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The Exponential Function Meaning in Mathematical Modeling Activities: A Semiotic Approach

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Date of publication: Junio 24th, 2018
Edition period: Junio 2018-Octubre 2018

To cite this article: Silva, K.A.P., & Almeida, L.M.W. (2018). The exponential function meaning in mathematical modeling activities: A semiotic approach. *REDIMAT – Journal of Research in Mathematics Education*, *7*(2), 195-215. doi: 10.4471/redimat.2018.2762

To link this article: http://dx.doi.org/10.4471/redimat.2018.2762

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REDIMAT, Vol. 7 No. 2 Junio 2018 pp. 195-215

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(*Received: 25 May 2017; Accepted: 24 June 2018;Published: 24 June 2018)*

Abstract

In this article we present a reflection about the meaning attribution to the mathematical object *exponential function* that emerges from two mathematical modeling activities. The theoretical framework of the text contemplates considerations on Mathematical Modeling and elements of semiotics as theorized by Charles Sanders Peirce. The empirical research refers to the development of two modeling activities carried out by different groups of students. We analyzed the interpretant signs produced by students working in groups on two activities. The analysis indicates that exponential function meaning in mathematical modeling activities is associated with the *familiarity* that the interpreter reveals to have in relation to the object; the interpreter's *intention* in signifying the object; the identification, by the interpreter, of the possibility to refer to the object in other circumstances or in future situations; and the *collateral experience* of the interpreter with the object. In addition, the significance of the exponential function in modeling activities is also associated with the specificities of the problem as well as with the pertinent systems of practices and contexts of use of this function.

Keywords: Mathematical modeling, Peirce's semiotics, meaning attribution, exponential function

2018 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2018.2762

REDIMAT, Vol. 7 No. 2 Junio 2018 pp. 195-215

El Significado de la Función Exponencial en Actividades de Modelización Matemática: Un Enfoque Semiótico

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(*Recibido: 25 Mayo 2017; Aceptado: 24 Junio 2018;Publicado: 24 Junio 2018)*

Resumen

En este artículo se presenta una reflexión acerca de la atribución de significado a la función exponencial de objeto matemático que emerge de dos actividades de modelación matemática. El marco teórico se basa en el enfoque de Peirce. La investigación empírica se refiere al desarrollo de dos actividades de modelado llevadas a cabo por diferentes grupos de estudiantes. Se analizan los signos interpretativos producidos por los estudiantes que trabajan en grupo. El análisis indica que el significado de la función exponencial en las actividades de modelización se asocia con la familiaridad que el intérprete tiene en relación con: el objeto, la intención del intérprete de significar el objeto, la identificación de la posibilidad de referirse al objeto en otras circunstancias o en situaciones futuras, y la experiencia colateral del intérprete con el objeto. Además, la importancia de la función exponencial en las actividades de modelización también se asocia con las especificidades del problema, así como con los sistemas pertinentes de prácticas y contextos de uso de esa función.

Palabras clave: Modelización matemática, semiótica Peirceana, atribución de significado, función exponencial

2018 Hipatia Press
ISSN: 2014-3621
DOI: 10.4471/redimat.2018.2762

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The discussion about meaning has been recurrent in different areas of knowledge such as philosophy, logic, semiotics, and psychology, among other areas interested in human cognition.

For our study, we take into account the notes that refer to the attribution of meaning in a semiotic sense. Semiotics is the science of signs, the signs of language. In this article we build upon the analysis of the signs to infer about the meaning. In dealing with the analysis of signs we base ourselves on the semiotic theory of Charles Sanders Peirce, particularly on his constructions and arguments about the signs that are created in the minds of students - the interpretants - and the inferences regarding the attribution of meaning, from these signs, to the object.

What, in general terms, is addressed in this text, concerns the search for evidence of the attribution of meaning to the mathematical object exponential function that emerges in the development of two mathematical modeling activities. Thus, the question that guides our research is: *how do the interpreting signs provide indications of attribution of meaning to exponential function in mathematical modeling activities?*

Our reflections are based on the analysis of the interpretants evidenced in the written records and students' speeches in the development of mathematical modeling activities carried out by students working in groups in Mathematical Modeling disciplines in two courses: graduate degree in Mathematics and postgraduate degree in Mathematics Education.

The Meaning in Peirce’s Semiotics

Charles Sanders Peirce (1839-1914) was an American semioticist, philosopher, and mathematician who from 1857 devoted much of his studies to the structuring of signs and their relation to the modes of meaning attribution.

In Peircean semiotics the sign has a triadic nature, being constituted by three components: the sign or representation, object, and interpretant. For Peirce, the object is what the sign refers to. The sign, according to Peirce (1972), has the function of representing an object to someone (an interpreter), creating another sign in someone’s mind. The interpretant is a new sign produced by the interpreter and corresponds to the interpretative effect that the sign produces in the interpreter's mind (Peirce, 1972; Peirce, 2005).

