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FORECASTING VALUE AT RISK (VAR) FOR EMERGING AND DEVELOPED MARKETS

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ABSTRACT

This paper explores different approaches to modelling and forecasting VaR, using both historical simulation and volatility-weighted bootstrap methods, where volatility is estimated using GARCH (1,1) and EGARCH (1,1). It examines the one day predictive ability of three historical simulation VaR models at the 90%, 95%, and 99% confidence levels for developed and emerging equity markets for the period 2011- 2017 that witnessed difficult and extreme market conditions. 870 scenarios of future returns are generated for each of the 500 days representing the out of sample period extending from March 2015 up to January 2017 in order to estimate the corresponding VaR for both markets. The GARCH (1,1) volatility-weighted model is accepted for both markets and is classified as the best performing model. The EGARCH (1,1) volatility-weighted model's results were inconclusive; in fact, the back-test was accepted at all confidence levels for the developed markets while rejected at the 99% confidence level for the emerging markets. The basic historical simulation failed in estimating an accurate VaR for the emerging markets.

Key words: Modeling Value at Risk (VaR); MSCI world index; MSCI emerging markets index; volatility-weighted bootstrap methods; GARCH models

RESUMEN

Este documento explora diferentes enfoques para modelar y pronosticar el VaR, utilizando tanto la simulación histórica como los métodos de bootstrap ponderados por volatilidad, en los que la volatilidad se estima utilizando GARCH (1,1) y EGARCH (1,1). Examina la capacidad predictiva de un día de tres modelos VaR de simulación histórica en los niveles de confianza del 90%, 95% y 99% para los mercados de valores desarrollados y emergentes para el período 2011-2017 que fueron testigos de condiciones de mercado difíciles y extremas. Se generan 870 escenarios de rentabilidad futura para cada uno de los 500 días que representan el período fuera de muestra que se extiende desde marzo de 2015 hasta enero de 2017 con el fin de estimar el VaR correspondiente para ambos mercados. El modelo GARCH (1,1) ponderado por volatilidad es aceptado para ambos mercados y está clasificado como el modelo de mejor desempeño. Los resultados del modelo EGARCH (1,1) ponderado por volatilidad no fueron concluyentes; de hecho, la prueba retrospectiva fue aceptada en todos los niveles de confianza para los mercados desarrollados, mientras que fue rechazada en el nivel de confianza del 99% para los mercados emergentes. La simulación histórica básica falló en la estimación de un VaR preciso para los mercados emergentes.

Palabras Clave: Modelado del valor en riesgo (VaR); índice mundial MSCI; índice MSCI de mercados emergentes; métodos bootstrap ponderados por volatilidad; modelos GARCH

JEL Classification: C12, C15, C58, G11

1. INTRODUCTION

Increased insecurity in financial markets is the main reason behind developing effective market risk's measures. Wide movements in market prices led to using risk measures that allow capturing and mitigating financial risk. Upper managements along with regulatory requirements demand allocating risk in order to make comprehensive investment decisions. For this reason, quantifying market risk became essential in the world of finance. A well-known tool called Value at Risk (VaR) for the specific area of market risk management became a widely used instrument in the 1990s although its origins go back to the 1952. The motivation behind estimating VaR was contributed to previous financial crisis that led the Basel Committee to set minimum capital requirements which can be computed through VaR. VaR is defined as the maximum loss amount given a specific confidence interval and a specific time horizon. VaR allowed quantifying the market risk to better compare risk limits. Additionally, VaR facilitates the formulation of hedging policies and evaluating the effect of a transaction on the portfolio net risk. If the definition of VaR is agreed upon, there is no consensus on how to calculate it. Today, no ideal model was derived to calculate VaR and risk managers are using different ways of calculation. Models for VaR calculation include the parametric approaches for exposures that assume a certain distribution such as the variance-covariance method and the non-parametric approaches such as historical simulation which can assume any distribution and looks at historical data, as well as the Monte Carlo simulation which involves developing a model for future returns (Jorion, 2007).

This paper models the VaR of a particular type of assets: the "MSCI world index" and the "MSCI emerging markets index". Both indices capture large and mid-cap representations across 23 developed markets and 23 emerging markets. The purpose behind choosing these two indices is to compare the different outcomes of VaR when applied on diverse markets during a critical period extending from 2011 to 2017 that was particularly representative of unusual market conditions and extreme events.

This paper attempts to evaluate three methods of calculating VaR which are categorized under the non-parametric approach. The first method used is the historical simulation which is a traditional approach extensively used by risk managers due to its simplicity. In 1998 Hull and White introduced an extension for the basic historical simulation that allows incorporating volatility into updating historical data. Hence, the second and third methods involve the use of the volatility-weighted bootstrap model, whereby volatility is computed using symmetric and asymmetric GARCH models, GARCH (1,1) and EGARCH (1,1) respectively. For each stock index, the parameters are estimated whereby the goodness of fit of these models is tested. The winning model based on the back-testing methodology reveals how diverse markets and selected time periods can affect the performance of VaR.

The paper is structured as follows. Section 2 is a literature review of the performance of different VaR models for emerging and developed markets. Section 3 reviews the methodology and defines the in sample and out of sample data while reviewing the specificities of the applied econometric models together with the back-testing methodology. Section 4 portrays the main findings where the parameters of each of the GARCH models are estimated 10 times, each 50 sub-sample, in order to have an accurate estimate of the volatility, and where 870 scenarios of future returns are generated for each of the 500 days representing the out of sample period to estimate the corresponding VaR. Also, this section assesses the results of the Kupiec back-test and the predictive ability of the chosen VaR models. Section 5 concludes and discusses the empirical findings.

2. LITERATURE REVIEW

Many studies tried to assess the accuracy of VaR models for different types of commodities. Some of them compare different types of VaR models to a certain type of asset, while others determine the power of a certain model when used on different types of assets and different time period of observations (Montero et al 2010).

A comprehensive study by Berkowitz and O'Brien (2002) examines VaR models for six US financial institutions. The results showed that VaR was highly inaccurate in some cases and losses suffered in banks exceeded the estimated VaR. The banks models examined were not able to adapt to changes in volatility. Their results suggest that simpler models such as GARCH can perform better than banks' structural models and could even be a replacement for these models. Same was echoed by Lucas (2000) who found that simpler univariate VaR models perform better than much more sophisticated models. Additionally, Jorion (2007) states that in the presence of volatility clusters, VaR estimates are more accurate when utilizing the GARCH models.

Angelidis et al. (2004) evaluated the accuracy of daily VaR, using a family of ARCH models, for five stock indices in Europe, Japan, and U.S. (CAC40, DAX30, FTSE100, NIKKEI225 and S&P500), with different distributional assumptions and different sample sizes. The results showed that using ARCH models, based on student's-t distribution or generalized error distribution, produces acceptable results. In contrary, using these models with normal distribution gave insufficient VaR estimates. Additionally, the sample size was seen to have important impact on VaR accuracy; for example, when using low confidence levels with a sample size less than 2000 observations, the probability values improved for GARCH(1,1) model.

Dimitrakopoulos et al. (2010) investigated the efficiency of VaR approaches for 20 stock markets, covering America, Asia and Europe, 16 of which represent the emerging market and 4 represent the developed market. The second part of their research was examining VaR approaches for the same stock markets in crisis period, hence they took the period 1997 - 1999, covering the Asian, Russian, and Brazilian financial crisis. Interestingly, for both markets, the same VaR models were seen to perform best. They noticed a certain pattern, whereby the majority of the VaR models tended to overestimate VaR for portfolios in emerging markets, when large sample size is used, and underestimate VaR for portfolios in developed markets, irrespective of the sample size chosen. Additionally, VaR models seemed to be affected less during crisis period in developed markets. Finally, the performance of parametric VaR models enhanced in post-crisis period in comparison to non-parametric models.

