

Biased innovations in the Harrod-Domar model

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Abstract. This paper presents an endogenous growth model where the aggregate production function is a Leontief (1941) and long run growth is completely explained through biased technological change. Under this framework we get two results: (i) if the income share of reproducible factors is high enough, in the long run the economy presents a positive balanced growth path; (ii) if the income share of reproducible factors is low, in the long run the economy behaves as a Harrod-Domar economy without long run growth.

Key words: endogenous growth, capital using and labor saving technological change.

JEL classification: O11, O31, O33.

Resumen. En este artículo se presenta un modelo de crecimiento donde la función de producción es del tipo Leontief (1941), la tasa de ahorro es endógena y el crecimiento de largo plazo es explicado por cambio tecnológico sesgado. En este entorno se obtienen dos resultados: (i) si la participación de los factores reproducibles en el producto es suficientemente alta, en el largo plazo la economía presenta una senda de crecimiento balanceado; (ii) si, en cambio, la participación de los factores reproducibles es baja, en el largo plazo no hay crecimiento y la economía se comporta al estilo Harrod-Domar.

Palabras clave: crecimiento endógeno, cambio tecnológico sesgado.

Clasificación JEL: O11, O31, O33.

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1. Introduction

In the early years of capitalism, a new way of production emerged where the capital income share was substantially higher than in the feudal production system. The institutional framework under which this phenomenon took place determined to a great extent the economic success of countries. Social organizations that let capitalists retain a greater share of the output enjoyed faster capital accumulation and, as a result, higher rate of technological growth.¹ Along with the changes in the functional distribution of income a process of biased technological change took place, and these facts together lead to the industrial revolution and the subsequent industrialization of the economies.²

In this article we provide a growth model that can account for these facts. We modify the Harrod-Domar model³ endogenizing the savings rate and allowing *capital using and labor saving* technological change. The model is one of biased innovations where factor income shares are determined politically or institutionally, that is, they are independent of market forces. Under this framework we get two results: (i) if the income share of reproducible factors is high enough, in the long run the economy presents a positive balanced growth path; (ii) if the income share of reproducible factors is low, in the long run the economy behaves as a Harrod-Domar economy.

In order to achieve long run growth, the income share of factors like land and unskilled labor should be low and the income share of reproducible factors like physical and human capital should be high. A similar result was obtained by Bertola (1993) who relates the growth rate of the economy to the functional distribution of income in an endogenous growth model. However, in his paper, the production function is AK and the technology is constant (there are no innovations), so the model cannot support a neoclassical steady state.

The set-up of the model is simple: every economy starts with a Leontief production function, which combines the fixed factor (L) and the reproducible factor (K), namely, $Y = A \min(K, \beta L)$ where A and β are technological parameters. Therefore, a change in A is a neutral technological change while a change in β is a biased technological change. For concreteness, we refer to the reproducible factor as capital. Once the stock of capital is equal to βL , capital productivity starts to decrease as well as the capital income share. For this reason, the economy cannot grow in the long run. Note that, under this framework, increasing total factor productivity increases output but does not generate incentives to accumulate capital. Therefore, technological innovations must be factor saving.

Nowadays, the Leontief production function is almost out of use and modern growth theorists do not give much importance to the Harrod-Domar model. Two main reasons may explain this fact: on the one hand, some undesirable

¹See Weber (1905); Hall and Jones (1999); Acemoglu, Johnson, and Robinson (2001); Engerman and Sokoloff (2002), among others.

²See Mathews, Feinstein, and Odlig-Smee (1982); Hansen and Prescott (2000); Allen (2005) and Zuleta (2007a).

³Harrod (1939) and Domar (1946).

predictions of the original model (perpetual growth of unemployment or perpetual growth of idle machinery) and, on the other hand, assumptions about the savings rate and about the behavior of the marginal productivity of factors. However, with the extensions we make, the so called undesirable elements of the original model are eliminated and, depending on the parameters of the economy, in the long run the economy may behave as if the production function were AK .

