Funding for longer lives. Retirement wallet and risk-sharing annuities

Longevity increases and population ageing create challenges for all societal institutions, particularly those providing retirement income, health care, and long-term care services. At the individual level, an obvious question is how to ensure all retirees have an adequate, secure, stable and predictable lifelong income stream that will allow them to maintain a target standard of living for however long the individual lives. In this paper we introduce and discuss the concept of retirement wallet representing the multiple income and service sources individuals and their families will have to fund for longer lives. We then address the main decumulation risks and options, including the adoption of a given longevity insurance strategy, of a programmed withdrawal strategy and of an investment strategy. The main payout options available for allocating assets accumulated in pension plans are discussed, particularly the role of traditional and innovative investment and longevity risk-sharing structures. We provide illustrative results for the price of innovative participating longevity-linked life annuities (PLLAs) that link benefits to the dynamics of both a longevity index and an interest rate adjustment factor using Spanish mortality and financial market data.

La longevidad aumenta y el envejecimiento de la población crea desafíos para todas las instituciones sociales, particularmente aquellas que proporcionan ingresos de jubilación, atención médica y servicios de atención a largo plazo. A nivel individual, una pregunta obvia es cómo garantizar que todos los jubilados tengan un flujo de ingresos de por vida, adecuado, seguro, estable y predecible que les permita mantener un nivel de vida objetivo durante el tiempo que dure la vida individual. En este estudio presentamos y discutimos el concepto de la *cartera de jubilación*, que representa las múltiples fuentes de ingresos y servicios que las personas y sus familias tendrán que financiar para una vida más larga. A continuación, abordamos los principales riesgos y opciones de decumulación, incluida la adopción de una estrategia dada de seguro de longevidad, de una estrategia programada de retiro y de una estrategia de inversión. Se discuten las principales opciones de pago disponibles para asignar los activos acumulados en los planes de pensiones, en particular, el papel de las estructuras de inversiones tradicionales e innovadoras y de riesgo compartido de longevidad. Proporcionamos resultados ilustrativos por el precio de las participativas rentas vitalicias innovadoras vinculadas a la longevidad (PLLA), que vinculan los beneficios a la dinámica del índice de longevidad y un factor de ajuste de la tasa de interés, utilizando la mortalidad española y los datos del mercado financiero.

Bizitza-luzeeraren etengabeko hazkundeak erronka berriak sortzen dizkie gizarte-erakundeei; bereziki, erretirokoei, arreta medikukoei eta epe luzeko arreta-zerbitzuetako diru-sarrerak ematen dituztenei. Maila indibidualari begiratuta sortzen den galdera begi-bistakoa da: Nola bermatu erretirodun guztiek bizi diren artean beren bizi-maila objetiboari eusteko moduko diru-sarreren fluxu egokia, segurua, egonkorra eta aurreikusteko modukoa izango dutela? Azterlan honetan, aurkeztu eta eztabaidatu egiten dugu erretiro-zorroaren kontzeptua; hain zuzen, pertsonek eta haien familiek bizi-luzetasuna finantzatzeko beharko dituzten diru-sarreren eta zerbitzuen iturri anitzak barnebiltzen dituena. Jarraian, erretiroan aurrezkiak baliatzearen arrisku eta aukera nagusiei helduko diegu, horren barnean kokatuta bizitza luzapenaren asegururako estrategia jakin bat izatea, erretiro-estrategia programatu bat izatea zein inbertsio-estrategia bat izatea. Horrez gain, eztabaidatzen ditugu pentsio-planetan metatutako aktiboak esleitzeko dauden ordainketa-aukera nagusiak ere, zehazki, inbertsio-egitura tradizional eta berritzaileen eta bizitzak luzatzearen arrisku partekatuko egituren zeregina. Bestalde, emaitza argigarriak azaltzen ditugu bizi-luzetasunari lotuta dauden biziarteko errenta berritzaile (PLLA) parte-hartzaileen prezioei buruz. Emaitzek bizi-luzetasunaren indizearen dinamikaren onurak eta interes-tasaren doikuntza-faktore bat uztartzen dituzte, horre-tarako, Espainiako hilkortasuna eta finantza-merkatuko datuak erabilita.

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Palabras clave: rentas vitalicias ligadas a la longevidad, riesgo compartido, etapa de pago de pensiones, cartera de jubilación, reforma de pensiones, mecanismos de obtención de capital

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1. INTRODUCTION

Increased longevity is probably one of the most important socioeconomic advances to happen to mankind. Longevity improvements initiated only about two centuries ago in the then advanced economies and has since spread around the world and across all socioeconomic groups in a non-uniform way (Ayuso, Bravo and Holzmann, 2017a,b). While a welcome development, longer lives and population ageing create challenges for all societal institutions (including the family), public and private, particularly those providing retirement income, health care, and long-term care services. At the individual level, an obvious question is how to ensure all retirees have an adequate, secure, stable and predictable lifelong income stream that will allow them to maintain a target standard of living for however long the individuals lives. In other words, how to guarantee an income and service stream that not only covers the basic expenditure needs but also is enough to achieve the full lifestyle individuals aspire in retirement. The answer to this question is not trivial and depends heavily on different factors such as the effects of institutions, government regulations and interventions (e.g., pension system architecture, role of occupational/personal pensions, DB/DC nature, minimum income guarantees), social networks (e.g., family structure and inter-

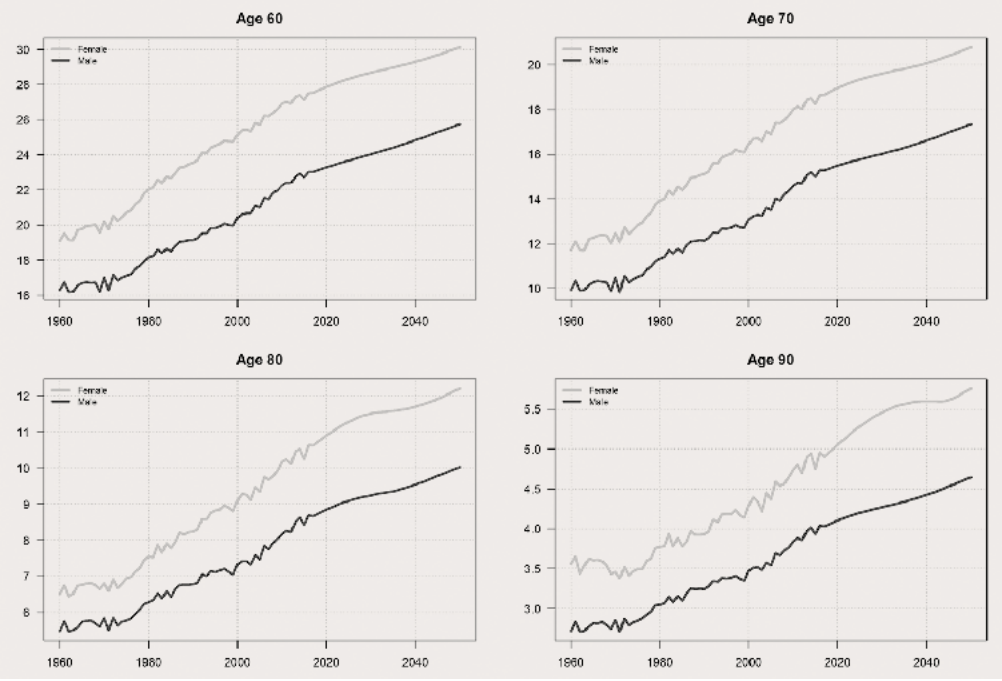
connectedness, informal care networks), on individual preferences (e.g., regarding continuing to work after retirement, bequest, lifecycle planning), family background and family shocks (e.g., inheritances, divorce) that prevent accumulation or accelerate decumulation, financial system development (e.g., the existence of efficient capital and insurance markets) or the risks involved in the generation of retirement income (e.g., investment, inflation, contribution, political, longevity, liquidity, behavioural). Depends also on general policy goals and constraints on fiscal policy, old-age poverty, tax neutrality over the life cycle, redistribution objectives, intergenerational fairness or the political economy of an ageing society. To fund for longer lives, people will ultimately rely on a combination of state, employer, social institutions, family, own savings (including housing wealth), continued labour income, and insurance sources, with weights determined by both personal and institutional circumstances.

