Muon flux measurements and the randomness of the data: a project work for honours´ physics degree students

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Abstract

The flux of muons of cosmic origin has been measured for the first time in Venezuela. Six independent measurements at different geographical locations above sea level (0 to 3600 m) were taken using a scintillation detector. The muons half-lifetime was measured; the mean obtained is $2.22\times10^{-6} \pm0.08$ [s]. A statistical analysis was undertaken to study the data from the detection events with the purpose of finding whether or not such data can be used as a random number generator. Its stocasticity cannot be guaranteed *a priori* since the actual physics of the complex detection process taking place inside the scintillator determine the data. The analysis consists in studying the correlation of the detection events using the autocorrelation $<s(t+d)>.s(t)-<s>^2)/<s^2>-<s>^2$. Also a simple algorithm was implemented to generate a uniform binary random sequence from the data. Muon measurement at a single location generated a string of approximately 32000 digits. The entropy of strings of different lengths, extracted successively from the first binary string is calculated and compared with the entropy of a uniform binary random string. The latter was generated using a standard pseudorandom number generator. We obtained values about 0.9934 for the relative entropy of both strings. Our conclusion is that the data collected can be reliably used as a good generator of random numbers. The measurement of the muon flux is an excellent project for a team of honours´ physics degree students taking Modern Physics courses. It provides firsthand experience with experimental Special Relativity and with detection techniques proper to high energy physics. It involves the cooperation and preparation of a true research group. We have complemented the standard flux measurement with the statistical analysis already described. This may be the first time that this type analysis has been undertaken with cosmic muons data. This project had the support of the Venezuelan Fundación Mercantil.

Keywords: Muon lifetime, Random Numbers Generation, Modern physics.

Resumen

El flujo de los muones de origen cósmico ha sido medido por primera vez en Venezuela. Seis mediciones independientes fueron realizadas en localidades geográficas de diferente altitud (entre 0 y 3600 m) sobre el Nivel del Mar utilizando un detector de centelleos. El valor promedio obtenido es $2.22\times10^{-6} \pm0.08$ [s]. Se realizó además un estudio estadístico de la data de los eventos de detección con el fin de saber si dicha data puede ser usada para generar números aleatorios. Su aleatoriedad no puede ser supuesta a priori ya que la naturaleza del proceso físico de detección en el centelleador determina la data. El análisis consistió en estudiar la correlación de los eventos utilizando la autocorrelación $<s(t+d)>.s(t)-<s>^2)/<s^2>-<s>^2$. Además, se diseñó un algoritmo simple para generar una secuencia binaria uniforme y aleatoria a partir de la data. La medición de los muones en una sola localidad genera una secuencia de aproximadamente 32000 dígitos. Se calcula entonces la entropía de secuencias de diferente longitud que son extraídas sucesivamente de la primera secuencia y se compara con aquella de una secuencia binaria aleatoria. Esta última es generada usando un generador pseudoaleatorio. La entropía relativa de ambas secuencias es aproximadamente 0.9934. Concluimos así que en efecto la data del decaimiento de los muones puede ser usada como un buen generador de números aleatorios. La medición del flujo de muones es un excelente proyecto para un equipo de estudiantes de licenciatura en física. Ello implica cooperación y preparación de un verdadero equipo de investigación. En este trabajo se complementó la medición standard del flujo de muones con el análisis estadístico arriba descrito. Este proyecto contó con el apoyo de la Fundación Mercantil venezolana.

Palabras clave: Tiempo de vida de los muones, Física Moderna, Generación de números aleatorios.