Since the incentives to save depend crucially on capital income share, the speed of capital accumulation is higher for economies where the capital income share is bigger. Similarly, technological change is costly, therefore the net income share of capital is smaller when technology is improving. For this reason, in economies where capital income share is small there are no incentives to make technological innovations; these economies are trapped in a steady state. This result has two implications. First, the bigger the income share of reproducible factors, the bigger the incentives to innovate.⁴ This implication is consistent with the institutional view of the industrial revolution; according to which, a critical factor to explain the economic performance in seventeenth-century England was the ability of the government to commit to private property rights and exchange instead of producing rules that benefit a small elite of land-owners (see North and Weingast, 1989). Second, labor is not a reproducible factor, so technologies in rich economies are likely to be less labor intensive than technologies in poor economies. Therefore, when technology is transferred from rich to poor economies (FDI), these countries are likely to experiment an increase in marginal productivity of capital and a decrease in labor demand. This implication is consistent with the increase in FDI and the subsequent behavior of wages and unemployment in many Latin-American countries from the second half of the 90's until now.

Our approach combines two strands of the literature: biased innovations and transition from stagnation to sustained growth.

In models of biased innovations, technological progress takes place through the adoption of new activities that demand less fixed factors per unit of output (Kennedy, 1964; Zeira, 1998; Boldrin and Levine, 2002; and Zuleta, 2006). The relevance of the biased innovations theory is supported by the evidence provided by researchers in economic history who show that during the industrial revolution there was *capital using and labor saving* technological change (Cain and Paterson, 1981). Economic literature also provides evidence that during the last few decades there has been *human capital using and raw labor saving* technological change (Krusell et al., 1997). Moreover, in both cases, the technological change was preceded by a change in factor abundance. The story behind these two facts is a story of biased technological change triggered by changes in factor abundance.⁵

⁴A similar result is obtained by Bertola (1993) who explains how the share of reproducible factors positively affects the savings rate.

⁵The idea of changes in productivity coming from investment is present in many articles on economic growth (Romer, 1986 and 1990; Aghion and Howitt, 1992; etc.). However, in this literature, it is usually assumed that technological innovations are neutral. In the same

To our knowledge, this is the first model of biased innovation that explains the transition from stagnation to sustained growth through the functional distribution of income.

The literature on transition from stagnation to growth is wide. The first unified theories view the transition from stagnation to growth as primarily driven by technological change (Galor and Weil, 2000; Hansen and Prescott 2002; and Doepke, 2004). A second set of theories point to institutions as a determinant of the quality of institutions and long run development success (Acemoglu, Johnson, and Robinson, 2000, among others). There is also a third approach which reconciles the previous theories: economic institutions affect economic incentives but also respond to changes in the economic environment (Engerman and Sokoloff, 2003). Our theory is in line with the third approach. In this paper, the ultimate mechanism that triggers the transition from stagnation to sustained growth is the functional distribution of income. However, we take factor shares as given, that is, we do not model the institutional or political changes. Nevertheless, our story is perfectly consistent with the institutional view. Indeed, one of the main consequences of the institutional changes was a change in the distribution of income.

Our paper differs from the previous literature in two main features: first, in our setting, innovations are factor saving; second, institutions affect economic activities only through the functional distribution of income.

The rest of the paper is organized as follows. In the next section the concept of *factor using and factor saving* technological change is explained in detail. In section 3 the model and its planner solution are presented. In section 4 we show the market equilibrium. In section 5 we discuss the implications of the model regarding capital taxation and foreign direct investment. Finally, some concluding comments are provided.

2. Capital using and labor saving technological change

2.1. Technological change and elasticity of substitution between factors

The technological change we analyze in this paper is both factor using and factor saving. If we consider a decentralized economy, we can express the optimal capital labor ratio (k^*) as a function of factor prices and some technological parameters. A *capital using and labor saving* technological change is a change in technological parameters such that, holding the factor prices constant, the optimal capital labor ratio is increased.

To illustrate this, take the Leontief production function $Y = A \min(K, \beta L)$. The optimal capital labor ratio is $k^* = \beta$. Thus, increasing β is the only way to have *capital using and labor saving* technological change.

