Comparison of the production behavior of regret-averse and purely risk-averse firms*

Comparación del comportamiento de producción de empresas aversas al arrepentimiento y aversas al riesgo

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Abstract

Previous studies focused on the comparison of the optimal output levels of regret-averse firms under uncertainty and firms under certainty. This paper extends the theory by further investigating the effects of regret-aversion on production. We compare the optimal output levels of regret-averse firms with purely risk-averse firms under uncertainty and firms under certainty. We first show that the linear-regret firms will surely produce more than their purely risk-averse counterparts and surely produce less than firms under certainty. Thereafter, we give sufficient conditions to ensure the regret-averse firms to produce more than the purely risk-averse counterparts and study the comparative statics of the optimal production. We also develop properties of regret-aversion on production

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by using a binary model. The findings in this paper are useful for production managers in their decisions on the production.

Keywords: *Production, regret aversion, risk aversion, uncertainty.*

JEL Classification: *D21, D24, D81.*

Resumen

Estudios previos se focalizaron en la comparación de los niveles óptimos de producción de empresas aversas al arrepentimiento bajo certidumbre e incertidumbre. Aquí se extiende la teoría al comparar los niveles óptimos de producción de empresas aversas al arrepentimiento con aquellas aversas al riesgo (con certidumbre e incertidumbre). Mostramos que empresas con arrepentimiento lineal producirán más que las empresas aversas al riesgo y menos que con certidumbre. Desarrollamos las propiedades de la aversión al arrepentimiento en la producción utilizando un modelo binario. Los resultados encontrados pueden ser de utilidad para las decisiones de producción.

Palabras clave: *Producción, aversión al arrepentimiento, aversión al riesgo, incertidumbre.*

Clasificación JEL: *D21, D24, D81.*

1. Introduction

Regret occurs when the ex-ante optimal decision becomes ex-post suboptimal. This kind of behavioral characteristic is commonly occurred and supported by many experimental literature (Loomes and Sugden, 1987; Loomes, 1988; Starmer and Sugden, 1993). To analyze the natural regret-averse preference, Loomes and Sugden (1982) and Bell (1982) introduce the regret theory that formally defines regret to be the disutility of not having made the ex-post optimal decision. The regret theory is then further extended by Sugden (1993), Quiggin (1994), and many others.

In traditional economic theory of competitive firms when output price is uncertain (Sandmo, 1971; Broll, 1992; Viaene and Zilcha, 1998), many studies consider purely risk-averse firms without taking the regret-averse preference into consideration. Investigating production behavior of a competitive firm under both risk-aversion and also regret-aversion, Egozcue and Wong (2012) and Wong (2014) use the additive separable utility function developed by both Braun and Muermann (2004) and Muermann *et al*. (2006). By doing so, Egozcue and Wong (2012) and Wong (2014) provide sufficient conditions under which the optimal output level of the regret-averse firm under uncertainty is less than that under certainty. Thereafter, several authors, for example, Niu *et al.* (2014), Egozcue *et al.* (2015), and Broll *et al.* (2016, 2017), obtain different sufficient conditions to assure the regret-averse firms to produce less than firms without uncertainty.

In this paper we extend the theory in the literature by further investigating the effects of regret-aversion on production. We compare the optimal output levels of regret-averse firms with those of purely risk-averse firms under uncertainty and firms under certainty.

We first show that the linear-regret firms, which have linear regret functions, will surely produce more than their purely risk-averse counterparts and surely produce less than firms under certainty. Thereafter, we give sufficient conditions to ensure regret-averse firms to produce more than purely risk-averse firms. We also examine the comparative statics of the optimal production such that the regret-averse firms will produce more when they are more regret-averse.

To give more insights, we consider a simple binary model in which the uncertain output price could be lower or higher with positive probability. Using the binary model, we prove that regret-averse firms could optimally produce more than purely risk-averse firms, especially when it is more likely that the output price is lower. This weakens the conditions set in the general case for the regret-averse competitive firm. The findings in this paper provide valuable complements of existing results and are useful for production managers in their decisions on the production.

The rest of this paper is organized as follows. Section 2 discusses the model and develops properties for the competitive firm when the price is uncertain and when the firm is not only risk-averse, but also regret-averse. Section 3 further studies properties of regret-aversion on production in a binary model. The final section concludes.

