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AN ESTIMATION OF THE INDUSTRIAL PRODUCTION DYNAMIC IN THE MERCOSUR COUNTRIES USING THE MARKOV SWITCHING MODEL

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RESUMEN: En este trabajo se aplicó una metodología basada en los métodos estadísticos Bayesianos inspirados en los esquemas de muestreo Monte Carlo por Cadenas de Markov, que simplifica el proceso de estimación y predicción del modelo de Markov switching. El objetivo general de este estudio es determinar simultáneamente: no linealidad, cambios estructurales, asimetrías y valores atípicos que son características presentes en muchas series financieras. La metodología se ilustra empíricamente utilizando series que miden la tasa de crecimiento anual de la producción industrial en los países del MERCOSUR. Se implementó el algoritmo de muestreo de [1] para estimar los parámetros del modelo. La estimación de los parámetros se realizó en términos de los valores esperados a posteriori y las desviaciones estándar a posteriori. Se utilizaron tres criterios para evaluar el modelo de predicción: raíz cuadrada del error cuadrático medio (RSME, por sus siglas en inglés), el test de Diebold-Mariano (DM) y el test estadístico T^{RC}. Estas medidas de bondad de ajuste mostraron que las estimaciones tienen errores pequeos. Se calculó el tiempo de ejecución del algoritmo, observándose alto desempeo. Como resultado del análisis de los datos, se concluye que no hay reducción de la volatilidad económica, no hay reducción en la profundidad de los ciclos económicos. En los puntos de ruptura, se observan valores atípicos y no linealidad en los datos. Se evidencia que no existen ciclos económicos en común para los países analizados.

Palabras Clave: Modelos Markov Switching, Algoritmos Monte Carlo por Cadenas de Markov, Producción Industrial en MERCOSUR.

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ABSTRACT: In this work the methodology applied is based on Bayesian statistical methods inspired by Markov chain Monte Carlo sampling schemes, which simplifies the estimation and prediction process of the Markov switching model. The general objective of this study is to determine simultaneously: non-linearity, structural changes, asymmetries and outliers that are characteristics present in many financial series. The methodology is empirically illustrated using series that measure the annual growth rate of industrial production in the MERCOSUR countries. The sampling algorithm [1] is implemented to estimate the parameters of the model. The parameters were estimated in terms of expected values a posteriori and standard deviations a posteriori. Three criteria is used to assess the prediction model: square root of the mean quadratic error (RSME), the Diebold-Mariano (DM) test and the T^{RC} statistical test. These measures of goodness-of-fit show, that the estimates have small errors. The execution time of the algorithm is calculated, observing high performance. As a result of the analysis of the data, it is concluded that there is no reduction of economic volatility and no reduction in the depth of economic cycles. At the breakpoints, atypical values and non-linearity in the data are observed. It also shows that there are no common economic cycles for the countries analyzed.

Keywords: Markov Switching Models, Algorithms Monte Carlo Markov Chains, Industrial Production in MERCOSUR.

1. Introduction

Markov Regime switching models have become very important in the last decades because they are used as an auxiliary tool for decision making in situations of uncertainty. It is a useful way to study industrial growth and analyze the financial statements of the economy in different countries of the world. They have recently been used in the modeling of financial series, with the aim of detecting non-linearity, asymmetries, change structures and atypical values in economies. In the statistical literature, there are not many formal works, that are unified in form try to simultaneously incorporate these characteristics. The studies are tried to characterize the long-term trend of econometric series: [2], [3], [4], [5], [6], [7], [8], [9], [10] and [11]. Hamilton [12] made several proposals to describe the asymmetric behavior of the business cycle in the United States using Markov switching models ([13], [14], and [15]). Granger and Terasvirta in [16] the soft transition regime change model are tried, the nonlinearity between different economic time series is evaluated, in particular autoregressive models of order p ($AR(p)$) are used; in [17] and [18] the analysis for choosing the appropriate lags in the $VAR(p)$ models is deepened. This work is part of the business cycle models for the economy of the United States and some countries of the European Union. Krolzig in [19] used a Markov regime model to analyze cointegration and economic cycles of the US, Japanese, Australian, Canadian, UK and German economies. Recently, [20] the analysis of exchange raterisk in solvency with regime change models is used; in [17] the Markov regime change models to describe the dynamics of exchange rates is used, in [21] a method to detect the presence of atypical values is proposed. When the series is generated by a general nonlinear model to identify and estimate atypical values, an autoregressive threshold model and autoregressive exponential models is used. In addition, they have developed a simulation study to illustrate the procedures and compares them with methods based on linear models and interpolated linear models. Kim and Nelson in [22] use a Bayesian approach to identify structural changes at point of unknown change, with a Markov switching model of the business cycle. In [23], Koop and Potter propose a Bayesian approach to evaluate evidence of nonlinearity in economic series. In [24], Koop and Potter propose an autoregressive threshold model, autoregressive soft threshold and autoregressive Markov switching models from the Bayesian perspective. In [25] Kim and Nelson propose model in state space form and a Markov switching model for series, that come from macroeconomics and finance. These authors made numerous applications such as: decomposition of time series in trend and cycle, established a new index of coincident economic indicators, and approaches to model uncertainty in monetary policy and methods for the detection of inflection points in the cycle economic. Recent computational advances facilitate the estimation of Markov switching models with representation in the state-space form, it is using mixtures of conditionally independent and Gaussian models. In [26] an efficient Markov Chain Monte Carlo sampling scheme is proposed, offersing a convenient overall structure to simultaneously treat these problems. In [1] the methodology proposed by [26] is applied to analyze the industrial production (IP) rate of the countries of the group $G7$ (Canada, France, Germany, Italy, Japan, UK and USA).

This work poses to adapt the proposal of [1] in the scenario of the MERCOSUR countries (Argentina, Brasil, Paraguay, Uruguay, and Venezuela). In order to achieve the objectives, the following model is considered:

$$\begin{aligned}
y_t &= z_t + v_t + \sigma_t K_{\delta_t}\delta_t + \sigma_t K_{a_t} a_t \\
z_t &= \phi_1 z_{t-1} + \ldots + \phi_p z_{t-p} + \sigma_t K_{e_t} e_t \\
\nu_t &= \nu_{t-1} + \sigma_t K_{o_t} o_t \\
\sigma_t &= \sigma_1 I_{\{t\leq\tau\}} + \sigma_2 I_{\{t>\tau\}} \\
\delta_t &= \delta_1 I_{\{t\leq\tau\}} + \delta_2 I_{\{t>\tau\}}
\end{aligned} \tag{1}$$

where y_t is the quarterly IP growth rate in annualized percentage points, and a_t, e_t, and o_t, are standard normal. Switching between the high- and low-growth states by $K_{\delta_t} \in \{0,1\}$ is determined. For identification purposes, it is assumed that $\delta_t < 0$ for all t such that $K_{\delta_t} \in \{0,1\}$, corresponds with the high-growth or low-growth state, respectively. The parameter δ_t is multiplied by the standard deviation of the innovation σ_t to be free scale. The innovation of variance changes once, at an unknown point τ, while δ_t is changed as it σ_t^2 changes. Permanent shifts in the mean growth rate by the timevarying process ν_t are captured. This variation is of the mixture type $K_{o_t} \in \{0, 0.3\}$, where the value 0.3 incorporates the idea that period-to-period changes in mean growth are unlikely to be very large. Additive outliers are allowed through $K_{a_t} = \{0.3, 5\}$, while innovation outliers are captured by $K_{e_t} = \{1, 3\}$ with $K_{e_t} = 1$ representing a regular innovation. The regime-switching process K_{δ_t} is first-order Markov with transition probabilities $p\left(K_{\delta_t} = j | K_{\delta_{t-1}} = i\right)$. Outliers and structural breaks are assumed to be independent of the regime-switching process, such that $p\left(K_{it} | K_{\delta_t, \sigma_t}\right) = p\left(K_{it}\right)$ for $i = a, e, o$ (see notation of [1]).

The Markov-Switching model has become popular. It is used when the asymmetry of the cycles business modeling grows, where the different states of K_{δ_t} correspond to periods of high and low growth (this is intended to coincide with expansions and recessions).

There are other methodologies that have dealt with the problem of analysis of MERCOSUR data series, among them the study of synchronization of economic cycles in MERCOSUR, classical business cycles in America, and asymmetries and common cycles in Latin America (see [27], [28], [29], [30], and [31]).

In [31] the existence of a common Markovian state is established in which the fluctuations of each economy are characterized by latent (unknown) process thant must be estimated, similar to the productive activity in each country. It was established that there are a number of investigations. Evidence of synchronization in the economic cycles of different groups of economies has been demonstrated; however, there are very few investigations that have been oriented to the analysis of the Latin America common economic cycle; the existing ones establish that it is not possible to speak of a common Latin American economic cycle. In this research, a Markov-Switching model regime is adjuested, through the methods of Bayesian Inference, to simultaneously determine: non linearity, structural changes, asymmetries and outliers, which are characteristics present in many financial series, in addition to checking if there is common cycle between these economics. The chronology of the common cycle is reconstructed from the smoothed probabilities of the model. Additionally, three criteria to evaluate model prediction: square root of the mean quadratic error (RSME), the Diebold-Mariano (DM) test and the T^{RC} statistical test.

The rest of the paper is organized as follows: in Section 2 the algorithms are developed, that is implemented; Section 3 defines the Markov Switching regime model used to model the MERCOSUR industrial production rate; in Section 4 the results are discussed and in Section 5 discussions and conclusions are made.