The interpretant, as Santaella (2007, p. 23) claims, corresponds to the "interpretative effect that the sign produces in a real or merely potential mind". In this sense, each sign, in the interpreter's mind, generates an interpretant which, in turn, acts as a *representamen* of a new sign, in a process of generation of interpretants in an *ad infinitum* cycle.

Winfried Nöth and Michael Hoffmann, interpreting the Peircean theory have done some reading on the role of the interpretant in Peirce’s Semiotics and confirmed that it is through looking at the interpretant that one can infer about the meaning of an object to the interpreter. According to Nöth (2008), the interpretant corresponds to the meaning of the sign or the interpretation of the sign by the interpreter. Hoffmann (2004, p. 198) claims that the main characteristic of Peirce’s Semiotic in the meaning attribution to the objects “is the interpretant role”.

The process adopted by Peirce to reconstruct or explain the meaning by means of signs, consists of an established group of conditions towards a given situation in which a definite operation would produce a definite result. Looking at the interpretant based on Peirce’s ideas of meaning, Silva & Almeida (2015), going through different studies on Peirce and his interpreters, concluded that evidence of attribution of meaning may be: the *familiarity* that the interpreter reveals to have in relation to the object; the interpreter’s *intention* in signifying the object; the identification, by the interpreter, of the possibility to refer to the object in other circumstances or in future situations; and the *collateral experience* of the interpreter with the object.

With regard to mathematics in particular, Wilhelmi, Godino & Lacasta (2007, p. 76), claim that the “meaning of a mathematical object is inseparable from the pertinent systems of practices and contexts of use of this object". In this way the meaning of a mathematical object is related to the activity in which this object is mentioned or is used. In this article we direct our attention to the meaning of mathematical objects in mathematical modeling activities.

Mathematical Modeling in Mathematics Education

Although different conceptualizations of mathematical modeling can be recognized, according to Blum (2002), when a mathematical modeling activity is developed it is important to consider a problem of reality as a starting point, by setting the activity as something in which

> The starting point is normally a certain situation in the real world. Simplifying it, structuring it and making it more precise – according to the problem solver's knowledge and interests – leads to the formulation of a problem and to a real model of the situation. […]. If appropriate, real data are collected in order to provide more information about the situation at one's disposal. If possible and adequate, this real model – still a part of the real world in our sense – is mathematised, that is the objects, data, relations and conditions involved in it are translated into mathematics, resulting in a mathematical model of the original situation. Now mathematical methods come into play, and are used to derive mathematical results. These have to be re-translated into the real world, that is interpreted in relation to the original situation. At the same time the problem solver validates the model by checking whether the problem solution obtained by interpreting the mathematical results is appropriate and reasonable for his or her purposes. If need be (and more often than not this is the case in 'really real' problem solving processes), the whole process has to be repeated with a modified or a totally different model. At the end, the obtained solution of the original real world problem is stated and communicated (Blum, 2002, p. 152-153).

To this structure of a mathematical modeling activity, Blum (2015) associates a schema, a cycle of mathematical modeling as indicated in figure 1.

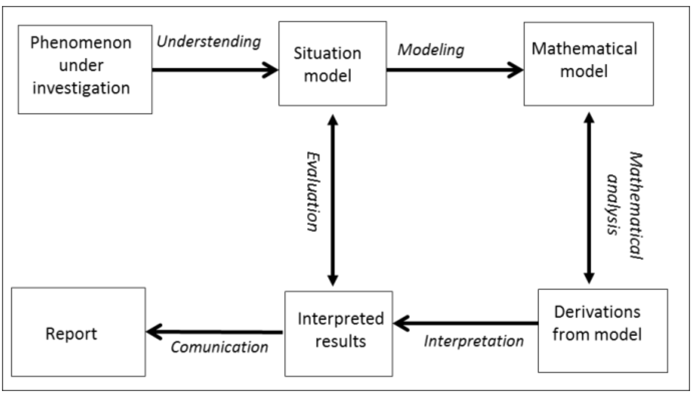


Figure 1. Modeling schema. Source: Blum (2015, p. 77).

Stillman, Brown & Geiger (2015, p. 95-96), also aligned with the development of an activity that follows this schema, consider that two essentially distinct aspects are relevant:

> The mathematical domain includes the mathematical model made of the situation, mathematical questions posed and mathematical artefacts (e.g., graphs and tables) used in solving the mathematical model. Mathematical outputs (i.e., answers) have then to be interpreted in terms of the idealised situation and the real situation that stimulated the modelling (i.e., back into the extra-mathematical domain). These outputs can then answer questions posed about the real situation or, if they are inadequate for this purpose, stimulate further modelling.