There is not much empirical work on the estimation of Leontief production functions. However, given the fact that rich economies have higher capital labor

way, many empirical studies try to describe the evolution of TFP assuming constant factors intensity (Barro and Sala-i-Martin, 1992, among others).

ratio than poor economies, if the relevant production function is a Leontief, then β must be higher in rich economies.

When we endogenize the capital intensity of technology (β) using a Leontief production function, the elasticity of substitution becomes a function of capital labor ratio and is not constant. To see this, consider the technical rate of substitution (TRS) and derive the elasticity of substitution (ε), in general, given by

$$\text{TRS} = \frac{\frac{\partial Y}{\partial L}}{\frac{\partial Y}{\partial K}} \quad \text{and} \quad \varepsilon = \frac{\text{TRS}}{k} \left(\frac{\partial \text{TRS}}{\partial k} \right)^{-1}.$$

In the case of the Leontief production function, in the optimum,

$$\text{TRS} = \beta \quad \text{and} \quad \varepsilon = \frac{\beta}{k} \left(\frac{\partial \beta}{\partial k} \right)^{-1}.$$

Thus, whenever the capital labor ratio positively affects the incentives for technological change, the elasticity of substitution is higher than zero. Out of the optimum, TRS is zero or infinity and the elasticity of substitution does not depend on k . We focus our attention on the case where $k = \beta$ because we want to study technological change as a response to changes in factor abundance. In this setting, whenever the capital labor ratio positively affects the incentives for technological change, the elasticity of substitution is higher than one.

2.1.1. The cost of changing technology

We assume there is a Leontief production function for any technology and technologies are differentiated by their capital intensity. The cost of a technology is paid by the capital owners. So, when firms want to improve their technologies, part of the assets (a) must be devoted to change β .

For simplicity, we assume a continuum of agents of measure 1 exists ($L = 1$), so in the aggregate $K = a - \beta$ and $\dot{a} = \dot{K} + \dot{\beta}$. Therefore, in continuous time, the restrictions faced by the social planner are given by $\dot{k}_t = u_t(y_t - c_t)$ and $\dot{\beta}_t = (1 - u_t)(y_t - c_t)$ where u_t is the share of the savings devoted to increase the quantity of a given type of capital good, $1 - u_t$ is the share of the savings devoted to increase the capital intensity of the technology, $k = \frac{K}{L}$, $y = \frac{Y}{L}$, and $c = \frac{C}{L}$.

Now, in the decentralized economy, if L_i is the amount of people consuming the good produced by firm i then $k_{\beta,i} = \frac{a_i}{L_i} - \beta_i$ where a_i and β_i denote the amount of assets and the technology of firm i . Since cost depends on size of the firm, increasing size has no technological advantages and firms are identical: $\frac{K_{\beta,i}}{L_i} = k = a - \beta$.

Finally, we are assuming homogeneous agents, so $l_i = L_i$ and the capital labor ratio for firm i is given by $k_{\beta,i} = \frac{a_i}{L_i} - \beta_i$. Therefore, the restrictions for the representative firm are identical to those faced by the planner.

3. The model

3.1. Endogenous savings rate

We assume optimizing agents with logarithmic utility function and infinite horizon. We also change the way in which the Leontief production function is presented, in order to identify the technology with only one parameter:

$$Y = A \min(K, \beta L).$$

For the moment, A and β are assumed to be constant.

The planner maximizes the consumers utility

$$U = \int \log c_t e^{-\rho t} dt,$$

subject to the resource constraint $\dot{k}_t = y_t - c_t$ and the transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} \frac{1}{c_t} k_t = 0$, where k is capital per worker; y , output per worker; and c , consumption per worker.