2. Computational Algorithms

In order to understand the model (1) and establish the algorithms, the notation of the models space state is introduced, as follows:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{h}_t^T \mathbf{x}_t + \gamma_t \mathbf{u}_t + \mathbf{g}_t \\ \mathbf{x}_t &= \mathbf{F}_t \mathbf{x}_{t-1} + \mathbf{\Gamma}_t \nu_t + \mathbf{f}_t \end{aligned} \tag{2}$$

where $\mathbf{y}_t$ is an observation vector, $\mathbf{x}_t$ is a vector of unknown states, $\mathbf{h}_t^T$ is a transition matrix of observations, $\mathbf{F}_t$ is a transmission matrix of the states, $\mathbf{u}_t$ is a vector of errors of observations, γ_t is a matrix associated with errors of observation, ν_t is a vector of the states errors, $\mathbf{\Gamma}_t$ is a matrix associated to the errors of the states, $\mathbf{f}_t$ and $\mathbf{g}_t$ are matrices associated with the dynamics of the system, and the errors $\mathbf{u}_t$ and ν_t are assumed to be independent and standard normal. The system matrices $\mathbf{f}_t$, $\mathbf{g}_t$, $\mathbf{h}_t$, γ_t, $\mathbf{F}_t$ and $\mathbf{\Gamma}_t$ are determined, up to a set of unknown parameters θ by the value of $\mathbf{K}_t$, where $\mathbf{K}_t = (K_1, \ldots, K_n)$ is an unobserved first-order Markov process such that $p(K_t|K_1, \ldots, K_{t-1}) = p(K_t|K_{t-1})$, this makes the observations $\mathbf{y}_{1:t} = (y_1, \ldots, y_t)$ mixtures of Gaussian. In [26] a two-step algorithm is proposed to estimate mixtures of Gaussian linear state space models, which are used in the intervention of models, that arise from econometric time series and nonparametric regression. Their main contribution is the proposal of a recursive algorithm, to treat mixtures of Gaussian linear dynamic models based on a Markov Chain Monte?Carlo sampling, which allows the posterior distribution to converge rapidly to the true distribution. The algorithm starts from the fact that:

$$K_t \sim p(K_t|y_{1:t}, K_{s\neq t}) \quad ; \quad t = 1, \ldots, n \tag{3}$$

without conditioning on the states. In addition, consider for $s \leq t$, $\mathbf{K}_{s:t} = (K_s, \ldots, K_t)$, and $\mathbf{y}_{s:t} = (y_s, \ldots, y_t)$. For the generation of the K_t, suppose that $(K_1, \ldots, K_{t-1})$ is previously generated, and that the probability density function $p(x_{t-1}|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t-1})$ have a conditional mean:

$$m_{t-1} = \mathbb{E}(x_{t-1}|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t-1}) \tag{4}$$

and the conditional variance:

$$V_{t-1} = \mathbb{V}ar(x_{t-1}|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t-1}) \tag{5}$$

Now, the hidden states K_t are generated, as follows:

$$p(K_t|\mathbf{y}_{1:n}, K_{s\neq t}) \propto p(\mathbf{y}_{1:n}|K_{1:n})p(K_t|K_{s\neq t}) = p(\mathbf{y}_{t+1:n}|\mathbf{y}_{1:t}, \mathbf{K}_{1:n})p(y_t|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t})p(K_t|K_{s\neq t}) \tag{6}$$

where $p(K_t|K_{s\neq t})$ is obtained from a known prior distribution; $p(y_t|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t})$ is obtained using a structure based on the Kalman filter (see [32], [33]), and the term $p(\mathbf{y}_{t+1:n}|\mathbf{y}_{1:t}, \mathbf{K}_{1:n})$ is obtained using $n - t + 1$ Kalman filter steps given the current values of $\mathbf{K}_{1:n}$. If K_t is a discrete vector, its conditional probability density function is multinomial and can be quickly determined. A more general way to find K_t is using the Metropolis Hastings algorithm ([34]), because it is only necessary to know the conditional density up to a multiplicative constant. Now, an efficient method based on a recursive algorithm is introduced to generate $\mathbf{K}_{1:t}$, which is obtained using Lemmas 1, 2, 3 and 4, defined in the article of [26]. The algorithm is summarized, as follows:

Algorithm 1: Sampling Algorithm ([26])

Step 1: Given the initial values of $\mathbf{K}_{1:n}$, Ω_t and μ_t is calculated for $t = n-1, \ldots, 1$, using the recursions established in Lemma 2 of [26]. In this Lemma, Ω_t are defined.

Step 2 : Given $\mathbb{E}(x_0)$ and $\mathbb{V}ar(x_0)$, the following steps are executed for $t = 1, \ldots, n$

- For each K_t, R_t, m_t and V_t is obtained from m_{t-1} and V_{t-1}, using the Lemma 3 of [26].
- Then, the $p(y_t|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t})$ is obtained, as in the Lemma 3, and $p(\mathbf{y}_{t+1:n}|\mathbf{y}_{1:t}, \mathbf{K}_{1:n})$ is obtained, as in the Lemma 4 of [26].
- Get $p(K_t|\mathbf{y}_{1:n}, K_{s\neq t})$ as follows:

$$(K_t|\mathbf{y}_{1:n}, K_{s\neq t}) \sim p(K_t|\mathbf{y}_{1:t}, K_{s\neq t}) \propto p(y_t|\mathbf{y}_{1:t-1}, \mathbf{K}_{1:t})p(\mathbf{y}_{t+1:n}|\mathbf{y}_{1:t}, \mathbf{K}_{1:t})p(K_t|K_{s\neq t}). \tag{7}$$

- Update m_t and V_t as in the Lemma 3 of [26], using the value of K_t.

In [26] a more efficient alternative algorithm in three steps is proposed, which provides a general way to simulate the Markov processes $\mathbf{K}_{1:n}$ without conditioning in the states $\mathbf{x}_{1:n} = (x_1, \ldots, x_n)$, they propose a Gibbs sampling scheme, which is summarized as follows:

Algorithm 2 : Modified Sampling Algorithm ([26])

Step 1: The Markov processes $\mathbf{K}_{1:n}$ are conditioned on the parameters θ and the data $\mathbf{y}_{1:n} = (y_1, \ldots, y_n)$, the $p(\mathbf{K}_{1:n}|\theta, \mathbf{y}_{1:n})$ is sampled.
Step 2: The states $\mathbf{x}_{1:n}$ are conditioned to $\mathbf{K}_{1:t}$, $\mathbf{y}_{1:n}$ and θ, the $p(\mathbf{x}_{1:n}|\mathbf{K}_{1:n}, \mathbf{y}_{1:n}, \theta)$ is sampled.
Step 3: θ is conditioned to $\mathbf{x}_{1:n}$, $\mathbf{y}_{1:n}$ and $\mathbf{K}_{1:n}$, the $p(\theta|\mathbf{x}_{1:n}, \mathbf{y}_{1:n}, \mathbf{K}_{1:n})$ is sampled.

Steps [1] and [2] are fixed for all models. Step [1] uses the algorithm of [26]. Step [2] a Gibbs sampling procedure is proposed in [35], it is used in conjunction with the [36] as an interesting alternative and the step [3] is dependent on the model. If the vector of parameters θ conditioned to the states $\mathbf{x}_{1:n}$ is sampled, only the standard results of Bayesian inference are usually required at least if conjugate priors are used. Alternatively, if θ is conditioned K_t and $\mathbf{y}_{1:n}$, the Metropolis Hastings algorithm is used in conjunction with the Kalman filter to evaluate the likelihood function, in which case step [2] in the algorithm is not necessary.

Consider the given state space model (2), then the sampling scheme given by [35] has a different approach to the Gibbs sampler, that is, all states are generated at once using the temporal ordering of the state space model. This allows all the necessary calculations to be performed using the Kalman filtering and the smoothed Kalman filter. This approach is more efficient, in the sense that the convergence of the parameters to the posterior distribution is faster and with small errors. First, preliminary results are generated and used in the algorithm. Let $\mathbf{y}_{1:n}$ the vector of observations, $\mathbf{x}_{1:n}$ the vector of states, $\mathbf{K}_{1:n}$ vector of the hidden Markov process and $\theta = (\theta_1, \ldots, \theta_p)$ the vector of parameters. To estimate the states, Lemmas 1 and 2 given in [35] are used. The algorithm for generating the state vector is followed:

Algorithm 3 : Sampling Algorithm ([35])

Step 1: Let:

$$m_{t|j} = \mathbb{E}(x_t|y_j) \quad , \quad V_{t|j} = \mathbb{V}ar(x_t|y_j) \tag{8}$$

For $t = 1, \ldots, n$, the conditional mean $m_{t|t}$ and the conditional variance $V_{t|t}$ are obtained using the Kalman filter proposed in [32] and [33].
Step 2: Using the Lemma 1 ([35]), $p(x_t|y_t)$ is generated:

$$x_{t+1} = F_{t+1}x_t + \nu_{t+1} \tag{9}$$

with m additional observations of x_t. If ν_{t+1} is diagonal then ν^i_{t+1} $(i = 1, \ldots, m)$ are independent and the observation update step of the Kalman filter can be applied m times to get:

$$m_t = \mathbb{E}(x_t|y_t, x_{t+1}) \quad and \quad V_t = \mathbb{V}ar(x_t|y_t, x_t) \tag{10}$$

In general, it is factorizedas:

$$\Gamma_{t+1} = L_{t+1} \triangle_{t+1} L^T_{t+1} \tag{11}$$

and using the Cholesky decomposition with L_{t+1} a lower triangular matrix with ones on the diagonal and $\triangle_{t+1}$ a diagonal matrix, then:

$$\tilde{x}_{t+1} = L^{-1}_{t+1} x_{t+1} \quad , \quad \tilde{F}_{t+1} = L^{-1}_{t+1} F_{t+1} \quad , \quad \tilde{\nu}_{t+1} = L^{-1}_{t+1} \nu_{t+1}. \tag{12}$$

Now, it is generated as:

$$\tilde{x}_{t+1} = \tilde{F}_{t+1}\mathbf{x}_t + \tilde{\nu}_{t+1} \tag{13}$$

so that, for $i = 1, \ldots, m$ is generated:

$$\tilde{x}^i_{t+1} = \tilde{F}^i_{t+1} x_t + \tilde{\nu}^i_{t+1} \quad (14)$$

where $\tilde{F^T}_{t+1}$ is the i-th row of $\tilde{F}_{t+1}$, $\tilde{x}^i_{t+1}$ and $\tilde{\nu}^i_{t+1}$ are the elements i-th of $\tilde{x}_{t+1}$ and $\tilde{\nu}_{t+1}$. The elements $\tilde{\nu}^i_{t+1}$ are $N(0, \triangle^i_{t+1})$ independents, where $\triangle^i_{t+1}$ is the element of the diagonal i-th of $\triangle_{t+1}$. For $i = 1, \cdots, m$ let:

$$m_{t|t,i} = \mathbb{E}(x_t | y_t, x^1_{t+1}, \ldots, x^t_{t+1}) \quad (15)$$

$$V_{t|t,i} = \mathbb{V}ar(x_t | y_t, x^1_{t+1}, \ldots, x^t_{t+1}) \quad (16)$$

and define $m_{t|t,0} = m_{t|t}$ and $V_{t|t,0} = V_{t|t}$. The observation update step m of the Kalman filter is now applied to the equation (14). For $i = 1, \ldots, m$ is generated:

$$\varepsilon_{t,i} = \tilde{x}^1_{t+1} - \tilde{F^T}_{t+1} x_{t|t,i-1} \quad (17)$$

$$R_{t,i} = \tilde{F^T}^i_{t+1} S_{t|t,i-1} \tilde{F}_{t+1} + \triangle^i_{t+1} \quad (18)$$

$$m_{t|t,i} = m_{t|t,i-1} + V_{t|t,i-1} \tilde{F}^i_{t+1} \varepsilon_{t,i} / R_{t,i} \quad (19)$$

and

$$V_{t|t,i} = V_{t|t,i-1} - V_{t|t,i-1} \tilde{F}^i_{t+1} \tilde{F^T}^i_{t+1} V_{t|t,i-1} / R_{t,i} \quad (20)$$

Therefore:

$$m_{t|t,m} = \mathbb{E}(x_t | y_t, x_{t+1}) \quad (21)$$

and

$$V_{t|t,m} = \mathbb{V}ar(x_t | y_t, x_{t+1}) \quad (22)$$

For precise details of the algorithms, in particular how the states and the indicator variables are generated, see appendices I and II ([35]).

