EVOLUTION OF THE COLOMBIA PESO WITHIN THE CURRENCY BANDS, NONLINEARITY ANALYSIS AND STOCHASTIC MODELING

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ABSTRACT

This paper studies the behavior of the Colombian Peso exchange rate against the US dollar for the period in which a currency band was prevailing (January 1994 – Sept 1999). A descriptive analysis, which encompasses monetary policy implementation, a description of the market structure and participants, as well as the evolution of the foreign exchange rate is presented. Because a linear relationship fails to account for the evolution of the nominal exchange rate in terms of real sector supply and demand and financial system's foreign exchange positions, neural networks techniques are used in order to get some insight on the non-linearity of the time series. Consequently, statistical tests - including non-linearity tests - are performed in order to further examine the properties of the returns data and an analysis of a three-volatility-regime stochastic model with mean reversion fitted to the data in prior work is expanded.

RESUMEN

Este trabajo analiza el comportamiento de la tasa de cambio del peso colombiano contra el dólar americano para el período de vigencia de la banda cambiaria (Enero 1994 a Septiembre 1999). Un análisis descriptivo, que incluye la instrumentación de la política monetaria, la estructura del mercado y sus participantes, así como la evolución de la tasa, es presentado. Debido a que una relación lineal con la oferta y demanda observada del sector real y los cambios en la posición propia de las entidades financieras no explica la evolución del tipo de cambio nominal, técnicas de redes neuronales son utilizadas para profundizar en el entendimiento de las propiedades no lineales de la series de retornos y un análisis de un modelo estocástico con tres regímenes de volatilidad y reversión a la media, estimado previamente, es extendido.

Key words: Exchange rates, non-linearity, stochasting modelling, monetary policy.
Clasificación JEL: G10, G18, E5, E42, E44.

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I. INTRODUCTION

This paper studies the behaviour of the Colombian Peso exchange rate against the US dollar for the period in which a currency band was prevailing (January 1994 – Sept 1999). Two main parts compose this work. On the one hand, a descriptive analysis is presented in part II, which encompasses monetary policy implementation, a description of the market structure and participants, as well as the evolution of the foreign exchange rate. On the other, as linear relationships fail to account for the evolution of the nominal exchange rate in terms of real sector supply and demand, as well as financial system's foreign exchange positions, neural networks techniques are used in order to get some insight on the non-linearity of the time series, and an analysis of a three-volatility-regime stochastic model with mean reversion fitted to the data in prior work (Brooks and Revéiz, 2002) is presented. Some statistical tests, including non-linearity tests, are performed in order to further examine the properties of the returns data. Section VI concludes.

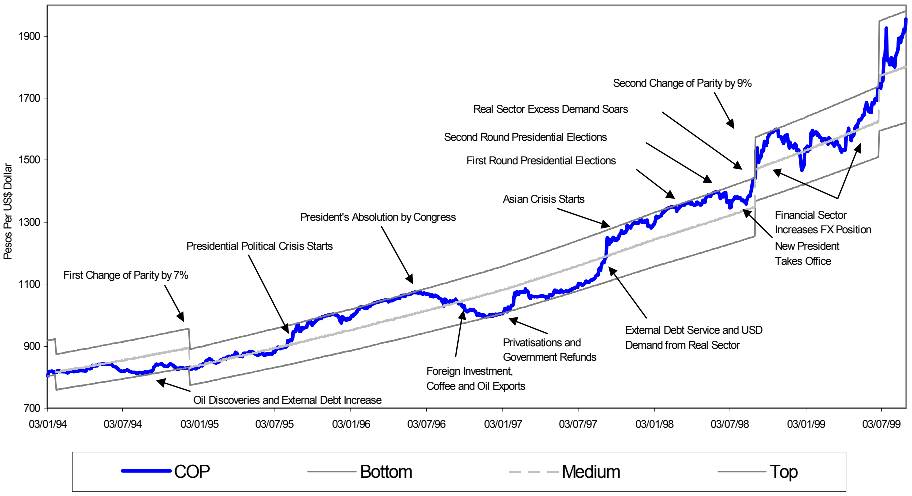
II. MARKET DESCRIPTION

1. Evolution of the nominal exchange rate

In this section, we provide a short summary of the evolution of the Colombian peso (COP) within its exchange-rate bands from 1994 onwards. Figure 1 charts the COP from January 1994 to September 1999.

Figure 1

Evolution of the Colombian peso (COP) within the foreign exchange band



During the period under study, the Colombian peso showed an overall tendency to devalue against the US Dollar as predicted by standard economic theory. However, since 1994, the COP has experienced two exogenous shocks that have resulted in temporary real or nominal appreciation of the currency. The first one occurred in 1994 when the exchange rate fell to 811.91 pesos per US Dollar, touching the bottom of the band. In that year, some major oil discoveries were made in Colombia, which could have modified the long-term equilibrium rate. Some changes were implemented by the Central Bank in terms of capital-flows controls. As a result, private external debt increased dramatically. Finally, the government granted mobile-phone contracts to both domestic and multinational companies. In order to pay for their share in the new companies, these firms brought US Dollars and converted them into Colombian Pesos. Given that these factors would have a structural impact on the long-term exchange rate, the Monetary Authorities decided to appreciate the prevailing currency band by 7%.

Notwithstanding this, the exchange rate changed direction during 1995. At the end of June, allegations that the president's election campaign in 1994 had been partially financed by the drug cartels, resulted in a sharp devaluation of the currency. On August 29th, the exchange rate was only 2.5 pesos below the top band.

Uncertainty in the foreign-exchange market prevailed until the middle of 1996 when the President was absolved by Congress. The dramatic appreciation of the currency that followed took the exchange rate to the bottom of the band at the end of 1996. With the end of the political turmoil, the currency's subsequent movements were driven mainly by the impact of direct foreign investment in the country (e.g. Spanish banks acquiring shares in the Colombian financial sector or Cemex, the Mexican cement producer, buying Cementos Samper), the privatisation process held by the government (e.g. banks and some hydroelectric plants) and the inflows from coffee and oil exports. The rate hit a low, within the band, of $994.02 per dollar. The Central Bank had to support the dollar by selling Colombian pesos in the market. This resulted in an excess of monetary supply in the economy.

During 1997, the excess of liquidity stimulated the private sector into buying dollars against pesos, in order to reduce the stock of external debt concentrated in the second half of 1997 (external loans taken out in 1994 with a maturity of 3 years as a result of the opportunities created by Resolución 21 of 1993). Because the domestic interest rate was low, firms had no incentive to renew their external debt. The Colombian peso lost approximately 12% against the dollar between August and September 1997. By the end of the year, the currency remained near the top of its band as concerns grew over the fiscal and current accounts deficits and the Asian crisis hit Latin America.

In February 1998, the exchange rate reached the upper limit of its band, around 1345 pesos per dollar. In order to defend the currency, the Central Bank sold approximately U$390 million and raised its 30-day interest rate. The peso appreciated slightly, moving away from the top of the band. However, a very close race between the two main presidential candidates in the first round of the elections (held on March 31st), resulted in new political uncertainty as one of the candidates was seen as a continuation of the existing government. Consequently, the exchange rate again hit the top of its band. This time, after selling more than 250 million dollars, the Central Bank decided to decrease the daily liquidity usually provided to the financial system. As a result, the interest rate increased markedly up to 80%, equivalent to a 60% real interest rate. The opposition party won the second round of the presidential elections, and the exchange rate moved away from the top of its band reflecting the market's renewed confidence in the structural changes (mainly, the reduction of the fiscal deficit).

