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Mathematics History and Cognitive Values on a Didactic Sequence: Teaching Trigonometry

Helenara Regina Sampaio Figueiredo[1], and Irinéa de Lourdes Batista[2]

1) Universidade Norte do Paraná. Brazil
2) Universidade Estadual de Londrina. Brazil

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Helenara R. Sampaio Figueiredo
Universidade Norte do Paraná

Irinéa de Lourdes Batista
Universidade Estadual de Londrina

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Abstract

This article summarizes the preparation and the results of a study, which aimed to evaluate the effectiveness of the historical-philosophical approach in teaching trigonometry in association with the detection of cognitive values, as presented by Lacey (1998). On this association lies the theoretical innovation of the work. To conduct the survey, we organized the activities in sequence as proposed by the Didactic Engineering, based on the history of trigonometry. The study was carried out with high school students of a public school in Londrina – PR, Brazil and it had a qualitative nature. We have concluded that the approach based on historical perspective with the observation of cognitive values expressed by the mathematics showed being useful for the learning of the Trigonometry and its incorporation into the student's knowledge.

Keywords: history and philosophy of mathematics, trigonometry, cognitive values.

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Historia de las Matemáticas y Valores Cognitivos sobre las Secuencias Didácticas: Enseñando Trigonometría

Helenara R. Sampaio Figueiredo
Universidade Norte do Paraná

Irinéa de Lourdes Batista
Universidade Estadual de Londrina

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Resumen

En artículo resume la presparación y los resultados de un estudio que tuvo como objetivo evaluar la efectividad del enfoque histórico-filosófico en la enseñanza de la trigonometría, junto con la detección de valores cognitivos, de acuerdo a Lacey (1998). En esta combinación reside la innovación teórica de este trabajo. Para realizar la encuestra, se realizó una secuencia de actividades de acuerdo a los supuestos del enfoque de la Ingeniería Didáctica, basándonos en la historia de al trigonometría. El estudio se realizó con estudiantes de secundaria de una escuela pública situada en Londrina, PR, Brazil, y tuvo un carácter cualitativo. Concluimos que el enfoque basado en una perspectiva histórica con la observación de valores cognitivos expresados por las matemáticas demostró ser efectivo para el aprendizaje de la trigonometría y su incorporación al conocimiento del estudiante.

Palabras clave: historia y filosofía de las matemáticas, trigonometría, valores cognitivos.

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As we began the research about the math teaching, we found articles and essays that addressed the topic trigonometry, for example, an article of Brito and Morey (2001, p. 65-70), which was a survey with teachers from Elementary school about the difficulties they encountered on the teaching of geometry and trigonometry concepts and how the teaching of these concepts has been proposed in textbooks over the last four decades of the twentieth century.

This research states that there has been a didactic transposition of low quality on the trigonometry teaching at high school, sometimes caused by too simplified content, which affects the student's learning. At the end of the article, the authors argue that the difficulties of the investigated teachers were closely related to school education of the 70s and 80s, a time characterized as disregard for the trigonometry.

The results quoted in the article have led us to reflect on the trigonometry learning as a whole and to consider it on a historical and philosophical view inserted in our school context. This approach allows us to view not only the content as something ready, systematized, but studying it as something built since ancient times, passing centuries to reach the systematization present in current textbooks.

The decision of taking a teaching methodology that considers the fundamental historical aspects to the understanding of the current status of the development of the scientific knowledge may contribute to overcoming difficulties, identified and understood, helping on the teaching and learning process. The history of mathematics, however, should not be interpreted as a simple methodological tool, but as a pedagogical approach to be adopted in the classroom.

Math educators like Sad (2004), Vianna (1995), Mendes (1997), Luccas (2004), Miguel (1993) and Nobre and Baroni (1999) have shown in their research the history of mathematics as a tool, which promotes attitudes and values, that can increase the motivation for learning, promoting a problem-action and contributing to their understanding of concepts and methods.

The historical reconstructions of mathematical content allow recognizing the cultural wealth of the people related to them, expanding its teaching value. Researches such as of Luccas (2004), for example, point to the importance of such studies for changes, adaptations and joint with other content as said D'Ambrosio (1999):

> Educational practices are based on culture, on learning styles and traditions and the history consists of the registration of these cornerstones. Therefore, it is almost impossible to discuss education without resorting to these records and interpretations thereof. This is also true as teaching these various subjects, especially mathematics, whose roots are confused with the history of humankind.