2. The Theory

In this section, we delineate the model of the competitive firm under uncertainty when the firm's preference is not only risk-averse, but also regret-averse. We provide sufficient conditions under which the optimal output levels of the regret-averse firms are larger than those of risk-averse firms. We also examine the comparative statics of the optimal output levels when the regret coefficient changes. We will use the term "proposition" to state new results obtained in this paper and "property" to state some well-known results or the inference drawn from the propositions obtained in this paper.

2.1. Model setting

We first briefly introduce the model. Consider a competitive firm which produces a single commodity with uncertain output price $\tilde{P}$ and the output level $Q \geq 0$, according to a special cost function $C(Q)$ satisfying $C(0) = C'(0) = 0, C'(\cdot) > 0$, and $C''(\cdot) > 0$.[1] The support of $\tilde{P}$ is $[\underline{P}, \overline{P}]$ with $\mathbf{0} < \underline{P} < \overline{P} < \infty$ and the firm's final profit is given by $\tilde{\Pi} = \tilde{P}Q - C(Q)$. To account for the regret that ex-post suboptimal decision has been made, Wong (2014) introduces the following bivariate utility function:[2]

$$V\left(\Pi, \Pi^{max} - \Pi\right) = U(\Pi) - \beta G\left(\Pi^{max} - \Pi\right) \tag{2.1}$$

Here, Π^{max} is the maximum profit that the firm could have earned if the realized output price is known in advance. Furthermore, if we have observed the realized output price P, Π^{max} would take the form that $\Pi^{max}(P) = PQ(P) - C[Q(P)]$ with $C'[Q(P)] = P \cdot U(\cdot)$ is a von Neumann-Morgenstern utility function with $U'(\cdot) > 0$ and $U''(\cdot) < 0$, accounting for the firm's risk-aversion. While $G(\cdot)$ is a regret function such that $G(0) = 0$, $G'(\cdot) \geq 0$, and $G''(\cdot) \geq 0$.[3] The parameter $\beta \geq 0$ is a constant regret coefficient, indicating the extent of the regret-aversion. The utility function $U(\cdot)$ is the same for all firms in our paper.

As a result, the production decision problem of the competitive firm reads:

$$\max_{Q \geq 0} E\left\{U\left[\Pi\left(\tilde{P}\right)\right] - \beta G\left[\Pi^{max}\left(\tilde{P}\right) - \Pi\left(\tilde{P}\right)\right]\right\} \tag{2.2}$$

Here, $E(\cdot)$ is the expectation operator with respect to the cumulative distribution function, $F(P)$, of the random output price $\tilde{P}$.

The first-order condition is then given by:

$$H(Q^*) = E\left\{\left\{U'\left[\Pi^*\left(\tilde{P}\right)\right] + \beta G'\left[\Pi^{max}\left(\tilde{P}\right) - \Pi^*\left(\tilde{P}\right)\right]\right\}\left[\tilde{P} - C'(Q^*)\right]\right\} = \mathbf{0}, \tag{2.3}$$

where an asterisk (*) indicates an optimal level. The second order condition is satisfied (Egozcue and Wong, 2012; Wong, 2014).

Since the seminal work of Egozcue and Wong (2012) and Wong (2014), several authors, for example, Niu *et al.* (2014), Egozcue *et al.* (2015), and Broll *et al.* (2016, 2017) compare the optimal output levels for regret-averse firms under uncertainty and under certainty. In this paper, we extend their studies to compare the optimal output levels between the regret-averse and purely risk-averse firms.

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1 We follow Egozcue and Wong (2012) and Wong (2014) to assume the strict convexity property of the cost function. This assumption reflects the fact that the firm's production technology exhibits decreasing return to scale.

2 We follow Egozcue and Wong (2012), Wong (2014), and others by using their notations. Braun and Muermann (2004) and Muermann *et al.* (2006) also use similar notations.

3 Egozcue and Wong (2012) and Wong (2014) impose this assumption to indicate that the more pleasurable the consequence that might have been, the more regret will be experienced. We modify the assumptions $G'(\cdot) > 0$ and $G''(\cdot) > 0$ set by Egozcue and Wong (2012) and Wong (2014) to be $G'(\cdot) \geq 0$, and $G''(\cdot) \geq 0$.