However, the new president's honeymoon did not last long. As the new government was being appointed and the new economic programme was being devised, the market perceived that the new fall in commodity prices and the Russian crisis would worsen the trade deficit in such a way as to make devaluation of the Colombian peso inevitable. The COP once again hit the top of its band at the beginning of September. Yet, on 3rd September, the Central Bank decided to devalue the central parity by 9%. That day, the COP lost approximately 6% against the USD. One argument in support of such decision is that as a result of the trade and fiscal deficits (above 5% and 4%, respectively) that Colombia was running, the equilibrium exchange rate could have depreciated. However, pressure on the exchange rate prevailed and was intensified as a result of the peace negotiations with the guerrillas that in turn raised the level of violence, and an increase in the cash net foreign exchange position of the financial intermediaries. US dollar positions were actively managed through forward, indexed forwards and options contracts in order to fulfil regulatory requirements regarding net foreign-exchange positions, and as limitations to the total amount of cash holdings were imposed by the Authorities in 1999 - see annex 1.

Speculation against the currency band, combined with poor economic performance (the worst recession since the 1930), global financial turmoil that directed capital to safe havens, and escalating violence obliged the authorities to eliminate the currency-band regime and the currency was allowed to float freely at the end of September 1999.

2. Monetary Policy and Traditional Exchange Rate Models

2.1 Operating procedures for the implementation of monetary policy

2.1.1. Monetary Policy

As mentioned before, the new constitution decreed that the main responsibility of the Central Bank, Banco de la República, was the preservation of the purchasing power of the Colombian people, that is lowering the inflation rate. In this context, the adoption of an explicit nominal exchange-rate band by the Colombian authorities in 1994[1] was in response to the implementation of a stabilisation model based on control of the monetary base in order to lower inflation. The case for an exchange rate band in Colombia was that allowing the nominal exchange rate to fluctuate

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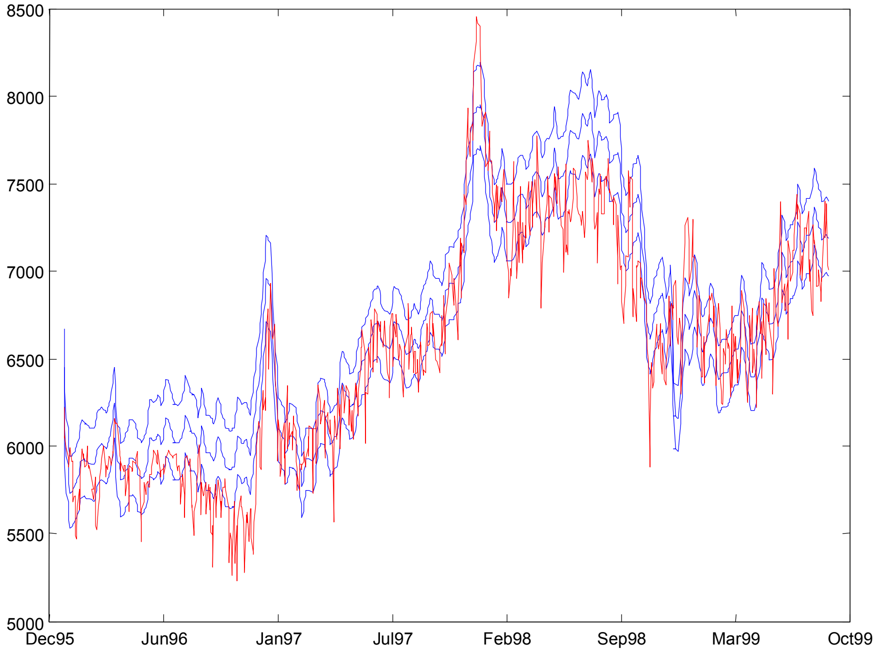
[1] An implicit band was established in November 1991.

within a band provides the Central Bank with monetary flexibility, while anchoring the market's expectations. In terms of the transition from political capitalism - in which the State served as an insurance system by offsetting the price cycles of the predominant export products - crawling bands allowed for a compromise between monetary flexibility (for the now independent Central Bank in order to fulfil its constitutional obligation) and some degree of certainty for the private sector and the Government.

In order to make this "model" operational, the Board of Directors set an inflation target each year that was consistent with the fundamentals of the economy. The latter was used to determine an appropriate annual growth rate for the Monetary Base and an adequate crawling rate for the exchange rate band. Thus, an estimate of M3 annual growth, consistent with this goal, was used to construct a monetary base "corridor" which in turn was used as a reference for carrying out Open Market Operations. The graph below shows the evolution of the monetary base and the corridors between January 1996 and August 1999.

Figure 2

Monetary base and Corridors from 02-Jan-1996 to Aug-31-1999
(MM Colombian pesos)



Observe that the monetary corridor is used only as a reference and that the monetary base can remain above or below the limits for small periods of time. Two additional features are worth nothing: during that period the monetary base was highly volatile and exhibited strong seasonality during the last two months of each year (demand for monetary base soars) and in the first month of each year the excess demand fell. As expected, the Colombian Peso appreciated in December and depreciated in January as the cost of holding USD first increased and then decreased as the interest rate varies.

The crawling rate for the exchange rate band was determined following the international convention to set the slope of the parity rate equal to the difference between the inflation goal and the expected international inflation rate in order to decrease inertia in the rate of inflation. Initially (1994), the slope of the band in Colombia was 11% and was usually reset at the beginning of each year along with the new macroeconomic program and inflation goal. For 1999, the crawling rate of the parity was 11%.

The width of the band from the central parity was ± 7% for the whole period that the band was in existence (January 1994-September 1999). The Central Bank committed to keeping the nominal exchange rate within those limits. In addition, in order to decrease the volatility of the interest rate, the Central bank set a floor and a cap on very short-term interest rates.

In summary, Banco de la República periodically set 3 bands – a Monetary Base Corridor, an Exchange-Rate Band and an Interest-Rate Range – in order to control inflation. Although this system gave the Central Bank a lot of flexibility, these 3 constraints on the main economic variables act against each other, forcing the Monetary Authorities to drop or modify one or more of its bands. Conflict between the Central Bank's foreign exchange and monetary goals could arise if, while selling securities to the financial system (monetary contraction), the Central Bank in its commitment to defend the bottom of the band had to buy dollars (resulting in monetary expansion). This could prevent the Central Bank from attaining its objective of lowering the inflation rate, as the amount of liquidity in the economy cannot easily be limited. Conflicts could also arise if the exchange rate touched the top of its band as dollar sales would diminish the monetary base. Obviously, conflicts between monetary objectives and interest-rate ranges could easily appear.

The constitutional obligation of the Banco de la República and the subsequent implementation of monetary policy by defining a monetary-base corridor, an interest-rate range and a currency-band simultaneously, gave rise to realignment expectations (i.e. redefinition of the band's parameters) by the economic agents as these contradict the long-run sustainability of the exchange-rate system. In an effort to avoid these contradictions, the Board had given priority to the monetary base (in line with the constitution) but the ambiguity of how, and under which conditions, this hierarchy would be applied added more uncertainty. In this context, realignment probabilities were affected by many factors: the level of reserves, other macroeconomic variables and also agents' perceptions of the effective and potential complexity of the political and economic arrangement.