As history is full of “teachings," we have been able to find a reliable and fundamental foundation to develop our research. The theoretical innovation of the study focuses on the use of cognitive values[1] (Lacey, 1998), recognized by the historical and epistemological reconstruction of the Trigonometry, and as a teaching tool to Didactic Engineering (ED). These values are, in turn, as instruments of mathematical comprehension. We highlight that the teaching sequence built for this project is intertwined with the objectives of this research, but it can go through reviews, ratings, can be re-signified, and reproduced under new perspectives and contextualization, so to be applied to the teaching practice.

The difficulties of the 23 students at the high school from a public school in the city of Londrina - PR were identified based on preliminary analysis to the construction and application of this didactic sequence to be used in the classroom. In this context, we describe briefly the development of the research, the conceptual/theoretical framework used, the activities applied and their results, relating mathematics teaching to the location as well as to the understanding of the following cognitive values: *empirical adequacy, consistency, fertility, generalization, explanatory power, simplicity*.

Conceptual Framework of Research

Cognitive Values on the History Mathematics

Mathematics is a consolidated area of expertise, with internal consistency. In the past, trigonometry, as a mathematical knowledge area, solved problems of physics and other sciences, enabling the mathematization of nature, as can be observed in the periodic phenomena and astronomy studies, expressing their cognitive values today, likewise.

We have attempted to place our discussion in the philosophy of science, from Hugh Lacey (1998) as a theoretical framework, since he discusses the scientific rationality regarding cognitive and social values, which is quite interesting from a pedagogical point of view. By exposing his work, *Values and Scientific Activity*, he refers to the epistemological process of selecting theories concerning commitment to a set of cognitive values.

According to Lacey (1998, p. 61), the cognitive values are constitutive of the Science; they are criteria to be met by a good scientific theory, emphasizing that such values are different from moral or social values.

> Scientists make judgments all the time. Evaluate their theories. They ask: Does the evidence confirm this theory? Does it enable new predictions and explanations of essential phenomena? Is it consistent with other theories already accepted? Are all of them questions about the acceptability of a theory on its cognitive value (epistemic, rational)? Are the evaluation criteria suggested by the questions - empirical adequacy, predictive power and explanatory, etc. – Are they called “cognitive values”? (Lacey 1998 apud Lacey, 2005, p. 42)

It presents a list of values that have played a role in choosing theories, at least in some moments of science, drawn from several references, being still completed with other cognitive values. For its preparation, Lacey (1998) states that the first task:

> It is interpretative and consists on the rational reconstruction of key-episodes theories choices and of theoretical controversies to ascertain the criteria that can be singled out and compared with the critical reflections of active scientists (Lacey, 1998, p. 66).

We have used the list referred to in the book of Lacey (1998, p. 61-66) in our research, which, according to the author, we reiterate, is not closed, once the criteria for a good theory and its interpretation may vary.

- Empirical adequacy: being empirically testable, having correspondence with reality, having primacy of experimental and quantitative data, accuracy, data accuracy.
- Consistency: having depth, theoretical coherence, explaining phenomena that matter to itself and other fields of knowledge.
- Fecundity: giving new questions, having predictive ability, being able to trigger new research programs and leading to the discovery of new phenomena.

- Generalization: having the property of extending to other data sets.
- Explanatory power: providing explanations for the phenomena in a broad range of areas, enabling the construction of a narrative that offers answers to various aspects.
- Simplicity: being in harmony, having elegance, conceptual clarity, having efficiency in use, likewise.

The discussion regarding the cognitive values in our study is based on a historical reconstruction of trigonometry. These values recognized in the mathematical activity developed by humanity are intertwined and cannot be delimited from each other with extreme accuracy. Although, we will attempt to show the context in which each of these values can be observed in our historical study and later on the activities prepared for the teaching sequence.

Thus, we consider the cognitive value empirical adequacy when mathematicians used trigonometry to solve real problems, increasingly seeking data accuracy, on the observations and experiments, as well as on the calculations of distances.

