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JOINT LOT SIZING AND SCHEDULING OF A MULTI-PRODUCT MULTI-PERIOD SUPPLY CHAIN

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ABSTRACT

The joint lot sizing and scheduling problem can be considered as an evolvement of the joint economic lot size problem which has drawn researchers' interests for decades. The objective of this paper is to find the effect of a capacitated multi-period supply chain design parameters on joint lot sizing and scheduling decisions for different holding and penalty costs. The supply chain deals with two raw materials suppliers. The production facility produces two products which are shipped to customers through distribution centers. A mathematical model is developed to determine optimum quantities of purchased raw materials, production schedule (MPS), delivered quantities and raw material and products inventory for predetermined number of periods. The model is solved to maximize total supply chain profits. Results showed that at high capacity and low holding cost, the supply chain tends to produce only one product each period, for limited capacity and high value of holding cost, the supply chain may produce the two products together each period.

Keywords: Joint Lot-Sizing and Scheduling, Supply Chain optimization, Integrated Supply Chain

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1. INTRODUCTION

In production entities, planning activities are done to allocate resources efficiently to satisfy customer demands while balancing contradicting objectives. The key operational planning decisions are those related to material purchasing, production, and delivery (SAWIK, 2016) as well as the inventory management decisions (CUNHA *et al.*, 2018; SENOUSSI *et al.*, 2016).

Taking each of these decisions independently result in conflicting issues that might harm supply chain performance (ZHAO; WU; YUAN, 2016). For this reason, and especially after mathematical tools have witnessed great improvement, many researchers are currently directed towards integrating the optimization of various supply chain decisions (GHARAEI; JOLAI, 2018). In this context, the problem of joint lot-sizing and scheduling attains its importance among different integrated decisions due its effects on supply chain performance.

The basic Joint Economic Lot size Problem (JELP) as defined by (BEN-DAYA; DARWISH; ERTOGRAL, 2008), and then adopted by (SADJADI; ZOKAEE; DABIRI, 2014) is to determine the order and delivery quantities for two echelons (vendor and buyer) supply chain. In a later review (GLOCK, 2012), JELP is defined as determining order, production and delivery (shipment) quantities for multi echelon supply chain minimizing total costs.

The basic JELP with various costs modelling is tackled by a number of researchers (ERTOGRAL; DARWISH; BEN-DAYA, 2007; LEE; FU, 2014; WANG; LEE, 2013; MARCHI *et al.*, 2016). Extensions of the basic JELP can be found in (VAN HOESEL *et al.*, 2005), were the decision variables were taken over a number of periods. Another extension was done by (POURAKBAR; FARAHANI; ASGARI, 2007; GHARAEI; JOLAI, 2018) as they solved supply chain designs with multi echelons and/or multi actor per echelon.

Another decision integration is to decide the lot size and schedule simultaneously. This approached is defined as lot size and scheduling problem where production sequence is determined integrally with the production quantities (HUANG; YAO, 2013), this definition is similar to the Economic Lot size and Scheduling Problem (ELSP). Another definition to the problem is introduced by (TORABI; FATEMI GHOMI;

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KARIMI, 2006; HARIGA *et al.*, 2013) which is to determine production quantities and delivery schedules in two echelon supply chains.

The two previous problems are considered integrally where the joint lot size and scheduling problem emerged, to decide on the quantities of raw material purchased, production, delivery, …etc. These are considered taken integrally with determining the production schedule. The production schedule may be to determine the sequence of production of various products as done by (MUNGAN; YU; SARKER, 2010; ZHAO; WU; YUAN, 2016; JIA *et al.*, 2016), or to determine which products to be produced each period i.e. determine the Master Production Schedule (MPS) as tackled by (SARIN; SHERALI; LIAO, 2014; CUNHA *et al.*, 2018; SENOUSSI *et al.*, 2016).

In this paper, the optimization of joint lot-sizing and scheduling problem is considered were the materials purchasing, production and delivered quantities are to be determined integrally with the MPS over a number of periods in multi-echelon supply chains. The optimization in present joint lot sizing and scheduling problem is made for different production capacities while investigating the change in optimum decisions at each production capacity level.

The rest of the paper is organized as follows: in section 2 the literature review of the joint lot size and scheduling problem, how the problem is developed and what solution methodologies are used, section 3 describes the definition of the problem, while in section 4 the proposed mathematical model of maximizing the supply chain profits is presented. Numerical Experiments and results with discussions are given in section 5 and 6 respectively while conclusions and recommendation of future work is given in section 7.

2. LITERATURE REVIEW

The scheduling decisions can be operational or tactical. In operational decisions, products' sequencing is made, while in tactical decisions, the MPS is developed (MUNGAN; YU; SARKER, 2010), studied the joint lot-sizing and scheduling problem in a two stages supply chain whose products suffer from continuous price reduction during its life cycle.

They found optimal lot-sizes for procurement and production, and delivery schedules that minimize total costs of raw materials (ordering and purchasing), and finished products (setup, production and holding). The results showed that on adopting

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the policy of smaller and more frequent deliveries, the considered costs are lower (JIA *et al.*, 2016), addressed the problem of unconstrained delivery consolidation for a manufacturer and multiple buyers supply chain.

They optimized lot-sizes at the manufacturer and hence their production starting times, replenishment lots for different buyers and suitable delivery schedules that minimize total costs per unit time including ordering, setup, and holding at the manufacturer and buyers. Through numerical experiments, it is demonstrated that adopting delivery consolidation in multi-buyer supply chains with SPT scheduling and capacity utilization approaches improves the supply chain costs (SAĞLAM; BANERJEE, 2018), formulated a mathematical model that integrates batch production schedules and shipment scheduling decisions to minimize setup costs, transportation costs and inventory holding costs per unit time for a two-echelon supply chain with multiple products.

In a common cycle approach, they determined the amounts produced, carried in inventory and shipped to the customers as well as production cycle length, shipment interval and number of shipments considering Shipment capacity. Results showed that when variable transportation costs are used, the optimal shipment schedule is lot-for-lot according to the demand.

A larger supply chain is considered by (ZHAO; WU; YUAN, 2016), as they considered an integrated supply chain composed of four echelon that delivers finished goods to customers having time-varying demand of a single product over a finite planning horizon. They determined optimally the batch size of finished goods, number of production cycles, setup time in each cycle, and raw material order times that minimize total operational costs. They proved the problem can be solved optimally for a time varying demand product.

Other researchers integrated the MPS decisions with lot sizes decisions such as: (SARIN; SHERALI; LIAO, 2014) discussed the problem of integrating lot-sizing and scheduling for different product families in the primary manufacturing phase in a pharmaceutical supply chain. Different pharmaceutical ingredients are to be scheduled on parallel capacitated bays for production in batches.

Changeovers between different production families necessitate setup times and costs. The objective is to minimize inventory holding and setup costs. Results showed

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the effectiveness of this modeling and the column-generation solution approach in remarkable reductions in computational times (CUNHA *et al.*, 2018), addressed the problem of integrating lot sizing of purchased raw materials with production scheduling of final products to fully meet customer demands in a chemical industry.