2.1.2. Implementation of the Crawling Band

The foreign-exchange band mechanism is usually defined by determining a reference for the nominal rate, a central parity, a crawling rate and the band's width. The following paragraphs describe the decisions taken by the Colombian monetary authorities with respect with these parameters, as well as their modifications through time.

2.1.2.1. *The Basket of Currencies*

In Colombia, between 70% and 80% of external trade was carried out in US dollars during the period under study (1994-1999) and approximately the same proportion of the stock and service of external debt was denominated in US dollars. For this reason, the reference for the nominal exchange rate was chosen to be the US dollar.

2.1.2.2. *The Central Parity*

Because it is very difficult to define a priori the equilibrium exchange rate, the Central Bank of Colombia, while creating the mechanism, decided that the exchange rate from the day before the system came into force should become the parity rate. Since then, the parity has been modified on several occasions before the band's elimination in September 1999. At the end of 1994, when oil discoveries in the country were made, the parity was appreciated by 7%, that is, the bottom of the former band became the middle of the new band as oil exports would imply increased US dollar incomes for the country. The second occasion was on September 3rd, 1998, an was the result of increasing pressure on the peso triggered by the Asian crisis and the worsening of the twin deficits (fiscal and current account). On this occasion, the parity was depreciated by 9%.

2.1.2.3. *The Crawling Rate*

Following the international convention to set the slope of the parity rate equal to the difference between the inflation goal and international inflation, the slope of the band in Colombia was set at 11% in 1994. It was usually reset at the beginning of each year with the new macroeconomic program and inflation goal. In 1998, the crawling rate of the parity was 13% and in 1999 it was 11% where it remained until the removal of the mechanism in September that year.

2.1.2.4. *The Band's Width*

As mentioned before, both external shocks and exchange-rate volatility have to be taken into account in order to determine the band's width. Historical data up to 1994 showed that a width of ±4% from the central parity would be adequate to absorb external shocks. However, the Banco de la República chose a width of ±7% as further developments were expected on the foreign-exchange market and the possibility of loose fiscal policy from the government could not be discarded. The band's width was not modified whilst the mechanism remained in place.

3. Foreign Exchange Market Description

Currency markets in Colombia encompass spot, next day and forward trades. In the period under study, OTC options were traded occasionally. Inter-bank COP trades value spot from 8:00 a.m. to 13:00 p.m. From 14:30 p.m. to 16:30 p.m. the convention was that any transaction would be settled on the next day. Nevertheless, this "Next-Day" market tended to be very illiquid. Therefore, the Central Bank intervened only in the spot market to protect the limits of the currency band. During this period, more than 80% of overall business was conducted through the *Citiinfo* system. The remaining trades were usually executed by telephone.

As the band was removed in September 1999, data was considered only up to August of that year. For the Intraday data - up to June 1998 - the market transacted actively in this system. From this point onwards, new trading systems came into the market and trading became fragmented until a new competitor trading system was chosen by the intermediaries for most of their transactions.[2]

3.1. Players, Trading Mechanisms and Settlement

Article 68 of the 'Resolución Externa 21 de 1993' defined the intermediaries of the foreign exchange market. Commercial and mortgage banks, investment banks, savings and loans institutions, commercial financing companies, the Financiera Eléctrica Nacional (Government's electrical energy financing institution) and the Banco de Comercio Exterior de Colombia (Government bank whose duty is the promotion of exports) were authorised to trade professionally.

However, operations were limited for savings and loans corporations and commercial financing companies unless the paid-up capital and legal reserve are at least the minimum required for the constitution of an investment bank. Article 74 states that intermediaries may not charge a commission when buying or selling currencies and authorises spot trading for next day settlement. Payment convention in the market is same day, as Central Bank intervention is settled in that manner.

In 1998, out of a total of some 150 potential intermediaries, approximately 75 reported their net foreign-exchange position to the Superintendencia Bancaria. On the other hand, the Citiinfo information system had approximately 100 member institutions, of which 55 participated in the market and 25 to 30 were highly active. Banks and investment banks were the most dynamic institutions in the market.

In the Citiinfo system, market participants included the prices at which they are willing to trade on either side of the market, as well as the dollar amounts. The name of each intermediary was posted with its quotes. The screen could be ordered either by best-selling or best-buying price. The price of the last trade was shown (without the agent's names or quantity), as well as the average rate for the day. A trader could deal in 3 distinct ways using this system. He could take or 'hit' the quotes on the screen, one

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[2] The Citiinfo system is indirectly linked to Citibank and although Chinese walls effectively existed, many institutions felt uncomfortable with this situation and favoured a change in system.

at a time or several at once. In this case, he would be paying the bid/offer spread for immediacy. Otherwise, he could quote his prices and amounts via the system and wait until another participant took or 'hit' them. This way, he didn't pay the bid/offer spread but he was taking the risk that the price will move against him or was hit on the other side. The system allowed participants to send public or private messages by using the intermediaries' codes (a single code exists to send a message to all participants). As each transaction had to be confirmed and settled separately from the information system, traders could trade confidentially using this tool.

By presenting the data in this way, the system provided the user with information not only on the trades but also on the behaviour of the quotes: trends, bid/offer spread variations, liquidity signals from many sources, namely the number of actively participating institutions, the amounts quoted, the gap between price quotes (from the best quote to the next, etc) and the bid/offer spread. Trading complexity increased, as players may tend to use, or create, this information to their advantage. For instance, a trader of a large bank (as the names of the institutions are transparent, size is a concern) wishing to sell may pressure the market up by quoting to buy small amounts at high prices. If he is successful, others will follow and then when the market rallies, he can unwind his position at a good price. These strategies appeared to be useful at the opening of the market, when traders did not have a definite view on the market's direction. Citiinfo's General Assembly agreed that operations could be reversed through the system up to ten minutes after execution.[3] However, given that the transactions were not settled automatically by the system, if two institutions decided to cancel a transaction before settlement, they could do so and the market would not know.

Article 77 of resolution 21 of 1993 authorises Banco de la República to intervene in the foreign-exchange market with the goal of 'avoiding undesired fluctuations in the exchange rate and the stock of international reserves in conformity with the policies established by the Board of Directors, directly or indirectly, either through the spot or the futures [futures, forward and options] markets' for the financial intermediaries, as well as the Nation through the Ministerio de Hacienda y Crédito Público (Finance and Public Credit Ministry). Banco de la República was a member of the Citiinfo information system as this same article authorises it to intervene through any procedure or mechanism used by the financial intermediaries to trade amongst themselves (inter-bank transactions).

In addition, article 79 compelled the Monetary Authorities to publish daily the rates at which they will accept offers to either buy or sell US dollars to the institutions mentioned in article 77. In addition, they must also indicate the rates at which they would be willing to buy or sell US Dollars for the next 10 working days. In practice, these rates were the lower and upper limits of the currency band. However, the Central Bank could also trade at other rates within that range during the day if it was considered expedient.

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[3] Mistakes such as hitting the wrong side of the market may occur.