The cognitive value consistency is analyzed when trigonometry showed its unity, its absence of contradiction with other theories, such as the study of pendulums in physics, and it is developed within the mathematics itself on the study of logarithms. The activities of astronomy, for instance, made it possible to demonstrate the consistency of trigonometric concepts and assured us that it is feasible to create a “system”, which branches and evolves temporally observing different perspectives to problem-solving.

Discoveries of phenomena and capacity of foresight showed the cognitive value fecundity. Historically, for example, we can see how much has been fruitful the knowledge concerning the vertical rods (gnomon) and studies of shadows as well, allowing that the mathematicians came to the systematization of the graphical representation of the trigonometric ratios, sine, cosine, and tangent. On the eighth century, the mathematical works of the Indians provided a revolution in the history of trigonometry.

Our subject has ceased having geometric character and now has greater algebraic development, becoming generalizable. Generalization is a critical cognitive value of trigonometry due to the importance of this discipline to other areas of knowledge. The invention of infinitesimal calculus foreshadowed the end of independent trigonometry and under development, for example. With the discovery and exploitation of the complex domain, all this theory has included in the analysis of mathematics, making the most consistent trigonometry in the theoretical corpus of mathematics and other disciplines.

The mathematician Leonhard Euler (1707-1783), at the end of the eighteenth century, presented the theorems of trigonometry as corollaries of the theory of complex functions (Kennedy, 1997, p. 1-3). We understand, before Kennedy's comment that flat trigonometry was much explored late in the eighteenth century because of a higher application, showing cognitive value explanatory power, while the spherical trigonometry was not as emphasized by mathematicians at that time.

Another development fostered by the trigonometry is the adoption of unit radius to the circumference, which has eased the astronomical calculations and the resolution of the trigonometric formulas. This step reflects the importance of its simplicity, because of its elegance, economy, and conceptual clarity, ability to be formalized and its efficient use. When algebra was incorporated into trigonometry, mathematicians had in their hands an instrument that explained the calculations easily before complex, achieving greater depth in problem-solving.

Didactic Engineering and Didactic Sequence

The Didactic Engineering (DE), a term used at the researches of the didactics of mathematics, which involves an experimental part, since early 1980s, is:

> seen as a research methodology, and is characterized by an experimental scheme based on educational achievements in the classroom, i.e., on the concept, realization, on the observation and on the analysis of teaching sequences. (Artigue, 1996, p. 196)

A Didactic Engineering is characterized as:

> a form of didactic work compared with the work of the engineer that, to carry out a project needed, supported by the scientific knowledge of its domain, it agrees to be submitted to a scientific control. (Artigue, 1996, p. 193)

Almouloud and Silva add:

> but at the same time, sees himself forced to work with much more complex objects than with the debugged objects of science and,

> thus, facing it, with all means available, problems that science does not want or cannot take into account. (Almounloud & Silva, 2012, p. 26)

According to Pais (2002, p. 108), the justification for the choice of using an DE is due to the fact that traditional instruments such as questionnaires, interviews, document analysis, are all insufficient to cover the complexity of the didactic phenomenon, reinforcing the need of linking to theory and reality of the classroom. One of the advantages of Didactic Engineering, according to Pais (2002, p. 99) “stems from this double anchorage, connecting the theoretical plan rationality experimental territory of educational practice.”

Another critical point highlighted is that the research expands its sense when guided by a philosophical concept (in our case, historical-philosophical) basing activities, ensuring greater meaning of didactic research and the viability of the relationship between teacher, student, and knowledge.

We can customize the term engineering for both the researcher and the teacher, being his work to choose or organize sequences of activities, which explore the domain of knowledge. These didactic sequences appear, likewise, as one of the primary objects of the Didactic Engineering.

The teacher's work, while developing or choosing a didactic sequence[1], should consider in an integrated manner: the domain of knowledge, previous knowledge of the student, the role of the teacher and his students. For this purpose, in each sequence, it is necessary a definition of the meaning of learning. The creation of a didactic sequence consists of an interactive process, in which the objective is the elaboration of a set of decisions so that processes are meaningful, and strategies are more effective. Considering the answers of these students and the conditions to which they are subjected.

Thus, the process involves: an analysis of the proposed situation, the organization's conditions, and the choice of strategies based on the analysis of the given instruction, of the determination of evaluation criteria, the preparation of issues that are in accordance with specific criteria and a review the whole process in the light of this evaluation.