3. The Markov Switching Model for Industrial Production Rate of MERCOSUR

The model given in equations (1) could be written as a state space model, as in the equations given in (2). Let $\mathbf{x}_t = (u_t, z_t, \ldots, z_{t-p+1})$, $\mathbf{u}_t = a_t$, $\nu_t = (o_t, e_t)$, $\mathbf{g}_t = \sigma_t \mathbf{K}_{\delta_t} \delta_t$, $\mathbf{h}_t = (1, 1, 0, \ldots, 0)$, $\gamma_t = \sigma_t \mathbf{K}_{a_t}$, and $f_t = 0$:

$$\mathbf{F}_t = \begin{pmatrix} 1 & 0 & 0 & 0 & & 0 & 0 \\ 0 & \phi_1 & \phi_2 & \phi_3 & \cdots & \phi_{p-1} & \phi_p \\ 0 & 1 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 1 & 0 & & 0 & 0 \\ & & \vdots & & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}$$

and

$$\mathbf{\Gamma}_t = \begin{pmatrix} \sigma_t K_{ot} & 0 \\ 0 & \sigma_t K_{e_t} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}$$

The parameters of the model are $\theta = (\phi_1, \dots, \phi_p, p, z_0, \dots, z_{-p}, \nu_0, \sigma_1, \sigma_2, \tau)$, in conjunction with the other parameters, governing the distribution of $\mathbf{K}_t = (K_{at}, K_{et}, K_{ot}, K_{\delta t})$. The prior for the lag length p assumes, that no lags are skipped; that is, if $\phi_j \neq 0$ then $\phi_i \neq 0$, $\forall i < j$. The support of p is given by $\{0, 1, 2, 3, 4\}$, with a prior probabilities proportional to $\{5, 4, 3, 2, 1\}$ (see [1]). The priori distribution for $(\phi_1, \dots, \phi_p)$, given p and $\sigma_i^2, i = 1, 2$, are given by:

$$\phi_1, \dots, \phi_p | p, \sigma_i^2 \sim N\left(0, \frac{\sigma_i^2}{5} V_p^{-1}\right) \tag{23}$$

where V_p is the covariance matrix of $(y_{t-1}, \dots, y_{t-p})$. This is considered as an empirical prior, which captures more or less the structure of the scale and the correlation of the parameters ϕ_i, $i = 1, \dots, p$; while for moderate and large samples is a probability function much more diffuse than the a prior of ϕ_i. This prior for ϕ_i is taken by the g-prior proposed by [37] and for this case, the matrix V_p is the covariance matrix of the covariates in the regression.

On the other hand, suppose that σ_1^2 and σ_2^2 are independent a prior, and each one is distributed as a Inverse Gamma, $\sigma_i^2 \sim IG(5\hat{\sigma}^2, 5)$, $i = 1, 2$ with 5 degrees of freedom, and $\hat{\sigma}$ is the residual variance of an model $AR(4)$ for $\mathbf{y}_{1:t}$. The priors for δ_1 and δ_2 are independent with $\delta_i \sim N(-2.2, 1)$, for $i = 1, 2$. The prior on τ is uniform on the central 70% of the sample, such that we have at least 15% of the observations before and after the structural change in σ_t and δ_t. Finally, the priors for $(z_0, \dots, z_p)$, and ν_0 are normal and independent, centered at zero and over-dispersed.

Let $\mathbf{K}_t^* = (K_{a_t}, K_{e_t}, K_{o_t})$, such that $\mathbf{K}_t = (K_{a_t}, K_{e_t}, K_{o_t}, K_{\delta_t})$, where $\mathbf{K}_t^*$ and $\mathbf{K}_{\delta_t}$ are independent. The support of $\mathbf{K}_t^*$ is given by the five triplets as follows: $(0, 1, 0)$, $(0, 3, 3)$, $(3, 1, 0)$, $(5, 1, 0)$, and $(0, 1, 0.3)$, assuming that outliers and innovation cannot occur simultaneously, and that outliers do not occur at times when the mean growth rate changes. The priori probabilities for the five possible states of $\mathbf{K}_t^*$ are set equal to (0.95, 0.01, 0.03, 0.005 and 0.005), with $n_{01} + \ldots + n_{05} = 500$, where $n_{01}, \dots, n_{05}$ are the sample occurrences of the state j for $\mathbf{K}_t^*$. Finally, a Beta distribution for transition probabilities $p_{00,\delta}$ and $p_{11,\delta}$ is supposed; that is $p_{00,\delta} \sim Beta(0.9, 25)$ and $p_{11,\delta} \sim Beta(0.85, 25)$.

In [1] an application of the algorithm of [26] is proposed to estimate the parameters of a Markov switching model, and that takes into account the outliers, the instability in the mean, the non-linearity, and the volatility of the growth and decrease. The model includes an autoregressive component and a structure change in the parameters σ_t and δ_t. The methodology by way of several examples is illustrated and the utility using empirical data of the growth rate of the industrial production of the countries of $G7$ is demonstrated. The algorithm of [1] is summarized, as follows:

Algorithm 4 : Sampling Algorithm ([1])

Step 1: Sample $(\mathbf{K}_{1:n} | \theta, K_t, \mathbf{y}_{1:n})$ using the algorithm of [26].

Step 2: Consider $y_t^* = y_t - \sigma_t K_{at} a_t$. Given $\mathbf{K}_{1:n}$ sample y_t^*, ν_t, and z_t, as in the algorithm of [35].

Step 3: Given $\mathbf{K}_{1:n}$, sample the probabilities of the states $\mathbf{K}_t^*$, as follows:

$$p(\mathbf{K}_t^* | \mathbf{K}_{1:n}) \sim Dir(n_{01} + n_{s1}, n_{02} + n_{s2}, n_{03} + n_{s3}, n_{04} + n_{s4}, n_{05} + n_{s5}) \tag{24}$$

where n_{sj} are the sample occurrences of the state j for $\mathbf{K}_t^*$.

Step 4: Given $\mathbf{K}_{1:n}$, sample $p_{00,\delta}$ and $p_{11,\delta}$ using a step from the Metropolis Hastings algorithm as in [38].

Step 5: Sample σ_1, σ_2 and τ conditional on $\mathbf{z}_{1:n}$, and $\phi_1, \dots, \phi_p$ as in [39].

Step 6: Sample δ_1 and δ_2 conditional on $\mathbf{z}_{1:n}$, ν_t and θ applying a standard conjugate analysis to

$$z_t^* = y_t^* - \nu_t - \phi_1 z_{t-1} - \ldots - \phi_p z_{t-p} = \sigma_t \delta K_{\delta t} + \sigma_t K_{et} e_t \tag{25}$$

By conditioning the variances σ_1^2 and σ_2^2, the values of δ_1 and δ_2 are generated separately based on the under-samples $\{z_t^*\}_{t=1}^{\tau}$ and $\{z_t^*\}_{t=\tau+1}^{n}$, respectively. An acceptance and rejection step imposes $\delta_j < 0$, $j = 1, 2$.

Step 7: Update the length of the delay p conditional on $\mathbf{x}_{1:n}$, τ, σ_1^2, and σ_2^2, as follows:

$$p(p|\mathbf{y}_{1:n}, \mathbf{x}_{1:n}, \tau, \sigma_1^2, \sigma_2^2) \propto p(\mathbf{y}_{1:n}|p, \mathbf{x}_{1:n}, \tau, \sigma_1^2, \sigma_2^2)p(p) \tag{26}$$

where the first term on the right side is the marginal distribution of $\mathbf{y}_{1:n}$ (conditional on $\mathbf{x}_{1:n}$, τ, σ_1^2, σ_2^2), which is analytically available and the second term is the a priori distribution of p.

The a priori distribution of the delay length is considered as a discrete uniform distribution $p = 0, \ldots, 4$.

Step 8: Update $(\phi_1, \ldots, \phi_p|\sigma_1, \sigma_2, p)$, using a standard conjugate analysis to z_t.

An improved version of algorithm 4, which facilitates computational calculations, is given below:

Algorithm 5 : Modified Sampling Algorithm ([1])

Step 1: Generate $\mathbf{K}_{1:n}$ according to the algorithm in [26].

Step 2: Let $y_t^* = y_t - \sigma_t K_{a_t} a_t$. Given $\mathbf{K}_{1:n}$, then y_t^*, ν_t and z_t are generated as in [35].

Step 3: Given $\mathbf{K}_{1:n}$ the probabilities of $\mathbf{K}_t^*$ is obtained, whose conditional distribution is Dirichlet; this is:

$$p(\mathbf{K}_t^*) \sim Dirichlet(n_{01} + n_{s1}, n_{02} + n_{s2}, n_{03} + n_{s3}, n_{04} + n_{s4}, n_{05} + n_{s5}) \tag{27}$$

where n_{sj} are the sample occurrences of the state j for $\mathbf{K}_t^*$.