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I. INTRODUCTION

Measuring the flux of muons of cosmic origin [1] is an excellent project for a team of honours´ physics degree students taking Modern Physics courses. It provides first-hand experience with experimental Special Relativity and with detection techniques proper to high energy physics. Below we present the results of a Project Work undertaken by a group of physics students taking two Modern Physics courses at Universidad Simón Bolívar of Caracas. The project aimed to measure the high-energy muons lifetime by measuring the muon flux at different locations in Venezuela. Special relativity is at the heart of most modern physics courses. Their concepts and main inferences although elegant are difficult to demonstrate to undergraduates using simple experiments. For instance, time dilation and space contraction usually demand either sophisticated setups or expensive equipment. Not many high-energy facilities exist in the so-called countries of the periphery. Therefore banking on a relatively straightforward and affordable experiment to witness firsthand the effects predicted by Einstein relativity theory is very relevant to modern physics courses. Fortunately, affordable and compact equipment exists nowadays, or can be assembled at modest cost, that would allow undergraduate students to measure the muon flux. Apart from obtaining the cosmic muons data, the participating students took the decision to study the randomness of such data. This additional task was undertaken because of the way that the decays are actually measured, and because the muons decay data is expected to follow a non-linear uniform distribution, the exponential distribution. The study of random numbers is of extreme importance for many computational tasks, and computers do not generate true random numbers, but instead pseudo-random numbers with some periodicity. There are well-known alternatives for random number generation both in Information Theory and computing. The most common alternatives are the so-called *linear congruential generators* based on equations such as $x_{n-1}=(ax_n)\ mod\ (m)$ [2]. With these one can obtain uniform random sequences. It is also of great interest to obtain random numbers from any distribution, as in the case of the exponential distribution generated by the decay of muons. One way of doing this is to use the *rejection method* [3] for random distributions which have an inverse. As a matter of fact it is well-known that natural decay processes of quantum objects (such as the muons) are indeed truly random, and after all muons decay is just another of such cases. Yet, one thing is the natural decay of an unstable object, another is the actual registration or detection of such decay, which is frequently based on some chain of physics interactions at the decay detector [4]. All that goes without mentioning the way the data is actually recorded as a file in the computer, which can also be troublesome. In this project we therefore also assessed whether or not the muon decay data obtained can be used as a true random sequence.

II. MUONS AS A TOOL TO VERIFY AND DEMONSTRATE EINSTEIN´S PREDICTIONS

Frequently found in high-energy processes muons are well characterized quantum objects. Being leptons they are far more stable than other particles of the same total energy. Interestingly enough they can also be produced in the interaction of cosmic rays (mainly high-energy protons from our sun) with molecules high in the Earth atmosphere; pions are thereby produced which then decay into muons (positive and negative) in a well-known process. The important fact is that muons are readily available on Earth, and even better that they are born as high-energy (~200 MeV) quantum objects with speeds comparable to the speed of light, i.e. as relativistic objects that pour down on us from the high altitude atmosphere. Unfortunately for the muons, and fortunately for us, they are also unstable and will eventually decay into other quantum objects. We only have to catch them before they decay!

Being relativistic objects of speed v their decay is affected according to the well-known Lorentz transformation, and the proper decay time τ' of the moving object is longer than it should be (τ) were the object at rest

$$\tau' = \frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\,. \tag{1}$$

In fact, the high speed of cosmic muons makes this relativistic effect easily appreciable provided one has the proper instrumentation to measure such lifetimes. For instance to have an apparatus that would allow one to measure the muon flux at a given time: to count how many of such muons decay and also their decay time.

III. RANDOMNESS OF MUON DECAY

Being unstable quantum objects muons undergo natural decay into other objects, and the time taken by them to decay is truly random. The decay is constrained by the shape of an exponential distribution whose time constant is the so-called half-life time τ of the muons. The counts N of the decaying muons can then be represented by the well-known time-dependent relation

$$N(t) = N_0\, exp\left(\frac{-t}{\tau}\right), \tag{2}$$

where N_0 is the maximum number of counts i.e. of decaying muon events (t is the time). This uniform distribution contains all the possible decay times and determines how many events can occur with a given decay time. As already mentioned the muon decay process provides an excellent framework to apply several randomness criteria. The abundance of data and its relative easy manipulation is also a good reason for applying such criteria. Two very useful tests for the randomness of a data are: the *Kolmogorov Complexity* [5] and the *Self-Correlation.*