Purchased materials are brought from different suppliers whose discount rates are different and depend on the purchased quantities. The production scheduling considers batch production of multi-stage production structure. The objective is minimizing total costs incorporating raw material purchasing, ordering, holding of both raw materials and final products, setup and production costs.

To highlight the importance of integrated scheduling and purchasing decisions, the authors solved the problem once on an integrated approach and compared the results to the independent (disintegrated) approach. Results have shown that the integrated approach outperforms the disintegrated one in all instances of their experimentation (SENOUSSI *et al.*, 2016), introduced the integration of vehicle routing to the joint lot-sizing problem in a supply chain composed of a single supplier production facility and multiple retailers performing under Vendor Managed Inventory (VMI) policy.

The authors considered vehicle capacity limitations, production capacities, and retailers' inventory capacities. The objective is obtaining optimal production, inventory and delivered quantities along with scheduling of production, vehicles and receiving retailers each period that minimize total supply chain costs. Numerical results show that the valid inequalities used improved the quality of the formulations. Also, the parameters influencing computational times are analyzed.

From the aforementioned review, it is obvious that the Joint Lot sizing and scheduling problem is gaining attention in the last few years, yet it is seldomly tackled with the effect of supply chain design parameters such as capacity and location. Most of the work reviewed solved the 2 echelons supply chain problem. (ZHAO; WU; YUAN, 2016) solved the problem for a larger supply chain, while in real life the integration of more members in the supply chain is increasing.

Thus, the objective of this paper is to study the effect of the production facility capacity on the Joint Lot Sizing and Scheduling decisions for three echelons supply

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chain containing two suppliers. In the following section a detailed problem definition is illustrated.

3. PROBLEM DEFINITION

In the supply chain under consideration, non-identical products are produced. Each product uses the same two different raw materials in its production yet with different ratios. The processing time of each product is different from one to another. The considered supply chain is a three-echelon-supply-chain as shown in figure 1. The production facility has two suppliers from which raw materials are acquired; the first supplier can supply both types of raw materials, while the second supplier can only supply one type of the raw materials.

The quality of materials received from both suppliers is consistent. Both suppliers dedicate part of their capacities for the production facility, and therefore, the facility is obliged to purchase a minimum quantity from each supplier for the whole planning period. The production facility can produce one or more of a batch of each product during the same period. If only one product is produced at any period, no changeover will take place.

If more than one product is produced during the same period, no changeover will be needed for the first product and changeovers will be done for the production of the next product. The production facility ships its production to a distribution center at which the products are either sent directly to customer(s) or kept as inventory to meet future demand of next periods.

The customers' demands are all confirmed orders of different products per period. Since no transportation costs are considered, all customers are assumed to be only one customer and its demand is the total demand from each product.

A Mixed Integer Non- Linear programming model is developed to maximize the supply chain profits with fixed selling price of each product at different periods. The costs considered are: material(s) purchasing cost, inventory cost for raw materials at the production facility and finished products at the DC, changeover cost at production facility, processing cost and penalty cost incurred for undelivered quantities to the customer.

It is required to determine the joint lot size (purchased quantities from each supplier, production quantities and delivery quantities) for the two products and the

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Master Production Schedule (MPS) that maximize the supply chain profit over number of periods composing a planning horizon.

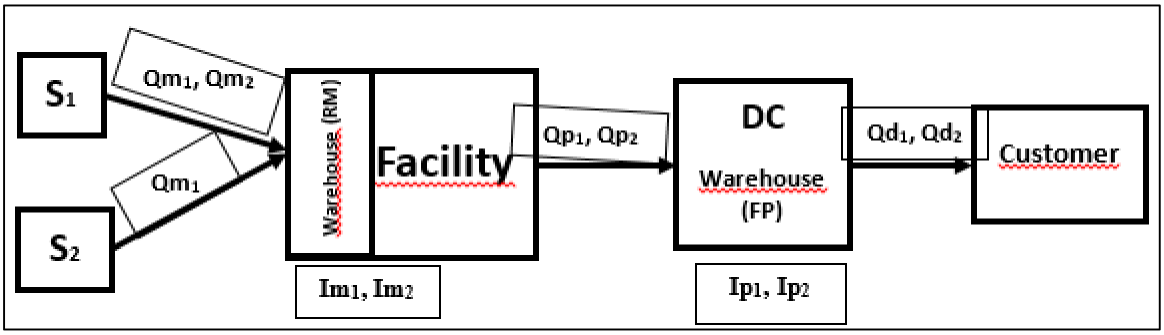

Figure 1: Supply Chain Structure

4. RESEARCH METHODOLOGY

4.1. Mathematical model

A mathematical model is developed to maximize the supply chain profits when optimum purchase, production and delivery quantities are determined jointly, in addition to determining the MPS.

4.1.1. Nomenclature

Indices

t: Periods (t = 1, 2, ..., T)

i: Items (raw materials) (i = 1, 2, ..., I)

j: Suppliers (j = 1, 2, ..., J)

n: Products (n = 1, 2, ..., N)

Parameters

S_n: Selling price per finished product 'n'

Cm_{ij}: Cost of one item of raw material 'i' from supplier 'j'

Cp_n: Cost of processing of product 'n' per unit time

Chm_i: Inventory holding cost of one item of raw material 'i' for one period

Chp_n: Inventory holding cost of one product 'n' for one period

Co: Changeover cost for each product except that at the start of the period

Cs: Penalty cost paid for each undelivered unit

Tp_n: Production time per product 'n' in the facility

Tm_{ij}: Production time of a single item of raw material 'i' at supplier 'j'

tc: Changeover time at the facility

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D_{tn}: Demand of product '*n*' at any period '*t*'

v_{tj}: Max. time capacity at supplier '*j*' dedicated to the production facility during period '*t*'

u_{tj}: Min. capacity at supplier '*j*' dedicated to the production facility during period '*t*'

W_t: Max. capacity available at production facility during period '*t*'

a_{in}: Amount of raw material '*i*' required for production of one product '*n*'

Decision variables

Qm_{tij}: Quantity of material '*i*' purchased by production facility from supplier '*j*' during period '*t*'

Qp_{tn}: Quantity of product '*n*' processed at facility during period '*t*'

Qd_{tn}: Quantity of product '*n*' delivered from DC to customer during period '*t*'

Ip_{tn}: Inventory level of product '*n*' at end of period '*t*' at the DC

Im_{ti}: Inventory level of raw material '*i*' at end of period '*t*' at the production facility.

L_{tn}: Binary Matrix where l_{tn} = 0 if the product '*n*' is not listed in the MPS in period '*t*' otherwise equal 1.

4.1.2. The Developed Model

The objective function is to maximize total supply chain profits which is given by total supply chain revenues minus total costs of purchasing, production, inventory holding for final products and raw materials, penalty and changeover.

Profit Model

The supply chain profit given in equation (1) is modelled as supply chain revenues from selling products to customers from the DC minus the costs incurred by the production facility and the DC. The revenue is modelled as the selling price 'S_n' multiplied by the delivered quantity 'Qd_{tn}' to the customer each period from each product. The first cost element is the cost of purchasing material '*i*' from supplier '*j*' and it is modelled by multiplying the material cost per unit 'Cm_{ij}' by the purchased quantity 'Qm_{tij}'. The processing cost of product '*n*' is calculated as the processing time 'tp_n' multiplied by the cost of processing 'Cp_n' multiplied by the sum of quantities manufactured during the planning horizon '*T*'.