According to the importance of teaching achievement, the Didactic Engineering distinguishes itself in two levels: the micro and the macro-engineering. The micro engineering researches are those, which have as a goal the study of a particular subject; they are located and consider the complexity of the class phenomenon. The macro-engineering researches are those, which allow expressing the complexity of micro-engineering researches as phenomena related to duration in the teaching-learning relationship.

The DE is characterized by the record of the studies done in question and its way of validating the results. This validation of the research is done primarily internally because it is based on the confrontation between a priori analysis, which is based on the theoretical framework and the later analysis, the validation is internal, whereas the one that is based on statistical methods is external, i.e., using comparative methods to validate its results.

The design of a project that follows the principles of engineering goes through well-defined stages and the time frame of its experimental process. According to Artigue (1996, p.196), we can distinguish four phases in the process of Didactic Engineering Methodology:

Phase 1: preliminary analysis: made through considerations of the general teaching theoretical framework and on the subject in question. It must consider, likewise, the epistemological analysis of the contents covered by the teaching; the analysis of the concept of the students, the difficulties that determine their evolution; of the field of barriers in which will be located the effective teaching achievement, and finally, state the specific objectives of the research.

Artigue (1996, p. 198), says that although the preliminary analysis match of bases for the design engineering, they may need to be resumed to face the difficulties during the work carried out by the investigator in the research.

Phase 2: a priori analysis of the situations proposed in the didactic sequence: the researcher, guided by preliminary analysis, defines some relevant variables regarding the system on which the school can act, and these are called variables of control. It is supposed to be relevant variables for the problem studied. Artigue (1996, p 202) distinguishes two types of variables of control: macro didactic (or global) and micro didactic (or local).

The a priori analysis includes a descriptive portion and another forecast, and it is an analysis focused on the characteristics of an a-didactic situation, which it was intended to be created and developed during the experiment.

As per Brousseau (1996), “when the student can apply some knowledge by himself to situations outside of the school context and in the absence of any intentional indication, such a situation is called “a-didactic.” In a priori analysis it is necessary:

- Describing each choice made and the characteristics of a-didactic situation arising from each option;
- Analyzing the weight of the cases for the student emerging from the possibilities of action, choice, decision, control, and validation that he will have during the trial, once operated the return;
- Predicting the possible behaviors of fields and showing in what the performed analysis allows to control the direction of these behaviors;

Furthermore, it must be assumed that, if such behaviors occur, they will result, clearly, from the knowledge application desired by the learning.

Phase 3: experimentation: It relies on the data set collected in the realization of the engineering, starting at the moment in which the researcher and the students, subject of the research come into contact. The data collected result from the following means: the recording of observations made during the trial, at several different moments of teaching; the external production student or class; the application of research instruments, through questionnaires and tests, individual or small groups.

Phase 4: A posteriori analysis and evaluation: At this stage, it is carried out the data processing that consists of the selection of relevant data relevant to a posteriori analysis. It is the confrontation of the two analyzes a priori and posteriori, which is based on the validation of the hypotheses in the research.

To Pais (2002, p. 98) it is possible, with the use of a Didactic Engineering, to enhance the reliability of the research, in particular for being connected to the reality of the classroom. In our study, it has been used a questionnaire as instrument of evaluation, to analyze the results of the application of the sequence, which follows the Propositional Model of Concepts (PMC), based on the benchmark of Meaningful Learning[2] , which we used to compare the answers of the students before the implementation of the didactic sequence and at the end of the activities in the classroom, when the same questions were asked.

Propositional Analysis of Concepts (PAC)

Artigue (1996, p. 209) states that in most of the researches relating to the Didactic Engineering, the authors do not get engaged in an accurate validation process, since the confrontation of the two analyzes, a priori and posteriori, displays distortions. The response to the hypotheses raised at work does not guarantee that, in a long-term, in fact, the learning becomes validated. Furthermore, Pais (2002, p. 103) points out that validation is one of the classic problems of Didactic Engineering.

We have sought, then, on the Meaningful Learning Theory the method of analysis called "Concepts of propositional analysis" (CPA), to validate the results of our research. This technique "is based on the psychological notion that the meaning that such a given concept has for a student is manifested through a set of pre-propositions incorporating the concept that the student elaborates" (Novak & Gowin, 1999, p. 156).

