Quasi-relaxation transforms in metallic specimens and meromorphic curves of quasi-relaxation

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Abstract

Into the study of quasi-relaxation, in the past researches it is have concluded that the condition of meta-stability in the metallic specimen is given by the plasticity explained by the plastic energy in the process of the quasi-relaxation [18], and [22]. It is calculate through of quasi-relaxation functional of this energy to obtain a spectra in the space $D(\sigma - \varepsilon, t)$, that induced the existence of functions $\varphi(t)$, and $\Psi(t)$, related with the fundamental curves of quasi-relaxation given by $\sigma(t)$, with their poles in $t = -1/k(\sigma_0 - \sigma_1)$, the which it is get in the maximum of stress given by $\sigma_0 = \sigma_1$. Also the tensor of plastic deformation that represents the plastic load during the application of specimen machine [1], not can be obtained without poles in the space $D(\sigma, t)$, corresponding to the curves calculated in [19], into the space $D(\sigma - \varepsilon, t)$, by curves that in the kinetic of the process of quasi-relaxation are represented by experimental curves in coordinates $lg\sigma - t$ [5]. This situation not can be eluded, since in this phenomena exist dislocations that go conform fatigue in the nano-crystalline structure of metals [12]. From this point of view, is necessary to obtain a spectral study related with the energy using functions that permits the modeling and compute the states of quasi-relaxation included in the poles in the deformation problem to complete the solutions in the space $D(\sigma - \varepsilon, t)$, and try a new method of solution of the differential equations of the quasi-relaxation analysis. In a nearly future development, the information obtained by this spectral study (by our integral transforms) will can to gives place to the programming through of the spectral encoding of the materials in the meta-stability state, that which is propitious to a nano-technological transformation of materials, concrete case, some metals.

Keywords

Deformation, Energy, Generalized Functional, Quasi-relaxation, Quasi-relaxation Transforms

Introduction

In the last 30 years, the experimental technique to the characterization of materials with the use of testing machines have experimented a big heyday. In the conventional machines of essays, where the specimen previously is loaded up to a initial level of the stress, after of that which the motorize system of the machine is disconnected, it is observe a spontaneous fall of stress. The kinetic of the fall of the stress is registered during all the process of the essays.

Similar experiment must be executed in a programmed specially machine, in the which during the essay of automatic manage it stays constant the longitude of the specimen; is to say, the condition of the essay in regime of quasi-relaxation can expressed in the following form

(1) $l \ = const$

or well,

(2) $\frac{d\,\varepsilon}{t\,d} = 0$

This condition define to the meta-stability as a state of constant deformation only in their plastic characteristics in the initial process of dislocations, where the energy of the nano-crystals accumulate the enough energy to maintainer the specimen in a stable range of recovering to original state, in a very short time interval [22]. In this respect, is necessary to realize a deep study of traces of deformation tensor in function of the stress tensor corresponding of plastic deformation and use a functional of energy that measures this recover energy due the nano-crystals. This come given for [], []:

(3) $$\int_M L\varepsilon(t)d\left(\varepsilon(t)\right) = \int_0^{+\infty}\left[\int_M d\varepsilon_{PT}\right]\varphi(\tau)\,e^{-dt}d\tau$$

Into the family of integrals of energy it is deduce with help of the integrals hereditary and the trace of the plastic deformation tensor, two integral transform in the spectral study of the quasi-relaxation including the poles of quasi-relaxation functions, and the phases of the material in the meta-stability process of the material, for example the spectra before of the enter of meta-stability conditions or after of this state. Through of the spectral encoding of this measured energy we can to realize diverse actions on the materials to introduce codes of memory and intelligence to nano-metric scale, and of this manage obtain special properties of the materials to diverse technological applications. Also the determination of these pair of transforms helps to reduce the compute methods of quasi-relaxation functions, as well as the determination of density of relaxation distribution that appears in the analysis of quasi-relaxation of all metallic specimens. We will give a short table of transforms to calculate the solutions of the differential equations of the quasi-relaxation in material engineering to more canonical loads.