Gencay, Selcuk, and Ulugulyagci (2003) studied VaR models for markets with high volatility represented by the Istanbul Stock Exchange (ISE-100) Index. They compared the traditional approaches of VaR such as GARCH, historical simulation and variance-covariance methods to the extreme value theory models. They concluded that the variance-covariance method performed the worst on any sample size. The GARCH(1,1) was seen to also perform bad except at a confidence level of 95%. On the other hand and at higher confidence levels, the extreme value VaR performed the best.

Maghyereh and Al-Zoubi (2006) were interested in estimating VaR for emerging stock markets in the MENA region specifically in Bahrain, Egypt, Jordan, Morocco, Oman, Saudi Arabia, and Turkey. For most indices the extreme value theory seems to give the best VaR estimates. However, the weak performance of the EVT in Morocco and Turkey markets was contributed to the low number of extreme values in such markets.

Choi and Min (2011) attempted to find the factors behind the different performances of VaR by using conditional and unconditional approaches. Their analysis was conducted on different set of financial data constituted of stock market indices, stock prices and exchange rate data. The results showed that the GARCH models can be improved if used with more flexible distribution. Thus, replacing the normal distribution with Student's-t or generalized T distributions will considerably improve the performance of VaR models and solve the underestimation problem accompanied with the GARCH-normal model at 99% and 99.5% confidence levels.

Huang and Tseng (2009) used the kernel estimator (KE) approach which is a non-parametric method of estimating VaR and an improvement of the extreme value theory. The kernel estimator (KE) allowed them to directly study the tail behavior of the asset return. The most reliable VaR estimates were the models of the KE approach for both developed and emerging countries, while the other approaches were found to overestimate VaR. Also, Giot and Laurent (2003) estimated VaR for three international stock indices for traders with short and long positions. They found that VaR models based on a skewed Student's-t distribution performed better than the ones based on normal distribution or on Student's-t distribution.

Andjelic et al. (2010) used the delta normal and historical simulation approaches to test the performance of VaR. They used a sample of data for stock indices representing the central and Eastern European countries (Slovenian, Croatian, Serbian and Hungarian stock indices) aiming at investigating VaR performance in developing countries by using different rolling windows with confidence levels of 95% and 99%. In stable conditions, the proposed approaches performed well at a confidence level of

95%, however in volatile market conditions, the tested approaches gave accurate results at a confidence level of 99%.

Driven by the fact that the available literature does not clearly indicate a superior model for VaR estimation, Miletic et al. (2015) also chose several stock exchange indices in the central and Eastern European emerging capital markets specifically in Czech Republic, Hungary, Croatia, Romania and Serbia, to study the performance of VaR models. They used symmetric and asymmetric GARCH models based on Student's-t distribution and normal distribution. They found that results vary significantly at different confidence levels and that GARCH type models perform better than RiskMetric and historical simulation models.

Many articles tackled the sample size issue to check whether it affects the accuracy of VaR estimates. For instance, Hendricks (1996) applied twelve approaches of VaR and the results showed that models used with longer observations period produce better outcomes. Same was the conclusion of Danielsson (2002) who found that VaR estimates are more accurate with longer time period. On the other hand, Hoppe (1998) in Angelidis et al. (2004) argued that a smaller sample size could result in a more accurate VaR. Frey and Michaud (1997) also stated that in order to capture the recent structural changes in the return data, due to the changes in trading behavior, a smaller sample size would be appropriate.

3. METHODOLOGY AND SAMPLE

The review of the available literature in the previous section reveals the absence of a superior model for the VaR calculation. Knowing the growing importance of VaR calculation in the world of finance, the need to continuously research further the models of VaR with different sample size and periods becomes more evident. Therefore, this paper attempts to evaluate VaR for two market indices, the "MSCI world index" and the "MSCI emerging markets index" for the period November 2011 - January 2017 that witnessed several volatile episodes including the plunge in oil prices in the second half of 2014. The tested models are all under the non-parametric approach and involves the basic historical simulation with equal weights, and the incorporation of volatility using GARCH (1,1) and EGARCH (1,1). Outputs will be compared and ranked from the most to least accurate.

3.1 Sample and Data Collection

"MSCI world index" and "MSCI emerging markets index" closing prices data are downloaded from www.msci.com from November 1, 2011 till January 31, 2017 totaling 1,371 daily observations. For each index the 1,371 observations are used to build 500 sub-samples each consisted of 871 observations resulting in a total of 871,500 daily observations. The mentioned sub-samples are constructed as a moving window, whereby to construct a new sub-sample, the first observation of the previous sub-sample should be deleted and the next observation following the previous sub-sample should be added. The data from November 1, 2011 till March 3, 2015 totaling 871 observations, which is considered as the first sub-sample, is used to compute VaR for March 4, 2015 that is the first day of the out-of-sample period then the second sub-sample from November 2, 2011 till March 4, 2015 is used to compute VaR for March 5, 2015 which is the second day of the out-of-sample period and so on. The data from March 4, 2015 till January 31, 2017 totaling 500 is used for the out of sample period to back-test VaR. For the purposes of VaR calculation the daily observations of the two indices are converted into daily returns using the following equation:

$$u_i = \frac{V_i - V_{i-1}}{V_{i-1}} \quad (1)$$

Where V_i and V_{i-1} are respectively the closing prices of the index at the end of day i and at the end of the previous day $i-1$

The descriptive statistics of the daily returns of the two indices are presented in Table 1.

Table 1. *Descriptive Statistics of the "MSCI World Index" and "MSCI Emerging Markets Index"*

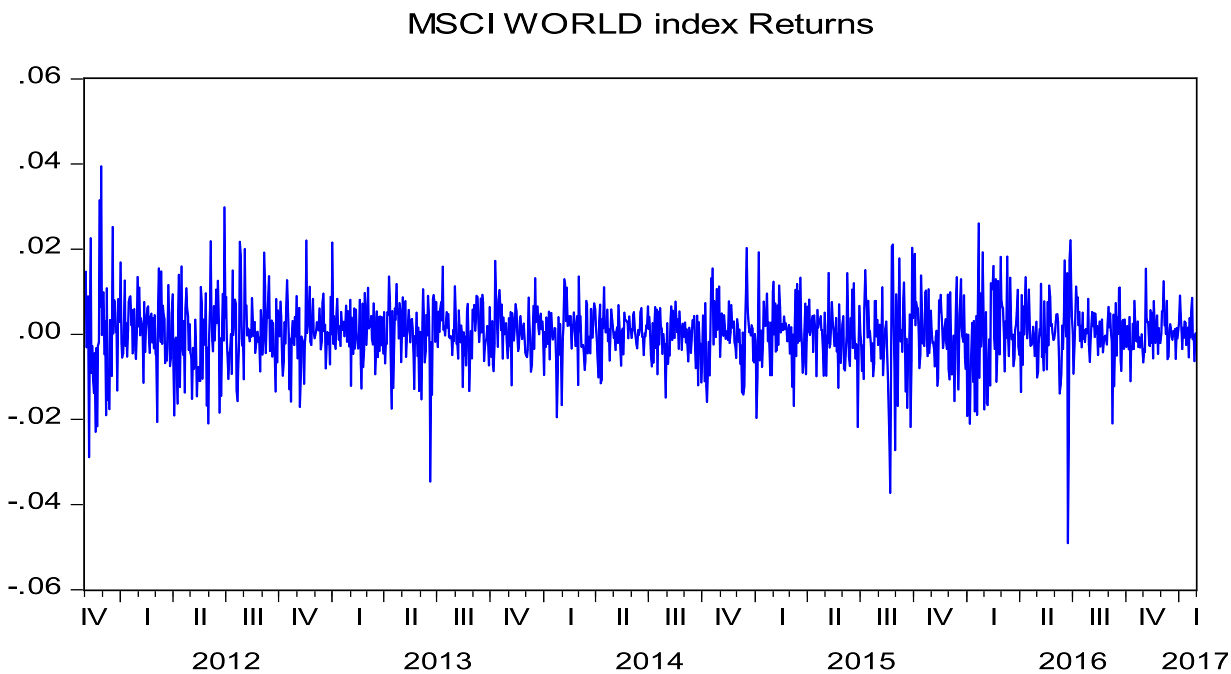
	MSCI world index	MSCI emerging markets index
Mean	0.000337	-0.00000365
Median	0.000498	0.000219
Maximum	0.039403	0.033796
Minimum	-0.049042	-0.049995
Std. Dev.	0.007504	0.009228
Skewness	-0.333058	-0.131926
Kurtosis	6.616137	4.751391
Jarque-Bera	771.7757	179.0698
Probability	0.000000	0.000000