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Products and materials inventory costs are calculated as the inventory holding cost per unit ('Chp_n' for products and 'Chm_i' for materials) multiplied by the sum of end of period inventory level ('Ip_{tn}' for products and 'Im_{ti}' for materials) during the planning horizon. The penalty cost is formulated as the difference between the required demand 'D_{tn}' and actual delivered quantities 'Qd_{tn}', multiplied by the penalty paid for each undelivered unit. The total supply chain changeover cost for all periods along the planning horizon is modeled as the cost per changeover '*Co*' multiplied by the sum of the number of products processed each period minus one to exclude the first product.

$$Z_{max} = \sum_{n=1}^{N}(S_n * \sum_{t=1}^{T} Qd_{tn}) - \sum_{j=1}^{J}\sum_{i=1}^{I}(Cm_{ij} * \sum_{t=1}^{T} Qm_{tij}) - \sum_{n=1}^{N}(Cp_n * tp_n * \sum_{t=1}^{T} Qp_{tn}) - \sum_{n=1}^{N}(Chp_n * \sum_{t=1}^{T} Ip_{tn}) - \sum_{i=1}^{I}(Chm_i * \sum_{t=1}^{T} Im_{ti}) - \sum_{n=1}^{N}\sum_{t=1}^{T} Cs * (D_{tn} - Qd_{tn}) - \sum_{t=1}^{T} Co * ((\sum_{i}^{I} l_{ti}) - 1) \quad (1)$$

Model Constraints

Total quantity of items purchased from supplier '*j*' at any period '*t*' lies between the minimum and maximum capacity limits determined by the suppliers. This is ensured by constraints (2) and (3) where the time needed by supplier '*j*' to produce quantity 'Qm_{tij}' for all materials '*I*' is greater than the minimum capacity in time units 'u_{tj}' and smaller than maximum time capacity 'v_{tj}' dedicated to the production facility.

$$u_{tj} \leq \sum_{i=1}^{I}(tm_{ij} * Qm_{tij}) \quad (2)$$

$$\sum_{i=1}^{I}(tm_{ij} * Qm_{tij}) \leq v_{tj} \quad (3)$$

$$\sum_{n=1}^{N}(Qp_{tn} * tp_n) + tc * (\sum_{i=1}^{I} l_{ti} - 1) \leq (w_t), \forall t \in T \quad (4)$$

Constraint (4) ensures that the production capacity is not violated, the sum of processing and changeover times at any period '*t*' cannot exceed the time capacity at the production facility 'w_t'. Total processing time of any product '*n*' at any period '*t*' is the multiplication of the quantity produced from this product 'Qp_{tn}' by the processing time for one item of this product. Total changeover time is the time of a single changeover '*tc*' multiplied by the number of products manufactured during the period '*t*' excluding the first product.

$$\sum_{j=1}^{J} Qm_{tij} \geq \sum_{n=1}^{N}(a_{in} * Qp_{tn}) \; \forall i \in I \;\&\; t \in T \quad (5)$$

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This constraint ensures that total purchased quantity of material *'i'* from all suppliers at any period *'t'* is greater than or equal to the required quantity from this raw material to produce 'Qp_{tn}' products in the same period. This is given in constraint (5) by having the purchased quantity of raw material 'Qm_{tij}' from all suppliers is greater than the amount of materials required for one product 'a_{in}' multiplied by the produced quantity of the same product 'Qp_{tn}'.

$$Qd_{tn} \leq D_{tn}, \ \forall\ t \in T,\ n \in N \quad \textbf{(6)}$$

Delivered quantity of product *'n'* at any period *'t'*; 'Qd_{tn}' is less than or equal to the customer's demand 'D_{tn}'. This constraint ensures that the delivered quantities may not exceed the customer demand.

$$Inp_{tn} = Inp_{(t-1)n} + Qp_{tn} - Qd_{tn}\ , (\forall\ t=2, 3, 4, \ldots, t\text{-}1) \quad \textbf{(7)}$$

Constraint (7) is a balance constraint, ensures that the inventory of final products 'Ip_{tn}' at any period *'t'*; is equal to inventory level at the end of the previous period '$Ip_{(t-1)n}$' plus remaining from production quantity 'Qp_{tn}' that is not delivered in the same period *'t'*.

$$Inm_{ti} = Inm_{(t-1)i} + Qm_{tij} - \sum_{n=1}^{N}(a_{in} * Qp_{tn}), (\forall\ t = 2, 3, 4, \ldots, t) \quad \textbf{(8)}$$

Constraint (8) shows that the inventory level of material *'i'* at any period *'t'*; 'Im_{ti}' equals its inventory level at the end of the previous period '$Im_{(t-1)i}$' plus amount of purchased quantity at period *'t'*; 'Qm_{tij}' minus the amount required for the production of this period 'a_{in}*Qp_{tn}'. The capacity of raw materials storage at the production facility is unlimited.

$$Qm_{t22} = 0 \quad \textbf{(9)}$$

Constraint (9) implies that Supplier (2) cannot produce the second raw material

$$Qp_{tn} = l_{tn} * Qp_{tn} \quad \textbf{(10)}$$

Constraint (10) shows that is to prevent not producing any quantity 'Qp_{tn}' from product *'n'*, yet it is in the MPS at the same period i.e. l_{tn} = 1.

$$l_{tn} \leq Qp_{tn} \quad \textbf{(11)}$$

Constraint (11) prevents having zero production quantity while the product is listed in the MPS

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5. NUMERICAL EXPERIMENTATION

The main objective is to study how the joint lot sizing and scheduling decision may change with the change in production capacity and inventory holding costs. In order to achieve this objective three different production levels are assumed; low in which the available capacity can only produce the demand from one product or slightly more, moderate capacity in which the capacity is sufficient to produce the demand of both products with slight shortages, and high capacity where the capacity is enough to produce both products with setup each period. Table 1 illustrates the values of the capacities and input parameters considered.

Table 1: Input Parameters for Numerical Experimentation

S_n	1500, 1500	Cs	100 & 1000	v_{tj}	500, 250
Cm_{ij}	200, 200, 200	Tp_n	1, 1.2	u_{tj}	50, 25
Cp_n	150, 150	Tm_{ij}	0.2, 0.2, 0.15	W_t	150, 250 & 350
Chm_i	20	tc	10	a_{in}	1, 1, 2, 1
Chp_n	25 & 100	D_{t1}	150		
Co	5000	D_{t2}	100		

The mathematical model is coded using LINGO 17.0 software which yielded the global optimum of the problem. LINGO is run using a workstation with Intel Xeon E3-1246 v3 (3.50 GHz) processor and 16 GB RAM, the run time varied drastically from few seconds to more than 100 hours for some instances.

6. RESULTS AND DISCUSSION

It is clear from table 2 that as the production capacity increases the supply chain profits increases due to the decrease in penalties paid and the increased delivered quantities. In cases of low and moderate capacities, the increase in penalty costs decreased the profits than high capacity case by an average 77.7% and 20.2% respectively, as at these two capacities shortage occur.

