Intersection of two surfaces in GeoGebra

Intersecção de duas superfícies no Geogebra

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Abstract

In this article our main objective is to illustrate how GeoGebra can be used to obtain the intersection of some surfaces of R^3, presenting several examples, and the procedures necessary to its construction. The intersection of the surfaces associated with the graph representations of bivariate functions is show in more detail. Some of the constraints of the application of these processes are presented, as well as their relation with some topics of algebraic geometry. Finally, some considerations are presented in the educational context where the material here presented can be used with students of mathematics courses in higher education.
***Keywords**: GeoGebra;* algebraic geometry*; surfaces.*

Resumo

Neste artigo o nosso principal objectivo é ilustrar como pode o GeoGebra ser usado para obter a intersecção de algumas superfícies de R^3, sendo apresentados vários exemplos, bem como os procedimentos necessários à sua construção. É tratado de forma mais detalhada a intersecção das superfícies associadas as representações do gráfico de funções bivariadas. Alguns dos constrangimentos da aplicação destes processos são apresentados, assim como a sua relação com alguns tópicos da geometria algébrica. Finalmente, apresentam-se algumas considerações no contexto educativo onde o material aqui apresentado pode vir a ser utilizado com estudantes em cursos de matemática no ensino superior.
***Palavras-chave**: GeoGebra; geometria algébrica, superfícies .*

1. Introduction

With the GeoGebra we can obtain the representation of surfaces in R^3. Examples of surfaces that we can define in GeoGebra is the graph representation of a bivariate function, a subset of points of R^3. An interesting problem in GeoGebra is how we can represent the intersection of two surfaces in tri-dimensional space. In some cases we have geometry software's that's permit obtain the intersection of two surfaces directly in other cases this problem is still open. However develop strategies in order to obtain intersection of two surfaces is an interesting issue to consider in educational by all the technological and enginery applications. As reported by Cox, Little & O'Shea

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When creating complex shapes like automobile hoods or airplane wings, design engineers need curves and surfaces that are varied in shape, easy to describe, and quick to draw. Parametric equations involving polynomial and rational functions satisfy these requirements. (2007, p.21)

Here we are interested in represent the intersection of two surfaces in the open software GeoGebra, presenting some geometrical and algebraically strategies to do it.

2. Intersections of quadrics in GeoGebra

In GeoGebra we can get in some cases the intersection between two surfaces, for example we have directly the intersection between some quadrics and a plane, also we have the intersection of some two quadrics. We have a tool and some standard commands, in GeoGebra, to obtain and intersection between a quadric and a plane, and between two spheres, however these commands only work when the intersection is a conic. For example we have the intersection between spheres but not have the intersection between: a sphere and a cylinder; a cylinder and a paraboloid; a paraboloid and a sphere. Indeed, the command *IntersectConic* fail in some cases that a intersection is a conic.

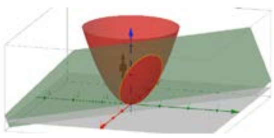
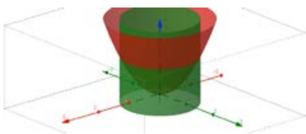
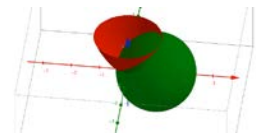
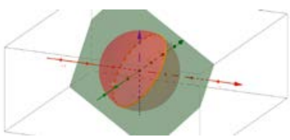
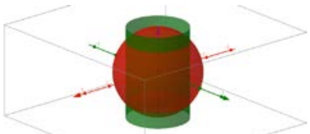
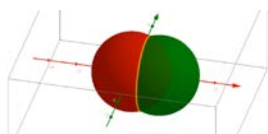
Command in GeoGebra		**Description**	
IntersectConic[<Plane>, <Quadric>]		Intersects the plane with the quadric.	
IntersectConic[<Quadric>, <Quadric>]		Returns a conic defined in case where the intersection is actually a conic.	
IntersectPath[<Plane>, <Quadric>]			
	z = x + y	1 = x^2+y^2	2 = (x-1)^2+y^2+z^2
z=x^2+y^2	 IntersectConic[z=x+y,z=x^2+y^2]	 IntersectConic[1=x^2+y^2+0 z^2,2 = x^2+y^2+z^2] . Undefined object	 IntersectConic[z=x^2+y^2, 2 = (x-1)^2+y^2+z^2] . Undefined object
x^2+y^2+z^2 =2	 IntersectConic[z = x + y, 2 = x² + y² + z²]	 IntersectConic[1=x^2+y^2+0 z^2,2 = x^2+y^2+z^2] Undefined object	 IntersectConic[2=x^2+y^2+z^2, 2 = (x-1)^2+y^2+z^2]