Materials and methods

Functional of Plastic Energy that Promote the Dislocations

The study of the resultant energy due to the meta-stables conditions that it is obtains in the quasi-relaxation phenomena establishes clearly their plastic nature for the suffered deformations on the specimen. Nevertheless their study can to require the evaluation of the field of plastic deformation on determined sections to a detailed study on the liberated energy in the produced dislocations when the field of plastic deformation acts. Thus, it is doing necessary the introduction of certain evaluations of the actions of the field to along of the dislocation trajectories in mono-crystals of the metals with properties of asymptotic relaxation. Thus we consider like specimens, mono-crystals of *Molybdenum* (Mo), subject to stress tensor that produce the plastic deformation given by the action of Ec. (3). By the *theorem of Bulnes-Yermishkin* [6], all functional of stress-deformation to along of the time must satisfy for hereditary integrals in the quasi-relaxation phenomena that

(4) $$\Gamma(\sigma-\varepsilon,t) = \int_0^{+\infty}\left[\int_0^t \sigma(t)d\varepsilon(t)\right]\varphi(\tau)\,e^{-dt}d\tau$$

Studies in mathematics [7], [8], [24], cans demonstrate that the integral Ec. (4) is a integral transform, if the expression between the brackets is a function with analytic properties that join with $e^{-\tau t}$, determine the kernel of the integral transform and the characterize like a quasi-relaxation transform. Our functional of energy are the evaluations of the field of plastic deformation considering the quantity of energy of liberated plastic deformation by the specimen for unit of time $\Delta G = \Gamma t$, in the generated dislocations in the specimen under the regimens of quasi-relaxation. If we consider the average energy of the longitude unit for line of dislocation, the integral Ec. (4), take the form for the Burgers vector b, and the initial reserve of elastic energy in the specimen [5]

$$\textbf{(5)}\quad \Gamma(\sigma-\varepsilon,t)=\int_0^{+\infty}\left[G_0-\frac{Gb^2\,\rho\,(t)}{2}\right]\varphi(\tau)e^{-\tau t}\,d\tau$$

During the unloaded of the specimen, stressed in the elastic field, these dislocations it is transforms in a density of initial dislocations. The realized videos on the evolution of the dislocation structure in *mono-crystals of Molybdenum* (Mo) (images of the transitory given by the Laplace transform involucrate in the functional Ec. (4), (see figure of mono-crystal evolution), during the load and unload, demonstrates the reversibility of the sliding character of the dislocations, and of variation of the density of these, in the field where the metal it is behavior elastically. The dislocations that already could exist in the stressed crystal, and that it is annihilated during the unload, we call dynamical dislocations. Like it is have demonstrate [5], practically all the elastic energy of the specimen it arises during the load and it is accumulate in form of the elastic fields of the dislocations, in base to the conservation law, in the volume unit of the deformed material during the load , it is describe through the following equation:

$$\textbf{(6)}\quad \frac{\sigma^2}{2E_r}=\frac{Gb^2}{2}\rho_d$$

where E_r, is the value of modulus of normal elastic relaxation, is to say, the valued energy considering the elasticity of the essay machine ρ_d , the density of the dynamical dislocations, expressed in the right part of the equation Ec. (6). Here is necessary observe that the property of the integral transforms to the obtaining of the spectral state of the quasi-relaxation phenomena, bounded by constants or coefficients that it is compute in the corresponding energy space of signals $\sigma(t)$, or $\varepsilon(t)$, is to say, in the space $L^2(D(\rho,\dot{\sigma}))$, it is reflexes in the norm of technology (given to nano-components like can be nano-crystal) given by [19] , and bounding the Langragian action given by Ec. (3), to know:

$$\textbf{(7)}\quad \|\Im(\sigma,\varepsilon)x(t)\|\leq\|\log\Im\sigma(t)\|^a\,\|\log\Im(x(t))\|^b$$

Where $\Im$ is the foreseen action in the *Theorem. 1. 1.*, and $x(t)$, is a particle of the material specimen M [19], with values in $a = b = 2$. The controls to $\Im(\sigma,\varepsilon)x(t)$ are given for $\log\sigma(t)$ and $\log\Im(x(t))$, [1], [6]. But the one that are isometries in the context of $\mathcal{L}^2(G)$, (space of measures obtained in the panel of control of the specimen-machine (see Figure 1).