Combining the first order conditions, we get

$$\frac{\dot{c}_t}{c_t} = \begin{cases} A - \rho, & \text{if } k_t < \beta, & \text{(a)} \\ -\rho, & \text{if } k_t > \beta, & \text{(b)} \\ 0, & \text{if } k_t = \beta. \end{cases} \quad (1)$$

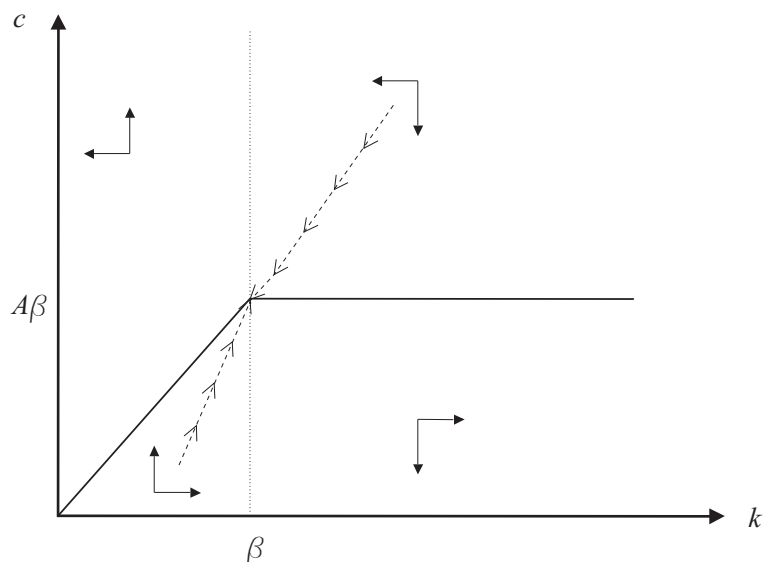
If $k_t = \beta$, the marginal productivity of capital is A (higher than ρ) for reductions in the stock of capital and 0 (lower than ρ) for increases in the stock of capital. Thus, when $k_t = \beta$, our representative agent does not have incentives to save or dissave and, as a result, consumption is constant. From (1.a) and (1.b) we can draw the phase diagram presented in figure 1.

The axes of figure 1 represent consumption and capital per worker. The vertical dashed line, represents the points where the optimal growth rate of consumption is zero. From the resource constraint, if the capital labor ratio is higher (lower) than β , the growth rate of consumption is negative (positive). In the same way, from (1) the growth rate of capital is equal to zero if consumption is equal to output, so using (1) we can derive the condition for the capital labor ratio to be constant. The solid line in figure 1 ($\frac{\dot{k}}{k} = 0$) represents the points where the growth rate of capital is zero. If consumption is above output then capital is reduced, if consumption is below output then there is capital accumulation.

From the resource constraint, (1), and figure 1 the economy has a steady state where consumption is equal to output and the capital labor ratio is equal to β .

For the sake of shortness, from now on, we call the Harrod-Domar steady state the point where $c = y = A\beta$.

Figure 1



3.2. Changing technology

Now we change the model assuming β may be increased paying a cost. We denote by u the share of savings devoted to capital accumulation and by $1 - u$ the share of savings devoted to technological change.

We assume every economy starts with the same technological parameters A and β_0 . Keeping β constant has no cost, but changing β has a positive cost which depends on the size of the change. Intuitively, β is a way to use capital and labor, so the higher the amount of inputs to be transformed the higher the cost that has to be paid. Changing K with β constant is increasing the number of machines of the same quality, while changing β with K constant is increasing the the quality of a given number of machines.

The cost of a technology β can be interpreted in three different ways: (i) the cost of inventing and implementing a technology, (ii) the cost of copying a new technology and building a similar capital good, and (iii) the price premium that has to be paid in order to acquire a higher quality capital good. In a market economy, the costs described in (ii) and (iii) are likely to be the same. If we assume that technology is not rival then the costs described in (ii) and (iii) are likely to be smaller than the one described in (i). However, if we assume technology is embodied in goods and is costly to reverse engineer (disembody) and appropriate, the difference between (i) and (ii) is substantially reduced (see Boldrin and Levine, 2002b). In any case, since we want to derive conclusions for developing countries, which are unlikely to be the source of new inventions, we can ignore interpretation (i) and assume capital goods of different qualities

are available in the market.

We include the cost of changing β assuming a production function of the form $\dot{\beta}_t = (1 - u_t)s_t$, where s is the amount of savings per capita.⁶ This function reflects the need to devote a share $(1 - u)$ of the output to develop or adopt new technologies and implies capital accumulation is given by $\dot{k}_t = u_t s_t$.