With the above definitions, we are ready to define regret-averse, purely risk-averse, linear-regret firms under uncertainty, and firms under certainty. A regret-averse firm is the firm who possesses the utility function $V\left(\Pi,\Pi^{max}-\Pi\right)$ defined in Equation (2.1). When the regret coefficient β is set to be zero in Equation (2.1), the regret-averse firm becomes a purely risk-averse firm. On the other hand, if the regret function $G(\cdot)$ in Equation (2.1) is linear, the regret-averse firm is linear-regret. Lastly, if the output price $\tilde{P}$ is fixed at its expected value, $E\left(\tilde{P}\right)$, firms are under certainty. We denote the optimal output levels and optimal profits by Q^*,Q_*,Q_n^*,Q^0 and $\Pi^*\left(\tilde{P}\right),\Pi_*\left(\tilde{P}\right),\Pi_n^*\left(\tilde{P}\right)$, and $\Pi^0\left(\tilde{P}\right)$, for regret-averse, purely risk-averse, linear-regret firms under uncertainty and firms under certainty, respectively.

For Q_*, we have:

$$E\left\{U'\left[\Pi_*\left(\tilde{P}\right)\right]\left[\tilde{P}-C'(Q_*)\right]\right\}=0 \tag{2.4}$$

Since $U'(\cdot)>0$, it is clear that

$$E\left\{U'\left[\Pi_*\left(\tilde{P}\right)\right]\left[\underline{P}-C'(Q_*)\right]\right\}<E\left\{U'\left[\Pi_*\left(\tilde{P}\right)\right]\left[\tilde{P}-C'(Q_*)\right]\right\}=0<E\left\{U'\left[\Pi_*\left(\tilde{P}\right)\right]\left[\overline{P}-C'(Q_*)\right]\right\}.$$

As a result, we get $\underline{P}-C'\left(Q_*\right)<0<\overline{P}-C'\left(Q_*\right)$.

Evaluating $H(Q)$ at $Q=Q_*$ and applying Equation (2.4), we get:

$$\begin{aligned}H(Q_*)&=\beta E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\left[\tilde{P}-C'(Q_*)\right]\right\}\\&=\beta Cov\left(G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right],\tilde{P}\right)+\beta E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\right\}\left[E\left(\tilde{P}\right)-C'(Q_*)\right]\\&=\beta E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\tilde{P}\right\}-\beta C'(Q_*)E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\right\}.\end{aligned}$$

Thus, as long as

$$\frac{E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\tilde{P}\right\}}{E\left\{G\left[\Pi^{max}(P)-\Pi_*(P)\right]\right\}}\geq C'\left(Q_*\right).$$

we have $H(Q_*)\geq 0$. Thereafter, according to Equation (2.3) and the second order condition, we have $Q^*\geq Q_*$.

To get the meaning of the above condition clearly, we define the following function:

(2.5) $$\Phi(P)=\int_{\underline{P}}^{P}\frac{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]}{E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\right\}}\,dF(P)\text{ forall }P\in\left[\underline{P},\overline{P}\right].$$

From Equation (2.5), it is evident that $\Phi'(P)>0$, $\Phi(\underline{P})=0$, and $\Phi(\overline{P})=1$. Thus, we can interpret $\Phi(P)$ as a cumulative distribution function of $\tilde{P}$. Hence, condition above can also be expressed as $E_\Phi\left(\tilde{P}\right)\geq C'(Q_*)$; that is, as long as the transformed expectation of uncertain output price, $E_\Phi\left(\tilde{P}\right)$, is larger than the marginal cost of production $C'(Q_*)$, regret-averse firms will produce more than their purely risk-averse counterparts. We will discuss this issue more in the following subsections.

2.2. Linear-regret competitive firm

Over here, we discuss whether competitive firms with linear regret functions will produce more than purely risk-averse firms. To do so, we consider a special case with $G'(\cdot)\equiv m$, a constant. In this case, we get $\Phi(P)=F(P)$ and $E_\Phi\left(\tilde{P}\right)=E\left(\tilde{P}\right)$. Thus, the condition $E_\Phi\left(\tilde{P}\right)\geq C'\left(Q_*\right)$ is the same as $E\left(\tilde{P}\right)\geq C'(Q_*)$. From Equation (2.4), we know that

$$Cov\left(U'\left[\Pi_*\left(\tilde{P}\right)\right],\tilde{P}\right)+E\left\{U'\left[\Pi_*\left(\tilde{P}\right)\right]\right\}\left[E\left(\tilde{P}\right)-C'\left(Q_*\right)\right]=0.$$

In addition, because $Cov\left(U'\left[\Pi_*\left(\tilde{P}\right)\right],\tilde{P}\right)\leq 0$, we get $E\left(\tilde{P}\right)-C'(Q_*)\geq 0$. Consequently, we can assert that for any linear-regret firm, we have $Q_n^*\geq Q_*$. The result is summarized in the following proposition:

Proposition 1. *The linear-regret firm will surely produce more than its purely risk- averse counterpart; that is,* Q_n^* *is greater than* Q_*.