After applied the questionnaire produced by the researcher according to the data he intends to collect, it is performed an analysis of the answers aiming to identify the pre-propositional statements formulated by the student. Our 1st questionnaire (PCAS 1) was comprised of 13 questions, not all of them present in the PCAS 2. Only the questions involving verification of the construction of knowledge, i.e., the changes caused by the didactic sequence, have been used in the two PCASs. Concerning the purpose of each of them, some have been implemented in the PCAS 1 (before the participation in the didactic sequence, in our case) and others PCAS 2.

In PCASs 1 and 2, the following questions have been implemented/analyzed: "Do you consider mathematics important?" (q.1); (q.5) "what kind of problem does the trigonometry calculations serve to solve to?" (q.6) "What reasons do you think led the mathematicians of the past, to study trigonometry?" and (q.7) "Do you think it's important to study the history of mathematics to learn it better?"

Question 3 is also present in the two PCASs, and it was made up of several alternatives, in which the cognitive values of mathematics were implicit. At the table below, we present the assumptions used in the research and the corresponding values:

Table 1.
Implicit cognitive values

When pointed out the following alternatives	Implicit cognitive values
It contributes to solving problems of both in the mathematics, as well as in other disciplines.	Consistency, fecundity.
It is applied to mathematics to solve real daily problems.	Empirical adequacy, explanatory power.
It attempts increasingly accurate results.	Certainty, accuracy.
Mathematics hardly makes sense to produce new discoveries.	Mathematics does not have fertility value.
Mathematical formulas make turn the calculations simpler.	Generalization, simplicity.
The contents studied today may be useful in the future.	Fertility.
Everything would be more accessible if there were no mathematics.	No cognitive value.
Even using mathematical calculations, the inaccuracy is constant.	No cognitive value accuracy.
The graphical representation helps students better understand mathematical concepts.	Generalization from the graphical representation.

Some other questions have been applied only in the PCAS 1, once they should recognize the students ' prior knowledge: (Question 4) “The subject trigonometry is part of the 8th-grade content. Have you studied trigonometry in this grade? What have been the main topics studied?”, (q.5) “When you think of trigonometry what terms have to do with this content: Sine cube angles, peccaries, adjacent complex numbers, perimeter, reason, Pythagorean theorem, degrees, power, divider?”

After acknowledging the propositions of the anterior and posterior students to the experiment, it is drawn up a table that shows, on one side (left), the proposals given by students before the instruction, and on the other (right), the propositions answered by students to the same questions after the instruction. In the middle of the table, there is located the main proposals presented in the instruction, that is, the questions or propositions created by the teacher or researcher for the interview.

It is worth noting that the model of the PCAS was presented at Research Group, in which we studied and discussed the book of Lacey (1998): “The Values and Scientific Activity”, and it made possible the validation of the questions of the questionnaire by the participants of the group, who are professors of several disciplines, such as physics, chemistry, geography, biology, and especially mathematics teachers. The questions, corrections, at last, considerations of colleagues of the research group contributed to the formulation of the questions of the data collection questionnaire. Our colleagues of mathematics validated the questions regarding the Elementary School Trigonometry content.

In the Classroom: A Didactic Sequence

For planning the didactical sequence, thus, we followed the steps set by the Didactic Engineering and to carry out the first of these steps (preliminary analyzes or previous), it has been carried out a literature review of researches on trigonometry. Textbooks of High School have also been required to observe how the topics of the trigonometry have been approached and structured.

Still, at this stage, another procedure has been used, the interview with three Mathematics teachers, who taught the content of trigonometry at High School at the Paranaense state system of education for over ten years. The questions were regarding the main difficulties that teachers watched related to the student learning.

The respondents reported that:

- Students cannot visualize the graphs of functions;
- Students do not have good geometric and algebraic performance;
- The fact that the students think that trigonometry at the rectangle triangle is the same as the trigonometric cycle hampers the learning of trigonometric functions;
- The students do not understand the study of the period, as well as they do not master the functions;
- Another difficulty is how to turn degrees into radians.

The lack of contextualization of trigonometry content is another factor highlighted as a problem for the students. We emphasize that the structure of trigonometric concepts currently taught in textbooks lacks historical information, which makes it difficult to contextualize for the student learning.