(i) The Kolmogorov Complexity: this is a function that allows to find the entropy or disorder for different chains of events. This function measures the information stored in a given entrance for a given string w of length l, a string extracted from the generator of events. It is convenient to define its length l as the order of the last entrance θ_l, therefore the string is:

$$w = \theta_1\theta_2 \ldots \theta_i \ldots \theta_l. \tag{3}$$

Once the probability P_i of a given entrance θ_i is calculated then the entropy H of the chain is defined as

$$H = \sum_{w\in s} -p(w)\log(p(w). \tag{4}$$

(ii) Self-Correlation: denoted C it that one to measure how alike are two parts θ_i , θ_{i+p} of a string of data:

$$C = \frac{<\theta_{i+p}\theta_i> - <\theta_i>^2}{<\theta_i^2> - <\theta_i>^2}. \tag{6}$$

This is measured along a whole chain

$$W = \theta_1 \ldots \theta_i \ldots \theta_N \ , \tag{7}$$

where N is the number of entries. With this function one can measure how independent is a given entry from another entry of the same chain separated p steps from the first one.

III. THE MEASURING EQUIPMENT

The muon flux was measured using a commercially available equipment of modest cost [6]. It basically consists of a plastic scintillations detector, a photomultiplier (PMT) and its associated power supply, plus the typical counting electronics module used in similar equipment of nuclear physics. The plastic scintillator (polyviniltoluene-based with fluor atoms embedded) and the PMT are located inside an anodized cylindrical alluminium container (some 12.5 cm long and 15 cm diameter). When a fast charged quantum object passes through the scintillation material it loses part of its kinetic energy thanks to ionization and quantum excitations of the molecules or atoms of the plastic material. This then causes the excitation of the fluor atoms present in the material. As the fluor atoms de-excite photons are emitted in the near UV and blue region of the spectrum within nanoseconds. For every 100eV deposited in the scintillator one photon is emitted.

To obtain the information of the decay rate one is only interested in the muons which decay inside the detector, whose total energy is down to about 160 MeV by the time that they reach ground. The decay rate of the muons is obtained thanks to the comparison of two clocks, one is activated via the detection of a muon that has actually *stopped* inside the plastic cylinder, by capturing the light emitted by the scintillator. The second timer is triggered when the muon decays into a high energy electron (plus a neutrino and an anti-neutrino), a fast electron that radiates light along its whole path in the scintillator. It is the distribution of the time differences between the two events that is used as the basis to measure the lifetime of the muons. As can be seen this a complex and indirect physics process that does not warrant a priori that the randomness of muons decay events is preserved. The PMT is energized by a high-voltage power supply and the signal pulses from it are cleaned and discriminated in an electronic module to separate the proper ones (pulses from the decaying muons) from other pulses emitted when other particles decay at the scintillator. The muon data is then conveniently stored. A computer connected to the electronic module and dedicated software is then used to recover and, process the muon data and display the results. It is very relevant that the power source fluctuations are minimal. Strong fluctuations in the voltage can cause anomalies in the data gathering process (leading to false detection signals).For very high muon flux the scintillator can become saturated with many scintillation events and resolution can be lost.

IV. RESULTS

A. Decay time measurements

The half-life τ of the muons was obtained studying the distribution of the decays times measured at the different geographical locations visited (about 12 hours of continuous monitoring at each location). The decay histograms for four of the locations of increasing altitude above sea level (ASL) are plotted in figures 1 to 4.