Source: Own elaboration

The mean daily return for the "MSCI world index" was 0.0337% with a standard deviation of 0.7504% compared to -0.000365% and 0.9228% respectively for the MSCI emerging markets index. Both markets exhibit very close maximum and minimum returns. The Jarque-Bera probability confirms the non-normality of both return distributions. This is further confirmed by the kurtosis values greater than 3 revealing a leptokurtic returns' distribution of both markets.

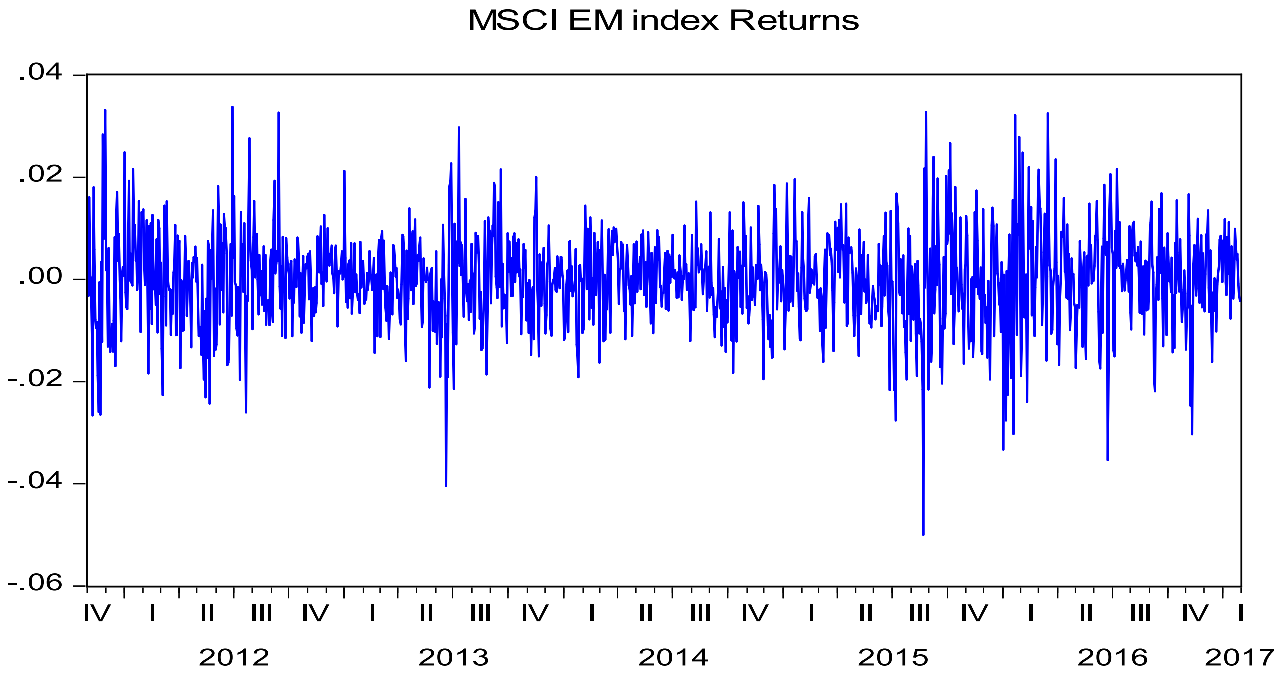
From the plot of return series in Figure 1 and Figure 2, persistence and volatility clustering are visible, which implies that the volatility can be forecasted.

Figure 1. *Time Series of "MSCI World Index" Daily Returns*



Source: Own elaboration

Figure 2. *Time Series of "MSCI Emerging Markets Index" Daily Returns*



Source: Own elaboration

The Augmented Dickey Fuller (ADF) test is used to check for stationarity. Table 2 summarizes the ADF test for the two indices' returns. The test statistics confirm the stationarity of data samples. Consequently, no transformation of the return series is needed.

Table 2. *Augmented Dickey Fuller Test*

		MSCI World Index		MSCI EM Index	
		t-Statistic	Prob.*	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-32.36838	0.0000	-30.22557	0.0000
Test critical values:	1% level	-2.566652		-2.566652	
	5% level	-1.941055		-1.941055	
	10% level	-1.616544		-1.616544	

*MacKinnon (1996) one-sided p-values.
Source: Own elaboration

3.2 Selected Models

3.2.1 Value at Risk

VaR is defined as the maximum loss amount given a specific confidence interval and a specific time horizon (Jorion, 2007). Mathematically it can be written as:

$$P(L(t) \leq \text{VaR}) = 1 - \alpha \quad (2)$$

with $0 \leq \alpha \leq 1$, and $\mathrm{L(t)}$ is the maximum probable loss that VaR at time t will not exceed with a probability of $(1 - \alpha)$.

Using the 500 sub-samples of data, 500 VaR estimates are calculated for each day of the out of sample period from March 4, 2015 till January 31, 2017 at different confidence levels: 90%, 95% and 99% for both the developed and emerging markets.

In order to estimate VaR, all models used are under the historical simulation approach known to be non-parametric.

3.2.2 Basic Historical Simulation

The main assumption for this method is that the returns are independently and identically distributed (IID), which means that returns are affected only by new information and they are time uncorrelated. Historical simulation approach assumes that the historical price changes will reflect the future price changes and it requires collecting as many historical data as possible to estimate the VaR. The historical simulation includes generating a set of data representing the daily changes in the market variable through a period of time. Using the past observed day-to-day variations in the values of the two selected market indices, the profit/loss probability distribution can be estimated for each index over a future period of time. The first sub-sample of each stock index, consisting of 871 days of observation from November 1, 2011 till March 3, 2015, is used to create 870 alternative scenarios for what can happen on day 872 (March 4, 2015). Scenario 1 assumes that the percentage changes in the value of the stock index are equivalent to what they were on day 1 and scenario 2 assumes that the percentage changes in the value of the stock index are equivalent to what they were on day 2, etc. The value under the ith scenario is calculated mathematically as follows (Hull, 2012):

$$V_{ith\ scenario} = v_n \frac{v_i}{v_{i-1}} \quad (3)$$

where v_i is the value of the stock index on day i; v_n is the value of the stock index on the last day of the chosen time period

Using equation (4), the return scenarios are calculated for each simulation trial resulting in 870 return scenarios for the reason of deducing the losses and gains expected on the first day of the out of sample period.

$$u_{ith\ scenario} = \frac{(V_{ith\ scenario} - v_n)}{v_n} \quad (4)$$

where:

$v_{it\ \ scenario}$ is the value of the stock index under the ith scenario

v_n is the value of the stock index on the last day of the chosen time period

The same procedure outlined above is repeated for each sub-sample in order to estimate VaR for 500 days from March 4, 2015 till January 31, 2017.

