

## A generalized index of market power

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**Abstract.** This paper analyses two approaches to measuring market power –the commonly used Lerner index and a range of exploitation measures–. It is argued that the Lerner index is designed to quantify market power from the supply side and the exploitation measures are designed to quantify market power from the demand side, and that the two approaches do not always behave in a symmetric way, since they do not always have the same bounds. To sort out these potentially undesirable properties, this paper proposes a new general index to measure market power, which is symmetrical in the sense that it is bounded between zero and one, regardless of whether the market power comes from the supply or the demand side. The index proposed allows for the presence of more than one firm and for the existence of conjectural variations.

**Key words:** market power, mark up, mark down, Lerner index, exploitation measures, industrial organization, conjectural variations.

**JEL classification:** D49, L10, L11.

**Resumen.** Este documento analiza dos enfoques para medir poder de mercado –el frecuentemente utilizado índice de Lerner y un conjunto de medidas de explotación–. Se argumenta que el índice de Lerner está diseñado para cuantificar el poder de mercado por el lado de la oferta y que las medidas de explotación están diseñadas para cuantificar el poder de mercado por el lado de la demanda, y que esos dos enfoques no siempre tienen los mismos límites. Para corregir estas propiedades potencialmente no deseables, este documento propone un nuevo índice general para medir poder de mercado, que es simétrico –estando restringido a valores entre cero y uno–, independientemente de si el poder de mercado proviene del lado de la oferta o de la demanda. El índice propuesto permite la presencia de más de una firma y la existencia de variaciones conjeturales.

**Palabras clave:** poder de mercado, *mark up*, *mark down*, índice de Lerner, medidas de explotación, organización industrial, variaciones conjeturales.

**Clasificación JEL:** D49, L10, L11.

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## 1. Introduction

This paper analyses two approaches to measuring market power –the commonly used Lerner index and a range of exploitation measures–, spelling out the central features of the main indexes within each approach. It is argued that the Lerner index is designed to quantify market power from the supply side and the exploitation measures are designed to quantify market power from the demand side, and that the two approaches do not always behave in a symmetric way, since they do not always have the same bounds.

To sort out these potentially undesirable properties, this paper derives a new general index to measure market power from profit maximization, which is symmetrical in the sense that it is bounded between zero and one, regardless of whether the market power comes from the supply or the demand side. The proposed index allows for the presence of more than one firm and for the existence of conjectural variations.

The use of the proposed index should increase awareness of the existence and measurement of market power from the supply and the demand side, while making more expedite the use of such measures in empirical estimations, regardless of the side of the market where the market power comes from.

The paper is organized as follows: the next section presents the theoretical framework in which the Lerner index and three alternative –and related– exploitation measures are derived from profit maximization, describing the main properties of each index. The following section proposes an index that overcomes some of the limitations of the standard measures considered before. The paper ends with the main conclusions.

## 2. Theoretical framework

This section derives measures of market power from profit maximization. In order to make general statements, assume that there are  $n$  firms that play Cournot to start with, and keep in mind that this assumption will be relaxed later on through the introduction of a conjectural variations coefficient. Assume also that all firms have identical cost structures.

### 2.1. Market power from the supply side

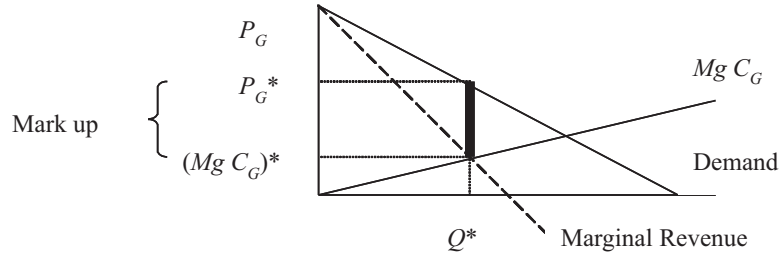
When market power is generated from the supply side (with prices greater than marginal costs, as in monopoly, monopolistic competition and –often– oligopoly), firms apply a mark up. This case is shown in figure 1.