3.3. The command optimum

The planner maximizes the consumers utility,

$$\max U = \int \log c_t e^{-\rho t} dt,$$

subject to the resource constraints

$$\dot{k}_t = u_t(y_t - c_t), \quad \dot{\beta}_t = (1 - u_t)(y_t - c_t), \quad (1 - u_t) \geq 0, \quad \text{and} \quad u_t \geq 0.$$

After combining the first order conditions (complete derivation in the appendix), we get

$$\frac{\dot{c}_t}{c_t} = \begin{cases} A - \rho, & \text{if } k_t < \beta_t, \\ \max\left(0, \frac{1}{2}A - \rho\right), & \text{if } k_t = \beta_t, \\ A - \rho, & \text{if } k_t > \beta_t; \end{cases} \quad (2)$$

$$\dot{k}_t = \begin{cases} A\beta_t - c_t, & \text{if } k_t < \beta_t, \\ \max\left(0, \frac{1}{2}\left(A - \frac{c_t}{k_t}\right)\right), & \text{if } k_t = \beta_t, \\ -\delta k_t, & \text{if } k_t > \beta_t. \end{cases} \quad (3)$$

If $k_t < \beta_t$ then the production function of the economy is $Y_t = AK_t$, so the marginal productivity of capital is equal to A and it is useless to reduce the need of labor, that is, increase β ($\frac{\partial Y}{\partial \beta} = 0$).

If $k_t > \beta_t$ then the production function of the economy is $Y_t = A\beta L_t$, so it is useless to increase the stock of capital ($\frac{\partial Y}{\partial K} = 0$). Moreover, investment is completely devoted to reduce the need of labor, that is, increase β ($u = 0$), and part of the capital is used to consume.

If $k_t = \beta_t$ then the increase in the number of capital goods must be equal to the increase in their quality. Capital, technology, and consumption grow at a positive rate if $A > 2\rho$, that is, $\frac{\dot{k}_t}{k_t} = \frac{\dot{\beta}_t}{\beta_t} = \frac{1}{2}\left(A - \frac{c_t}{k_t}\right)$. Therefore, in the long run consumption capital ratio is determined by $\frac{c_t}{k_t} = 2\rho$.

Note that in the AK model the consumption capital ratio is equal to the discount rate. The reason for the difference is that in the AK all savings are devoted to capital accumulation while in our model half of the savings are devoted to technology creation. Moreover, if we define the variable a as the amount of accumulated assets (including capital and technology) then the consumption asset ratio converges to the discount rate in the long run, namely, $\frac{c}{a} = \rho$.

⁶This assumption simplifies the analysis and does not drive the results of the model. A version of the paper where labor is the only input in the production of new technologies is available by request.

3.3.1. Different regimes, transition and balanced growth path

Recall that we assume every economy starts with the same technological parameters. In particular, the parameter β_0 is assumed to be given by nature. For this reason, capital scarce economies are likely to be in the case where $k < \beta_0$ and do not present technological change. In contrast, economies with high capital labor ratio ($k > \beta_0$) may present technological change ($u \neq 1$) and break the limits of Harrod-Domar. Thus, we have a model with two different regimes: one for poor economies and the other for rich economies. In this subsection we describe the law of motion of some key variables in the two regimes, characterize the transition path, and see if one or more steady states exist.

Regime 1 $k \leq \beta_0$

Under this regime, the share of the output devoted to capital improvement is zero ($u = 1$), so the law of motion of the variables k and c can be described by the following equations:

$$\frac{\dot{k}}{k} = A - \frac{c}{k}, \quad \text{so} \quad \frac{\dot{k}}{k} = 0 \rightarrow \frac{c}{k} = A \text{ or } c = y; \quad (4)$$

$$\frac{\dot{c}_t}{c_t} = \begin{cases} A - \rho, & \text{if } k_t < \beta, \\ \max\left(0, \frac{A}{2} - \rho\right), & \text{if } k_t = \beta. \end{cases} \quad (5)$$