Let $G_n(\cdot)$ be the regret function of the linear-regret firm. For any linear-regret firm, we have $G_n'(\cdot)\equiv m>0$. In addition, from condition (2.3), we obtain:

$$H_n\left(Q_n^*\right)=Cov\left(U'\left[\Pi_n^*\left(\tilde{P}\right)\right],\tilde{P}\right)+E\left\{U'\left[\Pi_n^*\left(\tilde{P}\right)\right]+\beta m\right\}\left[E\left(\tilde{P}\right)-C'\left(Q_n^*\right)\right]=0.$$

From the above formula, we can get $E\left(\tilde{P}\right)-C'\left(Q_n^*\right)>0$. For Q^0, we get $C'\left(Q^0\right)=E\left(\tilde{P}\right)$. Thus, we can conclude that linear-regret firms produce less than firms under certainty. The result is summarized in the following proposition:

Proposition 2. *Linear-regret firms will surely produce less than firms under certainty; that is,* Q_n^* *is smaller than* Q^0.

Combining Propositions 1 and 2, we can conclude that $Q^0 > Q_n^* > Q_*$. For any general regret-averse firm, we cannot assert that regret-averse competitive firms will surely produce less than firms under certainty without imposing enough necessary condition(s). In fact, some previous studies, including Egozcue and Wong (2012), Wong (2014), Niu *et al*. (2014), Egozcue *et al*. (2015), and Broll *et al*. (2016, 2017), have presented different sufficient conditions to assure that regret-averse firms produce less than firms under certainty.

2.3. Regret-averse competitive firm

In order to study the behavior of the regret-averse competitive firm, we need to compare the covariance $Cov\left(G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right],\tilde{P}\right)$ and the positive value of

$$E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\right\}\left[E\left(\tilde{P}\right)-C'(Q_*)\right],$$

which, in turn, determine the sign of the term $H\left(Q_*\right)$ that can be used to draw conclusion on the behavior of the regret-averse competitive firm. For this purpose, we first present the following lemma to determine the sign of $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right)$, given that $\tilde{X}\in[\underline{X},X]$ and $\phi(\cdot)$ is convex:

Lemma 3. *For any convex function* $\phi\left(\tilde{X}\right)$ *with* $\tilde{X}\in[\underline{X},X]$, *we have:*

1. *if* $\phi'(\underline{X})\geq 0$, *then* $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right)>0$;
2. *if* $\phi'\left(\bar{X}\right)\leq 0$, *then* $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right)<0$;
3. *if* $\phi'(\underline{X})<0<\phi'\left(\bar{X}\right)$, *and*
   (a) *if* $E\phi\left(\tilde{X}\right)\geq\phi(\underline{X})$, *then* $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right)>0$; *or*
   (b) *f* $E\phi\left(\tilde{X}\right)\geq\phi\left(\bar{X}\right)$, *then* $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right)<0$.

Proof of Lemma 3 is in the appendix.

Determining the sign of the covariance of one random variable and its function is an important problem with many applications. However, existing studies[4] are mainly designed for completely monotone functions. However,

[4] See, for example, Egozcue, *et al*. (2009, 2010, 2011a, 2011b, 2012, 2013) and the references therein for more information.

sometimes, the function studied is not monotone in the entire support. Lemma 3 gives an easy and concrete approach to determine the sign of the covariance of one random variable and its convex function that can be used in all the discussions in our paper, and thus, it enables us to develop properties for the regret-averse competitive firm.