Consider the following profit equation for a firm playing Cournot:

$$\Pi_i = P_G(q_i + Q_{-i})q_i - TC_i,$$

where

**Figure 1.** Market power from the supply side: monopoly, monopolistic competition and –often– oligopoly



- $\Pi_i$  = profits of firm  $i$ ,
- $P_G$  = per unit price of the good produced by firm  $i$ ,
- $q_i$  = quantity produced by firm  $i$ ,
- $Q_{-i}$  = quantity produced by all firms in the market, except firm  $i$ ,
- $Q$  = quantity produced by all firms in the market,
- $TC_i$  = total costs of firm  $i$ .

Since all firms are assumed to have identical cost structures, the sub-index  $i$  can be dropped. Profit maximization with  $n$  firms implies that

$$P_G(Q) + QP'_G(Q)\frac{q}{Q} = Mg C_G.$$

Thus the Lerner index –proposed by Abba Lerner (1934)<sup>1</sup>– with  $n$  firms, can be derived as

$$L_H = \frac{P_G - Mg C_G}{P_G} = \frac{s}{\eta},$$

where

- $L_H$  = Lerner index with more than one firm,
- $Mg C_G$  = marginal cost of producing the good,
- $H$  = Herfindahl market concentration index,
- $s$  = market share of a firm,
- $\eta$  =  $-\frac{\partial Q}{\partial P_F} \frac{P_F}{Q}$ , the negative of the price elasticity of demand,

and all other variables are defined as before. Given that all firms have identical market shares,

$$H = \sum_{i=1}^n s_i^2 = \frac{n}{n^2} = \frac{1}{n} = s \tag{1}$$

<sup>1</sup>This index has been criticized, among other reasons, because estimating it is complex, since it is difficult to obtain measures of marginal costs, and since prices may be affected by cyclical economic behaviour. Thus, in general, it is useful to think of the Lerner index –and of market power quantifications in general– as measuring average mark up.

and

$$L_H = \frac{H}{\eta}. \quad (2)$$

The Lerner index can also be adapted to the conjectural variations model described in Cabral (2000) by multiplying  $H$  in (2) by the conjectural variations coefficient  $\theta$ :

$$L_H^\theta = \frac{\theta H}{\eta} = \frac{\theta}{n\eta},$$

with  $0 \leq \theta \leq \frac{1}{H}$ , such that if

$\theta = 0$ , players play Bertrand and price equals marginal costs;

$\theta = 1$ , players play Cournot;

$\theta = \frac{1}{H}$ , players play collusion or cartel.<sup>2</sup>

Recall that since marginal costs are non-negative and since profit maximizing firms set marginal revenue equal to marginal costs, such firms always operate in the elastic region of the demand curve  $1 \leq \eta < \infty$ . Thus, in order to evaluate the bounds of this index, consider the following:

If  $\theta = 0$ ,  $L_H^\theta = 0$ .

If  $\theta = n$ , the least upper bound of  $L_H^\theta = 1$ , and the greatest lower bound of  $L_H^\theta = 0$ .

Besides,  $\lim_{|\eta| \rightarrow \infty} (L_H^\theta) = 0$  and, if  $\eta = 1$ ,  $L_H^\theta = 1$ . So,  $0 \leq L_H^\theta \leq 1$ .

In general, the  $n$  firms' Lerner index with conjectural variations has the following properties:

- i. It is designed to measure market power from the supply side.
- ii. It is bounded between zero and one.
- iii. The index is zero when there is no market power (perfect competition).
- iv. The maximum market power generates a value of one.
- v. The index is increasing in market power.

Bresnahan (1989) reviews a vast literature on measuring market power and provides a summary of Lerner index estimations, shown in appendix 1.