In Figure 1 we highlight the magnitude of the retirement planning horizons individual face by plotting the dynamics of observed and forecasted period life expectancy estimates computed for Spain for both male and female populations and selected ages from 1960 to 2050. Estimates were computed using a Bayesian Model Ensemble (Averaging) technique of six well know generalized age-period-cohort stochastic mortality models fitted to Spanish mortality data from 1960 to 2017 (Bravo, Ayuso and Holzmann, 2019; Bravo, 2019). We can observe a clear almost linear positive trend in life expectancy at all ages, with sex differentials slightly increasing for the oldest-old. As of today (2019), the remaining lifetime of a Spanish female (male) alive at age 60 is 27.76 (23.20), but by mid-century (2050) is forecasted to be 30.12 (25.74). However, empirical studies worldwide show that remaining lifetime is increasing faster than healthy life expectancy, which means that starting from around age 70 individuals and the society will have to provide for not only regular income but increasing health care costs and, most significantly, care costs, which for an increasing proportion of the older population will reach «ruinous» levels.

The demographic change underway in developed and developing countries, the increasing problems of traditional pay-as-you-go old-age social security systems, the fiscally driven public pension reforms, the move from non-funded (collective) defined benefit (NDB) schemes towards (individualized) funded and defined contribution (FDC) schemes, the decreasing generosity of public health care systems and of public annuities, with deep adequacy and poverty concerns in several countries and within certain groups of people (e.g., women, less-educated groups and migrants), the reduction in the traditional family support at old-age because of falling fertility rates, urbanisation and migration, all have increased the need for additional private savings to cover the old age income gap and to avoid relying on state-managed social transfers to counter the risks of poverty (Bravo & Holzmann, 2014; EC, 2018). Moreover, the current prolonged low interest rates and inflation environment, linked to low economic growth directly affects savings for retirement and makes it difficult for insurance and pension systems to sustain long-term guarantees and deliver appropriate retirement income. In the current interest rate scenario, it

will take several decades for DC pensions system to build up to a reasonable level which means time is also part of the equation.

Figure 1. **SPAIN | PERIOD LIFE EXPECTANCY FORECAST BY SEX FOR SELECTED AGES, 1960-2050**



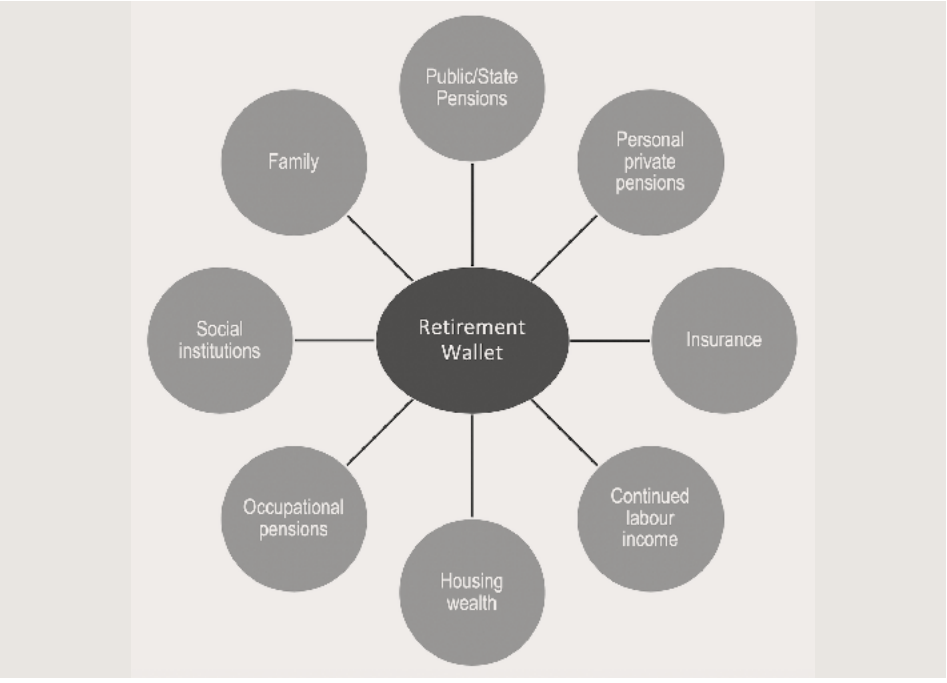
Note: Period life expectancy values computed using a Bayesian Model Ensemble (Averaging) technique of six generalized age-period-cohort stochastic mortality models fitted to Spanish mortality data from 1960 to 2017.
Source: Author's preparation.

In this paper we introduce and discuss the concept of retirement wallet representing the multiple income and service sources individuals and their families will have to fund for longer lives, comprising not only traditional public and private pension fund sources but also new important alternatives like equity release schemes and insurance mechanisms (Section 2). We then address the main decumulation risks and options, including the adoption of a given longevity insurance strategy, the design, advantages and limitations of programmed withdrawal strategies, and challenges posed by liability-driven investing (LDI) strategies (Section 3). The main payout options available for allocating assets accumulated in pension plans are discussed, particularly the role of traditional and innovative investment and longevity risk-sharing structures. We provide illustrative results for the price of innovative participating longevity-linked life annuities (PLLAs) that link benefits to the dynamics of both a longevity index and an interest rate adjustment factor using Spanish mortality and financial market data. Section 4 concludes.

2. FUNDING FOR LONGER LIVES: THE RETIREMENT WALLET

Planning for retirement individuals requires individuals to clearly understand their specific needs, vulnerabilities and preferences at old age, the likelihood of experiencing specific life events (e.g., experiencing changes in physical and mental health, losing partner, caring for spouse or other family members, changing housing, starting another career), their financial goals, the income (and services) sources they anticipate in the accumulation and decumulating phases and the risks they are willing to take. Typical old-age financial needs include having a minimum guaranteed income stream that smooths the transition from working life to retirement and protects from the eroding effect of inflation on the purchasing power of money, having an extra income to guarantee access to health-care and long-term care services (medicines, dental care, care at home, nursing home care), bequeathing (cash inheritance, housing wealth, grandchildren's education, funeral expenses, donations) or paying for life style activities (e.g., travelling). To fund for longer lives, people will ultimately rely on a retirement wallet comprising public and private pensions (state, linked to an employment relationship or occupational, based on contracts between individuals and private pension providers), private savings (dividends, coupon payments, cash withdrawals), housing wealth, continued labour income, insurance, family and social institutions (Figure 2).

Figure 2. **FUNDING FOR LONGER LIVES: THE RETIREMENT WALLET**



Source: Author's elaboration.