Step 4: Given $\mathbf{K}_{1:n}$, and assuming independence between $p_{11,\delta}$ and $p_{00,\delta}$, with the following priori distributions:

$$p_{00,\delta} \sim Beta(a, b) \quad ; \quad p_{11,\delta} \sim Beta(c, d) \tag{28}$$

and assuming that

$$\mathbf{K}_{1:n} \sim Binomial(n, \theta) \tag{29}$$

then

$$p_{00,\delta}, p_{11,\delta}|K_t \sim Beta(\mathbf{K}_{1:n} + a + c - 1, n - \mathbf{K}_{1:n} + b + d - 1) \tag{30}$$

To sample $p_{11,\delta}$, and $p_{00,\delta}$, using a step from the Metropolis-Hastings algorithm as in [38].

Step 5: Sample $(\sigma_1, \sigma_2|z_t, \phi_1, \ldots, \phi_p)$ as in [39]. Note that under the assumption of independence between τ, σ_1, and σ_2:

$$p(\sigma_1, \sigma_2, \tau|z_t, \phi_1, \ldots, \phi_p) \propto p(z_t, \phi_1, \ldots, \phi_p|\sigma_1, \sigma_2, \tau)p(\sigma_1)p(\sigma_2)p(\tau) \tag{31}$$

In addition it is assumed:

$$z_t \sim N(\mu_{z_t}, \sigma_{z_t}) \tag{32}$$

where:

$$\mu_{z_t} = \sum_{j=1}^{p} \phi_j z_{t-1} \quad ; \quad \sigma_{z_t} = \sigma_t \sigma K_{e_t} \tag{33}$$

$$(\phi_1, \ldots, \phi_p|p, \sigma_i^2) \sim N(0, \sigma_\phi) \tag{34}$$

$$\sigma_\phi = \frac{\sigma_i^2}{5} V_p^{-1} \quad ; \quad i = 1, 2 \tag{35}$$

$$\sigma_i^2 \sim IG(\alpha_0 \hat{\sigma}^2, \beta_0) \quad ; \quad i = 1, 2 \quad ; \quad \tau \sim U(a, b) \tag{36}$$

Note that V_p^{-1} represents the variance of the observations y_t and $\hat{\sigma}^2$ represents the variance of a model $AR(4)$, IG represents a Gamma Inverse distribution, $N(\mu, \sigma^2)$ represents a Gaussian distribution with mean μ, and variance σ^2, and $U(a, b)$ represents a Uniform distribution in the range (a, b). Let $\sigma_i = \sigma_t$, then is assumed:

$$\begin{aligned} p(\sigma_1, \sigma_2, \tau | z_t, \phi_1, \ldots, \phi_p) &\propto \prod_{i=1}^{p+1} \exp\left(-\frac{1}{2\sigma_{z_t}} (z_t - \mu_{z_t})^2\right) \prod_{j=1}^{p} \exp\left(-\frac{\phi_j^2}{2\sigma_\phi}\right) \sigma_t^{-(\alpha_0 \hat{\sigma}^2 + 1)} \exp\left(-\frac{\beta_0}{\sigma_t}\right) \\ &\propto \sigma_t^{-(\alpha_0 \hat{\sigma}^2 + 1)} \exp\left(-\frac{1}{2\sigma_{z_t}} \sum_{i=1}^{p+1} (z_t - \mu_{z_t})^2\right) \exp\left(-\frac{1}{2\sigma_\phi} \sum_{j=1}^{p} \phi_j^2\right) \exp\left(-\frac{\beta_0}{\sigma_t}\right) \\ &\propto \sigma_t^{-(\alpha_0 \hat{\sigma}^2 + 1)} \exp\left(-\frac{1}{\sigma_t} [\frac{\sum_{i=1}^{p+1} (z_t - \mu_{z_t})^2}{2K_{e_t}} + \frac{5}{2\sigma_t} V_p \sum_{j=1}^{p} \phi_j^2 + \beta_0]\right) \end{aligned} \tag{37}$$

Substituting $\sigma_{z_t} = \sigma_t K_{e_t}$, and $\sigma_\phi = \frac{\sigma_t^2}{5} V_p^{-1}$, is obtained:

$$p(\sigma_1, \sigma_2, \tau | z, \phi_1, \ldots, \phi_p) \sim IG\left(\alpha_0 \hat{\sigma}^2, \frac{\sum_{i=1}^{p+1} (z_t - \mu_{z_t})^2}{2K_{e_t}} + \frac{5}{2\sigma_t} V_p \sum_{j=1}^{p} \phi_j^2 + \beta_0\right) \tag{38}$$

Step 6: Simulate $(\delta_1, \delta_2 | z_{1:t}, \nu)$ as follows:

$$p(\delta_1, \delta_2 | z_{1:t}, \nu) \propto p(\mathbf{z}_{1:t} | \delta_1, \delta_2) p(\nu | \delta_1, \delta_2) p(\delta_1) p(\delta_2) \tag{39}$$

$$\delta_i \sim N(0, 1) \quad ; \quad i = 1, 2 \tag{40}$$

$$z_t \sim N(\mu_z, \sigma_z^2) \quad ; \quad \mu_z = \sum_{j=1}^{p} \phi_j z_{t-j} \quad ; \quad \sigma_z^2 = \sigma_t K_{e_t} \tag{41}$$

$$\nu_t \sim N(\mu_\nu, \sigma_\nu^2) \quad ; \quad \mu_\nu = \nu_{t-1} \quad ; \quad \sigma_\nu^2 = \sigma_t K_{o_t} \tag{42}$$

then the posterior distribution has the following form:

$$\begin{aligned} p(\delta_1, \delta_2 | z_{1:t}, \nu) &\propto \prod_{t=1}^{p+1} \exp\left(-\frac{1}{2\sigma_z} (z_t - \mu_z)^2\right) \exp\left(-\frac{1}{2\sigma_\nu} (\nu_t - \mu_\nu)^2\right) \exp\left(-\frac{1}{2} \delta_1^2\right) \exp\left(-\frac{1}{2} \delta_2^2\right) \\ &\propto \exp\left(\left[-\frac{1}{2} \delta_1^2 - \frac{1}{2} \delta_2^2\right]\right) \propto \exp\left(-\frac{1}{2} \delta_1^2\right) \exp\left(-\frac{1}{2} \delta_2^2\right) = p(\delta_1) p(\delta_2) \end{aligned} \tag{43}$$

Based on the above calculations, samples δ_1 and δ_2 according to their distribution are estimated, and using conjugated a priori standard distributions, as follows:

$$z_t^* = y_t^* - \nu_t - \phi_1 z_{t-1} - \ldots - \phi_p z_{t-p} = \sigma_t \delta K_{\delta_t} + \sigma_t K_{e_t} e_t \tag{44}$$

Conditioning on the variances σ_1^2 and σ_2^2 then δ_1 and δ_2 are generated separately from subsamples $\{z_t^*\}_{t=1}^{\tau}$ and $\{z_t^*\}_{t=\tau+1}^{n}$, respectively. A step of acceptance and rejection makes $\delta_j < 0$, $j = 1, 2$.

Step 7: Update the lag p length $p(p|\mathbf{x}_{1:t}, \tau, \sigma_1^2, \sigma_2^2)$, as follows:

$$p(p|\mathbf{y}_{1:n}, \mathbf{x}_{1:n}, \tau, \sigma_1^2, \sigma_2^2) \propto p(\mathbf{y}_{1:n}|p, \mathbf{x}_{1:n}, \tau, \sigma_1^2, \sigma_2^2)p(p) \tag{45}$$

It's known that:

$$y_t \sim N(\mu_y, \sigma_y^2) \quad ; \quad \mu_y = z_t + \nu_t + \sigma_t K_{\delta_t}\delta_t \quad ; \quad \sigma_y^2 = \sigma_t K_{a_t} \tag{46}$$

the lag p is distributed as a discrete Uniform, that is $p \sim U(0, \dots, 4)$, so:

$$\begin{aligned} p(p|\mathbf{y}_{1:t}, \mathbf{x}_{1:t}, \tau, \sigma_1^2, \sigma_2^2) &\propto \prod_{i=1}^{n} \exp\left(-\frac{1}{2\sigma_y}(y_i - \mu_y)^2\right) p(p) \\ &\propto \prod_{i=1}^{n} \exp\left(-\frac{1}{2\sigma_y}(y_i - \mu_y)^2\right) \propto \prod_{i=1}^{n} N(\mu_y, \sigma_y^2) \end{aligned} \tag{47}$$

Step 8: Updated $p(\phi_1, \dots, \phi_p|\sigma_1, \sigma_2, p)$ and a conjugate analysis is applied to z_t, that is, $p(z_t|\phi_1, \dots, \phi_p, \sigma_t^2, p)$ is calculated, knowing that:

$$p(z_t|\phi_1, \dots, \phi_p, \sigma_t^2, p) \propto p(\phi_1, \dots, \phi_p, \sigma_t^2, p|z_t)p(\phi_1, \dots, \phi_p|\sigma_t^2, p) \tag{48}$$

and

$$z_t \sim N(\mu_{z_t}, \sigma_{z_t}) \quad ; \quad \mu_{z_t} = \sum_{j=1}^{p} \phi_j z_{t-1} \quad ; \quad \sigma_{z_t} = \sigma_t \sigma K_{e_t} \tag{49}$$

$$(\phi_1, \dots, \phi_p|\sigma_t^2, p) \sim N(0, \sigma_\phi) \quad ; \quad \sigma_\phi = \frac{\sigma_i^2}{5} V_p^{-1} \tag{50}$$

$$\begin{aligned} p(z_t|\phi_1, \dots, \phi_p, \sigma_t^2, p) &\propto \prod_{i=1}^{p+1} \exp\left(-\frac{1}{2\sigma_{z_t}}(z_t - \mu_{z_t})^2\right) \prod_{j=1}^{p} \exp\left(-\frac{\phi_j^2}{2\sigma_\phi}\right) \\ &\propto \exp\left(-\frac{1}{2\sigma_{z_t}} \sum_{i=1}^{p+1}(z_t - \mu_{z_t})^2\right) \exp\left(-\frac{1}{2\sigma_\phi} \sum_{j=1}^{p} \phi_j^2\right) \\ &\propto \exp\left(-\frac{1}{2\sigma_{z_t}} \sum_{i=1}^{p+1}(z_t - \mu_{z_t})^2\right) \\ &\sim N(\mu_{z_t}, \sigma_{z_t}) \end{aligned} \tag{51}$$

4. Empirical Results

In this section, the results obtained from the estimation of industrial production of the MERCOSUR countries through a Markov switching model is showed. The data used to develop the methodology are obtained from the World Bank Group (WBG) (https://datos.bancomundial.org/indicador/NV.IND.TOTL.KD.ZG?view=chart) and it is shown in Figure (1). The variable analyzed is the industrial production rate, which is determined by percentage of production of the exploitation of mines and quarries, manufacturing industries, construction, electricity, gas and water, that are generated in these countries. The data are represented through annual time series from the year 1992 to the year 2012. The priors is used to initialize the model, as follows:

- Argentina: $p_{00} \sim Beta(0.9, 25)$, $p_{11} \sim Beta(0.85, 25)$, $K_0 = (1, 1, 1, 1)'$, $\delta_1 = 0.005$, $\delta_2 = 0.005$, $\alpha_0 = 0.1$ and $\beta_i = 1$.

