm} f'(Out_n), \quad (6)$$

where P_o , Out_o , Out_n and λ represent- the expected output of the o th output node, the actual output in the output layer, the actual output value in the hidden layer and the adjustable variable in the activation function, respectively. Note that f' denotes the derivative of f . The weights and biases in both the output and hidden layers are updated through error back-propagation. The weights w_{nm} and $\beta(n)$ biases are adjusted according to the following equations:

$$w_{nm}(k+1) = w_{nm}(k) + \gamma e_o Out_m \quad (7)$$

$$\beta_m(k+1) = \beta_m(k) + \gamma e_m \quad (8)$$

where k and γ denote the epoch number and the learning rate, respectively.

■ 3. Empirical research

In the field of stock market forecasting, historical data from the last trading day, including data on daily opening price, daily highest price, daily lowest price, daily closing price and daily turnover, are commonly used to predict the closing stock price of the next

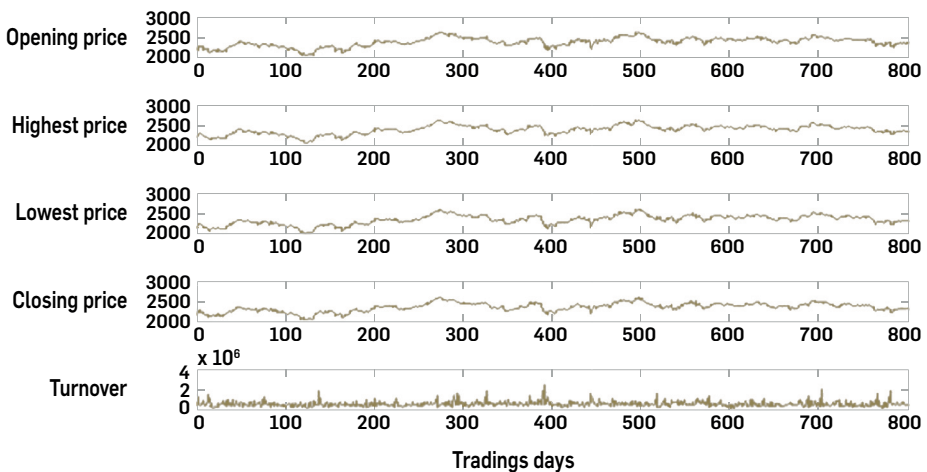
trading day (see Balachandler *et al.*, 2002; Leigh *et al.*, 2005; Atsalakis and Valavanis, 2009; Kao *et al.*, 2012). These historical indicators can be used as the input of an ANN trained with a BP algorithm, the output being the $t+1$ -closing price of the stock market.

In this article, three-layer feedforward ANNs trained with a BP algorithm and differing in the number of neurons in the hidden layer have been implemented in order to analyse their out-of-sample forecasting performance. In line with the criteria set out by Heaton (2008), the three architecture models considered, which differ according to the number of neurons in the hidden layer, are the following:

- Model I:** N neurons in the input layer, $(N-1)/2$ neurons in the hidden layer and one neuron in the output layer.
- Model II:** N neurons in the input layer, $2/3(N+1)$ neurons in the hidden layer and one neuron in the output layer.
- Model III:** N neurons in the input layer, $2N-1$ neurons in the hidden layer and one neuron in the output layer.

In order to evaluate the performance of the BP algorithm in the models presented above, five financial time series raw datasets have been compiled containing data from WIG20 on daily opening price, daily highest price, daily lowest price, daily closing price and daily turnover from 25th November 2011 to 16th February 2015. There are 800 data points in each dataset (Figure 2): the first 500 data points of the total sample have been used as the training sample while the remaining 300 data points have been retained for use as the testing sample for evaluating out-of-sample performance.

■ **Figure 2. WIG20 daily prices and turnover. 25th November 2011 to 16th February 2015**



SOURCE: AL RAJHI BANK WEBSITE SAUDI ARABIA ([HTTP://ALRAJHIBANK.COM.SA/EN/PAGES/DEFAULT.ASPX](http://alrajhibank.com.sa/en/pages/default.aspx))

The forecasting performance of the three competing models proposed has been evaluated using the following statistical criteria: Mean Square Error (MSE), Prediction Error (PE), Normalized Mean Square Error (NMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) as well as the Epoch Number (Epochs). MSE, PE, NMSE, MAE and MAPE are measures of the deviation between the actual and predicted values. Epochs represents the number of iterations required to reach the desired error. The smaller the values of MSE, NMSE, MAE, MAPE, PE and Epoch, the better the out-of-sample performance of the corresponding model.

Figures 3, 4 and 5 show the architecture of the three competing feedforward ANNs, and also provide some information on the out-of-sample PE and MSE (as well as the number of epochs) resulting from the corresponding ANN after being trained with a BP algorithm. NMSE, MAE and MAPE were calculated separately and they are listed in Table 1 together with PE and MSE.