3.2.3 Incorporating Volatility Updating into Historical Simulation

Hull and White (1998) elaborated an extension for the basic historical simulation which involves incorporating volatility in updating the historical return. Because the volatility of a market variable may vary over time, sometimes it is high other times low, they recommend modifying the historical data to reflect the variation in volatility. This approach uses the variation in volatility in a spontaneous way to estimate VaR by including more recent information. The first sub-sample of each stock index, consisting of 871 days of observation from November 1, 2011 till March 3, 2015, is used to create 870 alternative scenarios for what can happen on day 872 (March 4, 2015). Using this approach, the value of the stock index under the ith scenario becomes:

$$V_{it\ \ scenario} = v_n \frac{v_{i-1} + (v_i - v_{i-1})\sigma_{n+1}/\sigma_i}{v_{i-1}} \quad (5)$$

where:

v_i is the value of the stock index on day i;

v_n is the value of the stock index on the last day of the chosen time period;

σ_i is the estimate of the daily volatility on day i;

σ_{n+1} is the most recent estimate of the daily volatility

Similar to the basic historical simulation method, for the reason of deducing the losses and gains expected on the first day of the out of sample period, equation (4) will be used to calculate the return scenarios for each simulation trial resulting in 870 return scenarios. Hull and White (1998) replaced the return scenarios by the following equation:

$$u_{ith\ scenario} = \sigma_{n+1} \frac{u_i}{\sigma_i} \quad (6)$$

where:

u_i is the return on day i;

σ_i is the estimate of the daily volatility on day i;

σ_{n+1} is the most recent estimate of the daily volatility

Equation (6) allows calculating the return scenarios directly using the volatilities and the returns of the indices; hence calculating different price scenarios is irrelevant through this technique. However, equation (6) is used only to confirm the results obtained from equation (4), under the method of incorporating volatility updating into historical simulation. The same procedure outlined above is repeated for each sub-sample in order to estimate VaR for 500 days from March 4, 2015 till January 31, 2017.

As previously indicated, incorporating volatilities requires estimating daily variance using the Generalized Autoregressive Conditional Heteroskedasticity model GARCH(1,1), and the Exponential GARCH model EGARCH(1,1) described below.

3.2.4 Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

Engle (1982) introduced the Autoregressive Conditional Heteroskedastic model, ARCH, which permitted the conditional variance to vary over time as a function of past errors. Bollerslev (1986) generalized this model and developed the GARCH model by adding the lagged conditional variances. GARCH(p,q) can be written as:

$$\varepsilon_t | \psi_{t-1} \sim N(0, h_t) \quad (7)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \quad (8)$$

$$= \alpha_0 + A(L)\varepsilon_t^2 + B(L)\, h_t$$

with

$p \geq 0, \quad q > 0$

$\alpha_0 > 0 \quad \alpha_i \geq 0, \quad i = 1, \dots, q$

$\beta_i \geq 0 \quad i = 1, \dots, p$

Where ε_t indicate a real-valued discrete-time stochastic process, and ψ_t denote the information set (σ-field) of all information during time t.

The GARCH(p,q) regression model could be achieved, by letting the ε_t's be innovations in a linear regression:

$$\varepsilon_t = y_t - x'_t b \quad (9)$$

where

y_t: dependent variable;

x_t: a vector of explanatory variables;

b: a vector of unknown parameters

h_t can be expressed as a distributed lag of past ε_t^2's, when all the roots of $1 - B(z) = 0$ lie outside the unit circle:

$$h_t = \alpha_0(1 - B(1))^{-1} + A(L)(1 - B(L))^{-1}\varepsilon_t^2 \quad (10)$$

$$= \alpha_0(1 - \sum_{i=1}^{p} \beta_i)^{-1} + \sum_{i=1}^{\infty} \delta_i \varepsilon_{t-1}^2$$

The power series expansion of $D(L) = A(L)(1 - B(L))^{-1}$ allows finding the δ_i's.

Brooks (2008) stated that the lag (1,1) is adequate to capture the data's volatility clustering. Additionally, GARCH(1,1) is the most popular between the GARCH models since it calculates σ^2 based on the latest observation and the latest estimate of the variance rate; GARCH (1,1) model is defined by the following equation:

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (11)$$

With $\alpha_0 > 0,\ \alpha_1 \geq 0,\ \beta_1 \geq 0$

And for a stable model the following should be met: $\alpha_1 + \beta_1 < 1$

The following notation of GARCH(1,1) will be adopted in this study to calculate the variance for day t :

$$\sigma_t^2 = \gamma v_l + \alpha u_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (12)$$

where $\gamma v_l = \omega$ and the parameters ω, α ,and β are weights; α ,and β are the weights assigned to u_{t-1}^2 and σ_{t-1}^2 respectively.

γ can be calculated using equation (13):

$$\gamma = 1 - \alpha - \beta \quad (13)$$

v_l is the long-run variance and it can be calculated using the following equation:

$$v_l = \frac{\omega}{\gamma} \quad (14)$$

Parameters of the GARCH(1,1) model are estimated 10 times each 50 sub-sample using the maximum likelihood approach (equation 17) in order to get an accurate estimate of the volatility and to keep the computation time short. Therefore, the estimation period will slide down to each day of the out of sample period.

Despite the simplicity of GARCH(1,1) model, it doesn't allow the effect of a shock to be independent of its sign, whereas the stock market is known to have asymmetric response. In fact, the volatility increases due to a drop in price levels in stock market, however it may decrease even more due to a rise of the same magnitude in price levels. Actually, the GARCH(1,1) model includes only the squared residuals in its conditional variance equation, hence the signs of the residuals have no impact on the calculated conditional volatility. In finance, bad shocks are known to have larger effect on the volatility than good shocks, or a falling market will lead to a higher volatility than a rising market. The asymmetric news influence on the market variable is known as the leverage effect (Miron and Tudor, 2010). To solve this problem, we will use the EGARCH model which allows for asymmetric effect to be considered.

3.2.5 Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH)

Using the EGARCH(1,1) model that was first proposed by Nelson in 1991, the variance for day t is calculated using equation (15):

$$ln\,\sigma_t^2 = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \quad (15)$$

Whereby the error terms are presumed to be normally distributed with mean equal to $\sqrt{\frac{2}{\pi}}$, ω is the long-term average value and α represents the "GARCH" effect or the symmetric effect of the model. Including the parameter β allows capturing the persistence of volatility shocks. The parameter γ allows to determine if there is leverage effect. Hence, if $\gamma = 0$ then the model is symmetric. When $\gamma < 0$, negative shocks generate more volatility than positive shocks of same magnitude.

By nature the EGARCH model assures that the conditional variance σ_t^2 will always be positive even if the parameters are negative, since $ln\,\sigma_t^2$ instead of σ_t^2 is used to calculate the conditional variance. Hence, one advantage of the EGARCH model over the GARCH model is that the positive constraints of the parameters could be ignored. The parameters $\omega, \alpha, \beta, \gamma$ will be estimated 10 times as previously explained, similar to the symmetric GARCH model, using the maximum likelihood approach (equation 17).

3.2.6 Maximum Likelihood for Parameters Estimation

The maximum likelihood method is a technique that involves determining the parameters values of GARCH and similar models by maximizing the likelihood of historical data occurring. $f(y|\theta)$ represents the probability density function, where y is random variable conditioned on a set of parameters θ. This function is a mathematical description, whereby given an observed sample of time series, the process of generating data can be identified. From this process, the joint density of n observations, known as the likelihood function, is the product of the individual densities:

$$f(y_1 \dots, y_n|\theta) = \prod_{i=1}^{n} f(y_i|\theta) = L(\theta|y) \quad (16)$$

y is used to indicate the time series at time i and θ denotes the vector of model parameters. Furthermore, the parameters are constants and their estimation will be based on the observed data. We use the Log of the Likelihood Function (LLF) since it is relatively simpler to work with (Greene, 2003):

$$\ln L(\theta|y) = \sum_{i=1}^{n} \ln f(y_i|\theta) \quad (17)$$

3.2.7 Back-Testing Methodology

To evaluate the models of VaR, a back-test is done. This shows how well the model used for estimating VaR has performed if used in the past. Usually the number of times the actual loss exceeds VaR is considered as an exception. If exceptions occurred on 1% of the days, the current model for calculating a one-day 99% VaR will be accurate.