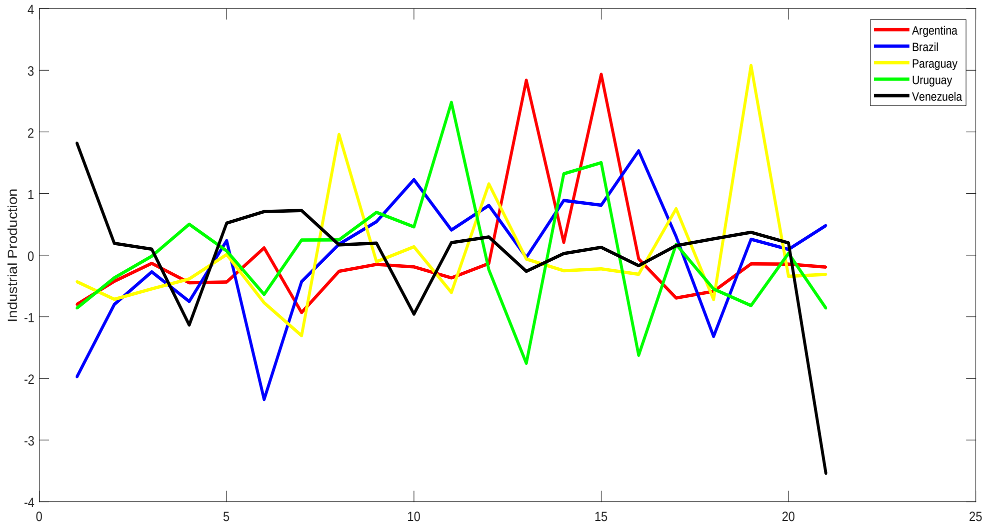

Figura 1. Industrial production of the MERCOSUR countries

- Brasil: $p_{00} \sim Beta(0.9, 25)$, $p_{11} \sim Beta(0.85, 25)$, $K_0 = (1,1,1,1)'$, $\delta_1 = 0.5$, $\delta_2 = 0.5$ $\alpha_0 = 5$ and $\beta_i = 0.3$.
- Paraguay: $p_{00} \sim Beta(0.9, 25)$, $p_{11} \sim Beta(0.85, 25)$, $K_0 = (1,1,1,1)'$, $\delta_1 = 0.5$, $\delta_2 = 0.5$, $\alpha_0 = 3$ and $\beta_i = 0.3$.
- Uruguay: $p_{00} \sim Beta(0.9, 25)$, $p_{11} \sim Beta(0.85, 25)$, $K_0 = (1,1,1,1)'$, $\delta_1 = 0.5$, $\delta_2 = 0.5$, $\alpha_0 = 3$ and $\beta_i = 0.3$.
- Venezuela: $p_{00} \sim Beta(0.9, 25)$, $p_{11} \sim Beta(0.85, 25)$, $K_0 = (1,1,1,1)'$, $\delta_1 = 0.5$, $\delta_2 = 0.5$, $\alpha_0 = 3$ and $\beta_i = 0.3$.

The algorithms under programming language $MATLAB$ are implemented, and on a $Siragon$ $Ultrabook\ UB - 3000$ with a processor $Intel(R)\ Core(TM)\ i3 - 2367M\ CPU$ @ $1.40GHz$, $4GB$ of RAM and Windows Operating System 8.1 of $64 - bit$ is executed.

The results of the estimation are shown in Table (1) in terms of a posterior means and standard deviations of unknown parameters, where it can be seen, that the mean and the variance of σ_1 and σ_2 are equal which means, that economically there is no reduction in volatility for all countries.

In addition it can be noticed that $|\delta_2| > |\delta_1|$, indicating there is no reduction of the depth of the economic cycle. It is defined as the difference between the average growth of expansions and recessions.

In Figures (2-6) four panels with the graphs of the estimated parameters are illustrated. In panel (a) the posterior average of the processes K_{a_t} (green) and K_{e_t} (blue) are illustrated, where K_{a_t} shows additive outliers and K_{e_t} shows the outliers of innovation. In panel (b) the posterior average of ν is illustrated, which indicates the changes in the average growth rate. In panel (c) the posterior average of the Markov switching process K_{δ_t} is illustrated, which allows to identify break points; and in panel (d) the standardized observations and the posterior average of $\mu = \nu_t - \sigma_t K_{\delta_t} \delta_t$ (blue) are illustrated.

The production rate of Argentina in Figure (2) is illustrated, where the model identifies high and low growth regimes, which is displayed in panel (c) between the time intervals $1994 - 1998$,

Tabla 1. The table presents posterior means of the parameters in the model (1) estimated for quarterly IP growth rates for the MERCOSUR countries over the period 1992 − 2012. Posterior standard deviations are given in parentheses.

Parameters	Argentina	Brasil	Paraguay	Uruguay	Venezuela
σ_1	4.999 (22.669)	0.016 (0.053)	0.029 (0.096)	0.050 (0.163)	0.007 (0.024)
σ_2	4.999 (22.669)	0.016 (0.053)	0.029 (0.096)	0.050 (0.163)	0.007 (0.0241)
δ_1	0.105 (0.882)	0.056 (0.946)	0.053 (0.864)	0.067 (0.628)	0.052 (0.950)
δ_2	-0.137 (1.067)	0.115 (0.904)	0.094 (0.818)	0.070 (1.174)	0.114 (1.269)
p	0.511 (0)	0.235 (0)	0.134 (0)	0.119 (0)	0.798 (0)
ϕ_1	-59.851 (308.949)	-0.050 (0.090)	0.020 (0.096)	-0.003 (0.191)	-0.105 (0.921)
$p_{00,\delta}$	0.020 (0.022)	0.040 (0.035)	0.013 (0)	0.011 (0)	0.020 (0.008)
$p_{11,\delta}$	0.010 (0)	0.020 (0)	0.013 (0)	0.020 (0)	0.016 (0.016)

2000 − 2005, and 2010 − 2012. These results are validated by the representation in the panel (a). In panel (b), it is observed that the rate of growth decreased until 1996 stabilizing from the year 1997 with very small changes. In panel (d), it is noted non-linear behavior of the data, a minimum is also observed in 2002, and a maximum in 2003.

The production rate of Brasil in Figure (3) is illustrated, where the model identifies the high and low growth regimes, see panel (c). In panel (a) the existence of the regimes of the last intervals is illustrated with better precision. In panel (b) the increases of growth rate up to 1995 is illustrated, and stabilized thereafter with very small changes. In panel (d) there are significant changes in the last periods, though stable between then in the mean. The nonlinearity of the data are also shown, and a minimum in 2009 and a maximum in the 2010 is observed.

The production rate of Paraguay in Figure (4) is observed. The model identifies the high and low growth regimes which can be visualized in panel (c), between the time intervals 1992 − 1994, 2003−2006, and 2008−2010. These results are validated by panel (a). In the panel (d) non-linearity of the data is observed. In panel (b) the growth rate increases up to 1994, stabilizing thereafter with very small changes.

The production rate of Uruguay in Figure (5) is observed, where the model identifies high and low growth regimes, which can be visualized in panel (c), between the time intervals 1992 − 1994, 1994 − 1996, 1998 − 2002, 2002 − 2004, 2004 − 2006 and 2010 − 2012. These results are validated in panel (a). In panel (b) the growth rate decreased until 1996, stabilizing from then on with very small changes despite the great variability of production rate. In panel (d) the non-linearity is observed, along with a minimum in 2002, and a maximum in 2005.

The production rate of Venezuela in Figure (6) is observed, where the model identifies the high and low growth regimes. This is visualized in panel (c) through the time intervals 2001 − 2004, 2004 − 2006 and 2007 − 2010. This is, are validated in the panel (a). In panel (d) the non-linearity of the data is observed, with a minimum of 2002 and a maximum of 2004. In panel (b), it is shown

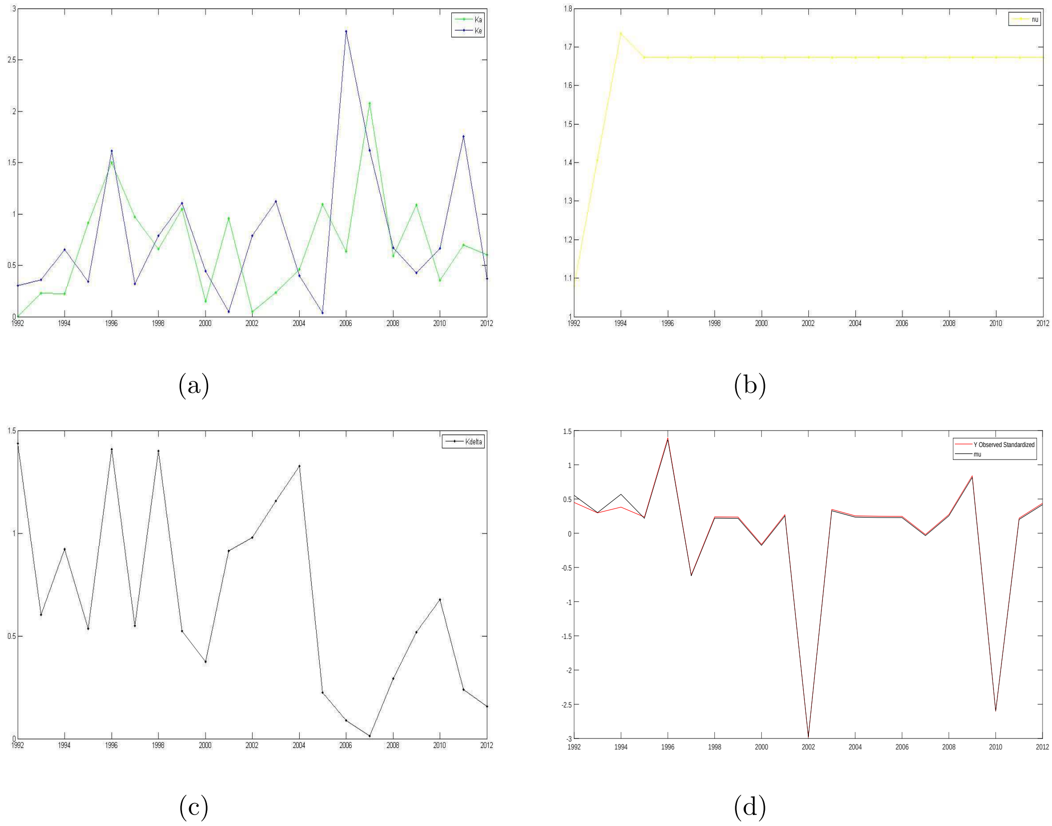

Figura 2. Estimation of K_{a_t}, K_{e_t}, ν, K_{δ_t}, and μ for Argentina.