Figure 3. Model I: architecture, out-of-sample PE and MSE, and number of epochs

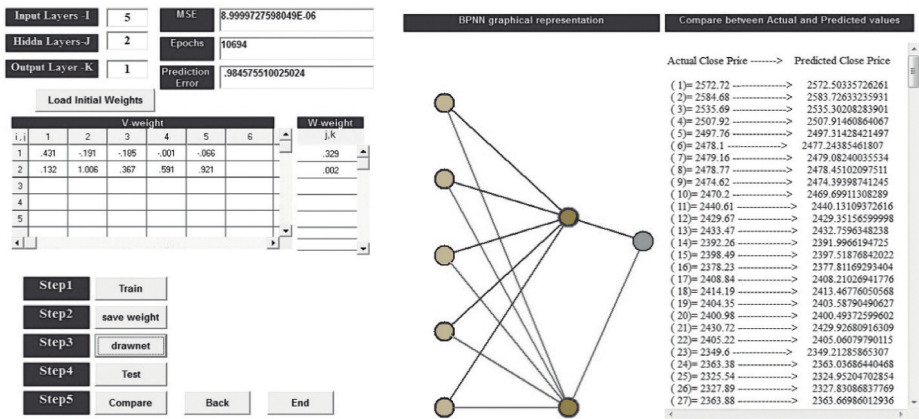


Figure 4. Model II: architecture, out-of-sample PE and MSE, and number of epochs

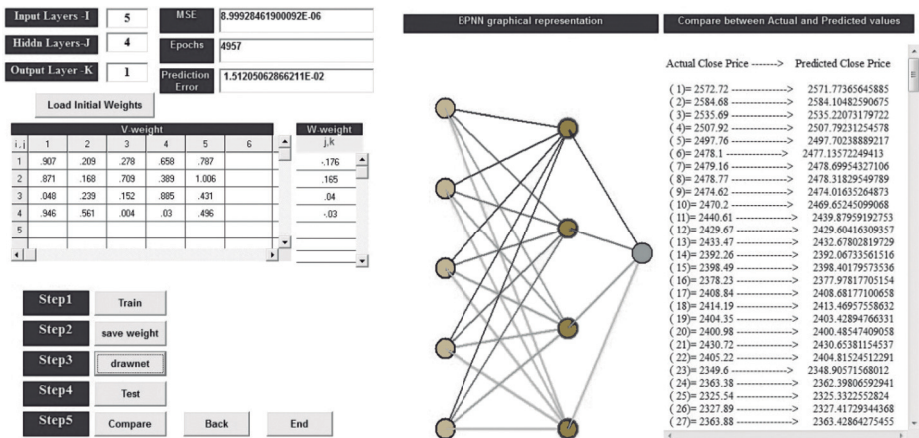


Figure 5. Model III: Architecture, out-of-sample PE and MSE, and number of epochs

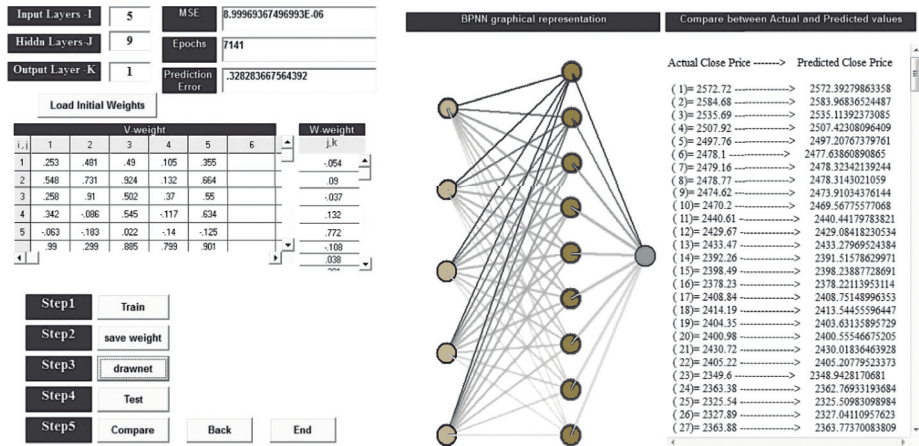


Table 1. Out-of-sample forecasting. Closing prices, 25th November 2011 to 16th February 2015

Criteria	Model I	Model II	Model III
No. Epochs	10694	4957	7141
PE	0.984555	0.015120	0.328284
MSE	8.99997E-06	8.999285E-06	8.999694E-05
MAE	0.518952	0.417958	0.489827
NMSE	9.283E-05	6.63E-05	8.33E-05
MAPE	0.000215	0.000173	0.000203

SOURCE: OWN ELABORATION

According to the results in Table 1, it can be seen that the training process of Model II took 4957 epochs, whereas the equivalent figure for Model I was more than double that, and 1.4 times greater for Model III. This significant decrease in the number of epochs leads to a significant decrease in time, which is one of the most important factors to consider when building prediction models. Consequently, after only 4957 epochs, Model II reached the desired approximation error, with a value of MSE = 8.999285E-06 and a value of PE = 0.015120. Note that these values are lower than the corresponding figures for Model I and Model III. Likewise, NMSE = 6.63E-05, MAE = 0.417958, MAPE = 0.000173, values which are also lower than those of Models I and III.

4. Conclusion

Stock market prediction is a difficult task that requires the continued improvement of forecasting models. Traditional techniques are complicated to implement and have not proved to be effective in stock market forecasting. ANN is regarded as more suit-

able for stock market forecasting than the traditional techniques and BP is one of the most popular neural network training algorithms for financial forecasting. However, there are two important decisions to make when using ANN with a BP training algorithm: selecting the number of hidden layers, and determining how many neurons should be included in each hidden layer.

This article compares the forecasting performance of three competing architectures of a feed-forward ANN with only one hidden layer. A BP mechanism was implemented for training those ANNs and, in order to evaluate their forecasting performance, data on WIG20 were used as an illustrative example. In light of the experimental results, we can draw the following conclusions:

- There is no reason to use more than one hidden layer in financial time series forecasting.
- Using too few neurons in the hidden layer will result in underfitting problems.
- Using too many neurons in the hidden layer can result in several problems, such as overfitting problems and an increase in the time it takes to train the network, among others.
- A three-layer feedforward BP algorithm topological architecture consisting of an input layer with N neurons, one hidden layer with $2/3(N+1)$ neurons, and an output layer with one neuron outperforms the other two competing architectures studied in this article since it produces smaller errors and is more accurate in stock market forecasting.

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