The most common test used for back-testing VaR is the Kupiec (1995) test which will be used in this study.

If the confidence level is X%, and if the model is accurate, then the probability that the actual loss exceeds VaR will be $p = 1 - X\%/100$

The number of exceptions will follow a binomial distribution:

$$P\left(\frac{N}{T}, p\right) = \binom{T}{N} p^N (1-p)^{T-N} \quad (18)$$

where:

N is the number of exceptions;

T is the number of trials;

p is the probability of failure.

Kupiec (1995) suggested the following log-likelihood Ratio (LR) to test the accuracy of VaR:

$$LR = -2ln[(1-p)^{T-N}p^N] + 2ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^N\right] \quad (19)$$

Equation (19) follows a chi-square distribution with one degree of freedom. There is a 5% probability that the chi-square variable with one degree of freedom will be more than 3.84. Hence, the VaR model is rejected whenever the LR is greater than 3.84. The LR value is large for either low or high number of exceptions, hence the VaR models are rejected in both cases were high or low failures occur. Additionally, the p values (probability of failure) are 0.1, 0.05, and 0.01 corresponding to VaR confidence levels of 90%, 95% and 99% respectively. The T value (number of trials) is 500, constant for all models since the out of sample period is equivalent to 500 days. Furthermore, the daily losses are taken into consideration and compared with the estimated VaRs, consequently the N values (number of exceptions) will be determined by counting the number of times the actual loss return exceeds the computed VaR on a given day.

4. FINDINGS

4.1 Basic Historical Simulation

As illustrated in Table 3, the number of violations where the actual loss exceeds VaR is greater for the emerging markets index than for the developed markets index. Particularly, at a 95% confidence level, the world index returns exceeded the VaR limits in 5.8% of the observations, while the corresponding percentage for the emerging index is 8.4%.

Table 3. *Number of Exceptions Using Basic Historical Simulation*

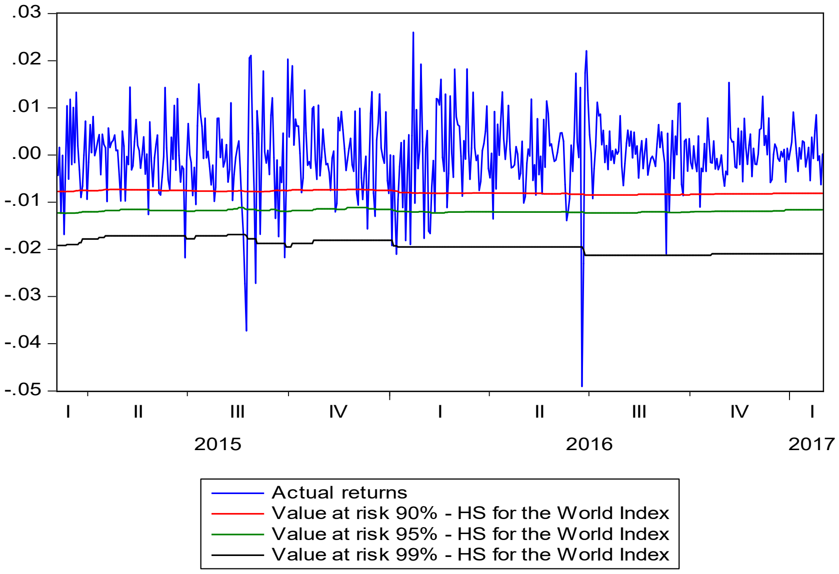
	90% CL VaR	95% CL VaR	99% CL VaR
Exceptions "MSCI world index"	64	29	9
Exceptions "MSCI emerging markets index"	66	42	13

Source: Own elaboration

Figures 3 and 4 illustrate the results for the whole out of sample period for both the "MSCI world index" and the "MSCI emerging markets index". Clearly, the emerging markets index exhibits a more volatile structure, hence the basic historical simulation method wasn't able to capture the large losses and VaR is exceeded in several occasions as presented in Table 3 where the number of exceptions is greater for the emerging markets index compared to the developed markets index. Also, VaR at 90% confidence level visibly displays the poorest performance and seems to understate the risk of both stock indices. On the contrast, the number of times the loss return exceeded the 99% VaR for the world index seems to be relatively limited.

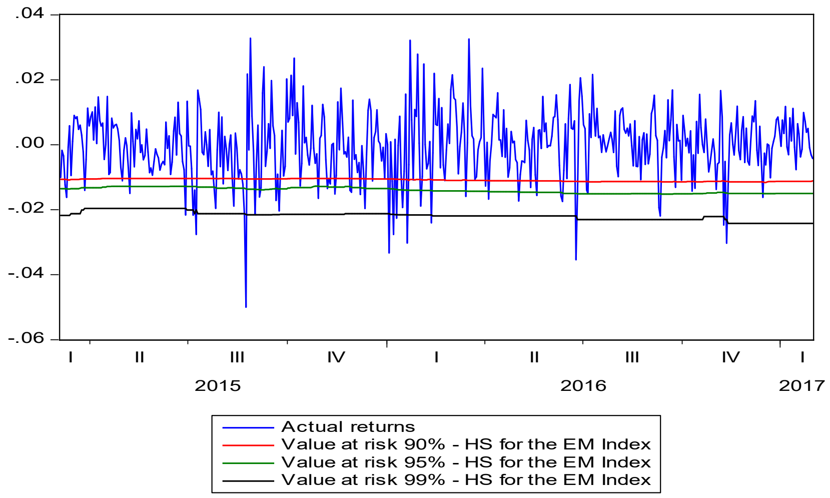
Using equally weighted observations fails in capturing the shifts in risk, which is a major disadvantage of the basic historical simulation.

Figure 3. *"MSCI World Index" Out-of-Sample Daily Returns Vs. VaR (Basic Historical Simulation)*



Source: Own elaboration

Figure 4. *"MSCI Emerging Markets Index" Out-of-Sample Daily Returns Vs. VaR (Basic Historical Simulation)*



Source: Own elaboration

4.2 Incorporating Volatility Updating into Historical Simulation

The parameters of the GARCH(1,1) and EGARCH(1,1) models are found using EViews 7. We assume that the probability distribution function of errors is normally Gaussian distributed knowing that the Student's t-distribution and the Generalized Error Distribution (GED) were also tested to optimize the predictive ability of both models, GARCH(1,1) and EGARCH(1,1). The Log Likelihood Function and the Akaike-Information Criterion were calculated in addition to a series of tests done on the standardized residuals (Exhibit 1 and Exhibit 2) which all confirm that the models' assumptions are respected and that both models are stable. In fact, the calculated means and standard deviations of the errors are all found to be close to zero and one respectively and the absence of ARCH effect was confirmed by applying the heteroskedasticity test and the serial correlation in the squared residuals is insignificant under the assumption of normal distribution of errors.

4.2.1 GARCH(1,1) Volatility-Weighted Historical Simulation

As previously mentioned, the GARCH(1,1) parameters are estimated 10 times each 50 sub-samples under the assumption of normal distribution of errors and are shown in Table 4.