that the rate of growth increased and decreased drastically up to 1995, stabilizing with very small changes. In addition, in panels (a) and (c) the model shows that in the industrial production rate grows and decreases over time. The exactitude of occurrence of the event is not specified very well, but may be due to the small amount of data that is available.

On the other hand, if these results of the MERCOSUR countries are compared with the study carried out by [1] on the countries of the $G7$, in these last countries data of 43 years divided quarterly is available, which allowed the model to capture breakpoints, outliers, and nonlinear behavior without any inconvenience in these economies.

The proposed model reconstructs the states of economies considered showing: the recession process in the decade of the nineties and the financial crisis of the end of the year 2008 and the beginning of the year 2009. Similarly as demonstrated in [27] and [28], the existence of an asymmetrical behavior of the business cycle regime with each country is evidenced, which allows us to infer that Latin American economies, work differently in expansion and contraction, showing different economic cycles for each country. It is important to emphasize that, as established in [27] in the case of a common economic cycle among the MERCOSUR countries, all economic cycles of the countries of the continent should react in a similar way as the Brazil cycle does in a synchronized way, that is to say in the same period or with little delay. This analysis does not show a joint response from all countries, but rather small associations between groups, which may be an

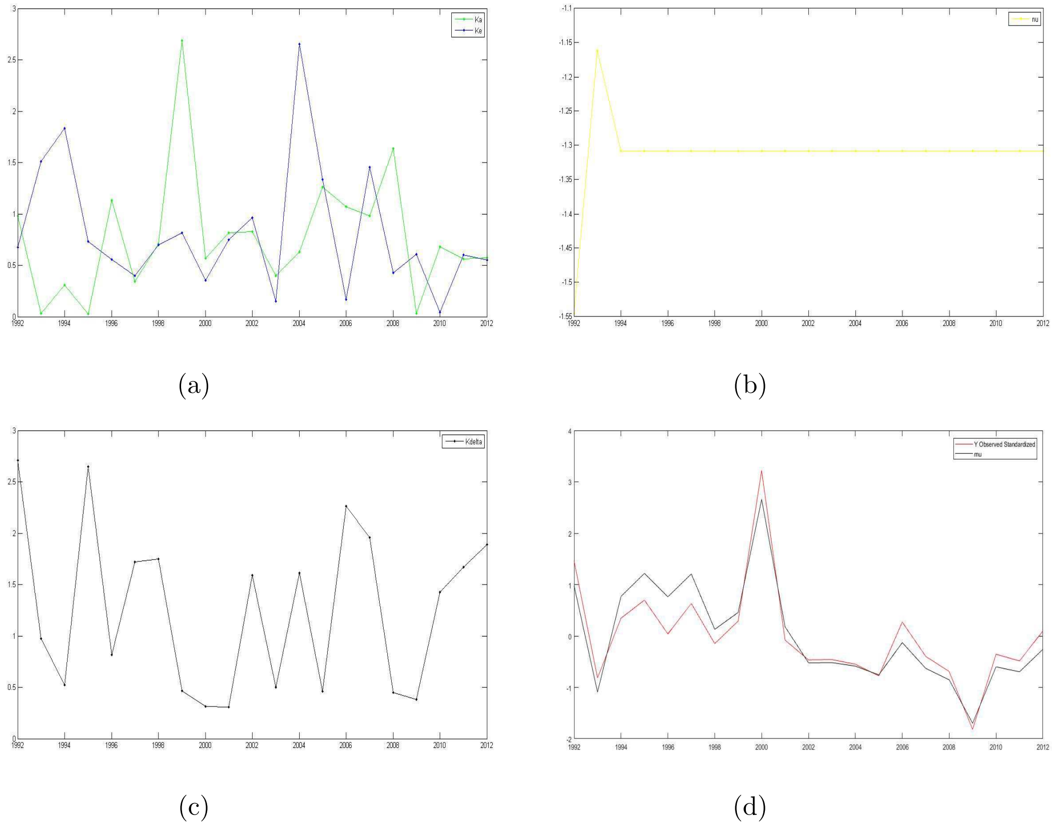

Figura 3. Estimation of K_{a_t}, K_{e_t}, ν, K_{δ_t}, and μ for Brasil.

indication of the lack of a common cycle. In [31] it was demonstrated that despite bilateral trade and financial links between nations and that there are important relationships in some pairs of countries, there is no evidence to suggest the existence of a common cycle in the region. Despite the fact that Brazil is the country with the highest level of production in South America, and one of the biggest in the world, it is expected it would have a more independent economic behavior than the smaller countries in the region, and more dependent on world economic activity. Additionally, the model proposed in this paper considers the inherent non-linearity of economic cycles, where real values are smoothed to reduce the importance of short-run erratic fluctuations. Also, to reconstruct the chronology of the common cycle from the smoothed probabilities obtained by the proposed algorithms, these results are similarly obtained in [29], where it is demonstrated that the Latin American economy experiences a great number of fluctuations, with the additional contribution that the methodology proposed in this work allows to capture breakpoints, outliers and determine non-linear behavior of the economic series simultaneously.

To validate the quality of the model estimate, several statistical indices are usually used as the evaluation criteria for forecasting models:

- The square root of the mean quadratic error (RSME) is used as a standard statistical

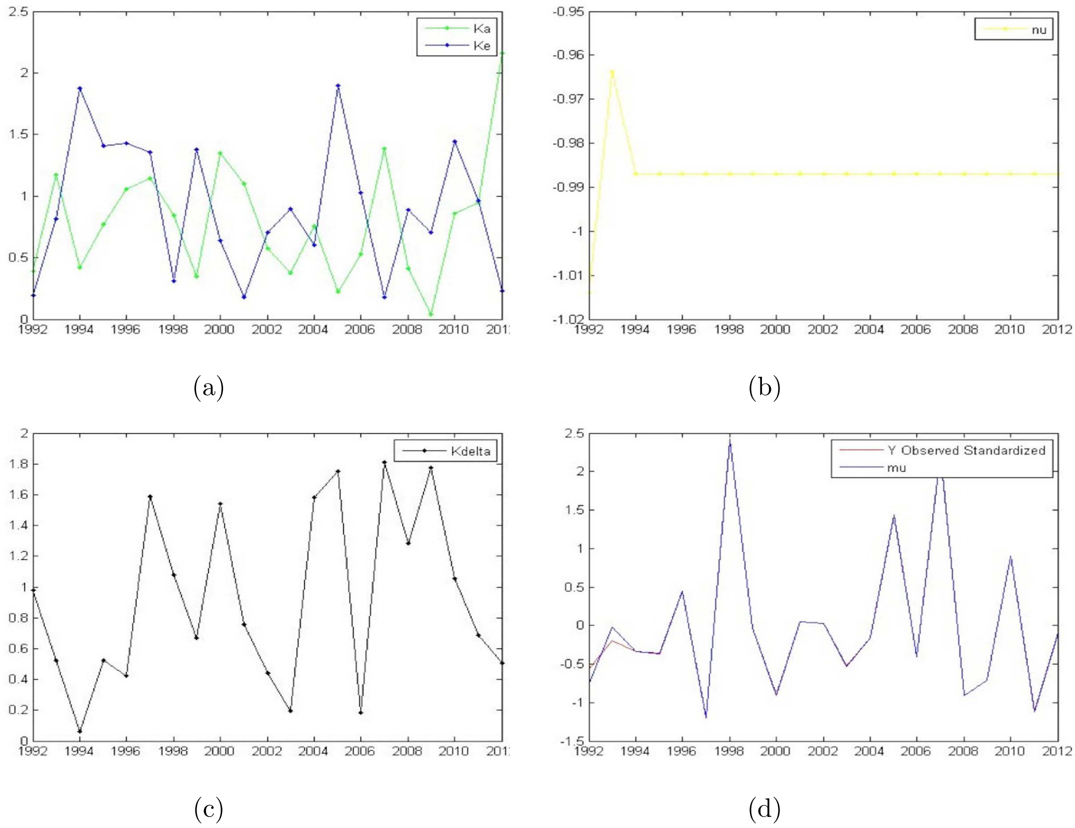

Figura 4. Estimation of K_{a_t}, K_{e_t}, ν, K_{δ_t}, and μ for Paraguay.

metric to measure model performance, and defined as:

$$RMSE = \sqrt{\sum_{t=T+1}^{T+h} \frac{(y_i - \hat{y}_i)^2}{h}} \tag{52}$$

where y_i is the value of the true time series, and $\hat{y}_i$ is the value of the estimated time series.