Table 1. **ASSETS IN PENSION FUNDS AND ALL RETIREMENT VEHICLES IN 2018**

OECD countries	Pension funds		All retirement vehicles
	in USD million	% of GDP	% of GDP
Australia	1.810.346	132,6	135,4
Austria	24.508	5,5	..
Belgium	37.531	7,3	..
Canada	1.459.272	85,6	..
Chile	193.110	70,2	70,2
Czech Republic	20.935	8,9	8,9
Denmark	154.373	45,4	199,0
Estonia	4.511	15,4	16,9
Finland	127.560	47,7	..
France	19.007	0,7	..
Germany	261.058	6,7	..
Greece	1.584	0,7	..
Hungary	5.876	3,9	5,3
Iceland	36.328	150,8	160,1
Ireland	115.073	31,6	33,5
Israel	203.224	57,4	..
Italy	153.430	7,6	9,8
Japan	1.398.144	28,2	..
Korea	191.066	12,0	..
Latvia	529	1,6	13,8
Lithuania	3.689	7,1	7,1
Luxembourg	1.883	2,8	..

Selected other jurisdictions	Pension funds			All retirement vehicles
	% change	in USD million	% of GDP	% of GDP
Albania	32,9	21	0,1	0,1
Armenia	50,2	329	2,6	..
Botswana	-3,7	7.358	40,8	..
Brazil	7,9	223.193	12,6	..
Bulgaria	5,7	7.881	12,5	12,5
Colombia	0,2	70.607	23,5	23,5
Costa Rica	13,9	11.527	20,2	20,2
Croatia	6,5	16.028	27,2	27,3
Dominican Republic	14,3	10.073	12,6	12,6
Egypt	10,0	3.757	1,5	1,5
El Salvador	6,6	10.648	40,9	..
Ghana	18,1	2.700	4,4	4,4
Guyana	21,5	309	8,3	8,3
Hong Kong, China	0,5	148.531	40,9	40,9
Indonesia	2,2	18.020	1,8	..
Isle of Man	6,6	14.321	..	..
Jamaica	14,6	4.750	30,2	30,2
Kenya	8,0	11.452	12,9	12,9
Kosovo	2,2	1.934	25,0	25,0
Malawi	37,6	944	13,7	13,7
Maldives	19,5	568	10,7	10,7
Namibia	9,5	10.213	80,2	91,3

.../...

.../...

OECD countries	Pension funds		All retirement vehicles
	in USD million	% of GDP	% of GDP
Mexico	168.311	14,1	..
Netherlands	1.514.345	171,0	..
New Zealand	54.481	27,4	27,4
Norway	39.834	9,8	..
Poland	42.112	7,5	..
Portugal	22.292	9,7	..
Slovak Republic	12.038	11,7	11,7
Slovenia	2.954	5,6	6,8
Spain	121.421	8,8	12,5
Sweden	22.610	4,1	90,6
Switzerland	888.799	126,9	..
Turkey	14.520	2,1	..
United Kingdom	2.809.112	104,5	..
United States	15.637.266	76,3	135,1
OECD Total	**27.573.129**	**53,3**	

Selected other jurisdictions	Pension funds			All retirement vehicles
	% change	in USD million	% of GDP	% of GDP
Nigeria	14,9	28.136	6,7	6,7
North Macedonia	13,2	1.228	10,0	10,0
Pakistan	10,3	185	0,1	..
Panama	3,7	557	0,9	..
Papua New Guinea	6,8	3.825	18,2	18,2
Peru	-1,8	45.469	20,7	20,7
Romania	19,4	12.176	5,2	5,2
Russia	1,4	81.456	5,5	5,5
Serbia	10,9	389	0,8	0,8
Suriname	3,5	467	13,6	..
Thailand	4,4	35.094	7,0	..
Ukraine	11,3	98	0,1	..
Uruguay	7,3	15.438	27,1	27,1
Total	**-4,1**	**799.686**	**10,3**	

Notes: «..» means not available.

Source: OECD Global Pension Statistics; French Asset Management Association; Bank of Japan; Bank of Korea; Swiss Occupational Pension Supervisory Commission; AIOS (for El Salvador and Panama).

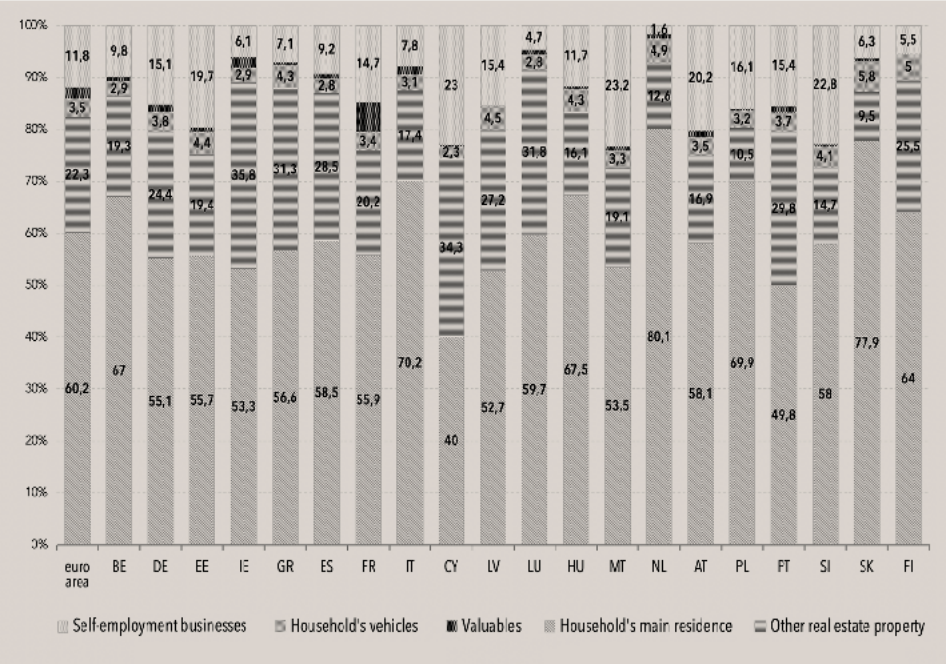
In most countries, public pension schemes (DB, DC, funded or unfunded) provide at least a basic income level, with varying generosity levels depending on the structure of the pension system and on financial system development. Empirical evidence shows that private pension plans financed through pension funds, pension insurance contracts, book reserves or other vehicles (e.g., bank or investment companies managed funds) are becoming more widespread, but there are still enormous differences in the coverage and significance of private pension provisions across jurisdictions even after accounting for the size of the population or domestic economy (Table 1) and the contribution amounts are insignificant in most cases (see, e.g., Household Finance and Consumption Survey (HFCS), ECB 2016).

Most DC scheme members have not contributed enough to receive even a modest income stream in retirement. For instance, pension funds held assets worth less than 1% of GDP in France or Greece while they held 171% of GDP in the Netherlands, 150.8% in Iceland or 132.6% in Australia. Assets earmarked for retirement in pension funds represented 53.3% of the GDP of the OECD area but just over 10.3% of the GDP of the other jurisdictions reported in Table 1. When considering the whole private pension system, i.e., including for other vehicles used to save for retirement (e.g., pension rights included in employers' books, pension insurance and retirement saving contracts), Denmark and Iceland have the largest amount of pension assets relative to GDP.