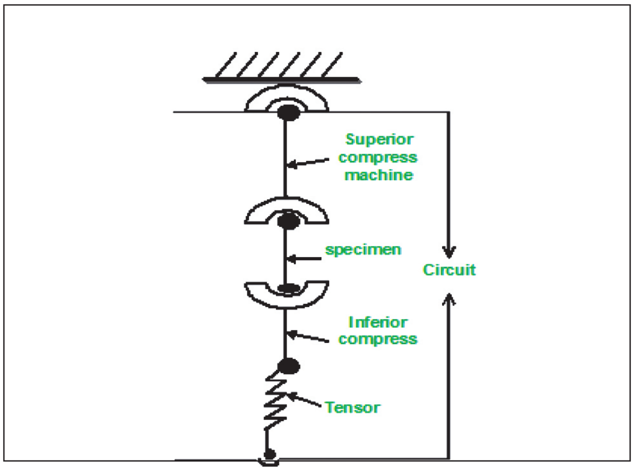

Figure 1. Specimen-machine to obtain the quasi-relaxation state of a material [5].

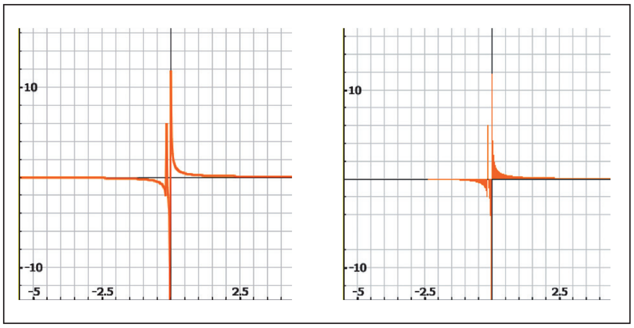

Figure 2. Spectral densities measured in space $L^2(D(\rho,\dot{\sigma}))$. $\Psi(0.2 + 0.025\exp(-t/7))$, is the spectral density of quasi-relaxation function of $\varphi(t) = 0.2 + 0.025\exp(-t/7)$. It is have used the *simulation program space-time 4.0*.

Pair of quasi-relaxation transforms

In to the our study is evident a bi-univocally correspondence between the quasi-relaxation function and their spectra through of the corresponding transformation that it is establish into of the space of the material. By functional analysis it is can prove the uniquely of this transformation, and using the characteristic of energy given by our generalized functional Ec. (3), that involucrate the Laplace transform [7] , [19], that expresses the action of the viscous-elasticity phenomena into of quasi-relaxation process, we can obtain un pair of integral transforms to this study.

Considering that in the research realized on quasi-relaxation in metallic specimens [15], [16], [19], [22] it has been achieved identify to the condition of meta-stability that define univocally a state of quasi-relaxation [5], through of plastic deformation expressed by the plastic work (*theorem II. 1*), that invert the system of machine-specimen to the deformation of the nano-crystalline structure of the metals risking the state of the curves (the history of deformation under stress σ, to along of time *t*) given by Ec. (4), whose quasi-relaxation function given by the distribution (distribution in the functional context), is

(8) $$\varphi(\tau)=\int_0^{\infty}\Psi(\tau)e^{-t/\tau}d\tau$$

where $\Psi(\tau)$, is the density of distribution of the times of relaxation or spectra of relaxation [1], [18].

Given that these applications conforms a class of functions $\varphi(t)$, such that [7]

(9) $$|\varphi(t)|\le M\left|e^{-\gamma t_0}\right|\le M\left|\frac{\gamma}{\zeta}\right|$$

to the integral operator $I_{\tau t}=\int\Psi(\tau)K(\tau,t)d\tau$, whose nucleus is $K(\tau,t)=e^{-t/\tau}$, and due to that

(10) $$\left[\int_V d\varepsilon_{PT}\right]\ge M\left|e^{-\gamma t}\right|,[7]$$

then exists two integral transforms of quasi-relaxation of the curves $\sigma(t)$, under the regime of plastic deformation given by (r), and with the following result:

Theorem (Bulnes F, Yermishkin V, and Stropovsvky Y) 3. 1[]. The nucleus $K(\tau, t)$, defined to operator $I_{\tau l}$, verify

$$\int_{Specimen}|K(\tau,t)|d\sigma(\tau)\le C_q\|\Omega\|_q\le 1$$

with $\Omega=\int_V d\varepsilon_{PT}=\varepsilon(t)-\varepsilon_0$. Then $\forall\ \varepsilon(t)=\Omega-\varepsilon_0$, the pair of transforms are:

(11) $$\varphi(t)=\int_0^{\infty}\Psi(\tau)e^{-t/\tau}d\tau$$

(12) $$\Psi(\tau)=\frac{\alpha}{b\sqrt{\rho_{dym}}}\int_{-\infty}^{\infty}\varphi(t)e^{t/\tau}dt$$

Proof. By [7], [18] and [24].