- The Diebold-Mariano Test (comparing predictive accuracy of two forecasts). The classical DM test is originally proposed by [40], and the routine of the classical version of DM test is defined, as follows. Let y_t denote the actual series and $\hat{y}_{i,t}^h$ denote the i^{th} competing $h-$ step forecasting series. Supposing the forecasting errors from i^{th} competing models are $e_{i,t}^h$, $i = 1, 2, \ldots, m$, where m is the number of the forecasting models. Then $h-$step forecasting errors, $e_{i,t}^h$, is:

$$e_{i,t}^h = y_t^h - y_{i,t}^h \quad ; \quad i = 1, 2, \ldots, m \tag{53}$$

The accuracy of each forecast is measured by the loss function:

$$L\left(y_i^h, y_{i,t}^h\right) = L\left(e_{i,t}^h\right) \tag{54}$$

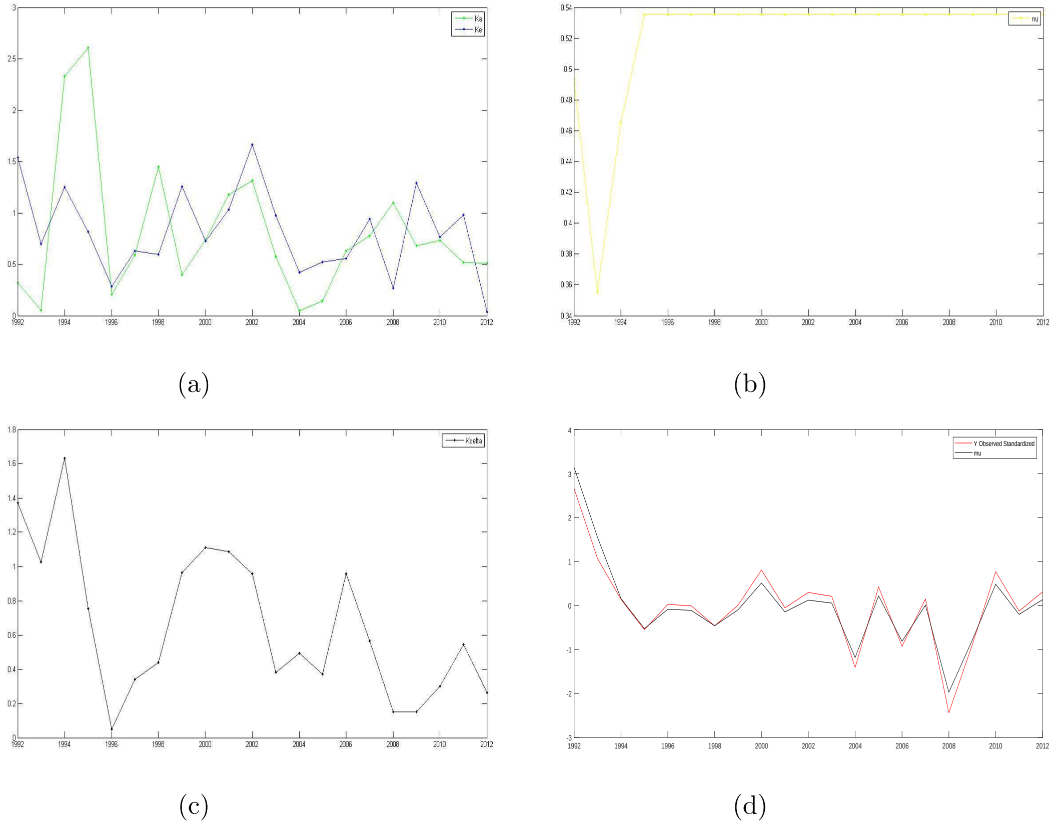

Figura 5. Estimation of K_{a_t}, K_{e_t}, ν, K_{δ_t}, and μ for Uruguay.

Let $h = 1$, there are lots of loss functions, and the most popular and usually adopted loss functions in power systems are the squared-error loss function and the absolute-error loss function:

- Squared-error loss function:

$$L_2(y_i, y_{i,t}) = L_2(e_{i,t}) = \sum_{t=1}^{T} \left(e_{i,t}^2\right) \tag{55}$$

- Absolute-error loss function:

$$L_1(y_i, y_{i,t}) = L_1(e_{i,t}) = \sum_{t=1}^{T} |e_{i,t}^2| \tag{56}$$

The squared-error loss and the absolute-error loss are both symmetric around the origin point. Furthermore, larger errors are penalized more severely by the squared-error loss one. To determine whether one forecasting model (say, the first model, model A) predicts more accurately than another (say, the second model, model B), the hypothesis of equal accuracy can be tested. The null hypothesis is given as:

- $H_0 : \mathbb{E}(L(e_{1,t})) = \mathbb{E}(L(e_{2,t}))$
- $H_a : \mathbb{E}(L(e_{1,t})) \neq \mathbb{E}(L(e_{2,t}))$, alternative hypothesis that one is better than the other

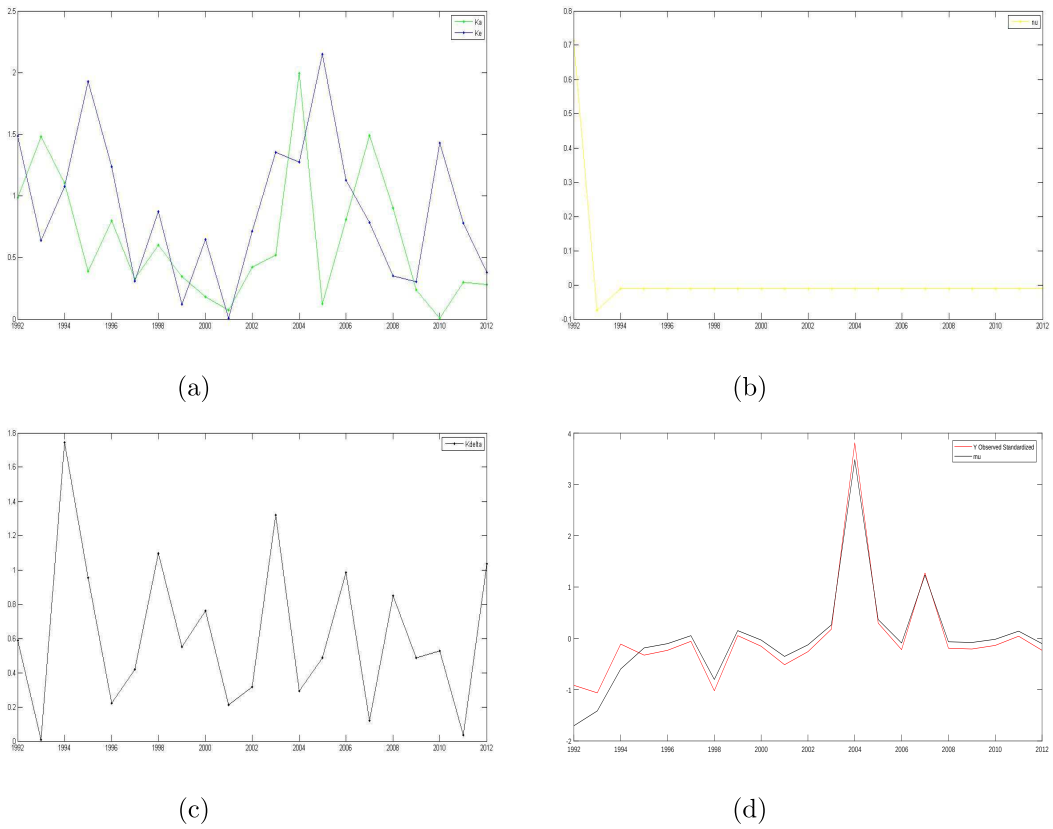

Figura 6. Estimation of K_{a_t}, K_{e_t}, ν, K_{δ_t}, and μ for Venezuela.

The Diebold-Mariano (DM) test is based on the loss differentials d_t:

$$d_t = L(e_{1,t}) - L(e_{2,t}) \tag{57}$$

Equivalently, the null hypothesis of equal predictive accuracy is shown as $E(d_i) = 0$. Then, let the sample mean loss differential $\bar{d}$, be:

$$\bar{d} = \frac{1}{T}\sum_{i=1}^{T} d_i \tag{58}$$

The DM test statistic is:

$$DM = \frac{\bar{d}}{\sqrt{\dfrac{2\pi \hat{f}_d(0)}{T}}} \tag{59}$$

where $2\pi\hat{f}_d(0)$ is a consistent estimator of the asymptotic variance of $\sqrt{T}d$, and where $\hat{f}_d(0)$ defined by:

$$\hat{f}_d(0) = \frac{1}{2\pi}\sum_{k=-(T-1)}^{T-1} I\left(\frac{k}{h-1}\right)\hat{\gamma}_d(k) \tag{60}$$

The $f_d(0)$ is the spectral density of the loss differential at frequenct 0, $\widehat{\gamma}_d(k)$ is the autocavariance of the loss differential at lag k and I is the weight indicator function; i.e,

$$\hat{\gamma}_d(k) = \frac{1}{T} \sum_{t=|k|+1}^{T} \left(d_t - \hat{d}\right)\left(d_{t-|k|} - \hat{d}\right) \tag{61}$$

and

$$I\left(\frac{k}{h-1}\right) = \begin{cases} 1 \text{ for } \left|\dfrac{k}{h-1}\right| \leq 1 \\ 0 \text{ otherwise} \end{cases} \tag{62}$$

- Test Statistic T^{RC} (for comparing nested models [41]). In the case, it is interesting to know if any of the models, $k = 1, 2, \ldots, m$, are better than the benchmark in terms of expected loss. Let:

$$d_{k,t} = L\left(\xi_t, \delta_{0,t-h}\right) - L\left(\xi_t, \delta_{k,t-h}\right) \quad , \quad k = 1, 2, \ldots, m \tag{63}$$

where $d_{k,t}$ denotes the performance of model k relative to the benchmark at time t, and these variables are stacked in the relative yield vector performances, $d_t = (d_{1,t}, \ldots, d_{m,t})$. Provided that $\mu = \mathbb{E}(d_t)$ is well defined, it is now formulate the null hypothesis of interest as:

$$H_0 : \mu \leq 0 \quad , \quad \textit{and our maintained hypothesis is} \quad \mu \in R^m \tag{64}$$

Under the assumption that model k is better than the benchmark if and only if $\mathbb{E}(d_{k,t}) > 0$. So we focus exclusively on the properties of d_t and abstract entirely from all aspects that relate to the construction of the $\delta - \textit{variables}$ (where δ be a finite set of possible decision rules). Thus d_t , $t = 1, \ldots, n$, is de facto viewed as our data.
In [41] proceeded by constructing the RC from the test statistic:

$$T_n^{RC} = \max\left(\sqrt{n}\bar{d}_1, \ldots, \sqrt{n}\bar{d}_m\right) \tag{65}$$

where:

$$\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_t \tag{66}$$

and an asymptotic null hypothesis of distribution based on:

$$n^{\frac{1}{2}}\bar{d} \sim N_m\left(0, \hat{\Sigma}\right) \tag{67}$$

where:

$$\mathbb{E}\left(\hat{\Sigma}\right) = \Sigma \tag{68}$$

These statistical tests, detailed above, allow to determine the model with the best predictive performance. The results of the evaluations of goodness-of-fit with these statistics in Table(2) are presented, observing that for all countries there is a low estimation error. The country with the highest estimation error was Paraguay but it remains in an acceptable range. In Table (3) the execution times of the algorithm of [1] is observed. The execution time of the algorithm for each country was relatively fast.