For the contrary, building up housing wealth through homeownership and mortgage repayment is by far the main way European households set aside for old age (ECB, 2016). In the Euro area countries, the household's wealth (excluding pension wealth, the present value of all future expected pension benefits) is primarily held in the form of real assets, which represent 82.2% of total assets owned by households (85.1% in Spain), with the remaining assets (17.8%) being financial. The largest component of real assets is the household main residence (HMR), representing 60.2% of total real assets, followed by other real estate property (22.3%). In the EU, roughly 70% of Europeans live in owner-occupied accommodation, ownership is higher in poorer countries and the proportion of home owners by age band has been steadily increasing with each successive generation. Empirical evidence also shows that homeowners are generally wealthier than their non-home owning counterparts, and this conclusion is valid across the income or net wealth distribution and across countries (Bravo, Ayuso & Holzmann, 2019).

Personal pensions and private homeownership are the two main assets individuals hold to finance (supplement) retirement consumption in an asset-based approach to welfare in which individuals accept greater responsibility for their own welfare needs. They both involve long-term saving and investment decisions over the life cycle, they are motivated by potentially competing objectives and generate different options and outcomes at old-age. Home homeownership provides a stream of housing services starting at time of house acquisition and represents wealth which could be liquidated in old age if needed. The asset serves both consumption and investment functions, which are assessed differently by households based on their personal preferences. Contrary to renting, home ownership is often regarded by individuals as an investment in asset-building, a better option to cope with inflation and to profit from house price appreciation. Access to affordable housing is crucial to any retirement income system since it contributes to reduce poverty by allowing retirees to maintain an adequate standard of living (Bravo, Ayuso & Holzmann, 2019). The question now is how to manage and access housing wealth in an efficient way to supplement your retirement wallet. In the next section we offer a catalogue of Equity Release Mechanisms (ERS), distinguishing between equity release is made possible while continuing to stay in their home or through a sale of the house and other design differences (e.g., time of release, owner of the property, amount of equity released).

Figure 3. **DECOMPOSITION OF REAL ASSETS BY ASSET CATEGORY, EURO AREA**



Notes: Shares of real assets types on total real assets by euro area country. The HFCS classifies real assets into five categories: the HMR, other real estate property, vehicles, valuables (valuable jewellery, antiques or art) and self-employment businesses.

Source: Own composition based on Household Finance and Consumption Survey (2016), 2nd wave.

The role of family in the retirement wallet is still significant in most countries, particularly in the form of a service annuity, but family support is expected to be reduced in the future due to smaller family sizes, fewer children to provide care, changing family composition and higher children's mobility, higher women labour participation (who traditionally provide care). In many countries, individuals also receive support (e.g., social assistance, care) from local/municipal social institutions (e.g., retirement community, senior living communities). Even though insurance is well established in most developed markets, evidence shows that, with few exceptions, it still plays a minor role in the retirement wallet of older people. Some of the most important risks and concerns individuals face during retirement (e.g., outliving one's wealth, longevity, health-care, long-term care, investment, inflation, interest rate, funeral expenses) are insurable risks and traditional and innovative solutions have been developing to address them in a cost-efficient manner. An ageing population is expected to increase the role for insurance in private market provision of retirement income and risk mitigating/sharing solutions, particularly in the decumulation stage.

Finally, contrary to traditional models of labour supply, including the standard versions of the lifecycle model which assume retirement is an absorbing state, empirical evidence suggests that the share of labour income from continued work after statutory retirement age is increasing and the trend is persistent, with labour force participation rates of 65 year-olds or more now surpassing 25% in many countries (e.g., Korea, Japan), including countries with relatively generous public pension (and health-care) benefits (Figure 4). Unretirement is more likely amongst individuals in better health, with a higher level of educational attainment or who have a spouse in the labour market, but one should not neglect involuntary retirement situations, financial illiteracy amongst individuals approaching retirement and unretirement due to financial constraints as possible causes for this trend.

The build-up and management of the retirement wallet will be different for each individual and there is no guarantee that individuals optimize consumption over their active and retirement period as predicted by the lifecycle hypothesis. Indeed, there is growing empirical evidence suggesting that the actual wealth accumulation, preservation and decumulation behaviour before and after retirement is often in conflict with lifecycle predictions, particularly when analysed considering the differentiated approach across the three main tiers of the population: (i) The lowest tier that typically does little saving and, as result, will have no capacity to dissave after retirement; (ii) The top tier that continues the accumulation of financial and non-financial asset after retirement and shows no sign of dissaving; (iii) The middle tier that seems to be the only one showing sign of life-cycle saving and dissaving, particularly those with no longevity insurance (public of private life annuity), but faces a number of constraints (e.g., illiquid housing wealth assets, the taxation of pensions, undeveloped financial and insurance markets).[1]

Several explanations have been put forward to explain why households deviate from the lifecycle hypothesis, saving in the top tiers more than predicted and often not decumulating their housing wealth (Figure 5). They include uncertainty regarding retirement income (e.g., investment risk), shocks that prevent accumulation (e.g., unemployment spells and scarring effects[2]), precautionary behaviour to face major family shocks (e.g., death of a spouse, divorce), uninsured future health care and long-term expenditures, evidence showing that individuals care about their descendants (intended bequests), behavioural and cultural biases, outdated social norms and psychological barriers, mental-accounting (precautionary) savings behaviour, the design and implementation of mandated earnings-related retirement schemes across countries including minimum income (and service) guarantees and related provisions, pensions taxation regimes that penalize accumulating or decumulation, low risk appetite and financial literacy, and the heterogeneity in longevity by income levels (Holzmann *et al.*, 2019).

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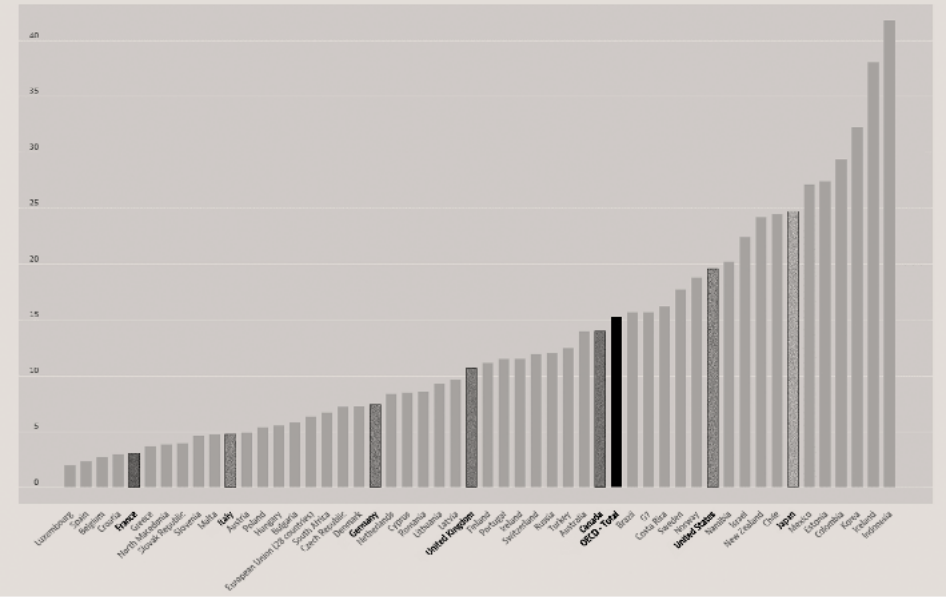
[1] See Holzmann *et al.* (2019) for an extensive literature review and conjectured research hypothesis.