Table 4. *GARCH(1,1) Estimated Parameters*

	The constant in the conditional volatility equation	The first coefficient of the ARCH component	The first coefficient of the GARCH component	Goodness of Fit (Normal Distribution)	
"MSCI world index" Time period chosen for parameters' computation	ω	α	β	**LLF**	**AIC**
02/11/2011 03/03/2015	0.00000191	0.078393	0.88170	3,112.24	-7.145379
11/01/2012 12/05/2015	0.00000323	0.08933	0.840825	3,148.897	-7.229649
21/03/2012 21/07/2015	0.00000271	0.079249	0.86175	3,145.979	-7.222941
30/05/2012 29/09/2015	0.00000390	0.118751	0.800846	3,135.359	-7.198525
08/08/2012 08/12/2015	0.00000428	0.120818	0.785989	3,164.989	-7.266641
17/10/2012 16/02/2016	0.00000352	0.12826	0.803475	3,130.197	-7.18666
26/12/2012 26/04/2016	0.00000311	0.119648	0.81939	3,125.102	-7.174948
06/03/2013 05/07/2016	0.00000321	0.14833	0.803015	3,100.618	-7.118662
15/05/2013 13/09/2016	0.00000463	0.172041	0.752104	3,104.452	-7.127476
24/07/2013 22/11/2016	0.00000438	0.173787	0.74732	3,129.433	-7.184903
"MSCI emerging markets index" Time period chosen for parameters' computation					
02/11/2011 03/03/2015	0.00000135	0.058404	0.921335	2,960.075	-6.795575
11/01/2012 12/05/2015	0.00000144	0.05273	0.925343	2,986.421	-6.856141
21/03/2012 21/07/2015	0.00000255	0.074926	0.887233	2,993.274	-6.871895
30/05/2012 29/09/2015	0.00000204	0.074955	0.896666	2,968.875	-6.815806
08/08/2012 08/12/2015	0.00000251	0.083301	0.88206	2,979.469	-6.84016
17/10/2012 16/02/2016	0.00000110	0.063885	0.924658	2,947.169	-6.765906
26/12/2012 26/04/2016	0.00000138	0.064866	0.919423	2,914.659	-6.691171
06/03/2013 05/07/2016	0.00000157	0.075318	0.909924	2,885.933	-6.625134
15/05/2013 13/09/2016	0.00000375	0.095363	0.86453	2,873.636	-6.596865
24/07/2013 22/11/2016	0.00000244	0.092289	0.882427	2,891.936	-6.638933

Source: Own elaboration

Based on the above estimated parameters the daily variances are calculated 871 times for each sub-sample using equation (12), which results in a total of 435,500 values of variances for each stock index. These volatilities are then plugged in equation (5) to generate various price scenarios. The outcome is 870 price scenarios on each day from March 4, 2015 till January 31, 2017 and a total of 435,000 scenarios for each stock index. Accordingly, the generated price scenarios are fitted into equation (4) for the purpose of estimating the return scenarios. On each day of the out of sample period, 870 return scenarios are created, which represents profit and loss distribution. Additionally, the return scenarios are deduced using equation (6). The results obtained from equation (4) and equation (6) led to generating equivalent return scenarios which further favors our results. The 90th, 95th, and 99th percentiles of the profit/loss probability distribution are estimated and represent the VaR confidence levels. We ended up with 500 VaR estimates covering the out of sample period, for each confidence level and for each stock index. The actual returns in the out of sample period are considered to determine the number of exceptions were the loss return exceeded VaR as a loss value. Table 5 depicts the number of exceptions obtained from computing VaR using the GARCH(1,1) volatility-weighted historical simulation.

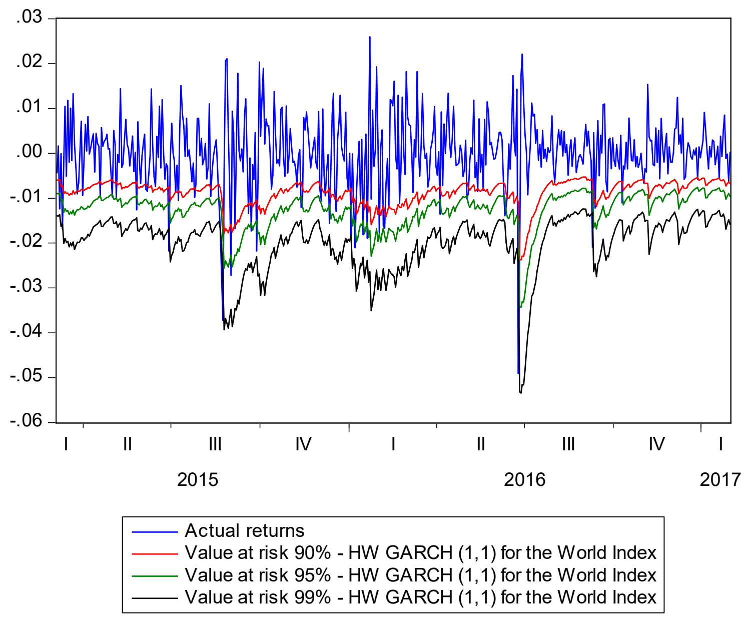
Table 5. *GARCH(1,1) Volatility-Weighted Historical Simulation: Number of Exceptions*

	90% CL VaR	95% CL VaR	99% CL VaR
Exceptions “MSCI world index”	54	25	7
Exceptions “MSCI emerging markets index”	51	33	8

Source: Own elaboration

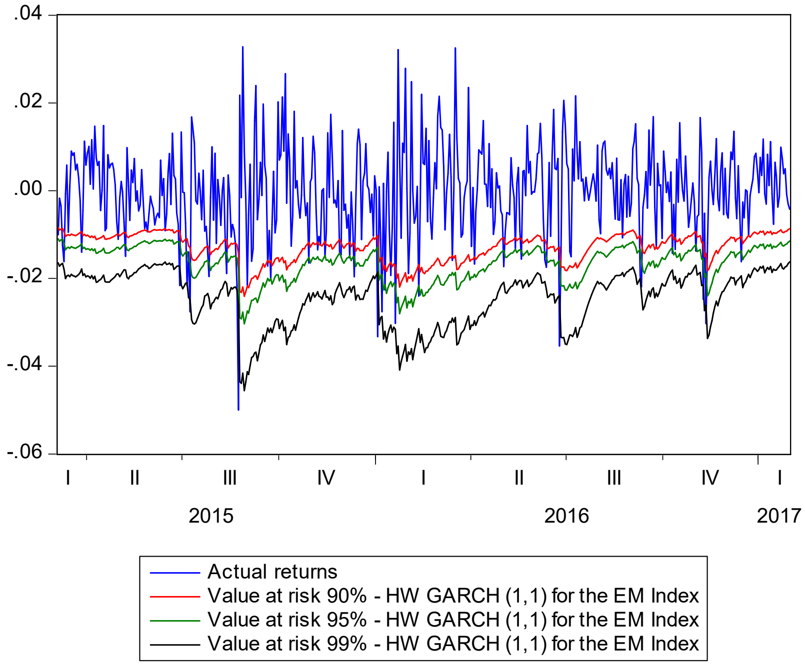
Incorporating GARCH(1,1) using historical simulation led to a decrease in the number of violations compared to the basic historical simulation, for both sock indices and at all confidence levels, however, this is to be confirmed by the Kupiec test. Figures 5 and 6 show the distribution of daily returns in comparison with VaR weighted by GARCH (1,1) at the three confidence levels. The VaR curves are slightly shifting downwards with a falling market, hence allowing a better capture of the changes in risk which justifies the decrease in the number of exceptions.

Figure 5. *“MSCI World Index” Out-of-Sample Daily Returns Vs. VaR (GARCH(1,1) Volatility-Weighted Historical Simulation)*



Source: Own elaboration

Figure 6. *"MSCI Emerging Markets Index" Out-of-Sample Daily Returns Vs. VaR (GARCH(1,1) Volatility-Weighted Historical Simulation)*



Source: Own elaboration

4.2.2 EGARCH(1,1) Volatility-Weighted Historical Simulation

The same methodology is also implemented at this level. Table 6 summarizes the EGARCH(1,1) parameters together with models fit, LLF and AIC.