Now, we turn back to study the regret-averse firm. We define $\phi(\tilde{P}) = G'\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right]$. Its derivatives ϕ' and ϕ'' can be shown to satisfy

$$\phi'(\tilde{P}) = G''\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right]\left(Q(\tilde{P}) - Q_*\right),$$

$$\phi''(\tilde{P}) = G'''\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right]\left(Q(\tilde{P}) - Q_*\right)^2 + G''\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right]Q'(\tilde{P}).$$

One could easily show that $Q'(\tilde{P}) = 1/C''\left(Q(\tilde{P})\right) > 0$ and $G''(\cdot) > 0$. If, in addition, we further assume that $G'''(\cdot) \geq 0$ [5], we can conclude that $\phi''(\cdot) > 0$, and thus, $\phi(\cdot)$ is a convex function. As a result, applying Lemma 3, we obtain the sign of the covariance $Cov\left(\phi(\tilde{P}), \tilde{P}\right)$. Together with the results that $\underline{P} < C'(Q_*) < \bar{P}$, $C'[Q(P)] = P$ and $C''(\cdot) > 0$, we have $Q(\underline{P}) < Q_* < Q(\bar{P})$ and $\phi'(\underline{P}) < 0 < \phi'(P)$. From the above discussion, we have the following result:

Proposition 4. *Assume that* $G'''(\cdot) \geq 0$. *Then, the regret-averse firm will produce more than its purely risk-averse counterpart if* $EG'\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right] \geq G'\left[\Pi^{max}(\underline{P}) - \Pi_*(\underline{P})\right]$.

For any linear-regret firm, the condition $EG'\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right] \geq G'\left[\Pi^{max}(\underline{P}) - \Pi_*(\underline{P})\right]$ holds automatically. As a result, Proposition 1 is a special case of Proposition 4. Thus, Proposition 4 is a proposition with more general results.

We discuss the intuition of Proposition 4 as follows: $EG'\left[\Pi^{max}(\tilde{P}) - \Pi_*(\tilde{P})\right] \geq G'\left[\Pi^{max}(\underline{P}) - \Pi_*(\underline{P})\right]$, it follows from Lemma 3 that $G'\left[\Pi^{max}(\tilde{P})\right]$ is positively correlated with $\tilde{P}$. Thus, the behavior of regret-aversion makes the firms concern more on the disutility from the discrepancy of its output level, $Q(\tilde{P}) - Q_*$, when $\tilde{P}$ gets higher realizations. This leads us to conclude that the regret-averse firm optimally adjusts its output level upward from Q_* to Q^* with $Q_* > Q^*$ to minimize regret.

We note that the above condition is sufficient , but it is not necessary. Thus, this leads to obtain the following property:

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5 This assumption has been made by many authors, see, for instance, Egozcue and Wong (2012), Wong (2014), Niu *et al.* (2014) and Broll *et al.* (2016, 2017).

Property 5. *As long as*

$$Cov\left(G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right],\tilde{P}\right)\geq -E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\right\}\left[E\left(\tilde{P}\right)-C'\left(Q_*\right)\right],$$

we have $Q_* > Q^*$. This implies that negative correlation between $G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]$ and $\tilde{P}$ is allowed, as long as they are not too negatively correlated, Q^* could still be bigger than Q_*. Thus, we conclude that whenever $E_\Phi\left(\tilde{P}\right)\geq C'\left(Q_*\right)$, even though $Cov\left(G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right],\tilde{P}\right)$ is negative but not too negative, the regret averse firm would produce more than purely risk-averse firm.

Broll *et al*. (2017) present sufficient conditions under which, the regret-averse firms produce more than firms without uncertainty. Recall that Q^0 is the optimal output level under certainty. Since $E\left(\tilde{P}\right)>C'\left(Q_*\right)$ and $C''(\cdot)\geq 0$, we have $Q^0 > Q_*$. Then, under conditions designed by Broll *et al.* (2017), we can also get $Q^* > Q^0 > Q_*$. However, to get $Q^* > Q^0$, Broll *et al*. (2017) not only require $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi^0\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}\left(\underline{P}\right)-\Pi^0\left(\underline{P}\right)\right]$, but also require the regret coefficient $\beta \geq \beta_0 > 0$. Here, $\Pi^0\left(\tilde{P}\right)=\tilde{P}Q^0-C\left(Q^0\right)$ and β_0 is a specified value. In this paper, we directly compare Q^* and Q_*, and thus, we allow the regret coefficient β to be any positive value; that is, under condition $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}\left(\underline{P}\right)-\Pi_*\left(\underline{P}\right)\right]$, any regret-averse firm will produce more than its purely risk-averse counterpart, even its regret coefficient β is very small.