Tabla 2. RSME, DM and T^{RC} estimated for the Markov switching model.

Measures	Argentina	Brasil	Paraguay	Uruguay	Venezuela
$RSME$	0.0850	0.0812	0.1159	0.0191	0.0177
DM	0.1691	0.239	0.886	0.5204	0.7203
T^{RC}	0.3481	0.3539	0.45103	0.1354	0.3066

Tabla 3. Execution time of the algorithm in seconds.

Countries	Argentina	Brasil	Paraguay	Uruguay	Venezuela
Time in seconds	8.6094	8.75	8.3281	9.2344	9.0938

5. Discussions and Conclusions

In this work a Bayesian methodology based on a Markov switching model is proposed, to estimate the growth rate of annual industrial production of the MERCOSUR countries. The model reflects how the economy of these countries has developed, how they have experienced ups and downs in the industry growth. And how they have a very variable annual industrial production. Through this model, the stocks of outliers and innovation in the economy of these countries are very efficiently identified. To obtain the results the algorithm of [1] is implemented. An estimated value for the mean and the standard deviation a posterior distribution of the parameters is obtained. Also, shown that the parameters σ_1 and σ_2 are equal, which indicates that economically there is no reduction of volatility; as well, $|\delta_2| > |\delta_1|$ is obtained, which indicates that there is no reduction in the depth of economic cycles. Additionally, a graphical representation of the parameters K_{a_t}, K_{e_t}, ν, K_{δ_t} and μ is made and through these graphical representations it is possible to identify: additive outliers, outliers of innovation, changes in the growth rate, breakpoints, asymmetries, and the non-linearity of the data.

The results show that there is no leading country that sets a pattern with the other countries, likewise, the presence of a common cycle among the participating MERCOSUR countries is not identified, as established in [27]. Although the coincidence of certain stabilized events in the cycles of these economies provides an idea of a joint movement, this alone does not constitute sufficient evidence of synchronization, because the growth rates of each country are very particular. The results show, that there is no leading country that sets standard among the economies studied. The analysis does not identify the presence of a common cycle among the participating MERCOSUR countries, coinciding with the results obtained in [27]. Additionally, the methodology used based on the state-space models and the flexible methods of Bayesian statistics, allows us to simultaneously identify the changes of regimes in non-linear series, the structural changes in the mean and volatility, which can be evidenced in Figures previously shown. In addition, three evaluation criteria to validate the model prediction are used: square root of the mean quadratic error (RSME), the Diebold-Mariano(DM) test, and the T^{RC} statistical test, as goodness-of-fit measures to calibrate the estimates, obtaining small errors. Finally, the execution times are estimated obtaining acceptable results.

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References

1. P. Giordani, R. Kohn and D. Van Dijk, "A unified approach to nonlinearity, structural change, and outliers", *Journal of Econometrics* **137** (2007) 112–133.
2. T. Beveridges and C. Nelson, "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle", *Journal of Monetary Economics* **7** (1981) 151–174.
3. Nelson, C & Plosser, C. (1982). *Trends and random walks in macroeconomic time series: some evidence and implications.* Journal of Monetary Economics, 10, 139-162.
4. J. Campbell and G. Mankiwi, "Permanent and transitory components in macroeconomic fluctuations", *American Economic Review Papers and Proceedings* **77** (1987a) 111–117.
5. J. Campbell and G. Mankiwi, "Are output fluctuations transitory quarterly", *Journal of Economics* **102** (1987b) 857–880.
6. A. Harvey, "Trends and cycles in macroeconomic time series", *Journal of Business and Economic Statistics* **3** (1985) 216–227.
7. M. Watson, "Univariate detrending methods with stochastic trends", *Journal of Monetary Economics* **18** (1986) 49–75.
8. P. Clark, "The cylical component of U.S. economic activity", *Quarterly Journal of Economics* **102** (1987) 797–814.
9. R. Engle and W. Granger, "Co-Integration and error correction: representation, estimation, and testing", *Econometrica* **55** (1987) 251–276.
10. R. King, C. Plosser, J. Stock, and M. Watson, "Stochastic trends and economic fluctuations", Working Paper No. 2229, 1987.
11. C. Kim and J. Piger, "Common stochastic trends, common cycles, and asymmetry in economic fluctuations", *Journal of Monetary Economics* **49** (2002) 1189–1211.
12. J. Hamilton, "A new approach to the economic analysis of nonstationary time series and the business cycle", *Econometrica* **57** (1989) 357–384.
13. M. Clements and H. Krolzig, "Can oil shocks explain asymmetries in the us business cycle?", *Empirical Economics* **27** (2002) 185–204.
14. T. Mills and P. Wang, "Plucking models of business cycle fluctuations: evidence from the G7 countries", *Empirical Economics* **27** (2002) 255–276.
15. C. Kim, J. Morley, and J. Piger, "Nonlinearrity and the permanent effects of recessions", *Journal of Applied Econometrics* **20** (2005) 291–309.
16. J. Granger and T. Terasvirta, *Modelling Nonlinear Economic Relationships* (Advanced Texts in Econometrics, Oxford: University Press, 1993).
17. P. Franses and D. Van Dijk, *Non linear time series models in empirical finance* (Cambridge University Press, 1 ed., 2000).
18. P. Franses, D. Vann Dijk and T. Terasvirta, "Smooth transition autoregressive models a survey of recent developments", *Econometric Reviews* **21** (2002) 1–47.
19. H. Krolzig, "Statistical analysis of cointegrated var processes with markovian regime shifts", *Institute of Economics and Statistics and Nuffield College* **25** (1996) 01–21.
20. P. D. Santomil, L. A. O. Gonzalez, M. V. Bua and S. F. López, "Análisis del riesgo de tipo de cambio en solvencia II", Universidad de Santiago de Compostela, 2012.
21. F. Battaglia and L. Orfei, "Outlier detection and estimation in nonlinear time series", *Journal of Time Series Analysis* **26** (2005) 107–121.
22. C. Kim and C. Nelson, "Has the US economy become more stable? A Bayesian approach based on a markov-switching model of the business cycle", *Review of Economics and Statistics* **81** (1999b) 608–616.
23. G. Koop and S. Potter, "Bayes factors and nonlinearity: evidence from economic time series", *Journal of Econometrics* **88** (1999) 251–281.
24. G. Koop and S. Potter, "Nonlinearity, structural breaks or outliers in economic time series?", in *Nonlinear Econometric Modeling in Time Series Analysis*, eds. W.A. Barnett, D.F. Hendry, S. Hylleberg, T. Terisvirta, D. Tjostheim and A. H. Wurtz (Cambridge University Press, 2000) pp. 61–78.
25. C. Kim and C. Nelson, *State-Space Models With Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications* (MIT Press, Cambridge, 1999a).
26. R. Gerlach, C. Carter and R. Kohn, "Efficient bayesian inference for dynamic mixture model", *Journal of the American Statistical Association* (2000) 819–828.

27. A. Hurtado and F. Builes, "Sincronización de ciclos económicos en el Mercosur: 1960- 2008",*Ecos de Economía: A Latin American Journal of Applied Economics* **14(31)** (2011) 7–35.
28. P. Mejía-Reyes, "Asymmetries and common cycles in Latin America: evidence from Markov switching models", *Economía Mexicana. Nueva época* **9(2)** (2000) 189–225.
29. P. Mejía-Reyes, "Classical Business Cycles in America: are national business cycles synchronised", *International Journal of Applied Econometrics and Quantitative Studies* **1(3)** (2004) 75–102.
30. C. M. Mondaca, "Estudio de sincronismos entre ciclos económicos mediante la transformada wavelet: análisis del caso Chile y Mercosur", Tesis Doctoral, Universidad de Valladolid, 2012.
31. A. Salamanca Lugo, "Sincronización de ciclos económicos: el caso de Colombia, Ecuador y Venezuela", *Cuadernos de Economía* **31(SPE57)** (2012) 179–199.
32. B. D. O. Anderson and J. B. Moore, *Estimation filtering and smoothing in stated space models with partially diffuse initial conditions* (Prentice Hall, Prentice Hall, 1979) pp. 286–316.
33. A. Harvey, *Forecasting, Structural Time Series Models, and the Kalman Filter* (Cambridge University Pres, Cambridge, 1989).
34. L. Tierney, "Markov chains for exploring posterior distributions", *Annals of statistics* **22(4)** (1994) 1701–1728.
35. C. Carter and R. Kohn, "On gibbs sampling for state-space models", *Biometrika* **81** (1994) 541–553.
36. J. Durbin and S. J. Koopman, "A simple and efficient simulation smoother for state space time series analysis", *Biometrika* **89** (2002) 603–616.
37. A. Zellner, "On assessing prior distributions and bayesian regression analysis with g-prior distributions", in *Bayesian Inference and Decision Techniques: Essays in Honour of Bruno de Finetti*, eds. P. K. Goel and A. Zellner (Amsterdam, North-Holland, 1986) pp. 233–243.
38. J. Geweke, *Contemporary bayesian econometrics and statistics* (John Wiley y Sons, New York, 2005).
39. L. Bauwens, M. Lubrano and J. Richard, *Bayesian Inference in Dynamic Econometric Models* (Oxford, 1999).
40. F. X. Diebold and R. S. Mariano, "Comparing Predictive Accuracy", *Journal of Business and Economic Statistics* **13**(1995), 253–263.
41. H. White, "A Reality Check for Data Snooping", *Econometrica* **68**(2000) 1097–1126.