While at lower holding cost the profits are higher by an average 2.2% for moderate and high capacities, while it has no effect as no inventory is kept at low capacity. Even at high capacity where the capacity is enough to produce total demand of both products each period and there is no need to keep inventory, yet inventory is kept from products as this will be illustrated using figures 2-7.

Table 2: Supply Chain Profits at different Capacities, Holding Costs and Penalty Costs

w (hours per period)	*Chp* (Unit cost per period)	*Cs* = 100 (unit cost per undelivered unit)	*Cs* = 1000 (unit cost per undelivered unit)

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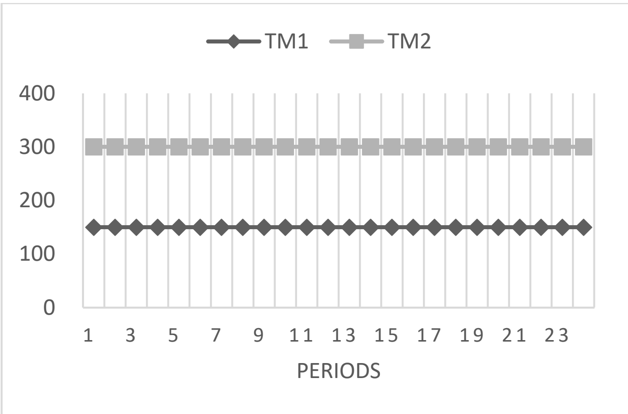
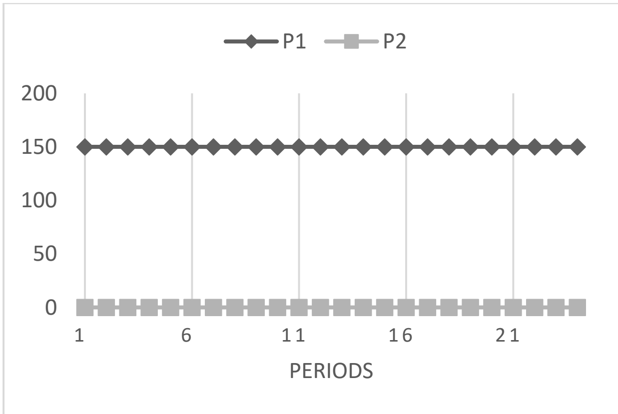
150	25	2460000	300000
	100	2460000	300000
250	25	4306430	3867265
	100	4176000	3690560
350	25	4817100	4817100
	100	4788000	4788000

6.1. Low Production Capacity

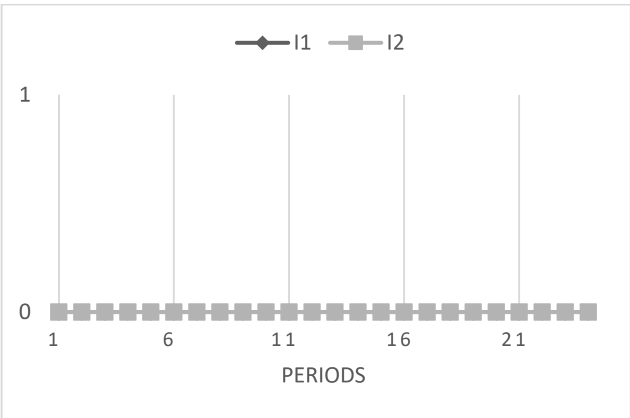
For the low production capacity, the decisions are the same for various holding and penalty costs, as there is enough capacity to produce the demand of product 1 which has higher demand and lower processing time. Figure 2 shows various decisions made in case of low production capacity at holding cost equal 100 units cost per unit per period and penalty cost 100 units cost penalty per each undelivered, it is clear that the production quantities are from one product (product 1) which consumes less capacity and has higher demand to minimize the penalty cost and hence maximize the profit.



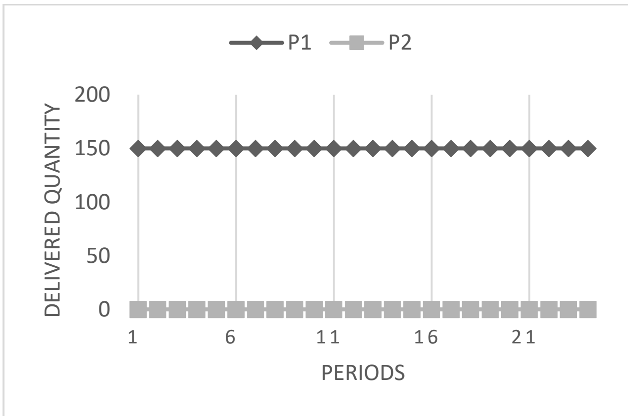
(a) Purchased Materials Quantities



(b) Production Quantities



(c) Products Inventory Levels



(d) Delivered Quantities

Figure 2: Various Decision Variables at Low Capacity and *Chp* = 100 and *Cs* = 100

6.2. Moderate Production Capacity

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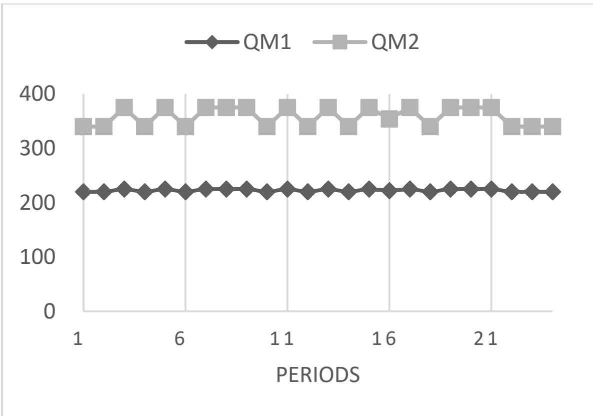
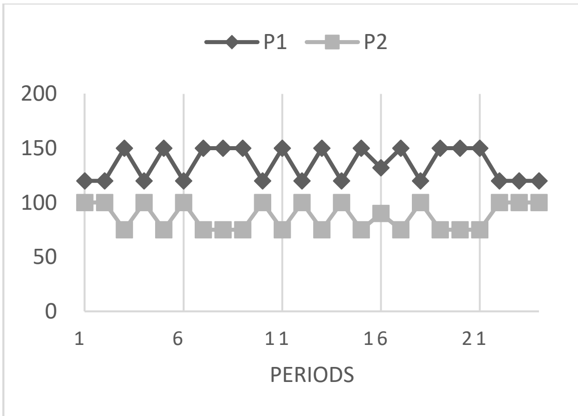
In the case of moderate capacity and having equal holding and penalty costs as shown in figure 3, there are three main production schedules altering in this solution. The first is producing full demand of product1 (150 units) and using the rest of the capacity to produce product2 (75 units with a shortage of 25 units) as in periods (3, 5, 7, 8, 9, 11, 13, 15, 17, 19, 20 and 21).