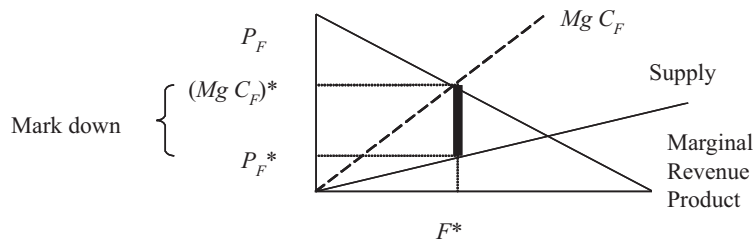
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<sup>2</sup>In this case, players replicate monopoly power.  $\theta = 1/H$  can also represent monopolistic competition if each firm with differentiated products, faces the specific demand for its variety as a monopolist.

2.2. Market power from the demand side

This section derives measures of market power from the demand side (with prices lower than marginal costs, as in monopsony, monopsonistic competition, and –often– oligopsony), assuming that all firms have identical production functions. This case, where firms apply a mark down, is shown in figure 2.

**Figure 2.** Market power from the demand side: monopsony, monopsonistic competition and –often– oligopsony



Consider the following profit equation for a firm playing Cournot (taking the demand of the factor or input by other firms as given):

$$\Pi_i = P_G Q(f_i) - P_F (f_i + F_{-i}) f_i,$$

where

- $f_i$  = amount of factor or input used by firm  $i$ ,
- $P_F$  = factor or input unit price,
- $F_{-i}$  = total amount of factor or input used in the market, except for firm  $i$ ,
- $F$  = total amount of factor or input used in the market,

and the rest of the variables are defined as before.

Profit maximization with  $n$  firms, implies that

$$MRP_i = P_F(F) + F P'_F(F) \frac{f_i}{F} = Mg C_{Fi},$$

where

- $MRP_i$  = marginal revenue product of firm  $i$ ,
- $Mg C_{Fi}$  = the marginal cost to firm  $i$ , as a demander of an input.

Since all firms have identical production functions, the sub-index  $i$  can be dropped and

$$Mg C_F = P_F(F) \left[ 1 + \frac{s}{\varepsilon} \right], \tag{3}$$

where

$$s = \frac{f_i}{F}, \text{ the share of the input used by the firm,}$$

$$\varepsilon = \frac{P_F(F)}{F} \frac{\partial F}{\partial P_F}, \text{ the price elasticity of supply,}$$

and all other variables are defined as before.

Note that under profit maximization with market power from the demand side, the marginal revenue product is equal to the marginal cost, and recall that by (1),  $s = H$ . Thus, from (3) it is possible to derive the following measures of exploitation:

### 2.2.1. Pigou's exploitation measure for $n$ firms

Arthur Pigou's (1924) exploitation measure for  $n$  firms can be derived from profit maximization as

$$PEM_H = \frac{Mg C_F}{P_F} = \left[ \frac{\varepsilon + H}{\varepsilon} \right],$$

where

$$PEM = \text{Pigou's exploitation measure,}$$

$$P_F = \text{factor or input unit price.}$$

This  $PEM$  for  $n$  firms can be adapted to the conjectural variations model by multiplying  $H$  by the conjectural variations coefficient  $\theta$  defined as before:<sup>3</sup>

$$PEM_H^\theta = \frac{Mg C_F}{P_F} = \left[ \frac{\varepsilon + \theta H}{\varepsilon} \right].$$

However, as opposed to the elasticity of demand, the elasticity of supply has the following characteristic:<sup>4</sup>

$$0 < \varepsilon < \infty.$$

Thus, in order to evaluate the bounds of this index, consider the following:

$$\lim_{\varepsilon \rightarrow \infty} (PEM_H^\theta) = 1 \quad \text{and} \quad \lim_{\varepsilon \rightarrow 0} (PEM_H^\theta) = 0, \quad \forall \theta > 0,$$

and so  $1 \leq PEM_H^\theta < \infty$ .

In general, the  $n$  firms' Pigou's exploitation measure with conjectural variations has the following properties:

<sup>3</sup>In this case, with  $\theta = 1/H$ , players replicate monopsony.  $\theta = 1/H$  can also represent monopsonistic competition if each firm with differentiated demand faces the supply of its variety of demand as a monopsonist.