Regime 2 $k > \beta_0$

Replacing u in the equation of capital accumulation and rearranging, we can find the value of $\frac{c}{k}$ that makes the growth of capital equal to zero:

$$\frac{\dot{k}_t}{k_t} = \frac{1}{2} \left(A - \frac{c_t}{k_t} \right), \quad \text{so} \quad \frac{\dot{k}_t}{k_t} = 0 \rightarrow \frac{c_t}{k_t} = A \text{ or } c_t = y_t; \quad (6)$$

$$\frac{\dot{c}_t}{c_t} = \max\left(0, \frac{A}{2} - \rho\right). \quad (7)$$

From (4), (5), (6), and (7), we have the following results:

1. One optimal path exists. If $\frac{A}{2} < \rho$, there is a steady state without growth, and if $\frac{A}{2} > \rho$, there is long run growth.
2. Rich economies may present technological change, but poor economies cannot.
3. Capital abundant economies have more capital intensive technologies (higher β), that is, better capital goods than capital scarce countries.
4. The level of total factor productivity and the discount rate determine whether or not the economy may converge to a steady state.

4. Decentralized economy

In this section we solve the model in the decentralized economy. We assume homogenous agents with log utility function. Firms are competitive and factor prices are exogenously given. Different technologies are available in the market and firms choose both the capital and technology.

Note that the decentralized equilibrium cannot be optimal because factor markets are not complete, factor shares are determined exogenously and factor prices are not equal to their marginal productivity. In this setting, the functional distribution of income is one of the main determinants of economic growth. Now, we consider two types of factors: reproducible and not reproducible. Among the not reproducible factors, the most important are land and raw labor. Therefore, the functional distribution of income is the political or institutional arrangement that settles a distributive conflict between land owners, capitalists, and workers.

Nowadays, the share of land is relatively small and we are used to thinking that the functional distribution of income is a problem of capitalists and workers.⁷ However, in the preindustrial era, land was a predominant factor and land income share was much bigger than it is now (see Hansen and Prescott, 2000). Moreover, one of the elements that helped trigger the Industrial Revolution was a new way of production where the entrepreneurs could retain part of the aggregate value, that is, a redistribution of income in favor of capital and against land.⁸

4.1. Consumers

The consumer maximizes utility subject to the resource constraints:

$$\max U = \int \log c_t e^{-\rho t} dt \quad \text{s.t.} \quad \dot{a}_t = w_t + a_t r_t - c_t,$$

where a is the amount of assets, r is the interest rate, and w is the wage. Then, $\frac{\dot{c}_t}{c_t} = r_t - \rho$. That is, the consumption growth rate is equal to the interest rate minus the discount rate.

4.2. Factor prices

The characteristics of the production function don't allow us to set factor prices equal to their marginal productivity. Therefore, capital income share (α) and labor income share ($1 - \alpha$) are determined by bargaining or by institutional

⁷Union wage bargaining is the common means of wage determination in continental Europe. Although union membership rates declined substantially, over the last two decades, union bargaining still determines approximately 90% of all wage contracts in continental Europe (see Cesifo Dice, 2005).

⁸The Industrial Revolution was not just a sequence of changes in industrial techniques and production but a social revolution with social causes as well as profound social effects (Perkin, 1971).

arrangements. For simplicity, we assume full employment and we take α as given. Therefore, the factor prices are determined in the following way:

$$\begin{aligned} w_t &= (1 - \alpha)A \min(k_t, \beta_t), \\ r_t &= \alpha \frac{A \min(k_t, \beta_t)}{a_t}. \end{aligned} \quad (8)$$

Firms invest in productive assets; therefore,

$$u_t = \begin{cases} 1, & \text{if } k_t < \beta_0, \\ 0, & \text{if } k_t > \beta_0, \\ \frac{1}{2}, & \text{if } k_t = \beta_t. \end{cases} \quad (9)$$

From (8) and (9), we can describe the dynamics of capital labor ratio and consumption.

$$\frac{\dot{k}_t}{k_t} = \begin{cases} A - \frac{c_t}{k_t}, & \text{if } k_t < \beta_0, \\ \frac{1}{2} \left(A - \frac{c}{k} \right), & \text{if } k_t \geq \beta_0. \end{cases} \quad (10)$$

$$\frac{\dot{c}_t}{c_t} = \begin{cases} \alpha A - \rho, & \text{if } k_t < \beta_0, \\ \max \left(0, \alpha \frac{A \min(k_t, \beta_t)}{a_t} - \rho \right), & \text{if } k_t \geq \beta_0. \end{cases} \quad (11)$$

If $\alpha < \frac{\rho}{A}$, a steady state without capital in the long run exists.