The second solution is producing the full demand of product2 (100 units) and using the rest of the production capacity to produce product1 (120 units with a shortage of 30 units) as in periods (1, 2, 4, 6, 10, 12, 14, 18, 20, 22, 24). The third solution found in period (16) only is a solution in which shortages is shared between the two final products; producing 132 units from product1 with a shortage of 18 units and 90 units of product2 with 10 shortage units.

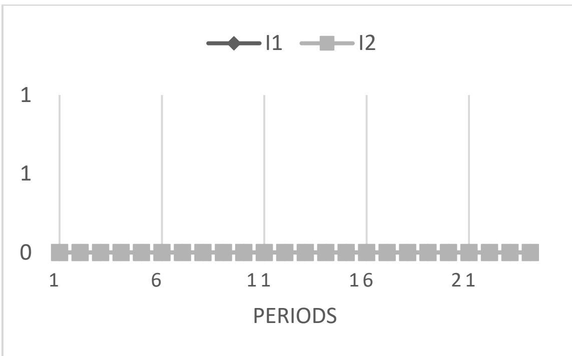
This schedule is not a unique optimal one, as when any of the three schedules is fixed for the all periods each yielded the same supply chain profit. Material purchased will be exactly equal the same amounts needed for production and the delivered quantities are the same as the production quantities.



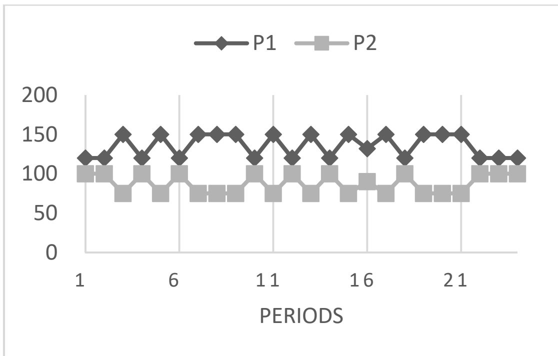
(a) Purchased Materials Quantities



(b) Production Quantities



(c) Products Inventory Levels



(d) Delivered Quantities

Figure 3: Various Decision Variables at Moderate Capacity and Chp = 100 and Cs = 100

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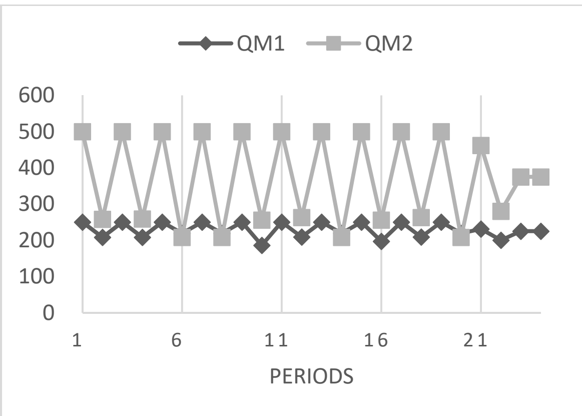
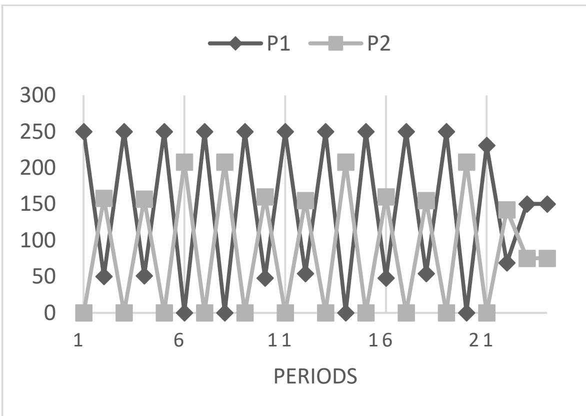
Similar decisions are observed on having the penalty cost higher than the holding cost as given in figures 4 & 5, the production decision favors the product that consumes less capacity (having less processing time). When the difference is low as in figure 4 the reached inventory levels is lower and shifting from one product to the other is more frequent. While on having high difference between the holding cost and penalty cost as in figure 5, the fill rate of product 1 is 100% and residual capacity is used to produce product 2.

Furthermore, a building of inventory is made from product 1 on the expense of not delivering product 2 in periods 3, 4 & 9 and having high shortage in period 8, the service level of product 2 in this period is only 23%. This enabled the supply chain to build inventory from both products as the capacity was used to produce only one product in 9 periods, which is used when the production quantity is less than demand. This production schedule enables the supply chain to have 100% service level in both products in 15 periods of the planning horizon (62.5% of the periods) which in return minimize the penalty cost.

The materials purchased quantities were exactly equal to the amounts needed for the production quantities. In figure 6 as the holding cost increase the same pattern of decision is made yet the 100% service level of both products was reached in 12 periods only and the favoring of product 1 is less as there 3 periods its service level was 93.3% and twice it was 97.3% (periods 5, 9 and 13). This is due to the fact that building inventory is becoming more expensive, so a tradeoff is made choosing to build less inventory from product 1.