<sup>4</sup>Profit maximization implies that  $MRP = Mg C_F$ . Since

$$MRP > 0 \quad \text{and} \quad Mg C_F = P_G \left[ 1 + \frac{s}{\varepsilon} \right],$$

profit maximizing firms can operate on the elastic and the inelastic regions of the supply curve.

- i. It is designed to measure market power from the demand side.
- ii. The measure is increasing in market power.
- iii. The measure is bounded by one when there is no market power (perfect competition).
- iv. The measure is unbounded as the market power rises.

### 2.2.2. Scully's rate of monopsonistic exploitation for $n$ firms

Scully's (1974) rate of monopsonistic exploitation index for  $n$  firms can be derived from profit maximization as

$$1 - RME_H = \frac{P_F}{Mg C_F} = \left[ \frac{\varepsilon}{\varepsilon + H} \right],$$

$$RME_H = \frac{Mg C_F - P_F}{Mg C_F} = \frac{H}{\varepsilon + H} = \frac{1}{\frac{\varepsilon}{H} + 1},$$

where

$$RME = \text{rate of monopsonistic exploitation,}$$

and all the other variables are defined as before.

This  $RME_H$  for  $n$  firms can be adapted to the conjectural variations model by multiplying  $H$  by the conjectural variations coefficient  $\theta$  as before:

$$RME_H^\theta = \frac{1}{\frac{\varepsilon}{\theta H} + 1}.$$

In the symmetric framework of this paper, this index can also be written as

$$RME_H^\theta = \frac{1}{\frac{n\varepsilon}{\theta} + 1}.$$

Given that  $0 \leq \varepsilon < \infty$ , in order to identify the bounds of this index, consider the following:

$$\lim_{\theta \rightarrow 0} (RME_H^\theta) = 0, \quad \forall \varepsilon > 0;$$

$$\lim_{\varepsilon \rightarrow \infty} (RME_H^\theta) = 0 \quad \text{and} \quad \lim_{\varepsilon \rightarrow 0} (RME_H^\theta) = 1, \quad \forall \theta > 0,$$

and so  $0 \leq RME_H^\theta \leq 1$ .

In general, the  $n$  firms' *RME* with conjectural variations has the following properties:

- i. It is designed to measure market power from the demand side.
- ii. It is bounded between zero and one.
- iii. The index is increasing in market power.
- iv. The index is zero when there is no market power (perfect competition).
- v. The maximum market power generates a value of one.

### 2.2.3. Boal's exploitation measure for $n$ firms

Boal's (1995) exploitation measure, generalized for  $n$  firms in Boal and Ransom (1997), can be derived from profit maximization as

$$1 + E_H = \frac{Mg C_F}{P_F} = \left[ \frac{\varepsilon + s}{\varepsilon} \right],$$

and, since  $s = H$ ,

$$E_H = \frac{Mg C_F - P_F}{P_F} = \frac{H}{\varepsilon}, \quad (4)$$

where

- $E_H$  = Boal's exploitation measure with more than one firm,
- $H$  = Herfindahl market concentration index,
- $\varepsilon$  = price elasticity of supply.

This exploitation measure for  $n$  firms can be adapted to the conjectural variations model by multiplying  $H$  in the index described in (4) by the conjectural variations coefficient  $\theta$ , where  $\theta$  is defined as before:

$$E_H^\theta = \frac{\theta H}{\varepsilon}.$$

In this symmetric framework, this index can be expressed as

$$E_H^\theta = \frac{\theta}{n\varepsilon}.$$

Since  $0 \leq \varepsilon < \infty$ , in order to identify the bounds of this index, consider the following:

$$\begin{aligned} &\text{If } \theta = 0, E_H^\theta = 0; \\ &\lim_{\varepsilon \rightarrow \infty} (E_H^\theta) = 0 \quad \text{and} \quad \lim_{\varepsilon \rightarrow 0} (E_H^\theta) = \infty, \quad \forall \theta > 0, \end{aligned}$$

and so  $0 \leq E_H^\theta < \infty$ .