As used in this paper, a mathematical model comprises “systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system” (Doerr & English, 2003, p. 112).

The introduction and use of the mathematical modeling in different levels of schooling and in different courses and subjects refers to the usage, application, and learning of Mathematics. It is in this use or application of Mathematics that the signs have an important role. In fact, signs are the means of access to mathematical objects and they indicate the attribution of meaning, whether for mathematics itself or for the phenomenon under study.

An important aspect for a teaching methodology to use modeling in the classroom is to orient a class management that considers that the group work is particularly suitable. The group is not only a social but also a cognitive environment (a co-constructive group work) (Blum, 2015).

In this way, if students engage actively in modeling and do it in groups, we have to consider that they may use or produce signs within these groups. From this point of view, we may not ignore that communication and meaning attribution are always intertwined and mediated by the signs they produce within these groups.

In this article we addressed our analyses about meaning attribution to the mathematical object *exponential function* performed by students when they are involved in two mathematical modeling activities developed by small groups of students.

Methods

In order to investigate how the signs provide indications of meaning attribution for exponential function in mathematical modeling activities, we articulate aspects from the presented theoretical framework and empirical data. We analyzed the development of two activities carried out in Mathematical Modeling disciplines of two different courses. Both courses were offered by a Brazilian public university, and contain the subject of mathematical modeling in their curriculum.

The first activity, *concentration of calcium in the river substrate according to the river depth*, was developed by 20 students (11 males and 9 females) of the 4th year of a Degree in Mathematics. The students developed this activity in small groups (pairs) in a period of four class hours that is, 200 minutes, in 2011. In this article we analyze the development of one of these pairs.

The second activity, *the evolution of consumption of cigarettes per inhabitant in the world*, was developed by 11 students (9 males and 2 females) of a Mathematics Education postgraduate course. In this case, four doubles and one trio of students developed the activity during about 100 minutes in 2012. We analyzed the development of the activity by one of these pairs.

One student, Paul was a member of both referred to courses and he is a member of the two groups whose signs we analyzed in this article. The activities were developed under the coordination / supervision of the authors of this article.

Information about the problem situations to be investigated by the students in pairs or trios was provided by the teachers, and the students carried out all the other procedures as indicated in the modeling schema presented in the previous section.

The investigation falls within the methodology of qualitative research. In qualitative research, observations, document analysis, and interviewing are the major sources of data for understanding the phenomenon under study (Bogdan & Biklen, 2003; Lesh, 2002). In the scope of qualitative research, Lesh (2002) characterizes the Research Experiment. According to Lesh (2002, p. 29), a Research Experiment "involves new ways of thinking about the nature of students developing mathematical knowledge and abilities". In this paper we use this approach, particularly, for the analysis of the signs used and produced by the students throughout the development of the mathematical modeling activities. The data were collected by means of written files, video recording, audio files, and interviewing.

Our inferences about meaning attribution to exponential function are based on the signs and interpretant signs produced by the students of the two groups analyzed.

The modeling Activities and Discussion about Meaning

The First Activity: Concentration of Calcium in the River Substrate According to the River Depth

This activity was developed by 20 students of the 4th year of a Degree in Mathematics. To investigate the meaning attribution to exponential function within this activity we present in this article the analysis of the activity development carried out by one of the groups to which we referred in the previous section. We consider the pair of students Paul and Mary and the signs they used and produced throughout the activity development.

In order to develop this activity in the classroom the teacher provided the students with information about the problem situation as indicated in Table 1 and Figure 2. To investigate the relationship between calcium concentration and phytoplankton production, the students performed a preliminary research on internet sites and books on the area. What the students learned from their research is that phytoplankton production requires a calcium concentration of 150mg / L, or 0.15mg / cm^3.

Table 1.
Calcium concentration in the Limoeiro river

River depth (cm)	Calcium concentration in the substrate (mg/cm^3)
30	2.958
90	2.316
150	1.641
210	1.264
270	0.893
330	0.697

Source: Borssoi, 2004

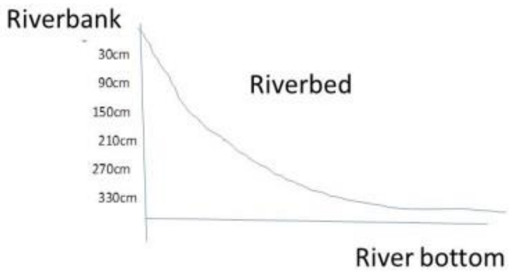


Figure 2. Graphical representation of the Limoeiro river depth

Paul and Mary considered the data provided by the teacher on the concentration of calcium in the river and the information they had obtained from their research. During the *Understanding* stage referred to in the modeling schema of Figure 1, what the two students wanted to understand and mathematize is the relationship between the depth of the river and the possibility of phytoplankton production according to these river depths. Particularly, the pair wanted to determine the maximum depth of the river at which phytoplankton production may still occur.