Table 1. Example of Quasi-relaxation integral transforms table considering the factor $\alpha/b\sqrt{\rho_{dym}}\approx 1$, and using the substitution $t'=t-\tau$.

φ(t)	Ψ(τ)	σ	U
$\varphi(t)=(G_1+G_2e^{-t/\tau_2})$	$\Psi(\tau)=\varepsilon_0(G_1+G_2)\delta(t')+\frac{\varepsilon_0G_1G_2}{\eta_2}U(t')$,	$\sigma=\varepsilon_0(G_1+G_2e^{-t/\tau_2})$	$U(t')$
$\varphi(t)=\varepsilon_0t_1(2\eta+Gt+\eta e^{-t/\tau})$	$\Psi(t)=\frac{\varepsilon_0}{t_1}\left\{3G+\frac{Gt'}{\tau}\right\}$	$\sigma=\varepsilon_0(2\eta+Gt+\eta e^{-t/\tau})/t_1$	$t/(t-t')$
$\varphi(t)=\int_0^{\infty}H(\ln\tau)e^{-t/\tau}d(\ln\tau)$	$\Psi(t)=\int_0^{\infty}L(\ln t)(1-e^{-t/\tau})d(\ln\tau)$	$\sigma=\varepsilon_0\int_0^{\infty}H(\ln\tau)e^{-t/\tau}d(\ln\tau)$	$J(t-t')$
$\varphi(t)=\int_0^{\infty}h(\lambda)e^{-\lambda t}d\lambda$	$\Psi(t)=\tau\int_0^{\infty}\varphi(t)e^{-\tau t}dt$	$\sigma=\varepsilon_0Ge^{-t/\tau}$	$\delta(t')$

Corollary 3. 1 (*Bulnes F and Stropovsvky Y*). All quasi-relaxation in the stress-deformation process is a response of relaxation type with singulariries in the limit after of obtains the meta-stable conditions in M^t.

Proof. First we must demonstrate that a quasi-relaxation function is a response of relaxation type. After, are necessary to show that this function have singularities in the limit after of entre the meta-stability conditions yet with the residual relaxation effects given by the term $e^{-\gamma t}$. Using the quasi-relaxation transform given by (11), and the hereditary integrals to a load given by $U(t-t')$, it is have that:

$$\varphi_c(\tau)=\int_0^\infty \sigma(\tau)e^{-t/\tau}\,d\tau$$
$$=\int_0^\infty\left[\int_{-\infty}^{t}\frac{d\varepsilon(t')}{td'}\varphi(t-t')dt'\right]e^{-t/\tau}\,d\tau$$
$$=\int_0^\infty\left[\int_{-\infty}^{t}\varepsilon_0[\delta(t)]\varphi(t-t')dt'\right]e^{-t/\tau}\,d\tau$$
$$=\varepsilon_0\int_0^\infty\varphi(t')U(t-t')e^{-t/\tau}\,d\tau$$
$$=\varepsilon_0 U(t-t_0)\int_{t_0}^{t}\varphi(t')\,e^{-t/\tau}\,d\tau,$$
$$=\varepsilon_0\varphi(t)$$

Thus is a function of relaxation type, and only is relaxation into a finite interval $[t_0,\ t]$. When it is carries the conditions of stress outside of this interval, we pass to the quasi-relaxation (we keep the load of deformation $\varepsilon_0 U(t-t_0)$). Here it is had used the identity:

$$\int_{-\infty}^{t}\varphi(\tau)\delta(\tau-t)d\tau=\varphi(t)U(\tau-t)$$

and also the fact of that

$$\int_{-\infty}^{t}\varphi(t')\delta(t'-t_0)dt'=U(t-t_0)\int_{t_0}^{t}\varphi(t')dt'$$

For other side, the singularities are confined in the negative part of real axis. This want to say in terms of quasi-relaxation, that in the meta-stable conditions only these singularities it is see reflected in the imaginary part of $\Psi(p)$. Relating of two quasi-relaxation spectra (is to say the $\Psi(\tau)$, and their complex extension), we have by the Carson transform [18], [24], that:

$$\Psi(p)=p\int_0^\infty\varphi(t)e^{-pt}dt \tag{13}$$

and their inverse:

$$\varphi(t)=\frac{1}{2\pi j}\int_{n-j\infty}^{n+j\infty}\frac{\Psi(p)}{p}e^{pt}dp \tag{14}$$