The assumptions built with this data –concerning to learning of the trigonometric functions– were the following:

- Students do not understand that the arches have linear and angular measurements;
- They do not understand the meaning of the study of trigonometry study and its graphical representation of trigonometric cycle;
- They do not realize the relationship between the content of trigonometry studied in 8th grade and the 2nd year of high school;
- They cannot apply the formulas of trigonometric functions when they face problem-solving tasks.

Moving forward with the 2nd phase, we analyzed a priori, in which we have tried to predict the possible difficulties of students learning and their performance in these activities. We have used a questionnaire (described in the previous section herein) to survey the students' prior knowledge about the trigonometry content in general and the triangle-rectangle, content taught in the eighth grade. We observed that most of them did not know how to solve such problems, which indicated the need to resume this content of the 8th grade through activities.

After the experimentation, to provide a subsidy to the validation, as well as to the completion of research, we have carried out a posteriori analysis, 4th phase of DE, from the performance of the students, the observations about the events during the application of the sequence. We have evaluated whether the hypotheses of the research were confirmed and whether students acquired new concepts.

While applying the propositional analysis of concepts we designed for each student and each activity the table in which we show: (i) the propositions given by a student before the instruction; (ii) the main propositions, which exemplify the instruction; (iii) the propositions that a student answers the same questions after the instruction. It allowed us to study the changes that produced in the student cognitive structure. Thus, PCAS applied the students supported us on designing the activities of the didactic sequence and on the analysis of the application of the results.

Activities of the Didactic Sequence

The 12 activities designed are based on the cognitive values that we want to take the student to realize, and at the same time, developing in each of them. Each proposed activity aimed to emphasize that mathematics has evolved along its construction and that several people contributed to its evolution. However, it is necessary always to seek alternatives to overcome the possible difficulties that may arise during the implementation of the activities to relate theory, methodology, and values.

We decided to start our work on trigonometry at the trigonometric cycle with a problem involving a situation of the mathematicians of the past, to instigate the student's curiosity to understand the needs of these people. In the activities, we use the term “historical context” so that the student could better monitor the class.

The educator can stimulate this contextualization through questions about the problem that assists the students in interpreting, and on the transcription to the mathematics language, although, the teacher should let the learner free to express his thoughts. The interaction is significant in this process. Educator and student dialogue a lot on the possibilities and difficulties that arose during the activities.

In activity 1, we attempted to resume to trigonometry triangle, rectangle, reviewing the trigonometric ratios, to highlight the importance of the graphical representation of the sine trigonometric ratios, cosine and tangent. Students drew with pencil and ruler triangles and segments and, then, made the calculations. They noted the similarity of the triangles and understood what the sine trigonometric ratios, cosine and tangent were. In this activity, the cognitive value generalization appears, due to the ability to be formalized systematically. We consider the graphical representation as a cognitive value of mathematics, capable of generalization.

We contextualized the content of trigonometry with problem situations involving current data for the student to observe the cognitive value explanatory power. This is a value that has been presented in all the activities since the trigonometry explains phenomena, explains the need for generalizing, and the contextualization, among other facts. With trigonometry it was possible to solve a current problem, reaching a final result.

To build and establish the cognitive value empirical adequacy, we carried out the application of Activity 2, which was performed with Styrofoam balls. Half of the ball represented the sky. We instigated the students to imagine themselves while watching the stars. With a thumbtack, they scored three points away that formed a triangle. A string passed by three points forming a triangle. Some questions were relevant, such as: how would we represent the plan of this triangle? Is it possible to do the calculations of distances? Is the triangle a rectangle? Do you know how this triangle is called? To each question, the students answered as we had expected. They drew a flat triangle and other spherical. One student said he found significant the activity because he did not imagine that trigonometry was so useful for calculations in astronomy.

In activity 3, it has been explained the historical context of the importance of strings for mathematicians of antiquity. The students identified the circle radius, the strings, and the arches. After the explanation of the arches, the students solved two exercises involving angles and quadrants. The evolution of the studies of the arches, its nomenclature showed that the cognitive value generalization was necessary for trigonometric calculations, creating a clearer and more understandable mathematical language.

In activity 4, we have explained the conversion of units from degrees to radians, with historical context on the degree and parts and apply some exercises, trying to reinforce the cognitive value simplicity.