Figure 1 - Several examples of use of command IntersectConic and the GeoGebra outputs.

Let see the case of intersection between a paraboloid and a sphere. Considering a R^3 orthonormal three-dimensional reference system, admit that we have a paraboloid, *a*, and a sphere, *b*, with at lest two common points in their surfaces. We can design a strategy to obtain the intersection of *a* and *b* using planes perpendicular to the axe of paraboloid, or other parallel line, these planes cut the quadrics, *a* and, *b* by two circumferences, c_a and c_b, the locus of the points of the intersection of these circumferences correspond to the curve intersection of *a* and *b*.

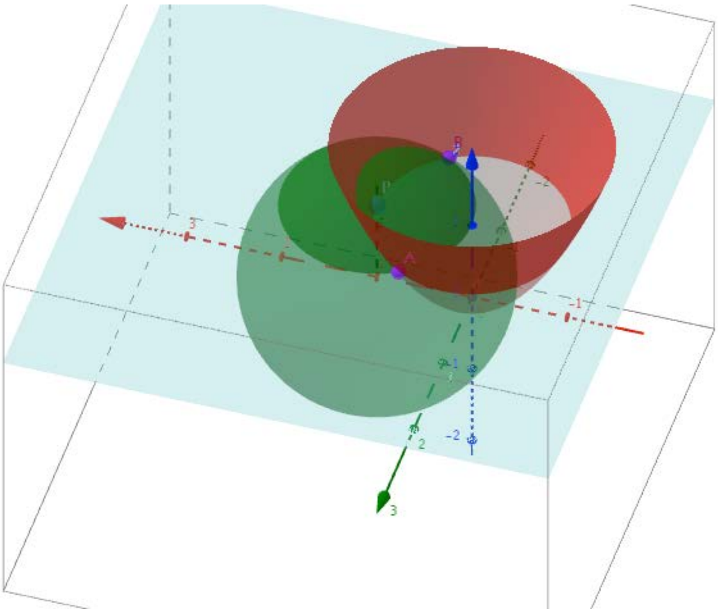


Figure 2 - Intersection between a paraboloid and a sphere.

For example, let *a* defined by $x^2 + y^2 - z = 0$ and *b* defined by $(x-1)^2 + y^2 + z^2 - 2 = 0$. We need to construct a diameter of *b* parallel to the axis of a, in this case the *z-axis*. In this example we only need a segment, *s*, parallel to *z-axis*, using the command:

s=Segment[Centre[b], (x(Centre[b]),y(Centre[b]),radius[b])].

Using a point, *P*, *P=Point[s]*, and a plane, *p*, perpendicular to segment *s* in P, *p=PerpendicularPlane[P,s]*. The intersection of *p* with *a* and *b* are the circumferences c_a and c_b . We can obtain these circumferences using, for example, the commands:

c_a*=IntersectConic[PerpendicularPlane[P,s],a] and*

c_b*=IntersectConic[PerpendicularPlane[P,s],b].* (1)

Then we obtain the points *A* and *B*, see figure 2, the intersection of c_a and c_b. Finally, we get the locus of the intersection of c_a and $c_{b,}$, that is the curve that correspond to the intersection between the paraboloid, *a*, and the sphere, *b*. We also can obtain this intersection using the trace attribute to points *A* and *B* when animate the point *P*.