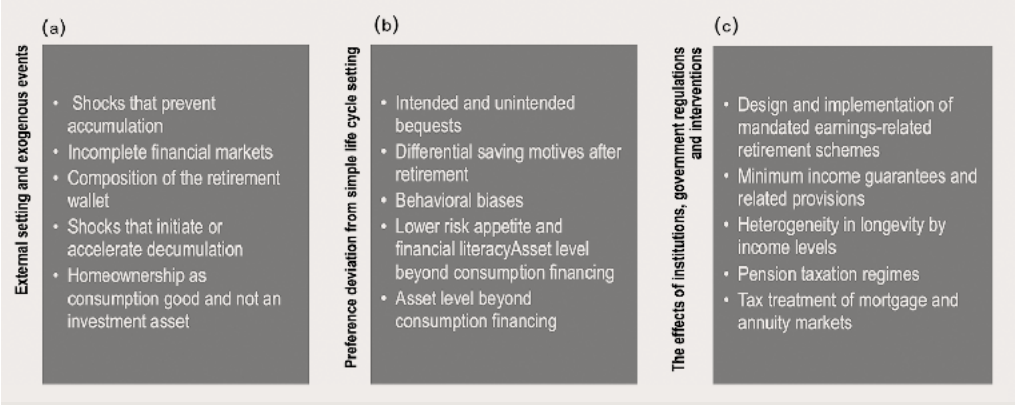
[2] See, e.g., Bravo and Herce (2019) and references therein.

Figure 4. **OLD-AGE LABOUR FORCE PARTICIPATION RATE (65-YEAR-OLDS OR MORE)**



Source: OECD, Labour Force Statistics; Note: 2018 or latest data available.

Figure 5. **DEVIATION FROM LIFECYCLE ACCUMULATION AND DECUMULATION**



Source: Based on Holzmann *et al.* (2019) with authors' additions.

3. RISK-SHARING OPTIONS FOR THE PAYOUT PHASE OF PENSIONS

The decumulation or payout phase of pensions is as important as the accumulation of financial, real and social wealth for retirement. Yet, while the accumulation phase of retirement saving has attracted most of the attention of industry and re-

search, the pay-out phase has been much less explored. Decumulation is the process of converting the retirement wallet into a flow of income and services for retirement. Decumulation requires individuals to decide upon a retirement strategy, comprising:

- A longevity insurance strategy, determining the provisions taken by individuals to guarantee they do not outlive their retirement wallet; This typically requires having a public or a private annuity, immediate or deferred, but recently other forms of longevity risk pooling have been proposed (e.g., ALDA, tontines, tonnuity, pooled annuity funds)
- A withdrawal strategy, specifying how to withdrawing cash from the retirement pot to finance regular consumption expenditures; This typically requires individuals to adopt simple rules (e.g., withdraw every year a fraction of the remaining life expectancy or a constant percentage of the retirement wallet) and must include a strategy to release equity from the housing wealth accumulated and/or inherited during working life, paying off outstanding debt and bequest arrangements.
- An investment strategy, determining how to maximize the retirement wallet's return; This involves defining a riskier or more conservative asset allocation strategy, depending on one's risk aversion. The benchmark for assessing the investment performance is not in this case in terms of an asset benchmark but in terms of a given liability cash flow stream (consumption expenditures), i.e., this is a liability-driven investing (LDI) strategy. Stated differently, in most cases managing a dedicated retirement pot requires assets to be invested with the primary objective of meeting current and future guaranteed or envisaged (targeted) consumption expenditures with an acceptable level of risk.
- Efficient and effective administration, protection of assets and appropriate value for money assessment.

3.1. **Key risk sources during decumulation**

It is important to be aware of the risks involved in the generation of retirement income from the retirement wallet. The key risks are listed in Table 2. There are several ways of dealing with such risks, including: (i) intra-generational risk pooling (the risks are pooled amongst individuals of a given cohort); (ii) Inter-generational risk sharing (the risks are shared between individuals of different cohorts); (iii) the hedging of risks using suitable hedging instruments; (iv) Risk mitigation by engaging in diversified investment strategies; (v) Adopting carefully designed default investment o withdrawal plans; (vi) Adopting effective regulation, e.g., against market conduct risk, credit risk and costs risk; (vi) Insurance, e.g., against individual longevity risk, health-care costs or long-term dependency risks, (vii) Public and private financial education programmes that promote rational informed choices. Unfortunately, many people do not understand the risks and, thus, cannot be expected to be able to manage these risks themselves. For them, auto-enrolment in a well-designed default decumulation strategy at retirement seems to be the best option.

Table 2. **KEY RISK SOURCES DURING THE DECUMULATION OF PENSIONS**

Risk	Definition
Individual longevity	Risk of outliving the retirement pot or experiencing a substantial reduction in retirement income, being forced to modify their standard of living (consumption) drastically.
Aggregate longevity	Risk that overall population lives longer than anticipated (systematic or aggregate longevity risk) forcing, e.g., a reduction in public pension benefits
Investment	The risk that portfolio investment performance is worse than expected or the risk that the LDI investment strategy do not generate income in a way that matches the desired pattern of consumption in retirement; Maybe a result of a bad product choice.
Inflation	The risk that a generalised rise in prices will result in an erosion of the real value of pensions payments and retirement income. For example, over 30 years a nominal fixed pension amount loses about 45% (78%) of its real purchasing power, when the inflation rate is 2% (5%) per annum.
Health	Risk that a sudden or increasing deterioration in the health of an individual significantly increases his or her health-care expenditure or requires expensive long-term care services.
Liquidity	Risk that accumulated retirement savings are not easily convertible (at least, not without a significant loss of principal) in liquidity, either for legal or contractual reasons.
Retirement timing risk	Uncertainty about when the scheme member will retire from labour market and/or begin to make withdrawals
Bequest	Most individuals have an altruistic approach to life and care about their closest relatives. They get satisfaction from knowing that their heirs will enjoy their inherited wealth once they die. Because of this, many parents want to leave their children their family home, leave money behind when they die, transfer some wealth to future generations or institutions.
Annuitisation	Mandatory annuitization may take place at the worst time, i.e., interest rates may be low at the point of annuity purchase generating lower than expected retirement income.
Political and regulatory	The risk that either public of private pension system providers may be forced to reduce their pension payments, because pension systems are financially unsustainable or as a result of a political decision and the risk that regulations change in an adverse way.
Taxes	Risk that a variation in the regulatory or tax environment will reduce the disposable retirement income, e.g., an increase in income tax rates or deductions, an increase in VAT taxes, an increase in capital market taxes.
Life events	Divorce, death of spouse/partner, etc.
Behavioural	Risk that pensioners behave in a way that is not considered to be rational, incapacity to make an 'informed choice' due to insufficient financial literacy and understanding of risks
Market conduct and credit risk	The risk that financial and non-financial service providers act in a way that disadvantages retirees and credit risk referring to the events after which companies or individuals will be unable to make the required payments on their debt or contract obligations.

Source: Author's elaboration based on Bravo and Holzmann (2014) and Blake (2016).

3.2. **The decumulation menu**

The main forms of retirement payout options available for allocating assets accumulated in DC pension plans include lump sum payments, programmed or phased withdrawals, life annuities and hybrid solutions. The possibility of taking accumulated financial savings as a cash lump sum is typically dependent both on the contractual arrangements defined by the pension plan and the tax rules in force in a particular jurisdiction. Lump sum payments offer retirees full flexibility in the use of accumulated savings, including spending on leisure activities (holidays, cruises, spending on hobbies or buying a car, boat, caravan, etc.), passing on part of their retirement pot to children or other family members, investing in new or additional property, paying off a mortgage on a house or other debts, or simply continue to pursue an investment strategy, benefiting from potential higher returns on equity markets and other assets. A major advantage of lump sum payments is the ability of retirees to «self-annuitize», at a time and on a basis that best suits their financial needs. However, However, lump sum payments also encompass significant disadvantages, particularly the fact that they do not provide any protection against individual or aggregate longevity risk, they expose retirees to investment, credit and inflation risks, they demand individuals to have the knowledge to manage their retirement accounts wisely and efficiently and require individuals to maintain a long-term financial discipline to minimize the risk of outliving their retirement pot.