Because the target depreciation, the width of the band and the central parity were given, the expected values for the currency band, for each year, could be approximately computed by all market participants. Nevertheless, Banco de la República reserved the right to change its parameters, or to eliminate the crawling band, at any time. This action was costly in terms of credibility. The Bank's commitment to defend the band was very strong as evidenced by the frequency of intervention performed in the limits of the band between 1994 and 1999.

No participant from the real sector of the economy was allowed to trade in or to observe the Citiinfo system screens. In 1998, Tesoro Nacional [National Treasury] and Empresa Colombiana de Petróleos, Ecopetrol [Government Petroleum Company], were given access to the system as observers. Other real-sector participants relied mostly on telephone information provided by intermediaries, information published daily (internet, bulletins) by the Central Bank and the Superintendencia Bancaria and a few relied on the services provided by Reuters and Telerate. However, hardly any bank quoted consistently via Reuters. Telerate started quoting a few times a day in 1997 and by 1998, quotes were provided through their system every five minutes. Still, their market was concentrated on the financial sector. Note that no futures market, in the shape of an organised exchange, existed during the period under study and therefore the only relatively liquid market was Citiinfo's.

Participation of the real sector has traditionally been by government entities such as the National Treasury, Ecopetrol and the energy generation and distribution companies and private or mixed sources, namely coffee and flower exporters, coal exporters (Cerrejón), pharmaceutical companies, telecommunications, etc. The privatisation and opening up of the economy undertaken under President Gaviria (1990-1994) resulted in increased imports (mainly automobiles, consumer products and food) and capital inflows from the privatisation of telecommunication, energy and financial institutions. Current account and fiscal deficits were partially offset by an increase in private and public debt and privatisations. The dollar amounts involved in these operations often surpassed the amount traded daily in the market, generating illiquid conditions and price volatility. As expected, the complexity of monetary policy design, planning and execution all increased under these conditions.

3.2. Citiinfo Spot Market

Until May 1996, the spot market traded until 16:00 hours. But, as trades were settled the same day, liquidity in the afternoon was limited. For this reason, the Next-Day market was formally introduced from 14:30 to 16:30 with next-day settlement and the spot market was open between 8:00 and 13:00 hours. Most of the statistics below are computed for the same day (spot) market because of its liquidity and because Central Bank intervention was performed in this market.

Table 1
Citiinfo Market General Statistic

Description	1994	1995	1996	1997	1998	1999*
Total Amount Traded (US$ Millions)	11578	27778	29221	42294	46609	15791
Daily Average Amount Traded (US$ Millions)	47.54	114.78	120.2	174.68	193.25	99.202
Average Daily # of Trades	140.71	304.31	275.23	364.67	278.47	129.74
Yearly Returns Spot Market (%)[ln(Close/Open)]	-1.755%	12.116%	-1.419%	16.362%	2.771%	14.028%
Total Yearly Returns (%)[Closing Price]**	2.797%	17.669%	1.396%	25.421%	17.848%	22.198%
Total Real Sector Supply (US$ Millions)	8166.9	12706	14578	16334	15824	8009.7
Total Real Sector Demand (US$ Millions)	8686.1	13405	14278	18255	18249	9342.9

* Data up to 31-Aug-1999
**Computed from daily closing prices (includes the Next-Day Market)

In Table 1 (above), returns were computed in each of two ways: using the closing and opening prices for each day; and using closing or average prices from two consecutive days. The latter measure effectively includes the spot market, the next-day market and any gap between closing and opening prices in the markets, whereas the former measure is exclusive to the spot market.

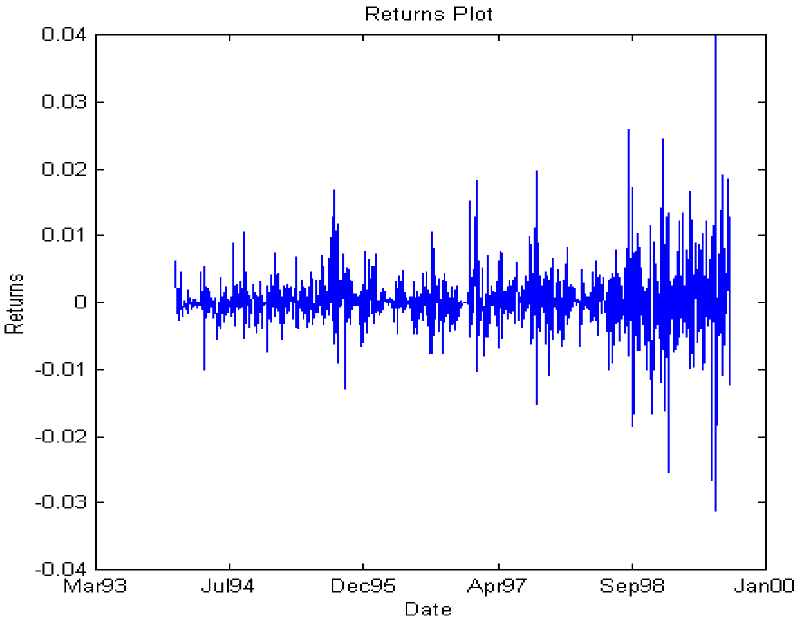
As the spot market is the most liquid (the next day market accounts for less than 3% of the amount traded in the spot market), one might expect that this market would account for most of the returns. However, it barely explains 50% of the depreciation for the period. The variance, however, is very similar for both measures. Table 2 presents various moments of the returns of the COP measured for the Citiinfo spot market and for the overall market (Citiinfo Spot and Next-Day Markets).

Table 2
Moments of COP Returns (Spot and Overall Market)

Market	Mean	Median	Standard Deviation	Variance	Kurtosis	Skewness
Spot Citiinfo	0.00030642	5.831e-005	0.004327	1.8723e-005	16.0309	0.59184
Overall	0.00064061	0.00035141	0.0044933	2.019e-005	23.4658	1.6023

Part of this effect is explained by a recurring depreciation gap at the beginning of every week that results from the depreciation of the currency band and the interest-rate differential. Gaps between closing and opening prices between markets may occur *de facto*, the market may open immediately higher or lower, or as the consequence of the price formation process in the first few minutes of trading. On average, the first trade is agreed upon 11 minutes and 40 seconds after market opening. Figure 3 presents the returns for the Citiinfo spot market.