3. Bivariate functions and Surfaces in R^3.

Considering the 3D window of GeoGebra we can represent there the map of a function with domain R^2. The map of these functions is represented by surface in R^3. In this section we will be use the graphical representation of these functions as a way to extend the capabilities of GeoGebra in order to obtain representations of the intersection of two surfaces in R^3.

Let $f(x, y)=x^2+y^2$ and $g(x, y)=y^2+y$, in this case the curve $\{(x, x^2, 0): x\in R\}$ was the projection in $z=0$ of the set of common points of the maps of *f* and *g*, and these points belong to the set $\{(x,x^2,x^4 +x^2): x\in R\}$ (Inselberg, 2009). For represent the intersections of the maps of *f* and *g* in GeoGebra we can use the command curve and type in the command line $curve[t, t^2, t^4 + t^2, t, -5, 5]$.

As we can see in figure 3 this procedure permits obtain the intersection of these tow surfaces. But how we can generalised these procedure without define explicitly the curve intersection of the surfaces?

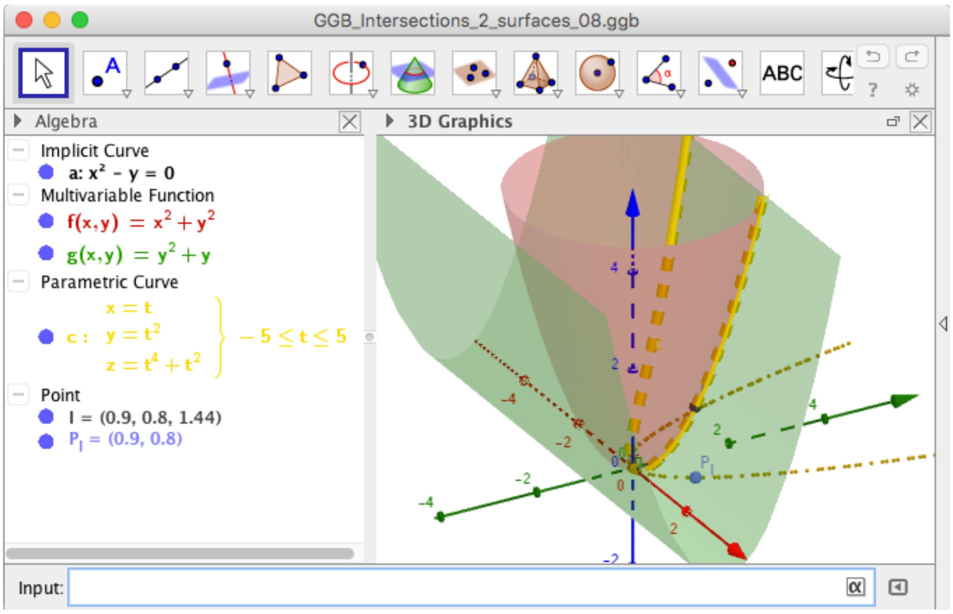


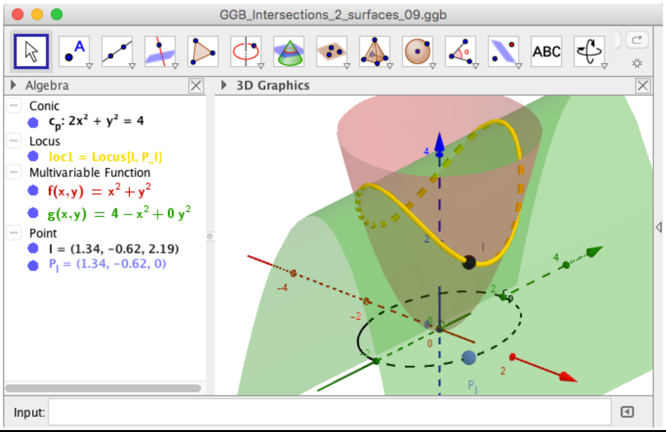
Figure 3 - $\{(x,y,z) \in R^3: x^3 +x^2 +y^2 - 4x = 0\}$, a connex and not limited set of points.