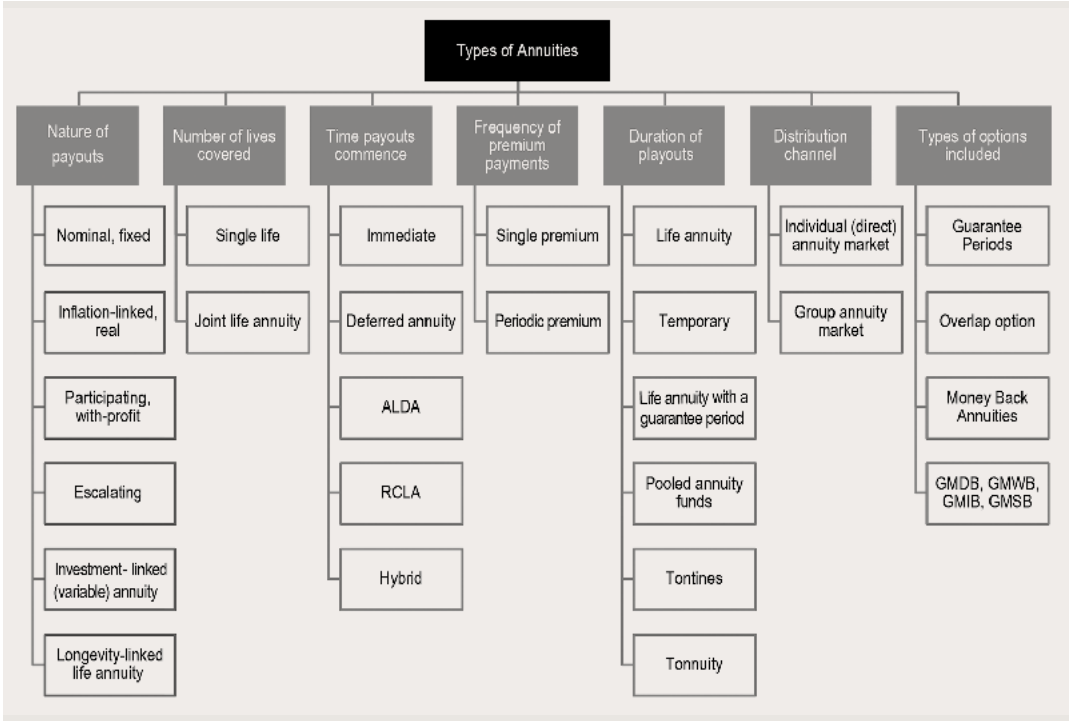
Under a programmed withdrawal strategy, retirees make periodic strategic and systematic withdrawals or lump sum payments from their retirement wallet to cover necessary expenses, instead of buying an annuity or receiving a single lump sum payment. The individual maintains the control and ownership of its assets, decides upon the investment strategy but there is no biometrical risk-pooling. The regular income flows may be the result of an explicit withdrawal rule or plan (e.g. the so-called 4% sustainable withdrawal rule, a fraction of the remaining life expectancy at the retirement age, possibly with lower and upper bounds, a constant amount) or simply be the result of discretionary actions. Although self-managed products are available, normally retirement withdrawal products are delegated management retirement products under which the account management activities are allocated to the asset management company. Programmed withdrawal has some advantages compared to annuity purchase (higher liquidity and flexibility to respond to unexpected consumption expenditures, retaining control over retirement assets, potentially higher pay-outs due to enhanced investment returns, possibility to allocate assets to inflation-linked investments, compatibility with the bequest motive, death benefit options), but also several drawbacks. The main disadvantages include the lack of protection against longevity risk, significant exposure to investment risk, there is no survival credit (mortality cross subsidy) when compared to buying an annuity contract, retirees are exposed to annuitisation risk if they decide to postpone annuity purchase to a later age, exposure to inflation risk and higher operating expenses when compared to the purchase of an annuity.

The most traditional pay-out solution for generating a predictable income stream in retirement is a life annuity. Annuity products offer protection against longevity risk and an extra return conditional on survival through pooling mechanisms but leave retirees with no control over assets and no flexibility in the use of accumulated assets, for instance, to address the bequest motive. There are many types of annuities that can be differentiated, for instance, by the nature of payment, by the number of people covered, by the duration of payments, by the time that payouts commence, by the frequency of premium payments, by the distribution channel and types of options included, among other features (figure 6). Importantly, the most commonly used type of annuities, nominal level annuities, provides certainty of income in nominal terms but offers no protection against inflation risk. Escalating nominal (real) annuities provide partial (full) protection against inflation but offer initially lower payments when compared to level annuities.

An annuity contract can be divided into two phases: the accumulation phase, when premiums are paid and capital builds up, and the decumulation phase when the benefits are paid out. The premium the insured (annuitant) must pay can be either a single, fixed periodic, or a variable periodic payment. The pay-out phase can follow the accumulation phase immediately (immediate annuity) or after a specified period (deferred annuity, advanced life deferred annuity – ALDA) or when your retirement pot is exhausted (Ruin Contingency Life Annuity – RCLA). While an immediate annuity is provided in exchange for a one-off lump sum, a deferred annuity is usually financed through regular premium payments. The amount the insurance company pays out can be conditional on the survival of just one (single annuity) or more than one individual, such as the spouse (joint and survivor annuities).

Regarding the duration of pay-outs, benefit payments can continue while the annuitant is alive (life annuity), up to a specified date (annuity certain), the earlier of the two (temporary annuity) or the later of the two (guaranteed annuity). The duration of pay-outs is the most important feature in connection with longevity risk. In the case of a guarantee period, the periodical payments will be made to the annuitant or to the heirs for a certain period (e.g. ten-years), regardless of whether the annuitant is alive. Guarantee periods as well as joint and survivor annuities are included to address the reduced bequest potential of a level annuity contract, since they continue to be paid out also when the annuitant passes away. Including these features comes, of course, at the expense of a lower survival credit (rate of return). Recent developments in this area include modern Tontines, a non-insurance contract structure (the insurance company's role is merely administrative) created to pool the longevity risk of a group of participants with the purpose to pay an income for life but with no guarantees, i.e., the pool of policyholders bears the aggregate longevity risk (see, e.g., Milevsky & Salisbury, 2015, 2016). Recently, a so-called Tonuity combining the appealing features of tontine and conventional life annuity has been proposed (Chen, Hieber & Klein, 2017).

Figure 6. **TYPES OF LIFE ANNUITIES**



Source: Author's elaboration.

The way the accumulated capital is paid out during the pay-out phase depends on the annuity type. The simplest is one which provides guaranteed constant nominal lifetime payment (nominal fixed annuity). Apart from that, variable annuities can adopt different forms. Annuity benefits can rise (or fall) at a prescribed fixed nominal rate that escalates with the age of the annuitant (escalating annuity); they can be indexed to inflation, thus providing a guaranteed income in real terms (inflation linked or real annuity); they can be linked to observed survival probability (longevity-linked life annuity); they can depend on the insurance company's surplus (participating or with profit annuity); or even reflect the performance of an underlying investment portfolio, usually represented by a family of mutual funds (investment-linked or variable annuity). In some annuities, pay-outs can also participate in mortality risk. In the case of with-profits (or participating) annuities, annuitants share both investment and longevity risk but gain the benefit of risk-pooling.