Initially, Paul and Mary noted that in Table 1 the information indicates that the calcium concentration decreases as the river depth increases. However, what seems to have been relevant to mathematize this decrease was the image in Figure 2. In fact, when the students were asked in an interview why they thought of exponential function in this modeling activity, one of the answers was:

> Mary: The figure of the river helped us to think of what mathematics we could use. We think it looks like exponential behavior because we already know the graph of this kind of function.

Thus, there seems to be an indication that Figure 2[1], in turn, led Paul and Mary to ponder that the decrease in the amount of calcium may have an exponential behavior. Therefore, it is a sign from which other signs are being produced by the students to refer to the exponential function. In this sense, we can consider that the exponential function corresponds, at this moment, to the interpretative effect that the sign produces in a real or merely potential mind, as claimed by Santaella (2007).

Other interpretants produced by the students to mathematize this exponential decrease had the purpose of obtaining the mathematical model associated to this exponential decay. Considering that both Figure 2 and Table 1 represent information on the problem, the pair of students seeks to identify characteristics of an exponential behavior in the data of the table. In the interview, Mary explains how they conducted their actions to model the situation.

> Mary: So we thought about what mathematics we should use ... We ended up taking an approach that started with an analysis of the data in Table 1 and from there we made the definition of the hypothesis for the relationship between the depth of the river and the amount of calcium.

This explanation from Mary refers to the procedure performed by the students and indicated in Figure 3.

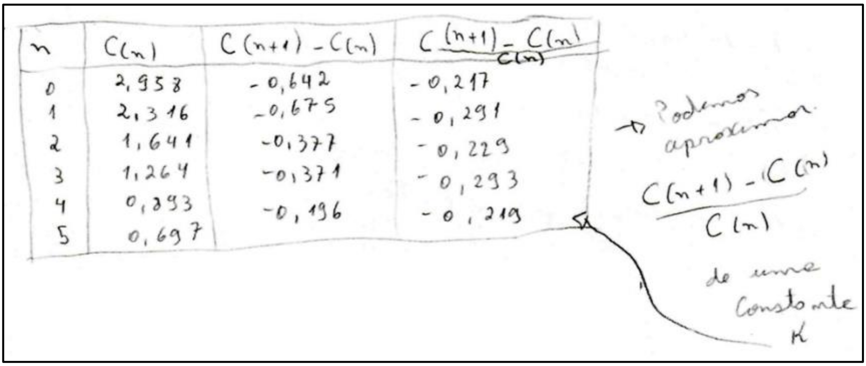


Figure 3. Hypothesis defined by Paul and Mary from analysis of the data in the table[2]

> Paul: Well ... we used a hypothesis that led us to solve an ODE.
> Professor: For this did you consider a rate of change?
> Paul: Yes... what we already knew of variation in quantities over time.

From the calculus performed by the students, as shown by Figure 3, the students defined the hypothesis that the rate of change of the calcium concentration in the substrate in relation to the river depth is proportional to the calcium concentration. This can be expressed by a first-order ordinary differential equation, indicated by dC/dp=$k \cdot C$ in which p is the river depth (in cm) and C(p) the calcium concentration (in mg/cm3) according to the depth p. To solve this ODE, Paul searched for help in his class notes and said to Mary:

> Paul: Mary, look [holding a note sheet] we have solved a differential equation just like this one in previous classes. Therefore, we know we'll get to an exponential model!

Paul's statement seems to be an indication of collateral experience with the object exponential function in other learning situations and so it is evidence of meaning attribution to exponential function, as Peircean semiotics establishes. The new signs produced by students make reference to the signs Table 1 and Figure 2 and they highlight the mathematical object that comes up in this modeling activity. These new signs are interpretants signs and they are, at this time, the idea that the interpreters (Paul and Mary) had from the original sign.

Analyzing the images captured by video, it is evident that Paul and Mary look for some kind of protocol in their notes to develop the activity using ODE. In this context, Manechine & Caldeira (2006, p. 3) claim that in the school context, "as the student becomes familiarized and learns certain universal signs, these become reference objects to the connection, relationship, and appropriation of new signs". In this case, the development of the 1st order separable ODE corresponds to a reference object to obtain the exponential model. In fact, the solution obtained by the students to the exponential function is $C(p) = \beta . e^{kp}$, as shown in Figure 4.