If $\frac{\rho}{A} < \alpha < 2\frac{\rho}{A}$, a steady state without long run growth but with a positive amount of capital exists.

If $\alpha > 2\frac{\rho}{A}$, there is technological change for $k \geq \beta_0$ and long run growth.

Summarizing, the value of α may determine whether or not an economy can converge to a steady state without growth. Given that the incentives to save depend on the return on capital, the higher the capital income share, the higher the incentives to save. For this reason, countries where the income share of reproducible factors is big are more likely to present long run growth. One implication of this result is that, in countries where the land owners are powerful and retain the mayor part of the output for themselves, capital accumulation and the subsequent industrialization is slower.

For the economies where there is long run growth (i.e., if $\alpha > \frac{2\rho}{A}$), the consumption capital ratio in the balanced growth path is given by $\frac{c_t}{k_t} = 2\rho + (1 - \alpha_t)$. Note that the consumption capital ratio is higher than the optimal because the interest rate is lower than the marginal productivity of savings. This result implies that, by taxing labor and land income and subsidizing capital income, the government can improve welfare.

5. Implications

The results of the previous sections have implications related to fiscal policy and international capital mobility. In the following lines we highlight these implications.

5.1. Taxation and long run growth

As we stated before, in our framework, capital income share (α) and labor income share ($1 - \alpha$) are determined by bargaining or by institutional arrangements. However, fiscal policy can be a useful tool to change the capital income share. Indeed, governments can impose taxes in order to finance redistributive programs (unemployment insurance, for example). In this case, fiscal policy reduces capital income. Now, if the government chooses to subsidize capital accumulation, the result would be the opposite, that is, an increase in the capital income share. In any case, it is clear that the capital income share is affected by the effective rate of capital taxation (τ_K). Therefore, it is possible to express α as a function of τ_K :

$$\alpha(\tau_K), \text{ where } \alpha'(\tau_K) < 1 \text{ and } \alpha'(1) = 0.$$

Replacing in (11),

$$\frac{\dot{c}_t}{c_t} = \begin{cases} \alpha(\tau_K)A - \rho, & \text{if } k_t < \beta_0, \\ \max\left(0, \alpha(\tau_K)\frac{A \min(k_t, \beta_t)}{a_t} - \rho\right), & \text{if } k_t \geq \beta_0. \end{cases}$$

Therefore, the value of τ_K may determine whether or not an economy can converge to a steady state without growth. For low levels of taxation the model presents long run growth, while for high levels of taxation the model behaves like the standard Harrod-Domar model.

5.2. Technological transfers and unemployment

Consider the model developed in section 4 and assume the parameters are such that $\frac{\rho}{A} < \alpha < 2\frac{\rho}{A}$. In other words, the economy has a steady state with a positive amount of capital and no economic growth.

Now, suppose the firms of the economy have access to a technology from abroad (β^*). Given domestic wages, firms have incentives to adopt the new technology because they can produce the same but using less labor, that is, they can make more profits using the new technology.

The adoption of the new technology has two consequences: (i) marginal productivity of capital increases and, as a result, savings become positive and the economy grows; (ii) the demand for labor falls, so fewer units of labor are used in the production process.

The model presented here assumes homogenous agents, so the reduction in units of labor implies a reduction in the working time of each worker. A future extension of this model can be the inclusion of endogenous heterogeneity to explore the interactions among innovations, employment, income distribution, and economic growth.

6. Conclusions

We revisit the Harrod-Domar model of economic growth. By endogeneizing the savings rate and allowing for biased technological change, some undesirable elements of the original model are eliminated.