In general, the  $n$  firms' Boal's exploitation measure with conjectural variations has the following properties:

- i. It is designed to measure market power from the demand side.
- ii. The index is increasing in market power.
- iii. The index is zero when there is no market power (perfect competition).
- iv. The index is unbounded as the market power rises.

Boal and Ransom (1997) collect information on a range of exploitation measures and express them in terms of  $E$ , the Boal's exploitation measure. Such information is summarized in appendix 2.

Thus, it is possible to conclude that there is an asymmetry between the Lerner index and the exploitation measures analyzed in this document, since the Lerner index is designed to measure market power from the supply side, while the exploitation measures are designed to quantify market power from the demand side. Besides, the Lerner index is bounded for both increases and decreases in market power, while the exploitation measures are not always bounded for increases in market power.

### 3. The generalized index of market power

To sort out the potentially undesirable asymmetries of the Lerner index and the exploitation measures, this paper proposes a Generalized Index of Market Power  $GIMP$ , which is symmetrical and bounded between zero and one, regardless of whether the market power comes from the supply or the demand side. The index proposed is

$$GIMP = \frac{|P - Mg C|}{\max[P, Mg C]},$$

where

$$\begin{aligned} P &= P_G \text{ if market power comes from the supply side } (P > Mg C), \\ P &= P_F \text{ if market power comes from the supply side } (P < Mg C), \\ Mg C &= Mg C_G \text{ if market power comes from the supply side } (P > Mg C), \\ Mg C &= Mg C_F \text{ if market power comes from the supply side } (P < Mg C). \end{aligned}$$

When market power comes from the supply side, the price is greater than the marginal cost and the index becomes  $GIMP_{SS}$ , which is the same as the Lerner index

$$GIMP_{SS} = \frac{P_G - Mg C_G}{P_G}.$$

Thus, the microeconomic derivation and the properties of the  $GIMP$  when market power comes from the supply side are the same as those of the Lerner

index. Besides, the *GIMP* with  $n$  firms and conjectural variations can be expressed as

$$GIMP_{SS(H)}^\theta = \frac{\theta H}{\eta}.$$

In this symmetric framework, this index can be expressed as

$$GIMP_{SS(H)}^\theta = \frac{\theta H}{n\eta}, \text{ with } 0 \leq GIMP_{SS(H)}^\theta \leq 1.$$

When market power comes from the demand side, price is lower than marginal cost and the index becomes  $GIMP_{DD}$ , which is the same as Scully's measure of exploitation

$$GIMP_{DD} = \frac{Mg C_F - P_F}{Mg C_F}.$$

The microeconomic derivation and the properties of the *GIMP* when market power comes from the demand side are the same as those of Scully's rate of monopsonistic exploitation *RME*. Besides, the  $GIMP_{DD}$  with  $n$  firms and conjectural variations can be expressed as

$$GIMP_{DD(H)}^\theta = \frac{1}{\frac{\varepsilon}{\theta H} + 1},$$

or, given symmetry, as

$$GIMP_{DD(H)}^\theta = \frac{1}{\frac{n\varepsilon}{\theta} + 1}, \text{ with } 0 \leq GIMP_{DD(H)}^\theta \leq 1.$$

In general, the  $n$  firms' *GIMP* index with conjectural variations has the following properties:

- i. It is designed to measure market power from supply or demand.
- ii. It is bounded between zero and one.
- iii. The index is increasing in market power.
- iv. The index is zero when there is no market power (perfect competition).
- v. The maximum market power generates a value of one.

The *GIMP*, as well as the best known indexes of market power used in the literature, has not been designed –and is not particularly well suited– to measure market power in the case of bilateral monopoly, as described in Pindyck and Rubinfeld (2001). These limitations also apply to other circumstances, such as the case of multisided markets –including the existence of non-internalized network externalities– as described in Rochet and Tirole (2004).

A summary of the features of the market power measures analyzed in this paper is presented in table 1.