In the above, we focus on the comparison of the optimal output levels of firms with $\beta > 0$ (regret-averse firms) and $\beta = 0$ (purely risk-averse firms). In the following, we turn to compare the optimal output levels of two firms with two different positive β's. To this end, we obtain the comparative statics of the optimal output levels when the regret coefficient β varies as shown in the following proposition:

Proposition 6. *Assume that $G'''(\cdot)\geq 0$. Then, the regret-averse firm's optimal output level, Q^* will surely increase with an increase in the regret coefficient β, if $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}\left(\underline{P}\right)-\Pi_*\left(\underline{P}\right)\right]..$*

Proof of Proposition 6 is in the appendix. One could interpret the regret coefficient β as "if the firm's regret coefficient β is larger, then the firm is more regret-averse". Thus, from Proposition 6, we get the following property:

Property 7. *Assume that $G'''(\cdot)\geq 0$. Then, the firm will produce more if it is more regret-averse, and if $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}\left(\underline{P}\right)-\Pi_*\left(\underline{P}\right)\right]$.*

3. A binary model

To get more insights, in this section we analyze a simple binary model in which $\tilde{P}$ takes on the low value, $\underline{P}$, with probability q and the high value, $\overline{P}$, with probability $1 - q$ with $0 < q < 1$. In the binary model, the right-hand sides of Equations (2.3) and (2.4) become

$$\begin{aligned} &q\left\{U'\left[\Pi^*(\underline{P})\right]+\beta G'\left[\Pi^{max}(\underline{P})-\Pi^*(\underline{P})\right]\right\}\left[\underline{P}-C'(Q^*)\right] \\ &+(1-q)\left\{U'\left[\Pi^*\left(\overline{P}\right)\right]+\beta G'\left[\Pi^{max}\left(\overline{P}\right)-\Pi^*\left(\overline{P}\right)\right]\right\}\left[\overline{P}-C'(Q^*)\right]=0 \end{aligned} \tag{3.6}$$

and

$$qU'\left[\Pi_*(\underline{P})\right]\left[\underline{P}-C'\left(Q_*\right)\right]+(1-q)U'\left[\Pi_*\left(\overline{P}\right)\right]\left[\overline{P}-C'\left(Q_*\right)\right]-0. \tag{3.7}$$

To develop some properties for the binary model, we now define the following threshold value:

$$q^+ = \left\{1-\frac{U'\left[\underline{P}Q^+-C\left(Q^+\right)\right]\{\underline{P}-C'\left(Q^+\right)}{U'\left[\overline{P}Q^+-C\left(Q^+\right)\right]\{\overline{P}-C'\left(Q^+\right)}\right\}^{-1} \tag{3.8}$$

where Q^+ is the quantity of output that solves

$$\Pi^{max}\left(\overline{P}\right)-\left[\overline{P}Q^+-C\left(Q^+\right)\right] = \Pi^{max}(\underline{P})-\left[\underline{P}Q^+-C\left(Q^+\right)\right].$$

Comparing Q^* and Q_* yields the following proposition.

Proposition 8. *Assume that* $G'''(\cdot)\geq 0$. *Then, in the binary model, the regret-averse firm produces more than the purely risk-averse firm; that is,* $Q^* > Q_*$, *if the probability that* $\tilde{P}=\underline{P}$ *is above the critical value,* q^+ *which is defined in (3.8).*

Proof of Proposition 8 is in the appendix.

We now provide the intuition for Proposition 8. To do so, we first consider a purely risk-averse firm. In this situation, when the price is more likely to get a lower value of the realization; that is, the probability that $\tilde{P}=\underline{P}$ is above the critical value q^+, the firm will optimally produce less so as to minimize the variability of its profit at date 1. Thus, the optimal output level, Q_*, is further away from $Q\left(\overline{P}\right)$ and closer to $Q(\underline{P})$, where $Q(\underline{P})$ and $Q\left(\overline{P}\right)$ are the ex-post optimal when $\tilde{P}=\underline{P}$ and $\overline{P}$, respectively. We now consider a purely regret-averse firm. Taking into account the disutility from the discrepancy of the output level $Q\left(\overline{P}\right)-Q_*$, regret-aversion in the objective function makes the regret-averse firm take care the situation when the actual price is high; that is, $\tilde{P} = \overline{P}$. Thus, the firm tends to produce more so that $Q^* > Q_*$ to avoid being regret.