Depending on the parameters of the economy, we derive the following results:

For economies with high marginal productivity of capital (A) and low discount rate (ρ), in the long run the economy may behave as if the production function were AK , that is, there is endogenous growth.

For economies with low marginal productivity of capital (A) and high discount rate (ρ), the economy behaves as in the Harrod-Domar model without technological change.

In the market economy, the possibility of long run growth depends on the capital income share (α). Economies where the part of the income that goes to capital owners is higher are more likely to present technological change and long run growth.

For low levels of taxation the model presents long run growth, while for high levels of taxation the model behaves like the standard Harrod-Domar model.

The adoption of capital intensive technologies by labor abundant economies (through FDI, for example) leads to a reduction in labor demand.

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Appendix

The planner maximizes the consumers utility subject to the resource constraints:

$$\max U = \int \log c_t e^{-\rho t} dt$$

subject to

$$\dot{k}_t = u_t(y_t - c_t), \quad \dot{\beta}_t = (1 - u_t)(y_t - c_t), \quad (1 - u_t) \geq 0, \quad \text{and} \quad u_t \geq 0.$$

So, the first order conditions are the following:

$$\frac{\partial H}{\partial c} = 0 = \frac{1}{c} e^{-\rho t} - \lambda_t u_t - \mu_t (1 - u_t), \quad (12)$$

$$\frac{\partial H}{\partial u} = 0 = (\lambda_t - \mu_t)(y_t - c_t) - (\phi_u - \phi_{1-u}), \quad (13)$$

$$\frac{\partial H}{\partial k} = -\dot{\lambda}_t = \frac{\partial y_t}{\partial k} (\lambda_t u_t + \mu_t (1 - u_t))_t, \quad (14)$$

$$\frac{\partial H}{\partial \beta} = -\dot{\mu}_t = \frac{\partial y_t}{\partial \beta} (\lambda_t u_t + \mu_t (1 - u_t)), \quad (15)$$

$$\mu_t \geq 0, \quad \lambda_t \geq 0, \quad (16)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} k_t = 0, \quad (17)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \beta_t = 0, \quad (18)$$

where λ_t and μ_t are the shadow prices of capital and technology, respectively.

Corner solutions and interior solution

Now, let us consider three possible cases.

Case 1 $k_t < \beta_t$

In this case the production function of the economy is $Y = AK$, so it is useless to increase β ($\frac{\partial Y}{\partial \beta} = 0$), (13) and (15) hold with inequality, and the share of output devoted to technological change is zero ($u = 1$). In this case,

$$\begin{aligned} \frac{\dot{c}_t}{c_t} &= A - \rho, \\ \dot{k}_t &= A\beta_t - c_t, \\ \dot{\beta}_t &= 0. \end{aligned}$$

Case 2 $k_t > \beta_t$

In this case the production function of the economy is $Y = A\beta L$, so it is useless to increase the stock of capital ($\frac{\partial Y}{\partial K} = 0$). Moreover, since we are considering only one good, part of the capital is used to consume and investment is completely devoted to increase β ($u = 0$). Equations (14) and (15) hold with inequality. In this case,

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= A - \rho, \\ \dot{k}_t &= -\delta k, \\ \dot{\beta}_t &= A\beta_t - c_t.\end{aligned}$$

Case 3 $k_t = \beta_t$

In this case we have interior solution. From (13), (14), and (15),

$$\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\dot{\mu}_t}{\mu_t}, \quad \text{so} \quad \frac{\partial y}{\partial \beta} = \frac{\partial y}{\partial k}. \quad (19)$$

To maintain the equality of (19), the equality $k_t = \beta_t$ must hold for every t . Therefore,

$$(1 - u) = u = \frac{1}{2} \quad \text{and} \quad \frac{\dot{k}_t}{k_t} = \frac{\dot{\beta}_t}{\beta_t} = \frac{1}{2} \left(A - \frac{c}{k} \right).$$

Consumption growth rate is just the difference between the marginal productivity of savings and the discount rate:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{2}A - \rho.$$

Therefore, in the long run consumption capital ratio is determined by $\frac{c}{k} = 2\rho$.