Variable annuities with guarantees have been developed to meet retiree demands more effectively than fixed annuities, namely claims for some upside market potential and increasing flexibility. The most popular product offering these advantages is the variable annuity (VA), a unit-linked product commonly sold with guarantees. The most common guarantees included in these contracts are: (i)

Guaranteed minimum death benefit (GMDB); (ii) Guaranteed minimum income benefit (GMIB); (iii) Guaranteed minimum withdrawal benefit (GMWB); (iv) Guaranteed lifetime withdrawal benefit (GLWB); (v) Guaranteed minimum accumulation benefit (GMAB).

3.3. **An innovative structure: Participating Longevity-Linked Life annuities**

Pension funds and annuity providers face uncertainty regarding financial returns and systematic longevity risk due to unexpected future mortality improvements. Although advances in longevity are not homogenous across socioeconomic groups, providing an efficient risk pooling mechanism that addresses the (individual) uncertainty of death through the provision of a lifetime annuity is one of the main mechanisms pension schemes are considered to redistribute income in a welfare-enhancing manner. Without such an instrument, individuals risk outliving their accumulated (financial, housing, pension) wealth or leaving unintended bequests to his/her dependents. Traditional (fixed, inflation-indexed) life annuities are a key instrument in mandated Defined Benefit (DB) pension schemes, in financial (FDC) and non-financial Notional (NDC) Defined Contribution schemes and in private pensions provided by insurance companies. Contrary to standard Modigliani life-cycle model of savings and consumption prediction, the voluntary market purchase of retirement annuities is in most countries very limited and decreasing and the actual saving/dissaving behaviour after retirement is often at odds with economic theory (Holzmann *et al.*, 2019). A number of demand side (e.g., perceived poor value-for-money, the existence of annuity alternatives, bequest motives, behavioural and informational limitations) and supply-side (e.g., the regulatory burden of annuity providers, with onerous capital requirements for unhedgeable risks (e.g., longevity risk) within Solvency II, nearly zero interest rate environment and significant interest rate risk exposure[3], long-term financial risk, the cost of loss control and loss financing longevity risk management solutions, limited reinsurance capacity to absorb massive exposure-to-risk) arguments have been put forward to explain this «annuity puzzle», i.e., to explain why the level of annuitization by individuals is much smaller than economic theory would suggest (see, e.g., Milevsky, M. (2013) and Holzmann (2015) for a review). Together with the development of capital market longevity-linked securities and their derivatives and innovative reinsurance designs, this has increased the attention towards new contract structures involving financial and longevity risk sharing mechanisms between the annuity provider and annuitants, and increased recommendations towards the use of deferred annuities, that reduce the cost of guarantees and potentially augment their attractiveness to policyholders.

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[3] See, e.g., Bravo and Silva (2006) and Simões, Oliveira and Bravo (2019) and Chamboko and Bravo (2016, 2019a,b) for single and multiple ALM interest rate risk immunization strategies for pension funds and annuity providers and for credit valuation problems.

A number of alternative index-type and indemnity-type mechanisms have been proposed in the literature to directly or indirectly share financial and longevity risks between annuity providers and individuals. They typically involve updating the annuity benefit according to observed mortality and investment developments. Depending on the contract design and underlying asset performance, future annuity benefits may decline with time, an undesirable feature that must be compensated at contract inception through lower prices or higher initial benefits (a risk premium). For instance, in investment-linked annuities payments fluctuate according to the actual return of the asset portfolio backing the contract. In traditional participating (with-profit) annuities payments depend on the providers overall performance regarding mortality, investments, and expenses. They provide guaranteed lifelong minimum annuity benefits in combination with participation in the insurer's positive surpluses. Investment guarantees may be in the form of a «technical interest rate» implicit within the actuarial structure of the product or explicit as a minimum annual return (Olivieri and Pitacco, 2019).

In participating longevity-linked life annuities (PLLAs) benefits are updated periodically based on the dynamics of both a longevity index, defined as the ratio between the expected survival probability and the survival rate observed in a reference population, and of an interest rate adjustment factor, defined as the ratio between observed and guaranteed financial returns (Bravo and Freitas, 2018; Bravo, 2019).[4] Alho, Bravo and Palmer (2012) investigate the consequences of introducing periodically revised annuities in NDC pension schemes and suggest updating benefits periodically based on the relationship between expected and observed period life expectancy. In this section we briefly describe the benefit structure and risk sharing design of immediate PLLAs. We then introduce the valuation setup via embedded longevity option decomposition. Consider an index-type participating longevity-linked life annuity (PLLA) along the lines proposed by Bravo and Freitas (2018). Under this contract, the annuity benefit is updated periodically based on both the observed survival experience of a reference pool and the investment performance of the financial assets backing the contract. Without loss of generality, let us assume that annuitants contribute equal amounts into the annuity fund and, in return, receive equal annuity benefit payments b_t at time t. Under this contract, the annual benefit at some future date $t_0 + k$, b_{t_0+k} will depart from the initial benefit b_{t_0} depending on the dynamics of both a longevity factor I_{t_0+k} and an interest rate adjustment (IRA) factor R_{t_0+k},

[4] A similar but narrower approach can be found in Denuit, Haberman and Renshaw (2011) in which only the systematic component of longevity risk is passed to annuitants and caps and floors can be introduced to limit the profit-loss share. A related approach is found in Bravo, Corte-Real and Silva (2009) in which annuity payments are updated only if observed survivorship rates exceed a given threshold.

$$b_{t_0+k} = b_{t_0} \times I_{t_0+k} \times R_{t_0+k}, \quad k = 1, \ldots, \omega - x \tag{1}$$

where I_{t_O+k} is a ratio between the expected survival probability and the survival rate observed in a reference population, defined by

$$I_{t_0+k} = \frac{{}_kp_{x_0}^{[F_0]}(t_0)}{{}_kp_{x_0}^{[F_k]}(t_k)} = \prod_{j=0}^{k-1} \frac{p_{x_0+j}^{[F_0]}(t_0+j)}{p_{x_0+j}^{[F_k]}(t_0+j)} \tag{2}$$

with

$${}_kp_{x_0+j}^{[F_0]}(t_0+j) = \prod_{j=0}^{k-1}\left[1 - q_{x_0+j}(t_0+j)\right] \tag{3}$$

denoting the k-year survival probability of some reference population cohort aged x_O at time t_O (computed at contract inception on a market or national population life table) and ${}_k p_{x_0}^{[F_k]}(t_k)$ is the corresponding k-year survival probability observed at time t_k and ω the highest-attainable age. In (3) $q_{x_O+j}(t_O+j)$ is the 1-year death probability of an individual aged $x_O + j$ at time $t_O + j$. The IRA factor $\mathrm{R}_{t_O\,k}$ is defined by

$$R_{t_0+k} = \frac{\prod_{j=0}^{k-1}(1+R_t)}{(1+i_{t_0})^k} \tag{4}$$

Where R_t denotes the observed net investment return in year t and i_{t0} is the (generally non-negative) guaranteed minimum interest rate set at time 0.