The singularities of $\Psi(p)/p$, must be considered like simple poles (due to that the $\varphi(t)$ must be character like response of type quasi-relaxation yet in the conditions to infinite). Then could be given by the sum of terms of the form $G_n e^{\gamma nt}$, $\gamma_n \leq\ 0$, where the sum could be finite or infinite. This can be written in terms by the like quasi-relaxation spectra:

$$\varphi(t)=\int_0^\infty h(\lambda)e^{-\lambda t}d\lambda \tag{15}$$

with $h(\lambda)=\Sigma_n G_n\delta(\lambda+s_n)$, (that is a of the solutions proposed in the table 1) and given that by the hereditary integrals a quasi-relaxation function is precisely the function of relaxation type obtained by $\varphi(t)$. A important characteristic of $\varphi(t)$, is that monotonic non-decreasing function and have not oscillatory terms. This goes agree to the observations of the curves of stress-deformation schematized in the figure 7. Then in the meta-stability conditions (after the simple relaxation), is to say with quasi-relaxation, the function $\varphi(t)$, takes the form, consider the complex extension of their spectra by (14):

$$\varphi(t)=\frac{1}{2\pi j}\lim_{\rho\to 0}\lim_{\eta\to 0}\left\{\int_{-\infty}^{-\sqrt{\rho^2-\eta^2}}[\Psi(s-j\eta)/(s-j\eta)]e^{(s-j\eta)t}ds+\right.$$
$$\left.j\int_{-a}^{a}\Psi(\rho e^{t\theta})e^{\rho e^{t\theta}}d^{\theta}+\int_{-\sqrt{\rho^2-\eta}}^{+\infty}[\Psi(s+j\eta)/(s+j\eta)]e^{(s+j\eta)t}ds\right\} \tag{16}$$

always with $a = (\pi/2) + \cos^{-1}(\eta/\rho)$. The second term of (16) yields $\Psi(0) = \Psi(\infty)$. The first and third terms can be combined; using the fact that Ψ, of p-conjugate equals the conjugate of $\Psi(p)$, to give:

$$\textbf{(17)}\quad \Psi(t) - \Psi(\infty) = \frac{1}{\pi}\int_0^{\infty}\left[\left(\frac{e^{st}}{s}\right)\lim_{n\to 0}\operatorname{Im}\Psi(s + j\eta)\right]ds$$

that which is:

$$\textbf{(18)}\quad \Psi(t) - \Psi(\infty) = \int_0^{\infty} h(\lambda)e^{-\lambda t}\,d\lambda$$

is to say, it is conserve the relaxation characteristic to our quasi-relaxation function, yet after the simple relaxation and with their singularities. This has proved the result.

Of (18), it is have calculated [24] that,

$$h(\lambda) = \pm\left(\frac{1}{\lambda\pi}\right)\lim_{n\to 0}\operatorname{Im}\Psi(-\lambda + j\eta)$$

This last spectrum appears in the study of the relaxation in polymeric materials like hard rubbers. But this is equivalent to spectra due to the plastic energy that is accumulate in metals in the quasi-relaxation process before of the dislocations, save the multiplicative coefficient of dislocations α [5].

Meromorphic curves in the quasi-relaxation spectra

The spectral function $\Psi(p)$, have cuts in the quasi-relaxation curves due to entre of dislocations phase due increasing of plastic energy in all crystals of the metal. These curves are analytic in all domain except in singular points $\lambda = 0$, or $p \to \infty$, when the quasi-relaxation spectra satisfies with help of the Laplace transform that $\Psi(p)\Phi(p) = p^2$, where the function $\Phi(p)$, is the fluency spectra corresponding to the fluency function that appear after of increasing too much to the sliding dislocations being possible detect the plastic deformation to macroscopic level. The factor p^2, define the duplicity of these actions through of relation between both functions [18], (see the corresponding Carson transform to the two functions $\varphi(t)$, and $\psi(t)$, and their functional relation, [24]).