(a) Purchased Materials Quantities



(b) Production Quantities

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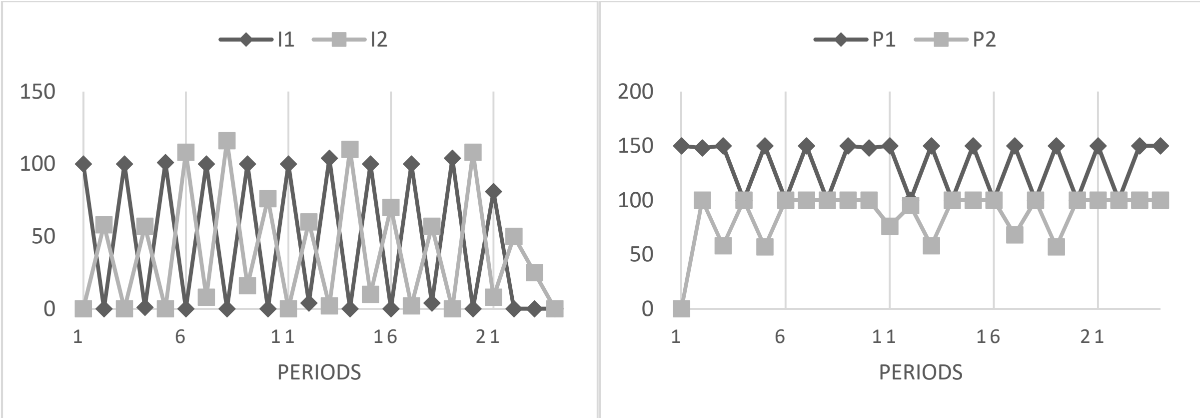
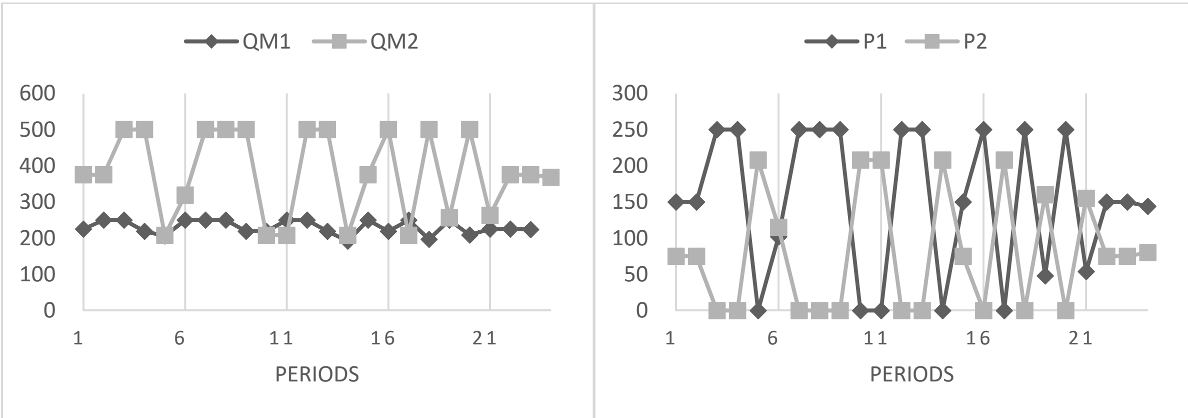
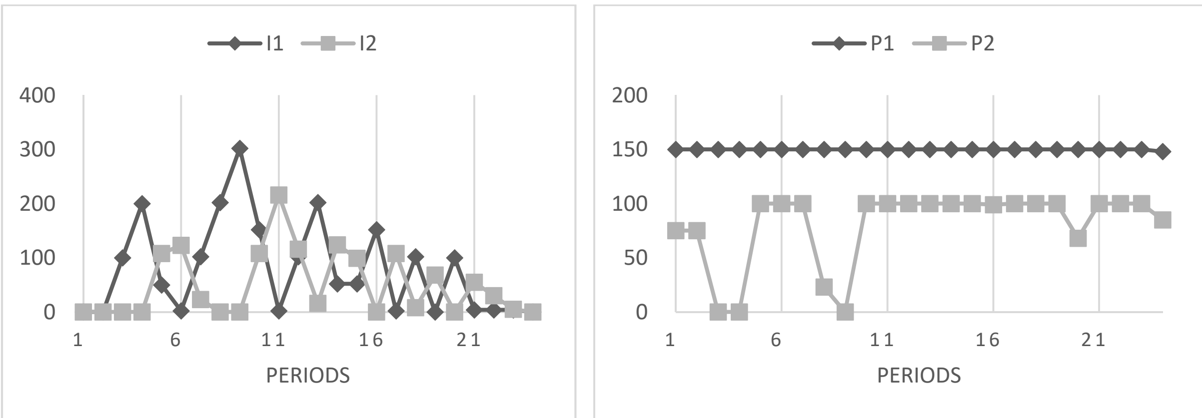


(c) Products Inventory Levels (d) Delivered Quantities

Figure 4: Various Decision Variables at Moderate Capacity and Chp = 25 and Cs = 100



(a) Purchased Materials Quantities (b) Production Quantities



(c) Products Inventory Levels (d) Delivered Quantities

Figure 5: Various Decision Variables at Moderate Capacity and Chp = 25 and Cs = 1000

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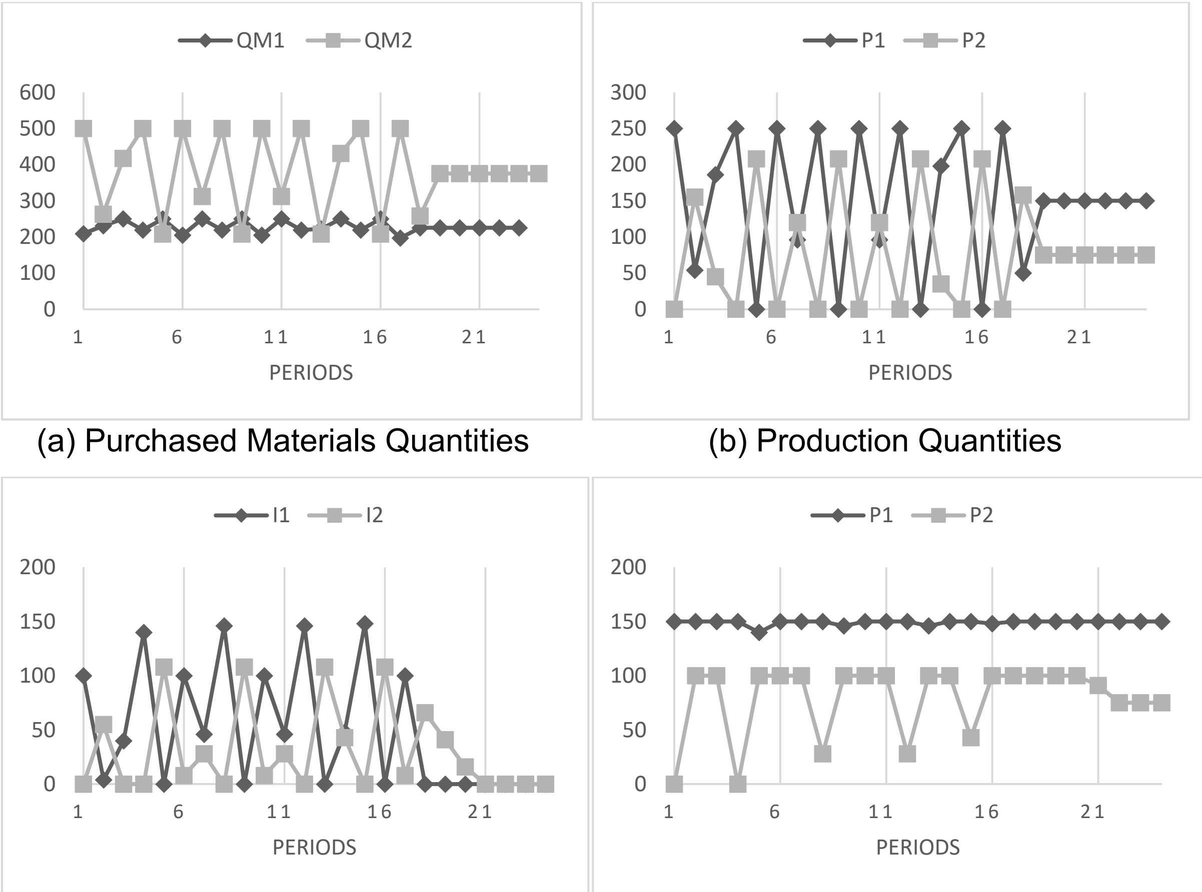

(a) Purchased Materials Quantities (b) Production Quantities

(c) Products Inventory Levels (d) Delivered Quantities

Figure 6: Various Decision Variables at Moderate Capacity and Chp = 100 and Cs = 1000

6.3. High Production Capacity

In the case of high capacity and low inventory holding costs as shown in figure 7, although it is possible to produce exactly the demand of both products each period, yet the production decision makes use of this low holding costs by producing one product in most periods. For 21 periods (87.5% of the periods) one product is produced and inventory quantity is kept for next period(s) where the other product is produced. During the remaining 3 periods both products are produced.

Two out of these three periods are the first two periods, in which the buildup of inventory is taking place using the excess available capacity. In period 13 both products are produced as there are not enough inventory to cover the demand of both products. This production schedule allows the reduction of the number of changeovers, and consequently reduce the changeover costs while maintaining 100% service levels for each product.