In activity 5, it has been explained to the students the historical context of Greek and Indian trigonometry. Then they observed the figure that represented the string and the one that represented a triangle-rectangle, noting the similarities and differences between the figures. In the activities 6 and 7, it has been explained the sine and complement sine (cosine) in the historical context and answered some exercises. This step led the students to realize the importance of the cognitive value simplicity.

From the historical context, the student could complete the activity 8 and perform the activity 9, with the introduction of the trigonometric functions. Each student received a CD, protractor, a ruler and a crayon to represent the functions sine, cosine, and tangent in a trigonometric cycle by manipulating in class this material.

In the activity 10 students filled tables with the summary of the changes in functions sine, cosine, and tangent, as well as the tables summarizing the signs of the functions. Then, noting the variations, graphically represented functions in the term.

In the activity 11, we have introduced the historical context, and some pendulums exercises to apply calculations of trigonometric functions. To fix the value consistency, logical connections and epistemic with one or more theoretical systems, the activity was carried out with studies of the pendulum. The application aimed to show the importance of mathematics, in particular trigonometry, regarding other fields of science such as physics.

The activity 12 had an extra exercise for the student to solve, such as task, trigonometry calculations applied to geography. The fecundity value once again was displayed. Finally, activity 13, the students listed a timeline of the most critical aspects of the history of trigonometry. In this activity students debated issues such as: Which was the oldest? The geometry or the trigonometry? In the last activity, students chose a chronological sequence of events that we consider most important in the history of trigonometry.

Discussion of the Results of the Application of the Sequence and PCASs

The answers of the students showed that our didactic sequence obtained a pedagogical result desired in the research, as the students recognized the cognitive the values of mathematics performing the participatory mode activities (motivated and engaged in learning), using the materials correctly and consulting historical texts.

As we already mentioned in this article, one of the first stages of the research, teachers were interviewed about the difficulties of students while studying trigonometry. These indicated that they had problems regarding the display of the graphical representation of trigonometric functions. We realized, then, that the graphic representation has become more critical for students after the application of the teaching sequence. It was the alternative that there was more change (76.67%) on the results of propositional analysis of concepts PCAS).

On the second question of the PCAS, the following question was asked to students: "In your opinion, does mathematics have a value that makes it important for study? Please, justify your answer." The answer of one of the students is that the history of mathematics facilitates the understanding of its origins, how it evolves over the world and contributes to the understanding of the calculations. They also argue that mathematics was required for the development of other disciplines, providing many discoveries. We note, in other students' answers, cognitive values of mathematics, such as explanatory power, accuracy, simplicity, consistency and empirical adequacy, meaning that there was an incorporation of these in their knowledge.

The fact that 53.33% of the students understood that various peoples and nations built trigonometry knowledge, it has to do with real problems, satisfied us because it humanizes Mathematics and because 46.67% of them pointed out that the formulas of trigonometry evolved, presenting more clarity and simplicity. We perceived the students about the fact that mathematics is not unchanging, you can always develop.

The importance of Trigonometry to the development of other areas of knowledge represented 36.67% of student answers, showing the cognitive value consistency. We emphasize that we would need more classes to apply the trigonometric functions in solving problems in other areas of knowledge. Even so, 30% of the students answered that trigonometry explains various phenomena and can be applied to solve problems.

We considered, to our analysis of the comprehension of the cognitive values, the answers of PCASs, but we emphasize that all the students delivered the material with all the activities solved at the end of the application, even students who did not need grades to accepted. This fact surprised us and also pointed to the value of the didactic sequence, reflected in the interest shown by the students.

Several references used in our research advocate the use of history of mathematics in the classroom by providing elements that help in learning how we can mention: "History increases motivation for learning; articulates mathematics and other sciences;" (Sad, 2004, p. 4) "It facilitates the investigation process of mathematical knowledge in the classroom;" (Mendes, 1997, p. 15) "See how theories and practices Mathematics were created, developed" (D'Ambrosio, 1996, p. 30). We realized that the arguments of students were similar to some authors.

We confirmed that utility when we realized that through the history of mathematics we developed our didactic sequence, which led the students to understand the importance of math/trigonometry to different people in different contexts and vice versa. This process of interaction is something interesting for the reflection on the role of mathematics in history for the promotion of values and attitudes in the students (Miguel, 2005, p. 61).