Coefficients of the asymmetric effect γ range between -21% and -5% indicating that negative shocks are more destabilizing than positive shocks. On the other hand, the observed values of the first coefficient of GARCH component β vary between 91% and 99% for the two stock indices which explains the high relative importance of the observations on the returns in determining the current variance rate. Table 7 and Figures 7 and 8 demonstrate how incorporating EGARCH(1,1) into historical simulation also resulted in a lower number of violations, compared to the basic historical simulation, for both stock indices and at all confidence levels. However, compared to the GARCH(1,1) volatility-weighted model, the outcomes of exceptions are inconclusive.

Table 6. *EGARCH(1,1) Estimated Parameters*

	The constant in the conditional volatility equation	The first coefficient of the ARCH component	The first leverage coefficient	The first coefficient of the GARCH component	Goodness of Fit (Normal Distribution)	
"MSCI world index" Time period chosen for parameters' computation	ω	α	γ	β	**LLF**	**AIC**
02/11/2011 03/03/2015	-0.451016	0.101003	-0.149391	0.9630160	3,142.06	7.211632
11/01/2012 12/05/2015	-0.808185	0.121118	-0.174131	0.9293450	3,178.617	7.295671
21/03/2012 21/07/2015	-0.633244	0.082618	-0.172772	0.9437810	3,178.559	7.295538
30/05/2012 29/09/2015	-0.651277	0.082557	-0.187581	0.9418250	3,170.970	7.278091
08/08/2012 08/12/2015	-0.933459	0.101108	-0.207546	0.9158040	3,201.957	7.349328
17/10/2012 16/02/2016	-0.670377	0.058454	-0.202982	0.9382490	3,171.616	7.279576
26/12/2012 26/04/2016	-0.5152	0.068294	-0.190747	0.9542960	3,165.643	7.265845
06/03/2013 05/07/2016	-0.563856	0.135315	-0.170907	0.9534670	3,120.962	-7.16313
15/05/2013 13/09/2016	-0.552897	0.098937	-0.18634	0.9518930	3,126.669	7.176251
24/07/2013 22/11/2016	-0.640584	0.134997	-0.171361	0.9462950	3,146.134	7.220998
"MSCI emerging markets index" Time period chosen for parameters' computation						
02/11/2011 03/03/2015	-0.086315	0.022871	-0.067451	0.9929510	2,981.045	6.841483
11/01/2012 12/05/2015	-0.218505	0.067677	-0.054669	0.9829150	2,997.839	-6.88009
21/03/2012 21/07/2015	-0.095404	0.005307	-0.080796	0.9905930	3,018.017	6.926477
30/05/2012 29/09/2015	-0.061144	-0.0259	-0.082128	0.9914350	3,002.524	-6.89086
08/08/2012 08/12/2015	-0.168643	0.022236	-0.079692	0.9842550	3,001.893	6.889408
17/10/2012 16/02/2016	-0.101029	-0.01635	-0.088045	0.9882010	2,979.528	6.837996
26/12/2012 26/04/2016	-0.088801	0.004657	-0.078066	0.9905390	2,944.507	6.757488
06/03/2013 05/07/2016	-0.10293	0.030997	-0.074351	0.9914520	2,908.235	6.674103
15/05/2013 13/09/2016	-0.16995	0.054703	-0.084836	0.9862590	2,889.595	6.631253
24/07/2013 22/11/2016	-0.191068	0.082169	-0.078883	0.9864750	2,905.647	6.668154

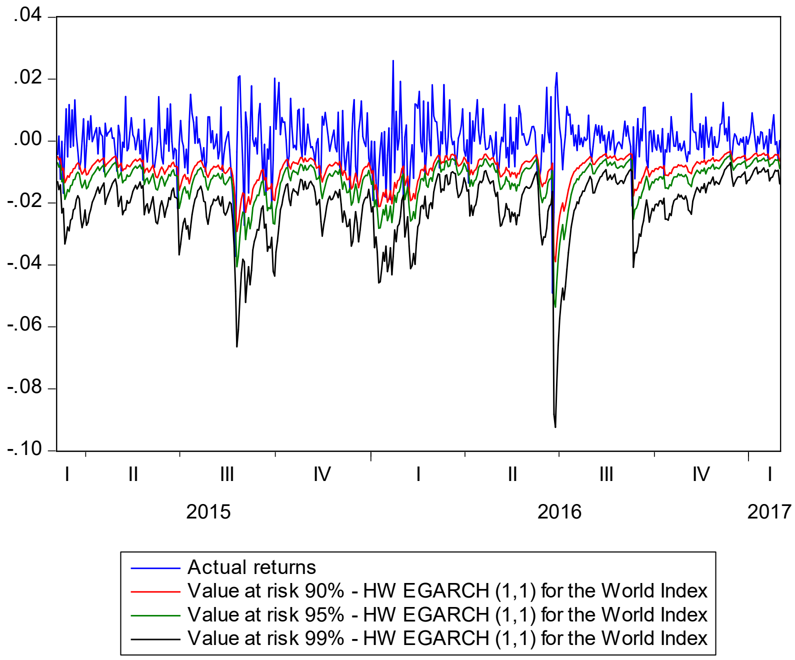
Source: Own elaboration

Table 7. *EGARCH(1,1) Volatility-Weighted Historical Simulation: Number of Exceptions*

	90% CL VaR	95% CL VaR	99% CL VaR
Exceptions “MSCI world index”	52	29	4
Exceptions “MSCI emerging markets index”	56	33	11

Source: Own elaboration

Figure 7. *“MSCI World Index” Out-of-Sample Daily Returns Vs. VaR (Volatility-Weighted Historical Simulation (EGARCH(1,1))*



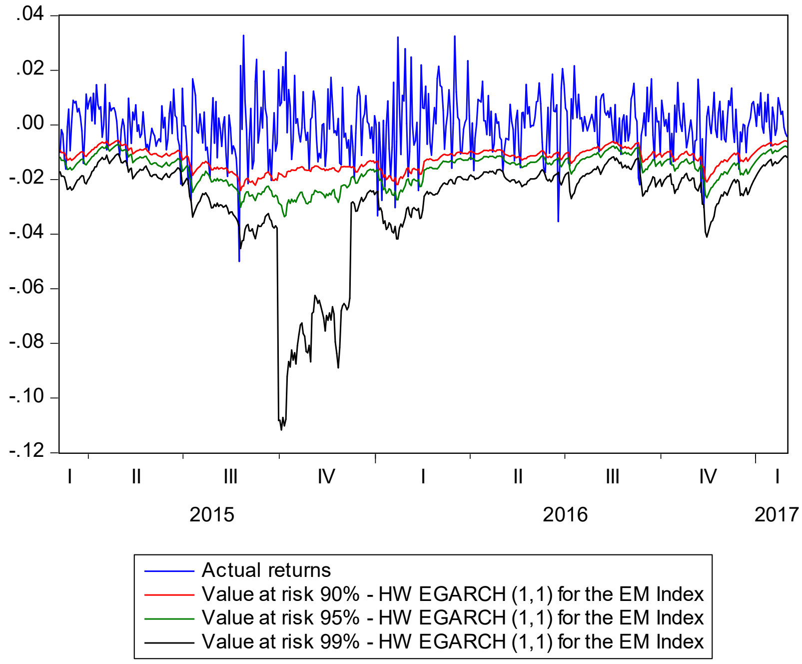
Source: Own elaboration

The VaR plots for the developed markets seem more coherent compared to the ones of the emerging markets. It is also apparent that VaR with a confidence level of 99% overrates the risk for the emerging markets during 2015 although it generated 11 violations hence underrating the risk during other different periods. Consequently, it cannot be concluded if EGARCH(1,1) volatility-weighted model yields accurate VaR estimates.