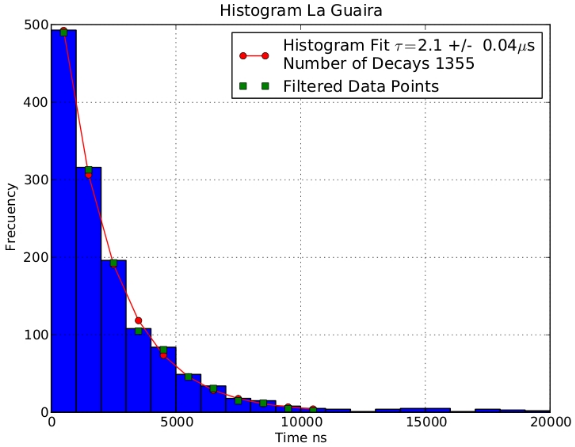

FIGURE 2. Decay Histogram at La Guaira, Edo. Vargas (8 m A.S.L.).

They correspond to 4 m, 480m, 1120 m and 1400m, respectively. It may be seen in the four figures that all the distributions are exponential, and follow the normalized form of the eq. (2) given above.

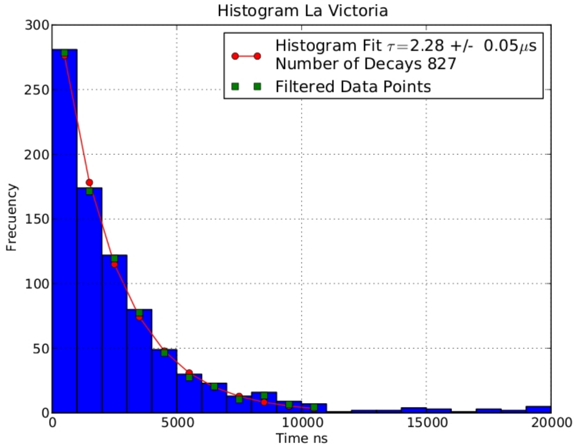

FIGURE 2. Decay Histogram at La Victoria, Edo. Aragua, Edo. Aragua (400 m A.S.L.)

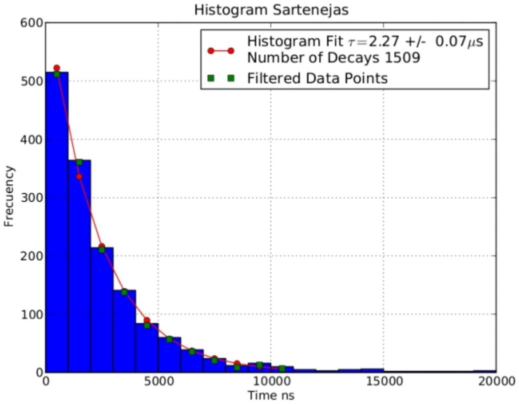

FIGURE 3. Decay Histogram at Valle de Sartenejas, Caracas (Universidad Simón Bolívar campus, 1110 m A.S.L.)

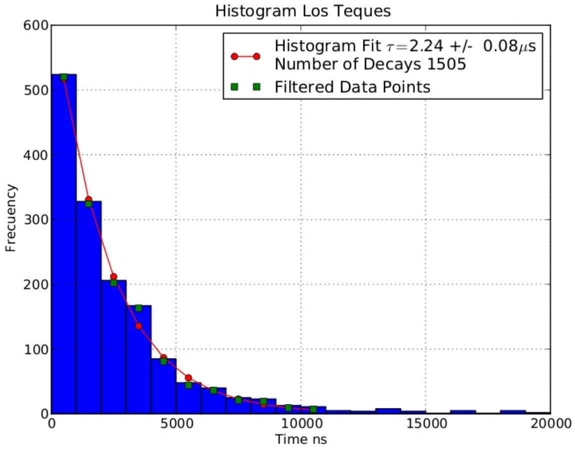

FIGURE 4. Decay Histogram at Los Teques (San Antonio de los Altos), Edo. Miranda (1450 m A.S.L.)

From six plotted distributions (similar to the ones in the figures 1 to 4) it was possible to obtain the mean value 2.22 ± 0.08 μs for the muon lifetime which compares well with the current accepted value 2.19 μs of the Data Particle Group [7].