We presented below, in Table 2, and divided the answers of 23 students on the questions applied in PCASs 1 and 2 that allowed confront the prior knowledge and after the experimentation (4th phase PCAS) (didactic sequence).

Table 2.
Changes in the structure of knowledge of the student, the PAC1 to PAC2

Q. 1 Do you consider mathematics important? Justify.
We noticed significant changes in 6 students: They justified the importance of mathematics.
Q. 3 Select x in the statements that you consider correct concerning Mathematics
They added more alternatives corresponding to an increased on the cognitive values: 16 students.
They added that mathematics searches increasingly accurate results: 4 students. They added the alternative that the graphical representation is important: 11 students.
From two students who had pointed out the alternative that everything would be better if there were no mathematics, 1 abandoned this alternative, joining the others who considered it to be important.
Q. 5 What kind of problem does the trigonometry calculations serve to solve to?
They justified the sort of problems that the trigonometry solves consistently: 19 students
Q. 6 What reasons do you think led the mathematicians of antiquity to study trigonometry?
They presented consistent reasons with the history of the mathematics that had been presented in the didactic sequence: 14 students
Q. 8 Do you think it is important to study the history of mathematics to learn better? Justify.
They went on considering important the history of mathematics in both PCASs: 17 students.
They continued finding unnecessary the history of mathematics: 2 students.
They justified even further the importance of the history of mathematics to their learning: 4 students.

It is significant to highlight that, when we considered adequate the justification of the student, we comprehended that the answer was correct before the knowledge built on the didactic sequence. There was no instruction on the content during the application of the PCASs.

Final Words

This article has been based on a historical-philosophical reconstruction of trigonometry content, focusing on the trigonometric functions, and on the relationships of mathematical facts with cognitive values (Lacey, 1998), to design a didactic sequence to take students to understand the importance of this science, motivating them to their learning. We have based on the students' conceptions about the trigonometric functions and cognitive values based on philosophers of science to carry it out, adapting them to the perspective of the Philosophy of Mathematics Teaching, featured in our research by the approach of "everyday teaching" as Bicudo describes it (1999, p. 25-27): “thematizing aspects of the educational doing, regarding to the teacher-student relationship, teaching, learning, assessment, curriculum, school, describing the ways by which this doing happens, analyzing them and reflecting on the meanings built.”

To validate the results of the didactic sequence used to bridge the scientific knowledge to the student, we used an instrument called propositional analysis of concepts (PC), based on the Theory of Meaningful Learning. The results of the analysis of PCAs showed significant changes in the values assigned to mathematics by students with new information being anchored to the concepts or existing propositions in cognitive structure of the individual, taking place, therefore, the learning Trigonometry.

The activities allowed the students to observe the values relevantly and solidify their knowledge about the nature of mathematical knowledge. Therefore, the understanding of these cognitive values grounded and justified, in a contemporary way, the presence of such a subject in the structure of the mathematics taught in Primary Education.

Thus, we believe that the research results supported the theoretical references advocates the use of the history of mathematics to take students to the learning content and values. On analyzing the results of our research, and the entire process involved in the construction of historical-philosophical approach, we have noted that the history of mathematics, has important educational potential. It is more than a teaching resource.

Notes

[1] For Pais (2002, p. 202) "a didactic sequence consists of many planned lessons and previously analyzed with the purpose of observing learning situations involving the concepts set out in didactic research."

[2] "Meaningful learning is one in which symbolically expressed ideas interact in a substantive and non-arbitrary manner with what the learner already knows. Substantive mean non-literal, not the foot-of-letter and non-arbitrary means that the interaction is not with any previous idea, but with some specifically relevant knowledge existing in the cognitive structure of the subject learning." (Moreira, 2010, p. 2)

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Helenara Regina Sampaio Figueiredo is associate professor at the Universidade Norte do Paraná, Brazil.

Irinéa de Lourdes Batista is associate professor at the Universidade Estadual de Londrina, Brazil.

Contact Address: Direct correspondence concerning this article, should be addressed to the author. **Postal address**: Universidade Norte do Paraná, Rua Marselha, 591, Jardim Piza; 86041-140, Londrina, PR, Brazil. **Email:** helenara.sampaio@yahoo.com.br , irinea@uel.br