Conclusions

The introduction of the integral transforms in the quasi-relaxation study helps to establish with major precision the limits of the existence of the quasi-relaxation of a material summated to a constant load in a interval to along of the time, before off to risk arrive to the accumulation plastic energy necessary

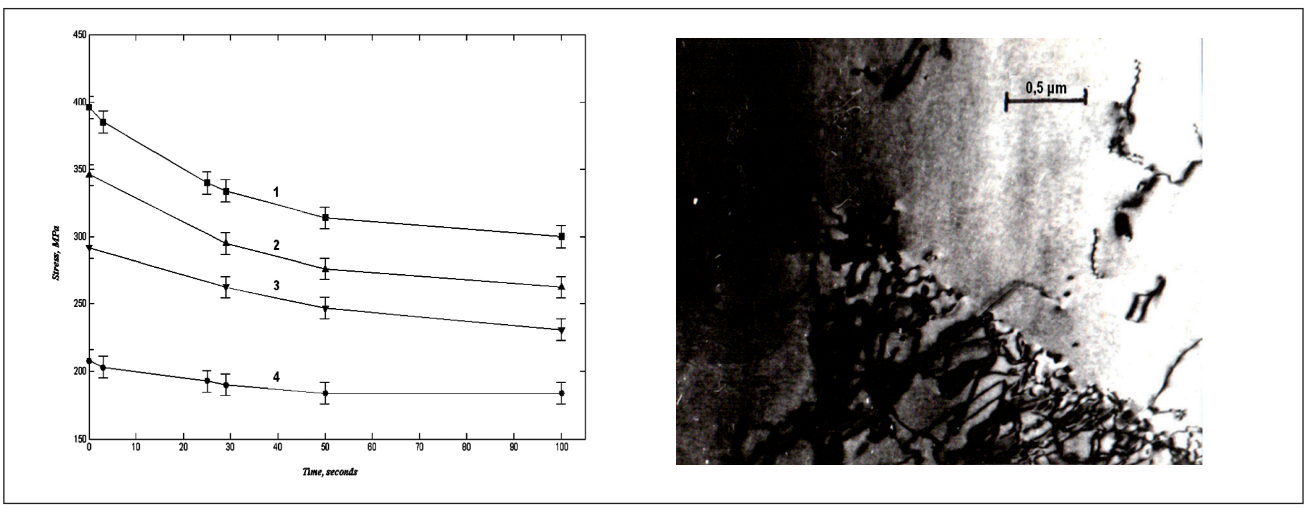

Figure 3. i). Quasi-relaxation curves for Molybdenum single crystal: 1.- $\sigma_0 = 396$ MPa, 2.- $\sigma_0 = 346$ MPa, 3.- $\sigma_0 = 292$ MPa, 4.- $\sigma_0 = 208$ MPa. Mo <100> {100}, at T= 293 °C, [19]. ii). Image of the electronic microscope of high voltage, HVTEM of Molybdenum single crystal in regime of quasi-relaxation.

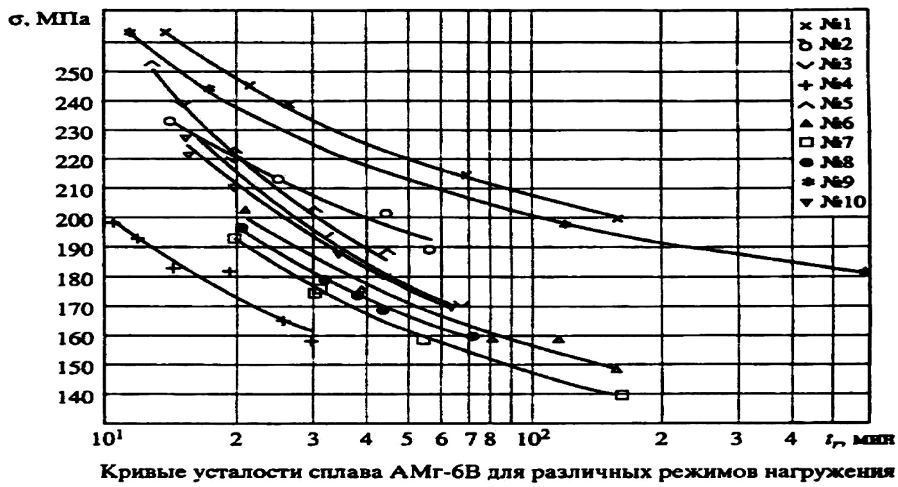

Figure 4. Meromorphic curves of quasi-relaxation. These are obtained by experiments of metal specimens under big stress obtaining [5]. These experimental results proved that our quasi-relaxation functions have ideal behavior very seemed to that obtained in the reality (compare these curve with the curves of the figure (4)), save the some singularities due to the entered of the hyperbolic part contemplated of the functions (31).