To determine parameters β and k of *C(p),* Paul and Mary used two mathematical procedures. First they chose two points from Table 1.The choice of the points was performed by running experiments and validating them during an exhausting process that can be visualized in the video recording in which the students run calculus using the calculator and erase times in a row, arriving at Model I represented by $C(p) = 3{,}3443 . e^{-0{,}004083p}$ as shown in Figure 5. In the interview Mary affirms:

> Mary: We knew that with two points it would be possible to determine the value of the parameters. Our question was which points in Table 1 we should choose.

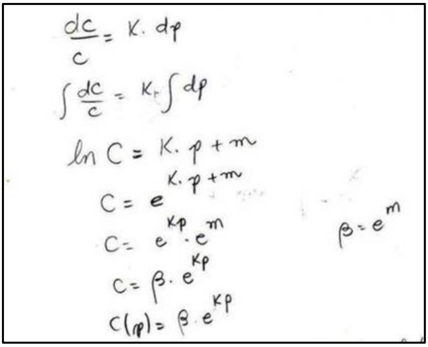


Figure 4. Signs produced in mathematization with ODE

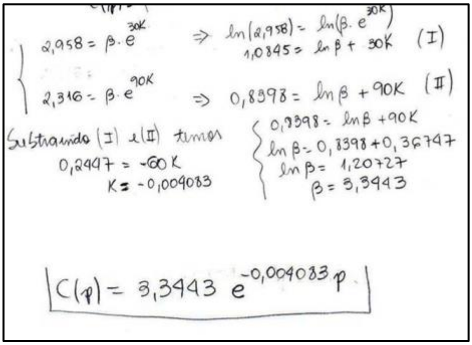


Figure 5. Signs produced to determine β and k in Model I[3]

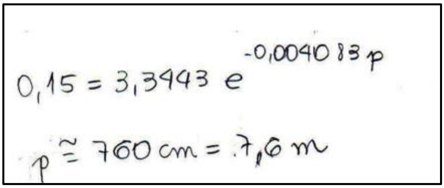


Figure 6. Solution to the problem using Model I

It is important to consider that in the activity the model obtained is not yet the solution to the problem that the activity was proposed to investigate. In fact, to determine the maximum depth of the river that still allows phytoplankton production, it was necessary to match the depth of the river to the minimum concentration of calcium that still allows this production (0.15 mg / cm^3). Thus, what Paul and Mary did with the obtained model was: $3.3443.e^{-0.004083p} = 0.15$. Thus the students obtained the answer *.p = 7.6m* as shown in Figure 6.

In the second approach Paul and Mary used the Least Squares Method (LSM), as shown Figure 7, to obtain *k* and β and obtained the mathematical model $C(p) = 4.047016e^{-0.0049270p}$ (Model II).

In this case, to determine the maximum depth of the river that still allows phytoplankton production, the equality resolved was $4.047016e^{-0,0049270p} = 0.15$, which shows that, according to this approach, the maximum depth is *p = 6.688m.*

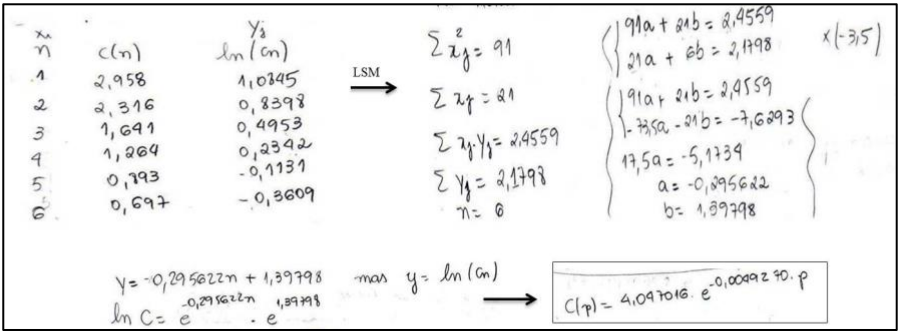


Figure 7. Signs produced to obtain Model II using the LSM

When asked, during interview, about this second resolution using the least squares method, Paul justifies:

> Paul: Well… firstly we solved the ODE equation and for that we used only two points to obtain the parameters. But we also wanted to use a method in which we could use all the points …Then, we came up with the least squares method. In fact, the least squares method you use to find a function or curve that best fits a set of points. And then we could use our set of points.

What is evident in these statements of the students is that they recognize in different signs the same mathematical object: the exponential function. This denotes that Mary and Paul are becoming familiar with the mathematical object exponential function. Thus, according to Peirce's assertions which we have already discussed, we have indications of attribution of meaning.

In the interview, when we asked Paul why they (Paul and Mary) were concerned with presenting the two models (Model I and Model II), he pondered that:

> Paul: Our models are different, but they are close to what we think about the behavior of calcium in the river, which decreases with the depth of the river. The exponential function in both models indicates this.

Paul's assertion is an indication that the attribution of meaning in this activity is also imbued with a student's intention to signify the exponential function object.