**Table 1.** Key features of the market power measures analyzed

Index	Type of monotonicity increased market power from supply	Greater lower bound as market power falls	Least upper bound as market power increases	Source of market power measured
Lerner	Positive	0	1	Supply
Pigou	Positive	1	Unbounded	Demand
Scully	Positive	0	1	Demand
Boal	Positive	0	Unbounded	Demand
<i>GIMP</i>	Positive	0	1	Supply and demand

#### 4. Conclusions

This paper has analysed two approaches to measuring market power –the commonly used Lerner index and a range of exploitation measures–. It has argued that the Lerner index is designed to quantify market power from the supply side, while the exploitation measures are designed to quantify market power from the demand side, and that the two approaches do not always behave in a symmetric way, since they do not have the same bounds.

To sort out these potentially undesirable properties, a new general index to measure market power has been proposed, which is symmetrical in the sense that it is bounded between zero and one, regardless of whether the market power comes from the supply or the demand side. Besides, the proposed index has been extended to be used when there are several firms in the market, and to allow for conjectural variations. However, the index is not particularly well suited to measure market power under some circumstances, for example the existence of bilateral monopoly or of multisided markets.

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**Appendix 1****Summary of existing empirical work on the Lerner index**

<b>Author</b>	<b>Industry</b>	<b>Lerner index</b>
Lopez (1984)	Food processing	0.504
Roberts (1984)	Coffee roasting	0.055/0.025 (a)
Appelbaum (1982)	Rubber	0.049 (c)
Appelbaum (1982)	Textile	0.072 (c)
Appelbaum (1982)	Electrical machinery	0.198 (c)
Appelbaum (1982)	Tobacco	0.648 (c)
Porter (1983)	Railroads	0.40 (b)
Slade (1987)	Retail gasoline	0.1
Bresnahan (1981)	Automobiles (1970s)	0.10/0.34 (d)
Suslow (1986)	Aluminum (interwar)	0.59
Spiller-Favaro (1984)	Banks “before” (e)	0.88/0.21 (f)
Spiller-Favaro (1984)	Banks “after” (e)	0.40/0.16 (f)

- a. Largest and second largest firm, respectively.
- b. When cartel was succeeding: 0 in reversionary periods.
- c. At sample midpoint.
- d. Varies by type of car; larger in standard, luxury segment.
- e. Uruguayan banks before and after entry deregulation.
- f. Large firms / small firms.

Source: Bresnahan (1989) p. 1051.

## Appendix 2

Summary of existing empirical work on Pigou's exploitation measure

Author	Industry	Pigou's exploitation measure (a)
Scully (1974)	Professional baseball	4-7
Medoff (1976)	Professional baseball	1-2 (b)
Boyd (1994)	Coal mining (c)	0.24 (d)
Sullivan (1989)	Nurses in hospitals	0.75-0.26 short run; close to zero long run; 0.04-0.13 total.
Hansen (1992)	Nurses in hospitals (California only)	Less than 0.05
Boal (1995)	Coal mining	0.15-0.53 short run; essentially zero in long run; 0.03-0.09 total.
Machin, Manning, and Woodland (1993)	Employment by residential homes (e)	0.04. 0.15 with correction for hires and quits.
Ransom (1993)	Senior college teachers	5 to 18 (f)
Van den Berg and Ridder (1993)	General labour market	0.13-0.15 (g)
Brown and Meedof (1989)	General labour market	0.01-0.03

a. Boal and Ransom adjust original estimates to the exploitation measure E.

b. Data from the Reserve Clause era. The Reserve Clause is a legal clause that teams adopted, which prohibited competition for the hiring players between teams.

c. From isolated Coal Mining towns.

d. Measure not significantly different from zero statistically.

e. Homes for the elderly in England.

f. Black and Lowenstein (1991) say Ransom's estimates should be upper bound, because movers may be paid more than MRP.

g. Allowing for heterogeneity among workers lower the estimations in comparison to other papers.

Source: Boal and Ransom (1997).