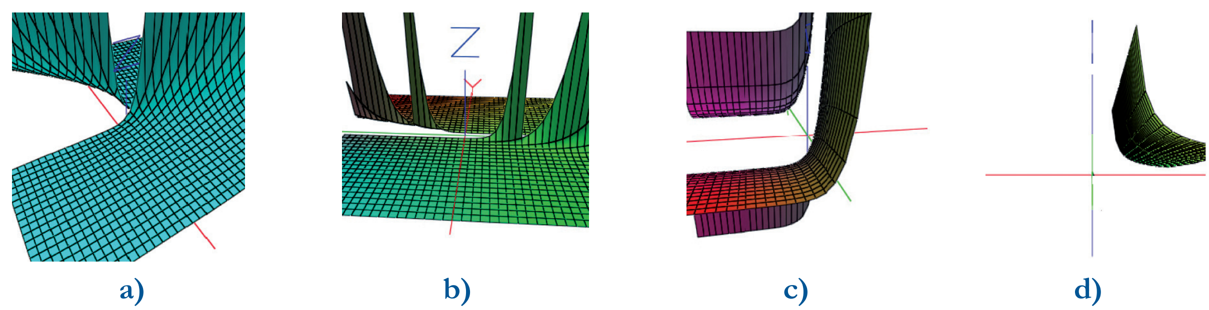

Figure 5. a). j(t, s) = 1 + 0.5exp(-t/s), with stress tensor s = U(t'), with t' = t - s. b). Surface of quasi-relaxation function j(t, s) = 0.02 + 0.0025exp(-t/s). Observes the curves of quasi-relaxation in the plane XZ, accord of the curves predict in figure 4, after of a fatigue to *Aluminum with Magnesium (Yerminshkin)*. c). Quasirelaxation surface with quasirelaxation curves in black such like the predicted in figure 2, with stress s = exp(-3t) + 1/s, in the plane YZ, with the fatigue time t = 50 seconds to AMr-6B. d). s = s_R + 1/lg t, with s_R = *Heaveside*(t) + 10/lg(s), Observes the curves of quasi-relaxation in the plane YZ, accord of the curves predict in figure 4, after of the fatigue to t = 50 seconds.

to the crystalline dislocations in metals and after of this stage. The quasi-relaxation spectra also give of support analytic information on the meromorphic behavior of the quasi-relaxation curves use to obtain information that can be codified in the field of the complex frequencies to their possible material decoding and with it open the possibility of manipulate in this meta-stable state to the material, being able to fill codes that can facilitate the transformation of the metals and their alloys [15], [22]. The quasi-relaxation conditions help to obtain special properties of the metals, likewise some alloys. For example, a of the alloys obtained to the program of spatial research in Russia, with the object of obtain metals with anti-corrosive memory and lightness (and the same time to support major temperatures to 650°, for example, in the turbines of the reaction airplanes) [10], are the experiments on a alloy of Zr + 2.5%Nb, after of the quasi-relaxation in the which it is observes fine segregations of the phase II, in a considerable count of effects of agglomeration. Using our transforms (13) and (14), we can demonstrate that the behavior of increasing of the analytic curves it is give this material like [5]:

(19) $\xi(t) = \xi_0 e^{\beta \sqrt[4]{t}}$

that which have behavior into of the family of curves, $\sigma(t) = \sigma_0 e^{-\gamma t}$. This is due to the big capacity of resistance of the material Nb [10]. In the Russian aerospace industry they are studying diverse metallic alloys to establish ranges of meta-stability in the deformation fields produced by the materials proposed [9], [10]. It wants obtain lightness materials and more resistant to the corrosion and deformations. Go of remains to mention that in the last researches, considering the analytic characterization proposed by Bulnes and Yerminshkin [5], [19], it is want like priority the development of precise methods on the multiple manipulations that we can realize of the metals in the meta-stable regime using the pre-disposition that these presents in accumulate energy to can realize actions like re-programming their nano-crystal structure and think in the possibility of obtain nano-technological transformations of the materials [], using their spectral encoding. Finally we believe that the use of inverse methods of the functional analysis in material physics can to help to development and obtaining of a complete theory of quasi-relaxation to characterization and transformation of materials of any nature [15], [19].

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