The Second Activity: The Evolution of Consumption of Cigarettes per Inhabitant in the World

This second activity was developed by 11 students of a Mathematics Education postgraduate course. In this case, four doubles and one trio of students developed the activity. We analyzed the development of the activity by one of these pairs, Paul and Carl.

In order to develop this activity in the classroom the teacher provided the students with information about the problem situation as indicated in Figure 8. This figure made it possible to see the evolution of per capita consumption of cigarettes per inhabitant in the world from 1950 to 2007. In 1950 the per capita consumption was 702 cigarettes a year; and in 1990 this consumption reached 1062 cigarettes per year; but then began to decline, reaching 844 cigarettes in the year 2007.

What the student proposed to study in this activity is: considering the decrease in cigarette consumption after the year 1990, could consumption be reduced again to reach the 702 cigarettes consumed per person, as it was in 1950?

From the mathematical point of view, the first problem of the student in this case is that from the data of the magazine from the year 1950 to 2000 the consumption was presented every 10 years. However, this does not happen in the final period in which the reported consumption is the year 2007 rather than 2010. Thus, the students used the hypothesis that from the year 2007 the consumption would continue to decrease and then determined the consumption in 2010.

The prediction of cigarette consumption for the year 2010 was made by the student from the observation that between 1990 and 2000 the decrease in annual cigarette consumption per person corresponds to approximately 13.75%; while for the period from 2000 to 2007, the percentage is reduced to 7.86%. Considering the hypothesis that the decrease of 7.86% in the final period was equally distributed among the seven years, they could consider that consumption decreased by 1.12% per year in this period. Assuming that this percentage was maintained between 2007 and 2010, they concluded that the number of cigarettes consumed per person in 2010 was 813 cigarettes, as indicated in the last line of Table 2.

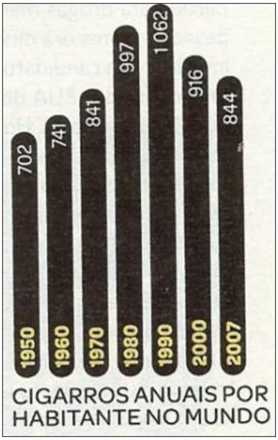


Figure 8. World cigarette consumption. Source: Super Interessante Magazine, August 2009, p. 35.

Table 2.
Number of cigarettes consumed per year per person

Year (t)	Number of cigarettes
1950	702
1960	741
1970	841
1980	997
1990	1062
2000	916
2010	813

In this activity the information provided by the teacher was not insufficient. The understanding process, as the modeling schema indicates, requires students to supplement this information so that the next step of the schema, modeling can be initiated.

This complementation of the data, producing the number of cigarettes consumed per person in the year 2010, is already indicative of the hypothesis that pervades the modeling of the students. In fact, the hypothesis that it is a decreasing phenomenon is already incorporated. What would be a problem for these students is determining in which year the consumption of cigarettes would again reach 702 cigarettes per person per year.

In order to continue their procedures, Paul and Carl produced a new sign from table 2. This is an interpretive sign that expresses the students' understanding of the situation (Figure 9). This interpretation of the students is an indication that "modeling activity allows the organization and elaboration of signs, i.e., the generalization of knowledge by semiotic representations and its interpretation" (Almeida, 2010, p. 409).

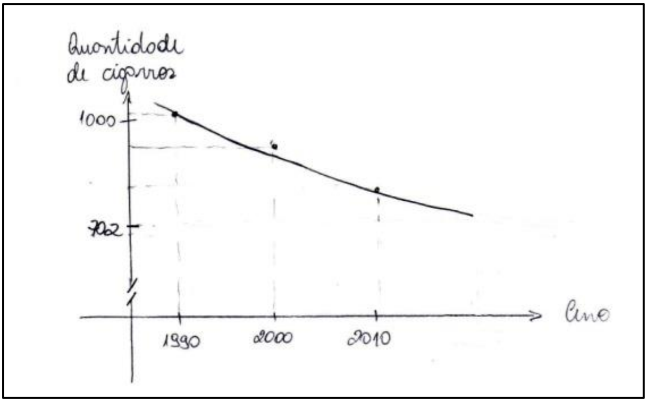


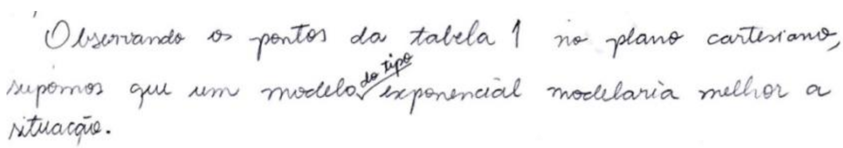
Figure 9. Graph produced by the students[4]

When the students were asked about the construction of this interpretant sign, they gave good explanations.