Figure 3
COP Returns for Citiinfo Spot Market



The autocorrelation function for spot returns is negligible, whereas it appears to be significant for returns computed using daily averages. Next-Day market settlement could result in depreciation via the interest-rate differential that may seem to be implicitly included in the spot market the next day (thereby increasing the depreciation rate). For both 1994 and 1996, depreciation was much lower than the crawling rate of the band determined by the Monetary Authorities (in 1994, the band appreciated by 7%), while in each of the other years Central Bank expectations were underestimated. As a result, a realignment of the band by 7% was implemented in 1998 and the system was eliminated in September 1999. The total logarithmic depreciation for the period (Jan-1994/Aug-1999) was 87% and the arithmetic depreciation was 139%.[4]

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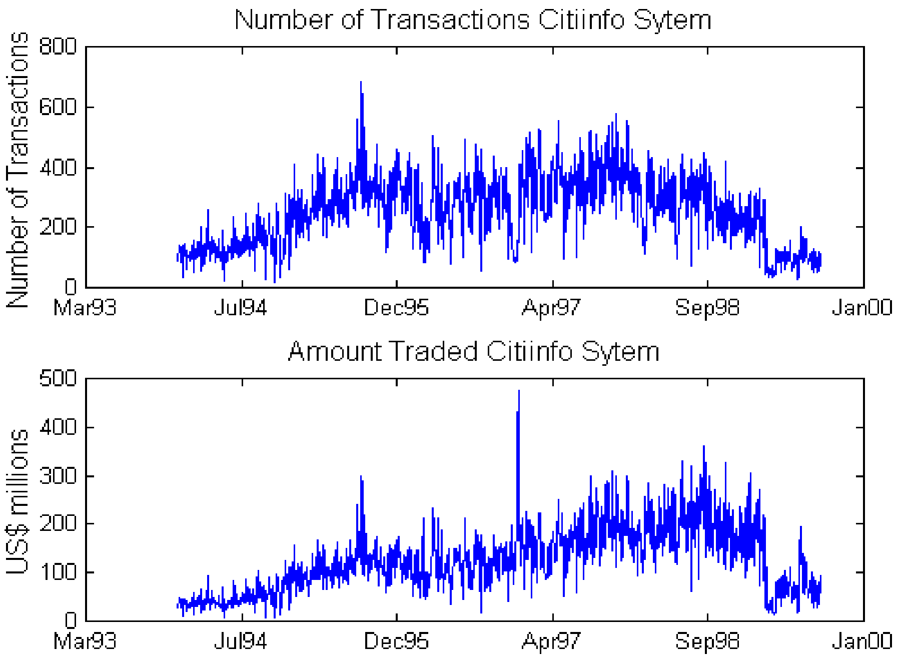
[4] Note that as the numbers are significant, the usual approximation does not hold. Returns are computed for a COP-based investor.

Electronic trading started in 1995. In that year, the amount traded increased by 140% and in 1996 remained around 29 billion dollars. In 1997, it grew by 45%, then by a furthern10% in 1998. In 1999, the implementation of a tax on all banking transactions to relieve fiscal deficit pressures resulted in a decrease of 49%.[5]

Figure 4

Number of transactions and amount traded via Citiinfo System


Number of Transactions Citiinfo Sytem

Amount Traded Citiinfo Sytem

Daily average trade mirrored total transacted amount, whereas the number of transactions declined in 1996, 1998 and 1999, respectively, perhaps as a result of Central Bank intervention at the limits of the currency bands during those years, which tends to occur in large amounts while inter-bank liquidity shrinks. The number of transactions and the daily market volume in the spot market are highly auto-correlated and slowly decaying (Figures 5 and 6).

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[5] The trading amount for the last four months of the year was computed as a linear projection of the amount traded in the first 8 months of the year.

Figure 5

Autocorrelation Function for Daily Number of Transactions

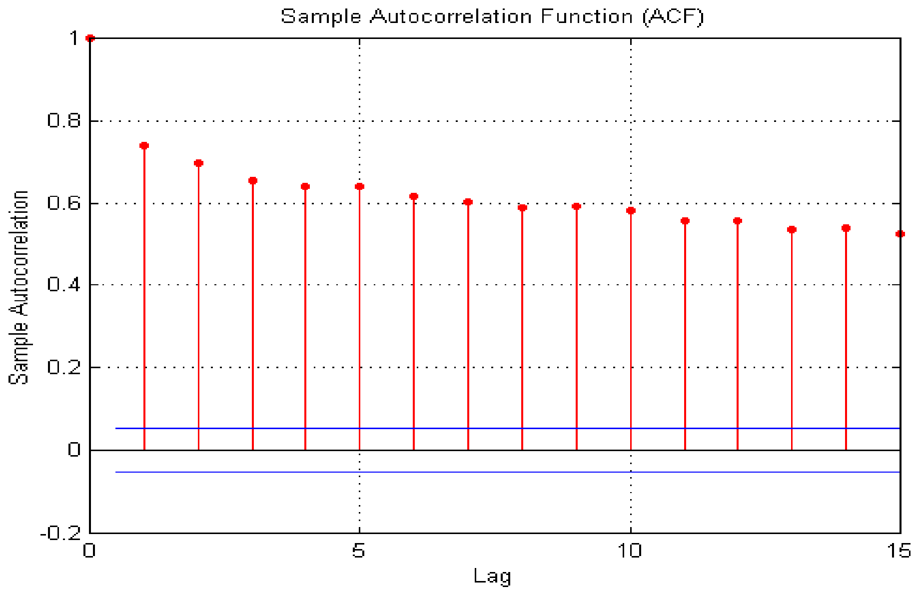
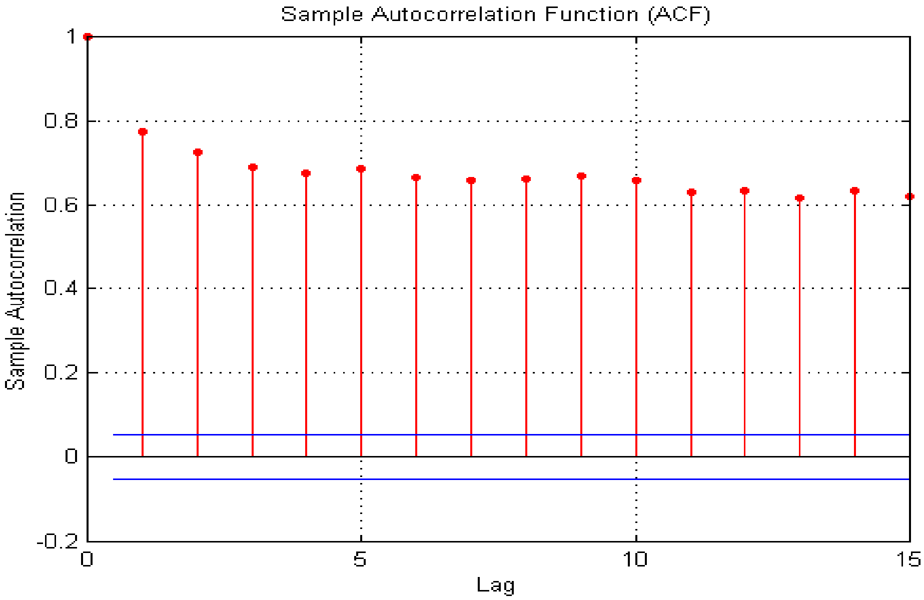


Figure 6

Autocorrelation Function for Daily Amount Traded



Partial autocorrelation functions decay geometrically in both cases, and both yield an autoregressive process with 9 lags according to the Bayesian Information Criterion. Dickey-Fuller tests without a deterministic part do not consistently reject the presence of a unit root.

Surprisingly, an excess of demand from the real sector in 1994 did not result in a depreciation of the currency, perhaps as a result of the expectations created by telecommunication contracts and forthcoming privatisations.

Table 3
Citiinfo Market Growth Statistics

Description	1995	1996	1997	1998	1999*
Total Amount Traded Growth	140%	5%	45%	10%	-49%
Daily Average Amount Traded Growth	141%	5%	45%	11%	-23%
Average Daily # of Trades Growth	116%	-10%	32%	-24%	-30%
Total Real Sector Supply Growth	56%	15%	12%	-3%	-24%
Total Real Sector Demand Growth	54%	7%	28%	0%	-23%

* Information up to September 1999, a linear projection was made for the last quarter of 1999.