In order to obtain the intersection of the graphical representation of *f* and *g*, we can use a similar strategy used in the last section. However in GeoGebra we can use another approach.

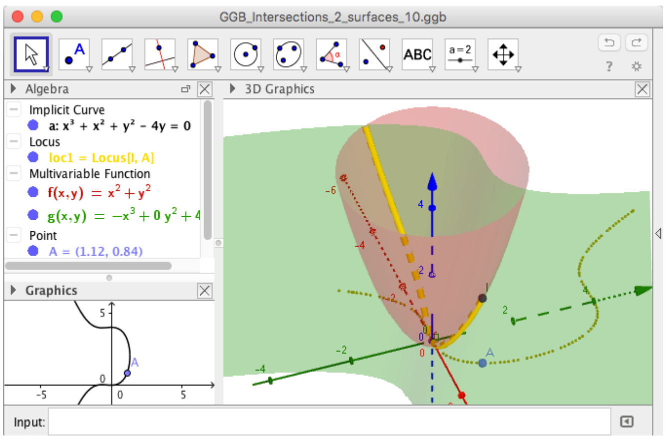
Begin to define the two functions of R^2 above R, f and g, directly in the command line. Let c the curve defined by the common points of the two maps. The set of points, here designed by c_p, that's satisfied the equation $f(x,y)-g(x,y)=0$ is the projection of c in the plane $z=0$. We can type $f(x,y)-g(x,y)=0$ in the command line of GeoGebra for obtain the representation of cp or, in alternative we can use the command

ImplicitCurve[f(x, y)−g(x, y)].

Let P_I a point in c_p, we can construct the point *I =(x(P_I),y(P_I),f(x(P_I),y(P_I)))*, for example, using the command line of GeoGebra, the locus of P when P_I travel in c is the intersection of the maps of f and g. In generally, we can visually obtain the intersection of the maps of f and g using the locus command. In figure 4 we can see two examples of the results in GeoGebra and the commands used for obtain the intersection of two surfaces.



f(x,y)=x^2+y^2
g(x,y)=4-x^2
c_p=ImplicitCurve[f(x, y) - g(x, y)]
P_I:=Point[c_p]
I:=(x(P_I), y(P_I), f(x(P_I), y(P_I)))
Locus[I, P_I]



f(x,y)=x^2+y^2
g(x,y)=4y-x^3
c_p=ImplicitCurve[f(x, y) - g(x, y)]
P_I:=Point[c_p]
I:=(x(P_I), y(P_I), f(x(P_I), y(P_I)))
Locus[I, P_I]

Figure 4 – Examples of intersection between a paraboloid and “cylinders”.

Note that, the process presented in this section use the command *ImplicitCurve*, apparently only works with polynomial functions in x and y. In order to obtain the intersection of two surfaces in a general way in GeoGebra will be need develop other algorithms, namely using the resultants methods to solve the implicitization curves problem, applying many of the theoretical mathematics achieved in algebraic geometry (Cox et al., 2007).

Conclusions

Here we show how we can use the capacities of GeoGebra in order to obtain the intersection of surfaces in *3d*. In certain cases, as we show, the curve result of the surface intersection was not totally represented, GeoGebra need more locus improvements, and eventually use other methods. Considering the intersection of surfaces defined by the map of bivariate functions we can get the intersection of two surfaces, however the implicit curve command have some limitations because was designed only to handle polynomial in two variables. The generalization of the procedures here discussed, are still under development, require the use of CAS, and will be object of our future work. Observe that still some limitations in Geogebra commands, namely related with the performance of the implicit curve and locus commands.

In an educational context, all the processes presented, once used in mathematics courses, could be a great contribution to a deep understanding of algebraic geometry issues with undergraduates students. Unfortunately, we have not found empirical studies of the use of GeoGebra in these topics. However, for us it seems important to present how geogebra can be used to obtain the intersection of surfaces in R^3, since there are still some open problems in these area.

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