If $R_t = i_{t0}\,\forall t$ and mortality improvements are as expected (i.e., $I_{t_O+k} = 1\ \forall k$), the arrangement resembles a classical life annuity with fixed-return, and fixed-benefit. If $R_t = i_{t0}$ and observed longevity improvements are higher (lower) than predicted i.e., $I_{t_O+k} < 1$ ($I_{t_O+k} > 1$) $\forall k$, annuity payments will decline (increase) along with the dynamics of I_{t_O+k}. If mortality improvements are as expected and investments perform above the guaranteed interest rate (i.e., $R_{t_O+k} > 1\ \forall k$), the extra return is returned to participants in the form of a higher benefit payment. If $I_{t_O+k} < 1$ and $R_{t_O+k} > 1$ the better than expected investment returns may at least partially compensate the negative impact of higher than expected mortality improvements. At annuity inception, the longevity and the IRA indexes are random variables and, hence, future annuity benefits are uncertain. This contrasts with traditional fixed life

annuity contracts that guarantee a constant benefit as long as the annuitant is alive, independently of longevity and financial performance developments, transferring thus all risks (financial and biometric) to the provider. Appropriate bounds to the longevity and IRA adjustment factors (or to the benefit amount) can in principle be introduced to offer partial guarantees, limit the volatility of annuity payments, to provide effective longevity insurance or to limit the profit-share (e.g., caps and floors, longevity corridors, partial participation mechanisms, a maximum age to apply the benefit adjustment). For instance, in Bravo and Freitas (2018) the authors suggest limiting the risk beared by policyholders by adding (possibly) time-dependent upper and lower barriers for the longevity index.

The valuation of a PLLA at time t_O can be obtained via longevity option decomposition. Following Bravo and Freitas (2018), the the fair value of a PLLA is decomposed into a long position in a classical fixed annuity $a_{x_0}^{[F_0]}(t_0)$ and a short position in an embedded European-style longevity floor $\mathrm{L}^F(t_O)$ with underlying I_{t_O+k}, constant strike equal to one unit of currency and maturity $\omega - x_O$ i.e.,

$$a_{x_0}^{PLLA}(t_0) = a_{x_0}^{[F_0]}(t_0) - L^F(t_0) \tag{5}$$

with

$$L^F(t_0) = \sum_{k=1}^{\omega - x_0} E^Q\left(B(0,k) \times {}_kp_{x_0}^{[F_0]}(t_0) \times \left(1 - I_{t_0+k}\right)^+ \middle| F\right) \tag{6}$$

where $B(t,T)$ is the discount factor, $a^+ = \max(a,0)$ and, without loss of generality, we consider an immediate PLLA contract with initial benefit $b_{t_O} = 1$ and a scenario in which observed longevity improvements are higher than predicted and investment performance matches the guaranteed interest rate.

In Table 3 for provide illustrative results for the price of non-participating PLLAs calibrated to the Spanish mortality data from 1960 to 2016 and for ages in the range 60-95.[5] Mortality data is obtained from the Human Mortality Database (2019).

[5] Detailed results can be obtained from the author upon request.

Table 3. **FAIR VALUE OF NON-PARTICIPATING PLLA AND EMBEDDED LONGEVITY FLOOR OPTION PRICES**

	Age						
	60	65	70	75	80	85	90
Pure premium of a fixed life annuity	24.36	19.85	15.61	11.75	8.40	5.69	3.66
Longevity Floor price quantiles							
2.5%	0.06	0.02	0.01	0.00	0.00	0.00	0.00
50%	1.55	1.16	0.80	0.50	0.28	0.13	0.06
97.5%	3.35	2.60	1.91	1.27	0.74	0.37	0.17
Longevity Floor price in % of a_{x_0} (basis points)	636	584	515	429	331	234	159
PLLA pure premium	22.81	18.69	14.81	11.24	8.13	5.56	3.60
PLLA price quantiles							
2.5%	26.16	21.29	16.72	12.51	8.86	5.93	3.77
97.5%	19.47	16.09	12.90	9.98	7.39	5.18	3.43

Notes: Guaranteed interest rate equal to 0%; Zero risk premium as reference life table; Market price of longevity risk set by λ=0.3 (Wang Transform parameter); Annuity payments capped at the initial benefit. See Bravo and Freitas (2018) for details on the calibration of the risk neutral simulation approach.

Source: Author's elaboration.

The setting comprises a risk-neutral, frictionless and continuous financial market in which the annuity provider invests the insurance premium in a portfolio of dividend-paying stocks (30%) and coupon bonds (70%), and a risk-free interest rate. We assume the yield curve dynamics is well captured by a two-factor equilibrium Vasicek (1977) model and the stock market index follows a standard geometric Brownian motion diffusion process. To account for the longevity risk premium in pricing the contracts, we compute cohort-specific risk-adjusted survival probabilities by using a risk-neutral simulation approach assuming the dynamics of mortality rates is well represented by the log bilinear Lee-Carter model under a Poisson setting, with time trend parameter modelled using a general ARIMA(p,d,q) model and risk neutral distribution of the innovations obtained using the Wang transform. The results are generated through 10.000 independent sample paths for both the survival probability of a cohort aged in at time 0 and the portfolio returns.

As expected, the fair value of a fixed annuity is smaller the older the policyholder at contract initiation, i.e., decreases with the reduction in the remaining life expectancy. Similarly, the longevity floor prices are increasing in maturity (decreasing

with the age of the policyholder at contract inception). For instance, for λ=0.3 the longevity option price for a 60-year old individual at the end of 2016 is 1.55, whereas for an equivalent contract starting at age 75 the price is 0.50. The embedded European-style longevity floor prices represent between 1.59% and 6.36% of the pure premium of a conventional fixed annuity. This means, for instance, that a 60-year old male individual entering into a non-participating PLLA contract should pay a pure single premium 6.36% lower (22.81) than that of an equivalent fixed annuity (24.36) to accept the chance of annuity benefits declining if observed survivorship rates are higher than predicted. For this representative case, the 95% confidence interval for the mean estimate of the fair value is [19.47–26.16] with mean estimate 22.81. These results are in line with those obtained by Bravo and Freitas (2018) using data for France, although in this later case the higher trend risk observed in the French population resulted in higher longevity option prices.

Participating longevity-linked life annuities include embedded longevity and financial options that allow the annuity provider to periodically revise annuity payments if observed survivorship and portfolio outcomes deviate from expected (or guaranteed) values at contract initiation. Contrary to standard fixed annuities in which the insurer bears all risk, PLLAs offer an efficient and transparent way of sharing biometric and financial market risks between annuity providers and policyholders. They are an interesting and promising product for the payout phase of pension schemes since the contract tackles some of the demand- and supply-side constraints that prevent individuals from annuitizing their retirement wealth and may contribute to help insurers writing new annuity policies. By linking the annuity benefit to the survival experience of a given underlying population and to the performance of the asset portfolio backing the contract PLLAs provide a direct mechanism to share financial and longevity risk and are an interesting alternative to manage systematic longevity risk in markets in which alternative risk management solutions (longevity-linked securities, reinsurance arrangements, capital allocation) are scarce and/or expensive.

4. **FINAL REMARKS**

In this paper we discuss the main accumulation and decumulation options individuals will have to fund for longer lives. We highlight the role of traditional public and private pension schemes but also claim the importance of developing solutions to efficiently release equity from the main asset families hold at retirement: their house. Pension plan designs range from those in which all the risk stays with the plan (and plan sponsor) to those that increasingly share the risks with the participant. The family, social institutions and insurance providers significantly contribute to address the specific needs at old-age. We the analyse the main challenges posed during the decumulation phase and give special attention to novel solutions sharing investment and longevity risk between policyholders and annuity providers. Illustra-

tive numerical results for the fair value of index-type participating longevity-linked life annuities (PLLA) are provided for Spain. Further research is also needed to design and valuate alternative methods to directly share longevity risk between the provider and annuitants.

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