In 1996, the real sector offered more US Dollars than it demanded, resulting in a pronounced appreciation of the peso during the second half of the year (Table 3). The graphs in figure 7 show the daily supply and demand from the real sector. Excess supply in 1996 was high in August because of privatisations or acquisitions (e.g. cement company 'Cementos Samper' and the financial institution Banco Ganadero) and during the last quarter when funds intended to pay for energy privatisations and financial institution privatisations or acquisitions showed up.

Table 4
Moments of Daily Net Real-Sector Supply and Demand

Mean	Median	Standard Deviation	Variance	Skewness	Kurtosis
-5.1061	-6.63	25.4278	646.5746	4.8485	68.7204

Daily average real-sector supply and demand is US$58.3 million and US$63.64 million, respectively. The net participation of the real sector is auto-correlated, with values lower than 30% .The BIC information criteria yield an AR(2) process. The presence of a unit root is rejected at the 1% significance level. Gaussianity is rejected by Hinich's test.[6]

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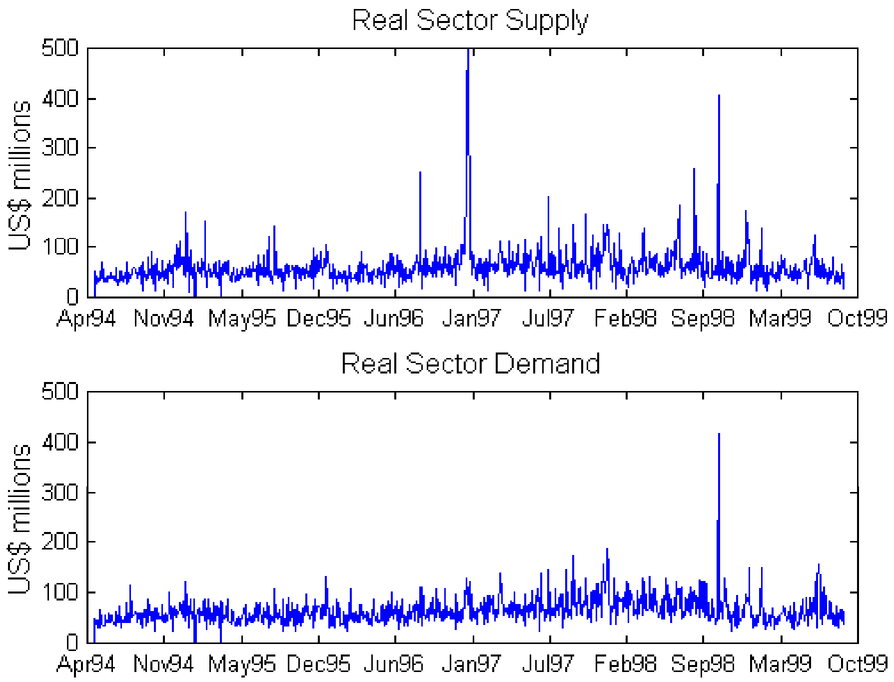
[6] This result is supported by the bi-spectrum plot.

Figure 7

Real-Sector Supply and demand



Daily net real-sector supply and demand has a mean US$ 5.1 million excess demand with a standard deviation of US$ 25 million (Table 4).

III. STOCHASTIC MODEL FOR THE COLOMBIAN PESO

To get a better understanding of the dynamics of the Colombian Peso, a stochastic model of the exchange rate in the presence of currency bands in the tradition of Krugman's (1991, 1992,1997b) canonical model is constructed and estimated. Any model of the Colombian Peso must account for the stochastic part of the exchange rate as well as the effective complexity that arises from the regularities created by the band's properties, namely its crawling rate, higher volatility near the central parity and its width.

1. The Model

Brooks and Revéiz A. (2002) model these features by incorporating a drift component, a volatility-switching factor related to the position of the exchange rate within the bands and a pair of reflecting barriers, respectively. The additional characteristic of this model is that a central-parity reversion factor is included in the stochastic differential equation (SDE). This modification reflects the fact that if the band is credible and the currency is trading close to the limits, expectations of future intervention result in a correction of the trend by the agents. Moreover, mean reversion could reflect interventions by the Central Bank to push the currency towards the Central Parity. Note that intra-marginal intervention could be used by the Authorities to reduce intra-day or daily volatility or to drive the rate towards the mid-point of the band. In the model presented, the former is embedded in the diffusion coefficients that form part of the input to the model.

The resulting specification of the model is:

$$dX(t) = A[\,X(t), \mathrm{Medbd}(t)\,, t\,].\, dt + B[\,X(t), t].\, dW(t)$$

$$A[\,X(t)\,, t\,] = \mathbf{a}.\, X(t) + \mathbf{c}.[X(t) - \mathrm{MedBd}(t)]$$

$$B[\,X(t)\,, t] = \begin{array}{ll} b1.\, X(t) & \text{if } A[\,X(t)\,, t\,].\, dt + B[\,X(t), t].\, dW(t) \in \text{Regime 1} \\ b2.\, X(t) & \text{if } A[\,X(t)\,, t\,].\, dt + B[\,X(t), t].\, dW(t) \in \text{Regime 2} \\ b3.\, X(t) & \text{if } A[\,X(t)\,, t\,].\, dt + B[\,X(t), t].\, dW(t) \in \text{Regime 3} \end{array}$$

with the following constraints on the limits of the band and behaviour within:

$$\begin{array}{ll} \mathrm{MaxBd}(T) \Rightarrow X(0) + \int_{t=0}^{T} A[X(t),t]dt + \int_{t=0}^{T} B[X(t),t]dW(t) \succ \mathrm{Maxbd}(T) & \\ X(T) = \quad X(0) + \int_{t=0}^{T} A[X(t),t]dt + \int_{t=0}^{T} B[X(t),t]dW(t) & \\ \qquad \mathrm{MinBd}(T)) \Rightarrow X(0) + \int_{t=0}^{T} A[X(t),t]dt + \int_{t=0}^{T} B[X(t),t]dW(t) \prec \mathrm{Minbd}(T) & \end{array}$$

A[X(t), t] includes the drift, {**a**. X(t)} and the Central Parity Reversion factor, {**c**.[X(t) – MedBd(t)]}. The coefficients **a** (drift) and **c** (reversion) are unknowns to be estimated.

B[X(t), t] is a diffusion function and **dW(t)** is an increment of a Brownian motion that represents the innovation term in the SDE. The diffusion function consists of 3 independent diffusion coefficients (**b1**, **b2** and **b3**) linked to a corresponding regime for the exchange rate. In order to define these regimes, the currency band is arbitrarily divided into four intervals of equal size. The intervals that include the limits are defined as regime 1 and regime 3 (top and bottom, respectively) while the two inner intervals are defined as regime 2.

Finally, constraints are imposed on the dynamics of the exchange rate in the form of limits on the band to preclude the path followed by X(t), from the first equation above, crossing above Maxbd(T) or below Minbd(T) in time T. In practice, Central Bank intervention will ensure that the exchange rate will not surpass these limits.