> Paul: The graph is important! I think it is interesting you see at each point what is happening. Because we - I do not know if I can generalize - we really needed to see the result, to view it in some way, i.e., how this is happening, you have something more visible. I think the graph somehow makes this possible…the behavior, because you say 'Oh! There is that point there, it seemed to be somewhat discrepant, what is happening?' good, but at that point this is happening. The graph allows you to explore more data. I think it helps a lot in this sense [to see the data behavior].

> Carl: And also with the graph we were convinced that the decline could not be linear. And that's why we thought the exponential function would be the best in this case.

The fact the students used the graph to represent the data indicates a familiarity with the handling of this sign, particularly the need '*to see at every point what's going on*'. According to Peirce (2005, p. 164), it is "the familiarity that a person has with a sign that makes this person able to use it or interpret it". Accordingly, we can infer that for Paul and Carl the table and graph are interrelated to characterize the mathematical object that emerges in the activity development.



(Looking at the points in table 1 in the graph we can assume that an exponential model can be used to represent the situation)

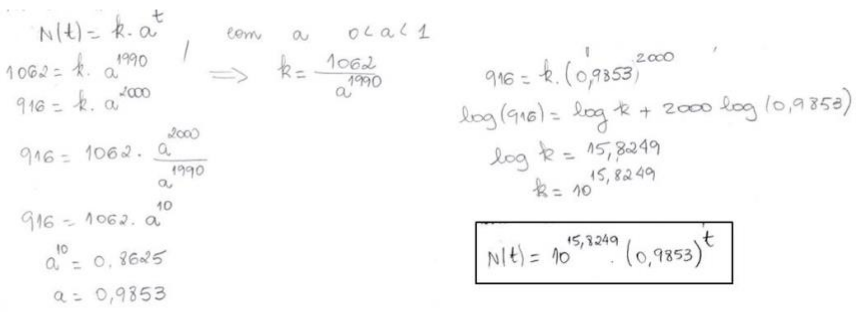


Figure 10. Mathematical model obtained by the students

In this case the students considered the graphical representation in Figure 9 to assume the hypothesis that an exponential function could be associated with the decrease in the number of cigarettes consumed per year per person. Figure 9 becomes a sign for future referrals in the development of the activity. In fact, interpretive signs are constructed to solve the problem.

To obtain the model in this situation the students start from the general form of the exponential function $N(t) = ka^t$, where *t* is the time in years; N(*t*) represents the number of cigarettes consumed according to time; and *k* and *a* correspond to parameters to be obtained. To determine the values of *k* and *a* the students choose the points (1990, 1062) and (2000, 916) in Table 2 to obtain the model as shown in Figure 10.

The choice of the two points was justified by the students:

> Carl: It is simple to deduce the mathematical model using two points. This is what we use with students in basic education. We wanted to do it in a way that could be done with our future students.
>
> Paul: Well ... I know I could have used ODE, and the least squares method, but thinking about my future professional activity as a high school teacher, and the possibility of introducing this activity in my classes, I thought choosing two points would suffice. In addition, the validation of the model performed indicates that the model is adequate.

Two aspects are relevant in this assertion from Carl. First, it seems that we can recognize here the identification, by the interpreter, of the possibility to refer to the object in other circumstances or in future situations. In fact, as a teacher in training, Carl associated his action with his future professional activity. Moreover, the assignment of meaning in this case seems to be associated with the fact that the "meaning of a mathematical object is inseparable from the pertinent systems of practices and contexts of use of this object" (Wilhelmi, Godino, & Lacasta, 2007, p. 76). In fact, these student procedures are indicative that the meaning attribution to the exponential function may be associated with student's educational and professional context: a student of a postgraduate course in the area of Mathematics Education and a secondary school teacher.

In order to obtain a solution to the problem, which is to determine the year in which the number of cigarettes consumed per person reaches that of 1950, that is, 702 cigarettes, Paul uses the deduced model $N(t) = 10^{15,8249}(0.9853)^t$, and he calculates $10^{15,8249}(0.9853)^t = 702$. From this it follows that the number of cigarettes consumed per year will again be 702 cigarettes per person in the year 2018, according to the procedures of the student in Figure 11.

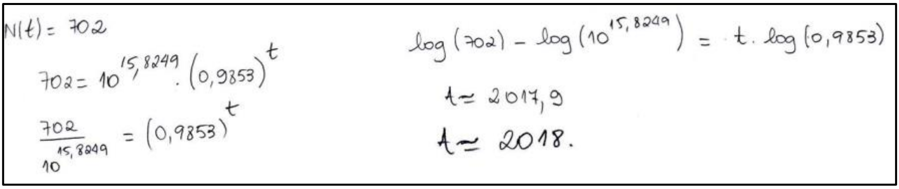


Figure 11. Resolution of the problem in the activity on cigarette consumption

The meaning of the exponential function in this case seems to be being awarded in the course of the interrelationship between the graph, table, and the students' collateral experience with the sign.