B. Randomness considerations

From a chain of 32000 events extracted from the measurements taken at the Mucuchíes location the entropies S_μ of five different chains of data were evaluated. The entropies S_c for similar chains of data generated using the random (Rand) generator of a well-known commercial computer software were also obtained. The last column of Table I show both the entropies and the ratio of the evaluated entropies. The results in Table I shows that the entropies from both sources agree to a good extent, showing that the muon experimental data we obtained is indeed sufficiently random as expected, and therefore it can be used as a random number generator.

TABLE I. Comparison of Entropy between muon data and random number generator.

l	$S\mu$	S_R	$\frac{S\mu}{S_R}$
1	0.984731	0.985815	0.998901
2	1.95697	1.97163	0.997127
3	2.96537	2.95745	1.00268
4	3.82274	3.94326	0.969435
5	4.92374	4.92908	0.998918

We also considered the self-correlation criterion already defined in Section II to test the randomness of the experimental muon data. The correlations C_μ *and* C_R were obtained using eq. (6), where the first correspond to the muon data, and the second to the computer random generator. The results are shown in Table II. Although the results are quite different all correlations values are small. For the muon data the correlation is zero with an error that most of the time is smaller than 1/100. Once again we conclude that the muon data can be used reliably as a random generator.

TABLE II. Comparison of correlations between muon data and computer random number generator.

p	Muons C_μ	Computer C_R
0	1	1
1	0.00412887	0.00275596
2	0.000699886	-0.000591843
3	0.00768965	-0.000841252
4	0.00800838	0.0040018
5	0.000169248	0.00275694
6	0.0125874	-0.00225169
7	0.0129062	0.00270242
8	0.00246568	0.00533136
9	0.00583774	0.000322385
10	0.00093788	-0.00346974

C. MUON FLUX DATA

The variation of the flux of muons with height is given by the number of decays (events). Our experimental data for the muon flux are shown in Table III. It may be seen that as expected the muon flux increases with height. The tendency to increase seems to be approximately linear but we do have a noticeable departure at the two lower altitudes locations. We can only say that these particular measurements were not done with enough care and they are being repeated.

TABLE III. Muon decay at different heights.

Locations in Venezuela	Height above sea (m)	Events
La Guaira	4	53
La Victoria	480	64
Valle de Sartenejas (USB)	1180	67
San Antonio de los Altos	1450	102
Pico el Ávila	2135	358
Mucuchíes	3600	673

VI. CONCLUSIONS

Measuring the half-life time and the flux of muons of cosmic origin is an excellent project for physics students majoring in physics. The project can now be undertaken at modest cost with commercially available equipment. It is important to take at least three to five measurements at the same number of locations of different altitude above sea level. In this way one can observe the variation of muon flux with altitude and obtain a reasonable average lifetime. It is not recommended to take measurements at altitudes nearing the 4000 m with the equipment we used, as the muon flux can be too much for this particular kind of detection equipment. We spent some time deciphering the way in which the data is coded and recorded by the accompanying software of the commercial muon detector we used (instructions can be sent via e-mail on request). The measurement of muon decay at different locations should be planned carefully so that the data obtained can be readily compared; for instance a minimum of 10-12 hours recording at each location.

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REFERENCES

[1] Melissinos, A. C, Napolitano, J., *Experiments in Modern Physics*, 2nd. ed., (Academic, Amsterdam, 2003).
[2] Gentle, J. E., *Random Number Generation and Montecarlo Methods*, 2nd. ed. (Springer, Berlin, 2003)
[3] Robert, C.P., Casella, G. *Monte Carlo Statistical Methods*2nd. ed. (Springer-Verlag, New York,2004)
[4] Coan, T. E., Ye J., 2003, *Muon Physics, TeachSpin Manual*. www.teachspin.com
[5] Cover, T. M., Thomas J. A. *Elements of Information Theory* (Wiley, New York, 1991).
[6] Teachspin, Muon Physics Experiment, www.teachspin.com
[7] Particle Data Review D, Particle Data Group, Lawrence-Berkeley National Laboratory http:/pdg.lbl.gov