By setting **c**=0 we obtain a general Geometric Brownian Motion process with drift, reflecting barriers and regime-switching volatility. By additionally setting **b1**=**b2**=**b3**=**b** we would obtain a GBM process with drift and reflecting barriers.

2. Numerical Approximation: Euler Scheme

A numerical approximation is used to estimate the solution to the above SDE. The scheme is as follows.

Following Kloeden, Platen and Schurz (1997), we consider an Ito process $X = \{ X(t), t_0 <= t <= T \}$ that satisfies the stochastic differential equation:

$$dX(t) = A[X(t) , t]. dt + B[X(t), t]. dW(t)$$

where the functions A[X(t) , t] and B[X(t), t] are the drift and diffusion coefficients respectively and the initial value of X is known

$$X(0) = X_0$$

For a given discretization of the time period [0,T] into n intervals of equal length k= T/n, an Euler approximation is a continuous-time stochastic process $Y= \{ Y(t), t_0 <= t <= T \}$ satisfying the difference equation:

$$Y_{n+1} - Y_n = A(t_n, Y_n)(t_{n+1} - t_n) + B(t_n, Y_n)(Wt_{n+1} - Wt_n)$$

for n= 0, 1, 2,..., N-1 and with $Y_n = X_0$.

The sequence of values for the Euler approximation can be computed in a similar way to those for the deterministic case. However, for the calculations, Brownian motion increments $\Delta W = (Wt_{n+1} - Wt_n)$ have to be generated. These have a Normal distribution with mean zero and standard deviation equal to one.

3. Grid search algorithm using the Method-of-Simulated-Moments (MSM)

Smith and Spencer (1992) estimate and test exchange-rate target zones for the DM/ITL spot rate using the method-of-simulated-moments. In the MSM procedure, the parameters are estimated by matching the sample moments of the historical data with the moments of the simulated series by minimising a loss function of the difference between the two.

The main argument to support the use of this technique is that the unobservability of the fundamentals precludes any direct estimation of a non-linear regression model linking the exchange rate to the fundamentals. It also precludes any maximum-likelihood estimation or analytical calculation of the moments of the exchange rate. Moreover, simulation estimation must be used, as the analytical expressions for the moments used in the estimation may not be known.

Stochastic solutions are evaluated by their sampling characteristics. Therefore, the objective of an optimisation in this context is to obtain a set of parameters (drift and/or mean-reversion coefficient) that generate a sample of paths that will have similar statistical properties to those of financial series. In this framework, we will consider an adequate solution to be the combination of parameters that more closely replicates the proportion of days spent by the exchange rate in each regime. In order to accomplish such a goal, we define a loss function, **Ω,** that sums the absolute deviation from the target proportions under each regime for ***M*** runs of the model and ***T*** time steps. In this procedure, the Ω function is computed ***S*** times for each set of parameters. Then, its mean is calculated and recursively the algorithm tries to minimise the mean with a set of constraints that relate to the values that the parameters can take as well as the deviation from each regime. Formally the process is as follows.

Define the regime function *Z[.]* as:

$$Z^{i}_{m,t} = \begin{matrix} 1 & \text{if } \mathrm{X}_{\mathrm{m,t}} \subset \text{Regime i} \\ 0 & \text{otherwise} \end{matrix}$$

where ***m*** refers to the path, ***i*** to the regime and ***t*** to the time step for a given path. $\boldsymbol{X_{m,t}}$ is the value of the exchange rate for path **m** at time **t**.

The function $\Omega_{\mathbf{a,c}}$ is then computed for each set of parameters **a** and **c**, ***S*** times as:

$$\Omega_{a,c} = \sum_{i=1}^{I} \left[\frac{\sum_{m=1}^{M}\sum_{t=1}^{T} Z^{i}_{t,m}}{(TxM)} - H^{i} \right]^{2}$$

where $\boldsymbol{H^i}$ is the historical proportion of days that the COP spent in regime ***i*** in a given period, ***T*** is the number of time steps and ***M*** is the number of paths.

The optimisation can then be written as a minimisation of the mean of the loss function $\Omega_{\mathbf{a,c}}$ subject to certain constraints in the absolute difference between *Z[.]* and H^i.

Minimise

$$\mu_{\Omega_{a,c}} = \frac{\sum_{s=1}^{S} \Omega^{s}_{a,c}}{S}$$

subject to

1. $| Z[.] - H^i | < p$
2. $l_1 < c < l_2$,
3. $d_1 < a < d_2$

where $mW_{a,c}$ is the mean of the loss function $\Omega_{\mathbf{a,c}}$. $Z^i / (TxM)$ is the proportion of time steps that the simulated exchange-rate spends in the regime ***i*** and $\boldsymbol{H^i}$ is the historical percentage of days the rate was in regime *i* during the period under study. **a** is the drift and **c** is the parity reversion coefficient and λ_1, λ_2, π, δ_1 and δ_2 are predetermined coefficients.

Note that the volatilities used for the diffusion term in the stochastic differential equation are given - historical data was used to compute them. Constraints 2 and 3 are used to reduce the estimation time and to ensure that the drift term is “close” to the crawling rate of the band and that the mean-reversion coefficient is positive. It is clear that c must lie between 0 and 1 and it can be expected to be small: c = 1 would imply that the exchange-rate is permanently close to the central parity, that is, similar to a crawling-peg mechanism. For each run, the level of the exchange rate is initialised at random between the bands and the estimation of the loss function for each combination of parameters to be tested involves enough runs to ensure that any bias that could arise from the selection of an initial value is eliminated.

The drawbacks of the methodology used to estimate the solutions result from the discrete approximation to the continuous time stochastic process and the use of a simple grid search algorithm, which may not reach a global minimum. The results of the model are presented in section V.D. in conjunction with the analysis of the other Colombian market data such as volume, real sector flows, etc.

IV. TECHNICAL DESCRIPTION OF SELECTED TESTS

1. Information Tests (Entropy H(k))

As Aparicio (1998) states, when measuring the entropy of a system we want to determine the amount of information that it contributes to the output, when combined with an input signal. This information is defined as the difference between the uncertainty in the input and in the output.

Press et al. (1989) illustrate the concept of entropy with a generalisation of the “twenty question” game where, by means of a succession of questions (either multiple-choice or binary), the participant tries by a process of elimination to find the correct answer. Questions are mutually exclusive and exhaustive. The authors assert that the value of an answer is a positive function of the number of possibilities that it eliminates, namely $-ln(p)$, where p is the remaining fraction of possibilities.

The value of a question with *I* possible answers (i=1,…,I) and the fraction of possibilities consistent with the i^{th} answer is p_i (with $\Sigma p_i = 1$) will be the expectation value of the answer, denoted H:

$$H = -\sum_{i=1}^{I} p_i \, ln(p_i)$$

H is the entropy of the distribution given by the probability measure.

Following Aparicio (1998), for a discrete source x, Shannon defines H(x) as:

$$H(x) = -\sum_{i=1}^{N} p_i^{(x)} \, log(p_i)^{(x)}$$

for $\{x_1,\ldots,x_n\}$ having probabilities $\{p_1^{(x)},\ldots, p_N^{(x)}\}$. The entropy function can be considered to be a measure of general disorder or variability in the data, while the variance is a measure of 'linear variability'. For details, the interested reader is referred to Frank and Stengos (1995).