From Proposition 6 and the proof of Proposition 8, we can easily get the following result:

Proposition 9. *In the binary model, if* $G'''(\cdot) \geq 0$, *the regret-averse firm will produce more if the firm is more regret-averse; that is,* $dQ^* / d\beta > 0$, *if the probability that* $\tilde{P} = \underline{P}$ *is above the critical value* q^+ *which is defined in (3.8).*

4. Conclusion

Egozcue and Wong (2012), Wong (2014), Niu *et al.* (2014), Egozcue *et al.* (2015), and Broll *et al.* (2016, 2017), have investigated several sufficient conditions under which the regret-averse firms will produce less than firms under certainty. In this paper, we compare the optimal output levels among linear-regret, regret-averse, and purely risk-averse firms under uncertainty and firms under certainty. We first show that different from the findings in the literature demonstrating that the regret-averse firms produce less than firms under certainty only under some sufficient conditions, the linear-regret competitive firm will surely produce less than firms under certainty without imposing any condition. We also show that the linear regret competitive firm will surely produce more than its purely risk-averse counterpart.

Thereafter, we show how to determine the sign of the covariance between a random variable and its convex function in a general case. This property enables us to derive the properties for the regret-averse firms. We show that under some conditions, more regret-averse firms will produce more outputs and the regret-averse competitive firms will produce more than purely risk-averse counterparts.

Last, we set the uncertain output price to take either low or high values with positive probability. Under this simple binary model, we find the possibility that the regret-averse firm may optimally produce more than the purely risk-averse firm, especially when the low output price is very likely to prevail. This weakens the conditions set in the general case for the regret-averse competitive firm to produce more than its purely risk-averse counterpart or when the firm is more regret-averse.

Together with the existing findings in the literature, the findings in this paper draw a clear picture on the optimal production decisions for the regret-averse firms. Our findings could be useful for production managers in their decisions on the production.

Appendix

Proof of Lemma 3

Proof: If $\phi'(\underline{X}) \geq 0$, then $\phi'(X) > \phi'(\underline{X}) \geq 0$. In this situation, the minimum is obtained at $\underline{X}$ and $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right) > 0$, and thus, Part 1 is proved.

Similarly, if $\phi'\left(\bar{X}\right) \leq 0$, then $\phi'(X) < \phi'\left(\bar{X}\right) \leq 0$. In this case, the minimum is obtained at $\bar{X}$, and thus, we have $Cov\left(\phi\left(\tilde{X}\right),\tilde{X})\right) < 0$. Part 2 is proved.

If $\phi'(\underline{X}) < 0 < \phi'\left(\bar{X}\right)$, then there exists a unique X_0 such that $\phi'\left(X_0\right) = 0$. Consequently, for $X < X_0$, we have $\phi'(X) < \phi'(X_0) = 0$, while for $X > X_0$, we have $\phi'(X) > \phi'\left(X_0\right) = 0$. Hence, $\phi(X)$ is decreasing (increasing) in X for all $X < (>) X_0$, and reaches a unique minimum at X_0. Now, when we assume $E\phi\left(\tilde{X}\right) \geq \phi(\underline{X})$, then there must exist a unique value $X_1 \in \left(X_0, \bar{X}\right)$, such that $E\phi\left(\tilde{X}\right) = \phi\left(\underline{X}_1\right)$. As a result, for any $X < X_1$, we have $\phi(X) < \phi\left(X_1\right)$ and if $X > X_1$, we have $\phi(X) > \phi\left(X_1\right)$. Thus, we obtain $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right) = E\left[\left(\phi\left(\tilde{X}\right) - E\phi\left(\tilde{X}\right)\right)\left(\tilde{X} - X_1\right)\right] > 0$.

On the other hand, when we assume that $E\phi\left(\tilde{X}\right) \geq \phi\left(\bar{X}\right)$, then there must exist a unique value $X_2 \in \left(\underline{X}, X_0\right)$ such that $E\phi\left(\tilde{X}\right) = \phi\left(X_2\right)$. As a result, for any $X < X_2$, we have $\phi(X) > \phi\left(X_2\right)$ and if $X > X_2$, we have $\phi(X) < \phi\left(X_2\right)$. Thus, we get $Cov\left(\phi\left(\tilde{X}\right),\tilde{X}\right) = E\left[\left(\phi\left(\tilde{X}\right) - E\phi\left(\tilde{X}\right)\right)\left(\tilde{X} - X_2\right)\right] < 0$, and all the assertions in Lemma 3 are obtained.