4.3 Kupiec Test and Log-likelihood Ratio Outputs

Comparing the violations’ outcome of each model is inconclusive as to which model yields the most accurate VaR. The Kupiec test is therefore utilized by calculating LR using equation (19). Results of the Kupiec test for the developed and emerging markets index are presented in Table 8.

Figure 8. "MSCI Emerging Markets Index" Out-of-Sample Daily Returns Vs. VaR (Volatility-Weighted Historical Simulation (EGARCH(1,1))



Source: Own elaboration

Table 8. *Kupiec Test for Developed and Emerging Markets*

Models Applied			"MSCI world index"		MSCI emerging markets index"	
	VaR CL	95% Critical value (Chi-square distribution with one degree of freedom)	LR	Test Outcome	LR	Test Outcome
Basic Historical Simulation	90%	3.84	4.04	Reject	5.22	Reject
	95%	3.84	0.64	Accept	10.19	Reject
	99%	3.84	2.61	Accept	8.97	Reject
Incorporating volatility to historical Simulation using GARCH (1,1)	90%	3.84	0.35	Accept	0.02	Accept
	95%	3.84	0.00	Accept	2.46	Accept
	99%	3.84	0.72	Accept	1.54	Accept
Incorporating volatility to historical Simulation using EGARCH (1,1)	90%	3.84	0.09	Accept	0.77	Accept
	95%	3.84	0.64	Accept	2.46	Accept
	99%	3.84	0.22	Accept	5.42	Reject

Source: Own elaboration

Kupiec test results unveil that both the GARCH(1,1) and the EGARCH(1,1) volatility-weighted historical simulation models are classified as the best performing models at all confidence levels for the developed markets index, while the basic historical simulation model is ranked the worst in terms of accuracy as it is rejected at a 90% confidence level.

5. DISCUSSION AND CONCLUSION

The GARCH(1,1) volatility-weighted model was the only model accepted at all confidence levels for the emerging markets index, whereby the basic historical simulation failed to produce an accurate VaR at a 90%, 95% and 99% confidence levels. The EGARCH(1,1) volatility-weighted model is ranked as the second best model since the 90% and 95% VaRs are accepted while the 99% VaR underestimates the risk of emerging markets index. The superiority of the GARCH(1,1) model can be related to its ability in taking into account the volatility changes in a natural way and generating VaR estimates that include more recent information. Such findings confirmed by those of Dimitrakopoulos et al. (2010), who state that the filtered historical simulation which is a mix between the GARCH model and the traditional historical simulation and the extreme value method- peaks over threshold are the most successful VaR models for both emerging and developed markets.

On the other hand, the ability of the GARCH(1,1) and the EGARCH(1,1) in incorporating information into the historical simulation VaR allowed to obtain VaR estimates that surpass the maximum loss in the historical data. This is in agreement with the conclusion reached by Hull and White (1998). The basic historical simulation was ranked as the worst method since it overlooks the volatility changes, which is a main drawback.

Interestingly, VaR models performed differently for the developed and emerging markets indices in some cases. While the basic historical simulation VaR estimates are accurate using 95% and 99% confidence levels for the developed markets index, this method failed at all confidence levels for the emerging markets index. Similarly, EGARCH(1,1) volatility-weighted model led to rejecting the 99% VaR for the emerging markets index and accepting the same model for the developed markets index. This implies that VaR models may act differently depending on the attributes of the market chosen which corroborates with the results reached by Andjelic et al. (2010) who proposed that VaR models which perform well in developed markets do not necessarily in developing and illiquid markets. Finally, our results are in line with Jorion (2007) who stated that when there are volatility clusters, VaR estimates are more accurate when utilizing GARCH models.

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Exhibit 1: *Residuals Analysis of GARCH(1,1) Assuming a Normal Distribution of Errors*

			Null Hypothesis		
"MSCI world index" Residual Analysis for each sub-sample	**AVG**	**STDEV**	**"There is no serial correlation in the squared residuals"**	**"Errors are normally distributed"**	**"There is no ARCH effect"**
02/11/2011 03/03/2015	-0.032536	1.002737			
11/01/2012 12/05/2015	-0.034637	0.999799			
21/03/2012 21/07/2015	-0.034492	1.001031			
30/05/2012 29/09/2015	-0.047173	1.0001			
08/08/2012 08/12/2015	-0.046767	0.998998	Accepted	Rejected	Accepted
17/10/2012 16/02/2016	-0.056566	0.999604			
26/12/2012 26/04/2016	-0.049559	1.000391			
06/03/2013 05/07/2016	-0.04869	0.99876			
15/05/2013 13/09/2016	-0.047147	0.999115			
24/07/2013 22/11/2016	-0.047752	0.998953			
"MSCI emerging markets index" Residual Analysis for each sub-sample					
02/11/2011 03/03/2015	-0.02702	1.003626			
11/01/2012 12/05/2015	-0.020922	1.00362			
21/03/2012 21/07/2015	-0.025783	1.000915			
30/05/2012 29/09/2015	-0.038121	1.00054			
08/08/2012 08/12/2015	-0.036427	0.999717	Accepted	Rejected	Accepted
17/10/2012 16/02/2016	-0.045983	0.998672			
26/12/2012 26/04/2016	-0.035669	1.00219			
06/03/2013 05/07/2016	-0.035623	0.999113			
15/05/2013 13/09/2016	-0.0295	1.000936			
24/07/2013 22/11/2016	-0.02648	1.0002			

Exhibit 2: *Residuals Analysis of EGARCH(1,1) Assuming a Normal Distribution of Errors*

			Null Hypothesis		
"MSCI world index" Residual Analysis for each sub-sample	**AVG**	**STDEV**	**"There is no serial correlation in the squared residuals"**	**"Errors are normally distributed"**	**"There is no ARCH effect"**
02/11/2011 03/03/2015	0.002836	1.005313	Accepted	Rejected	Accepted
11/01/2012 12/05/2015	0.000146	1.002045			
21/03/2012 21/07/2015	-0.000917	1.001545			
30/05/2012 29/09/2015	0.000617	0.999885			
08/08/2012 08/12/2015	-0.006545	1.000092			
17/10/2012 16/02/2016	-0.004191	0.998202			
26/12/2012 26/04/2016	0.002645	1.002761			
06/03/2013 05/07/2016	0.005644	1.00363			
15/05/2013 13/09/2016	0.00576	1.002256			
24/07/2013 22/11/2016	0.000634	1.000868			
"MSCI emerging markets index" Residual Analysis for each sub-sample					
02/11/2011 03/03/2015	0.014417	1.012125	Accepted	Rejected	Accepted
11/01/2012 12/05/2015	0.020225	1.013081			
21/03/2012 21/07/2015	-0.003619	1.007475			
30/05/2012 29/09/2015	0.017266	1.02589			
08/08/2012 08/12/2015	0.010538	1.009677			
17/10/2012 16/02/2016	-0.0114	1.003296			
26/12/2012 26/04/2016	0.063195	0.98456			
06/03/2013 05/07/2016	0.024841	0.994022			
15/05/2013 13/09/2016	0.021192	1.011055			
24/07/2013 22/11/2016	0.000926	1.003354			

Note: Exhibit 1& 2 show the residual analysis of EGARCH(1,1) model assuming a normal distribution of errors. The calculated means and standard deviations of the errors are found to be close to zero and one respectively. Moreover, the absence of ARCH effect was confirmed by applying the heteroskedasticity test and the serial correlation in the squared residuals is insignificant under the assumption of normal distribution of errors. However, the errors are found to be not normally distributed.