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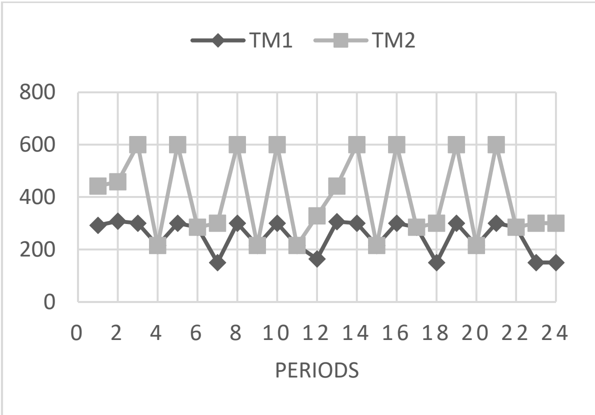
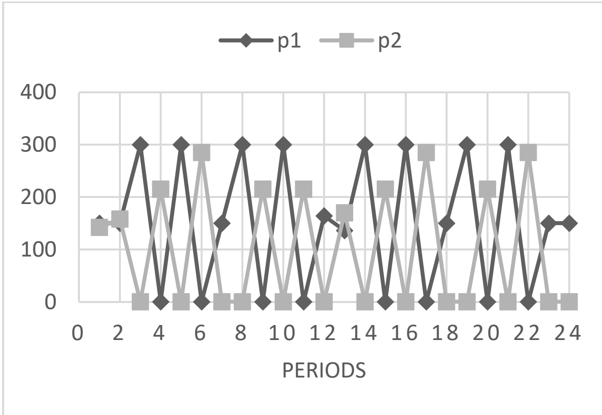
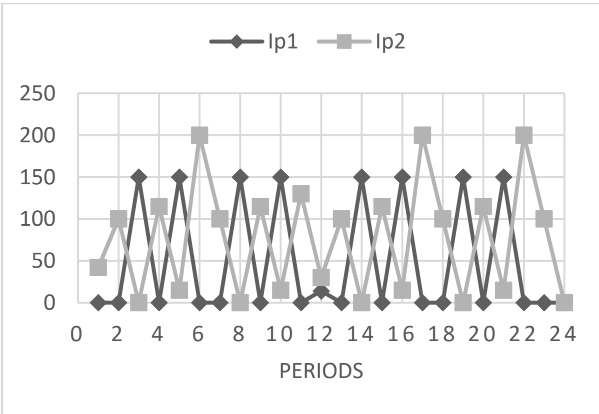
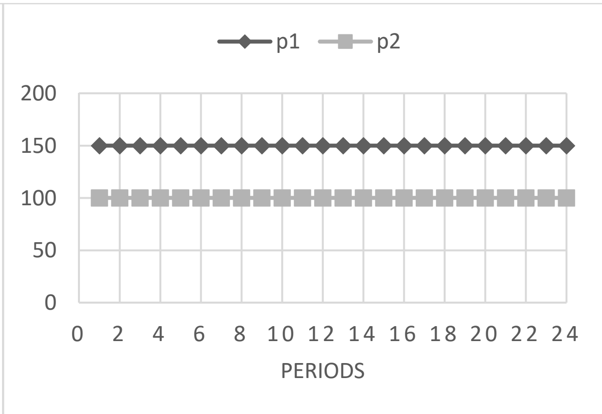
As in previous cases the material purchase follows the production schedule and no materials inventory is kept.





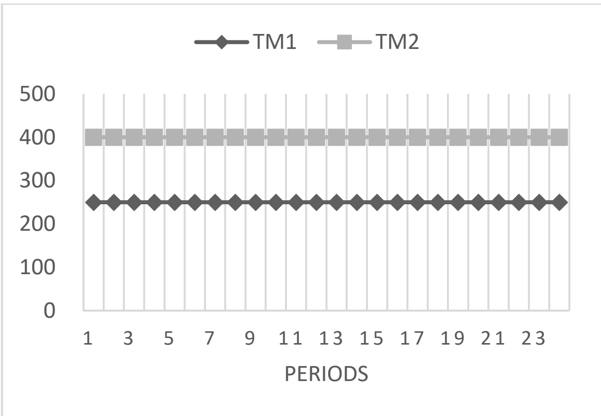
(a) Purchased Material Quantities (b) Production Quantities

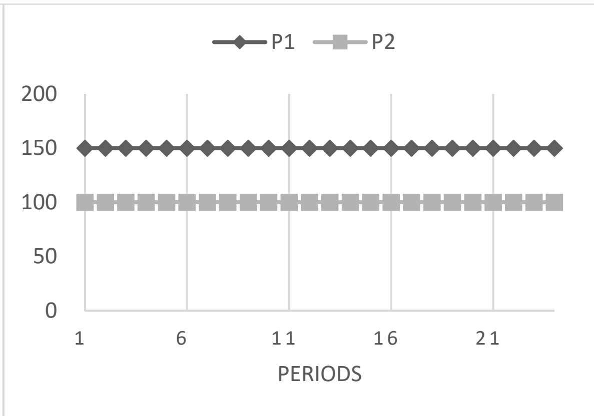




(c) Products Inventory Levels (d) Delivered quantities

Figure 7: Different Decisions variables at each period at w=350, Chp=25, Cs=100





(a) Purchased Material Quantities (b) Production Quantities

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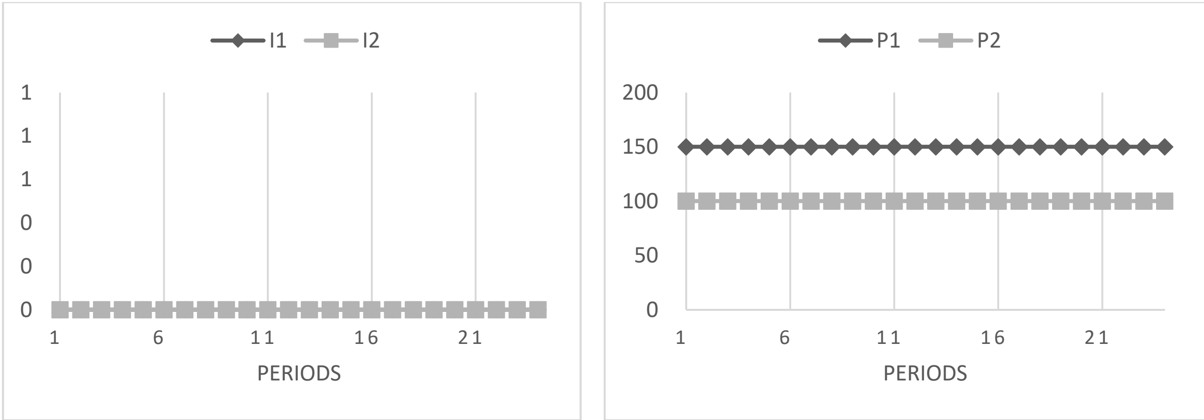

(c) Products Inventory Levels (d) Delivered quantities

Figure 8: Different Decisions variables at each period at w=350, Chp=100, Cs=100

The high production capacity and high inventory holding cost case shown in figure 8 both products are produced in quantities equal to the demand each period and hence all produced quantities are delivered, and no inventory is kept from products. The materials purchased each period equal to the amounts needed to produce the demanded quantities.