Proof of Proposition 6

Proof: From the first-order condition in (2.3), we get

$$H\left(Q^*,\beta\right) = E\left\{\left\{U'\left[\Pi^*\left(\tilde{P}\right)\right] + \beta G'\left[\Pi^{max}\left(\tilde{P}\right) - \Pi^*\left(\tilde{P}\right)\right]\right\}\left[\tilde{P} - C'(Q^*)\right]\right\} = 0,$$

when $Q = Q^*$. Applying the implicit function theorem and the second-order condition, we obtain

$$sign\left(\frac{dQ^*}{d\beta}\right) = sign\left(\frac{\partial H}{\partial \beta}\right) = sign\left(E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right) - \Pi^*\left(\tilde{P}\right)\right]\left[\tilde{P} - C'(Q^*)\right]\right\}\right).$$

On the other hand, from the first-order condition in (2.3), we know that

$$E\left\{G'\left[\Pi^{max}\left(\tilde{P}\right) - \Pi^*\left(\tilde{P}\right)\right]\left[\tilde{P} - C'(Q^*)\right]\right\} = -\frac{1}{\beta}E\left\{U'\left[\Pi^*\left(\tilde{P}\right)\right]\left[\tilde{P} - C'(Q^*)\right]\right\}.$$

From Proposition 4, we know that when $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right] \geq G'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*(\underline{P})\right]$, we have $Q^*>Q_*$, and thus, we have $E\left\{U'\left[\Pi^*\left(\tilde{P}\right)\right]\left[\tilde{P}-C'(Q^*)\right]\right\}< E\left\{U'\left[\Pi_*\left(\tilde{P}\right)\right]\left[\tilde{P}-C'(Q_*)\right]\right\}=0$ and

$$sign\left(\frac{dQ^*}{d\beta}\right)=sign\left(-\frac{1}{\beta}E\left\{U'\left[\Pi^*\left(\tilde{P}\right)\right]\left[\tilde{P}-C'(Q^*)\right]\right\}\right)>0.$$

As discussed above, when $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}(\underline{P})-\Pi_*(\underline{P})\right]$, the regret-averse firm optimally adjusts its output level upward from Q_*. In this situation, increasing the regret coefficient β would surely intensify this effect. As a result, with an increase in β, Q^* will also increase.

Proof of Proposition 8

Proof: Applying the implicit function theorem, Equation (3.7), and the second-order condition, we obtain

$$\begin{aligned}sign\left(\frac{dQ_*}{dQ}\right)&=sign\left(U'\left[\Pi_*(\underline{P})\right]\left[\underline{P}-C'(Q_*)\right]-U'\left[\Pi_*\left(\bar{P}\right)\right]\left[\bar{P}-C'(Q_*)\right]\right)\\&=-\frac{1}{q}sign\left(U'\left[\Pi_*\left(\bar{P}\right)\right]\left[\bar{P}-C'(Q_*)\right]\right).\end{aligned}$$

Since $\bar{P}>E\left(\tilde{P}\right)>C'(Q_*)$, we get

$$\frac{dQ_*}{dq}<0.$$

From Proposition 4, we know that when $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}(\underline{P})-\Pi_*(\underline{P})\right]$, we have $Q^*>Q_*$. Further, we note that in the binary case, the condition $EG'\left[\Pi^{max}\left(\tilde{P}\right)-\Pi_*\left(\tilde{P}\right)\right]\geq G'\left[\Pi^{max}(\underline{P})-\Pi_*(\underline{P})\right]$ is equivalent to

$$(1-q)\left(G'\left[\Pi^{max}\left(\bar{P}\right)-\Pi_*\left(\bar{P}\right)\right]-G'\left[\Pi^{max}(\underline{P})-\Pi_*\left(\bar{P}\right)\right]\right)\geq 0$$

Since $G''>0$, this is further equivalent to

$$H(q)=:\left[\Pi^{max}\left(\bar{P}\right)-\Pi_*\left(\bar{P}\right)\right]-\left[\Pi^{max}(\underline{P})-\Pi_*(\underline{P})\right]\geq 0$$

Recall that $\Pi_*\left(\tilde{P}\right)=\tilde{P}Q_*-C\left(Q_*\right),$ we have:

$$H'(q)=:\left(\underline{P}-\overline{P}\right)\frac{dQ_*}{dq}>0.$$

Solving $H\left(p\right)=0$ yields Q^+ while fixing $Q=Q^+$ in Equation (3.7) yields q^+. Then, for all $q>q^+$, we have $H\left(q\right)>0,$ and thus, for all $q>q^+$, we have $Q^*>Q_*,$ and thus, the assertions of Proposition 8 hold.

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