When asked to justify the use of the exponential model for this situation Paul argues that:

> Paul: We knew the function must be decreasing. We even thought of doing a linear fit, but a linear function could have negative values and this is not true for the situation of smoking. Besides this, with the exponential function we had guaranteed it would be even positive and so, asymptotic, but we did not know if this asymptote would be greater or less than 702; the table data, however, indicate that it certainly would be less than 702. We also had to take into consideration the fact that the function was decreasing and guarantee that parameter a must be between zero and one.

Paul's arguments provide indicative attribution of meaning to exponential function. In fact, Paul's statement is an interpretant sign that reveals his understanding of exponential function characteristics considering his different representations (table, graph, and algebra). It is in this sense that the statement of Peirce (2005, p. 222) that "meaning interpreted meaning of a sign" could be observed.

Discussion and Results

Analysis of the signs produced by the students in the development of the two activities allows us to have indications of how the interpretive signs provide indications of meaning assignment for exponential function in mathematical modeling activities. In our research we consider the thoughts of Santaella (2007, p. 37), that "the analysis of the interpretants must be based on the careful reading of both the aspects involved in the foundation of the sign and in the aspects involved in the relations of the sign with its object".

When studying calcium concentration in the Limoeiro River, the *understanding* of the phenomenon investigated, to a certain extent, is guided by the students' analysis of the data provided by the teacher. This action makes possible the production of signs from which the modeling arises, aiming at the construction of the mathematical model. The stage of mathematical analysis referred to in the Blum schema (2015) in this case is what gave the students the solution to the proposed problem.

The interpreting signs are being produced by the students with the intention of obtaining the solution. The meaning for the exponential function has been constructed to the extent that different methods and procedures are used to understand the phenomenon through an exponential function.

Collateral experience and familiarity are articulated in the development of the modeling activity, since there is an intention to obtain a "better" mathematical model to solve the studied problem. For each mathematical model deduced, the students perform *mathematical analysis* to obtain a solution interpreted with the phenomenon.

In the activity of the analysis of the consumption of cigarettes the route taken by the students was guided only by information given to them by the teacher. In this case, the steps *Understanding and Modeling* of the schema were those that required the most effort from the students. In fact, Table 2 and Figure 10 are interpretive signs produced by the students from which the meaning for the exponential function would be consolidated for these students.

In this case the interaction between the pairs of students in the development of the activities was fundamental so that the appropriate signs were produced by the students. It is precisely in this sense that "a sign is only a sign because it is interpreted by somebody, by the interpreter and it creates a new sign in their mind, the *interpretant*, which is in reality the idea that the interpreter had of the original sign" (Miskulin et al, 2007, p. 5).

We can conclude that from a mathematical point of view the meaning attribution to the exponential function is also associated with the specificities of the activity. In each activity obtaining the exponential function was oriented by specific characteristics of the problem under study. In the case of the calcium concentration of the Limoeiro River, the exponential function was found as the solution of an ordinary differential equation. In the case of the reduction in the annual number of cigarettes, the representation of the data in the Cartesian plane is what led to the formulation of the hypothesis that the decrease could be exponential.

Thus, particular conditions in each case directed construction of interpretant signs, in line with that considered by Wilhelmi, Godino & Lacasta (2007, p. 76); "the meaning of the mathematical object is inseparable from the pertinent systems of practices and contexts of use".

In both activities the evidence of meaning attribution also reflects familiarity with the object, the intention to signify the object, and the collateral experience with the object, according to Peirce's considerations concerning the meaning revealed in the interpretant signs.

What the analysis of the two activities also indicates is that, although the phenomena studied in the two activities are different, the data present the same behavior. Thus, producing mathematical models for the two situations considering the specificities of each one, is an indication of what Perrenet & Zwaneveld (2012) point out in relation to the fact that in modeling activities the mathematics is 'only' a part of the whole process.

Notas

[1] These data were collected in a previous research and are reported in Borssoi (2004).
[2] In this figure, performed by Brazilian students, the expression 'Podemos aproximar C_{n+1}-$C_{n'}$/C_n de uma constante k' is translated as: 'We can approximate C_{n+1}-C_n/C_n of a constant k'.
[3] In this figure, performed by Brazilian students, the expression 'subtraindo I e II temos' is translated as: 'subtracting I and II, we can write'.
[4] In this figure, performed by Brazilian students, the expression 'Quantidade de cigarros' is translated as: 'Quantity of cigarettes'. The word 'ano' is translated as 'year'.

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