7. CONCLUSIONS

The results showed that the Joint Lot Sizing and Scheduling problem is modelled and solved optimally for a large number of planning periods (24 periods). The MPS depends hugely on the available capacity at the manufacturing facility while, the purchased material is done as a lot-for-lot to fulfill production needs each period. In case of low capacity, only one product is produced, whatever the costs were, as there isn't enough capacity to produce both products.

The produced product is the one with higher demand to reduce the penalty costs. In moderate and high capacities, the holding cost has a great impact on the decision; as the holding cost decreases the tendency to produce only one product each period, minimizing changeover costs, and keep inventory to satisfy demand in future periods increases.

Another factor affected the decisions which is the ending inventory at the last period, since its optimum value is zero, the solution resulted in steady production of both products in the last periods to assure that no ending inventory is kept at the end of the last period. The effect of having variable demand, different suppliers' quality and lead times and more real bill of materials for the product family can be researched in the future.

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DOI: 10.14807/ijmp.v10i5.959

REFERENCES

BEN-DAYA, M.; DARWISH, M.; ERTOGRAL, K. (2008) The joint economic lot sizing problem: Review and extensions, **European Journal of Operational Research,** v. 185, n. 2, p. 726–742. doi: 10.1016/j.ejor.2006.12.026.

CUNHA, A. L. *et al.* (2018) An integrated approach for production lot sizing and raw material purchasing, **European Journal of Operational Research**. Elsevier B.V., v. 269, n. 3, p. 923–938. doi: 10.1016/j.ejor.2018.02.042.

ERTOGRAL, K.; DARWISH, M.; BEN-DAYA, M. (2007) Production and shipment lot sizing in a vendor-buyer supply chain with transportation cost, **European Journal of Operational Research**, v. 176, n. 3, p. 1592–1606. doi: 10.1016/j.ejor.2005.10.036.

GHARAEI, A.; JOLAI, F. (2018) A multi-agent approach to the integrated production scheduling and distribution problem in multi-factory supply chain, **Applied Soft Computing Journal**. Elsevier B.V., n. 65, p. 577–589. doi: 10.1016/j.asoc.2018.02.002.

GLOCK, C. H. (2012) The joint economic lot size problem: A review, **International Journal of Production Economics**. Elsevier, v. 135, n. 2, p. 671–686. doi: 10.1016/j.ijpe.2011.10.026.

HARIGA, M. *et al.* (2013) Scheduling and lot sizing models for the single-vendor multi-buyer problem under consignment stock partnership, **Journal of the Operational Research Society**. Nature Publishing Group, v. 64, n. 7, p. 995–1009. doi: 10.1057/jors.2012.101.

HUANG, J. Y.; YAO, M. J. (2013) On the optimal lot-sizing and scheduling problem in serial-type supply chain system using a time-varying lot-sizing policy, **International Journal of Production Research**, v. 51, n. 3, p. 735–750. doi: 10.1080/00207543.2012.662604.

JIA, T. *et al.* (2016) Optimal production-delivery policy for a vendor-buyers integrated system considering postponed simultaneous delivery, **Computers and Industrial Engineering**. Elsevier Ltd, n. 99, p. 1–15. doi: 10.1016/j.cie.2016.07.002.

LEE, S. D.; FU, Y. C. (2014) Joint production and delivery lot sizing for a make-to-order producer-buyer supply chain with transportation cos', **Transportation Research Part E: Logistics and Transportation Review**. Elsevier Ltd, n. 66, p. 23–35. doi: 10.1016/j.tre.2014.03.002.

MARCHI, B. *et al.* (2016) A joint economic lot size model with financial collaboration and uncertain investment opportunity, **International Journal of Production Economics**. Elsevier, n. 176, p. 170–182. doi: 10.1016/j.ijpe.2016.02.021.

MUNGAN, D.; YU, J.; SARKER, B. R. (2010) Manufacturing lot-sizing, procurement and delivery schedules over a finite planning horizon, **International Journal of Production Research**, v. 48, n. 12, p. 3619–3636. doi: 10.1080/00207540902878228.

POURAKBAR, M.; FARAHANI, R. Z.; ASGARI, N. (2007) A joint economic lot-size model for an integrated supply network using genetic algorithm, **Applied Mathematics and Computation,** v. 189, n. 1, p. 583–596. doi: 10.1016/j.amc.2006.11.116.

SADJADI, S. J.; ZOKAEE, S.; DABIRI, N. (2014) A single-vendor single-buyer joint

ISSN: 2236-269X
DOI: 10.14807/ijmp.v10i5.959

economic lot size model subject to budget constraints, **International Journal of Advanced Manufacturing Technology**, v. 70, n. 9–12, p. 1699–1707. doi: 10.1007/s00170-013-5382-2.

SAĞLAM, Ü.; BANERJEE, A. (2018) Integrated multiproduct batch production and truck shipment scheduling under different shipping policies, **Omega (United Kingdom)**, n. 74, p. 70–81. doi: 10.1016/j.omega.2017.01.007.

SARIN, S. C.; SHERALI, H. D.; LIAO, L. (2014) Primary pharmaceutical manufacturing scheduling problem, **IIE Transactions (Institute of Industrial Engineers)**, v. 46, n. 12, p. 1298–1314. doi: 10.1080/0740817X.2014.882529.

SAWIK, T. (2016) Integrated supply, production and distribution scheduling under disruption risks, **Omega (United Kingdom)**. Elsevier, n. 62, p. 131–144. doi: 10.1016/j.omega.2015.09.005.

SENOUSSI, A. *et al.* (2016) Modeling and solving a one-supplier multi-vehicle production-inventory-distribution problem with clustered retailers, **International Journal of Advanced Manufacturing Technology**, v. 85, n. 5–8, p. 971–989. doi: 10.1007/s00170-015-7966-5.

TORABI, S. A.; FATEMI GHOMI, S. M. T.; KARIMI, B. (2006) A hybrid genetic algorithm for the finite horizon economic lot and delivery scheduling in supply chains, **European Journal of Operational Research**, v. 173, n. 1, p. 173–189. doi: 10.1016/j.ejor.2004.11.012.

VAN HOESEL, S. *et al.* (2005) Integrated Lot Sizing in Serial Supply Chains with Production Capacities, **Management Science**, v. 51, n. 11, p. 1706–1719. doi: 10.1287/mnsc.1050.0378.

WANG, S. P.; LEE, W. (2013) A joint economic lot-sizing model for the hospital's supplier with capacitated warehouse constraint, **Journal of Industrial and Production Engineering**, v. 30, n. 3, p. 202–210. doi: 10.1080/21681015.2013.805700.

ZHAO, S. T.; WU, K.; YUAN, X. M. (2016) Optimal production-inventory policy for an integrated multi-stage supply chain with time-varying demand, **European Journal of Operational Research**. Elsevier B.V., v. 255, n. 2, p. 364–379. doi: 10.1016/j.ejor.2016.04.027.