2. Unit Root Test (Dickey Fuller t-Tests)

Shocks to stationary time series are temporary as their effect will gradually be dissipated and long-term forecasts will converge to the unconditional mean of the series as a consequence of its reversion to the mean, to the fact that it has finite and time-invariant variance and a decaying autocorrelation function. On the other hand, non-stationary time series do show mean-averting properties, time dependency in the variance and sample autocorrelations that diminish slightly only very gradually (Enders, 1995).

Following Cuthbertson (1996), consider the AR(1) model for time series yt

$$y_t=\alpha_o + \alpha_1 y_{t-1}+\varepsilon_t \qquad \alpha_1>0$$

where ε_t is a stationary white noise series. If $\alpha_1<1$, y_t is a stationary I(0) series.[7] On the other hand, if $\alpha_1=1$, y_t is I(1) and must be differenced once in order to obtain a stationary series:

$$\Delta y_t=\alpha_o +\varepsilon_t$$

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[7] A series is I(1) if it needs to be differenced once to yield a stationary series.

By subtracting Δy_t from both sides in the first equation and rearranging, we obtain:

$$\Delta y_t = \alpha_o + \Theta y_{t-1} + \varepsilon_t$$

where $\Theta = \alpha_1 - 1$. Clearly, if $\alpha_1 < 1$ *then* $\Theta < 0$.

Dickey and Fuller (1979) developed procedures for testing for unit root in time series. The null (H_0) and alternative (H_1) hypotheses for the Dickey Fuller τ-ratio tests, τ, τ_μ and τ_τ are presented below:

i) Test for a random walk against a stationary autoregressive process of order 1

H_0: $\Delta y_t = \varepsilon_t$
H_1: $\Delta y_t = \Theta y_{t-1} + \varepsilon_t$

ii) Test for a random walk against a stationary autoregressive process of order 1 with drift

H_0: $\Delta y_t = \varepsilon_t$
H_1: $\Delta y_t = \alpha_o + \Theta y_{t-1} + \varepsilon_t$

iii) Test for a random walk against a stationary autoregressive process of order 1 with drift and time trend.

H_0: $\Delta y_t = \varepsilon_t$
H_1: $\Delta y_t = \alpha_o + \Theta y_{t-1} + \lambda t + \varepsilon_t$

The test statistic is defined as:

$$\text{test} - \text{statistic} = \frac{\hat{\theta}}{se(\hat{\theta})}$$

Using Monte Carlo simulation techniques, Dickey and Fuller (1979) computed these critical values for the three equations and found that they depend on the form of the regression and the sample size. In each case, the null hypothesis of a unit root is rejected in favour of the stationary alternative if the test statistic is more negative than the critical value.

These tests can also be augmented by the addition of p lags of the dependent variable to the estimated equation. This allows for possible autocorrelation of residuals in the regression. The reader is referred to Brooks (1996, 1997) and Enders (1995) for further technical details. Drawbacks of the Dickey-Fuller methodology arise from the fact that the underlying distribution theory assumes that the errors are statistically independent and have constant variance and the low power of the test for small sample sizes.

3. Normality and Gaussianity Tests

A Kolmogorov-Smirnov test was performed to examine the differences between the cumulative probability distribution functions of the data series and a Gaussian normal distribution in order to determine whether the two data sets are drawn from the same distribution. A significance test is performed which compares the cumulative distribution functions of the data series, and calculates the maximum distance D between them.

4. Hurst Exponent

The Hurst Exponent (Hurst, 1951) provides information on the length of memory of the series. Time series behaviour can be characterised in terms of the Hurst Exponent H as follows:

	$H=0.5$	random walk
If	$0<H<0.5$	mean reverting
	$0.5<H<1$	mean averting (trending)

Computation of the Hurst Exponent is discussed below. The first step consists of computing the range of partial sums of deviations of the time series from its mean rate of change, rescaled by its standard deviation, the R/S statistic. Following Garvidis (1998), denoting a time series by r_t and its average and biased standard deviation by $m(N,t_0)$ and $S(N, t_0)$, respectively:

$$m(N,t_0) = \sum_{t=t_0+1}^{t_0+N} \frac{r_t}{N}$$

$$S(N,t_0) = \left\{ \frac{1}{N} \sum_{t=t_{0+1}}^{t_0+N} \left[r_t - m(N,t_0) \right]^2 \right\}^{1/2}$$

the partial sums of deviations of r_t from its mean and the rescaled range are defined as:

$$X(N,t_0,\tau) = \sum_{t=t_0+1}^{t_0+\tau} \left(r_t - m(N,t_0) \right) \qquad \text{for} \quad 1 \le \tau \le N$$

$$R(N,t_0) \equiv \max_{\tau} X(N,t_0,\tau) - \min_{\tau} X(N,t_0,\tau)$$

The R/S statistic for time scale N is the ratio of $R(N,t_0)$ and the biased standard deviation:

$$[R/S](N)=\frac{\sum_{t_0} R(N,t_0)}{\sum_{t_0} S(N,t_0)}$$

If a scaling law exists for [R/S](N), we can write $[R/S](N) \approx aN^H$ where a is a constant and H is the Hurst Exponent.

5. Non-Linearity Tests

5.1. ARCH Test

Engle (1982) proposed a set of stochastic processes named autoregressive conditional heteroscedasticity models (ARCH) which introduce persistence in prices assuming that the variance is autoregressive, and, by definition time varying. Testing for ARCH in the residuals of a postulated linear model (normally an autoregressive model chosen on the basis of some information criteria statistic for financial returns) is done as follows:

1. Run the linear regression with the chosen information criterion and save the residuals, $\hat{e}_t$.
2. Square the residuals, $\hat{e}_t$ and regress them on p own lags: $\hat{e}_t^2 = a_1\hat{e}_{t-1}^2 + a_2\hat{e}_{t-2}^2 + \ldots + a_p\hat{e}_{t-p}^2 + \varepsilon_t$
3. The test statistic is defined as TR^2, the number of observations muliplied by the coefficient of correlation from the regression in 2, and is distributed as $\chi^2(p)$ under the null of no ARCH(p) effects.

5.2. Tests Based on Phase Space Reconstruction

We are interested in determining whether a time series is a realisation of a (nonlinear) chaotic deterministic dynamical system or random noise. In order to compute the statistics presented in this section, the appropriate series of state vectors (m-histories), $X(t_i)$, of delay coordinates in the m-dimensional phase space must be computed. There are several techniques that can be used for embedding in state space. One technique is to embed with delay coordinates:

$$X^m(t_i)=\left[X(t_i),X(t_i+\tau),\ldots,X(t_i+(m-1)\tau)\right]^\tau$$

where $t_i = i\delta t,\ i=1,\ldots,N,$ and δt is the sampling time, m is called the embedding dimension and τ is the delay time.

In this context, a point in phase space characterises the instantaneous state of a dynamical system and a phase space trajectory results from a sequence of such states. For a chaotic deterministic system phase space trajectories will converge to an invariant subset of phase space, the attractor of the system.

Farmer and Sidorowich (1988) state that, if the time series is produced by a non-linear dynamical system 'on a moderate-dimensional attractor, and the [embedded] dimension m and the delay time τ are chosen properly, then the resulting vector $X(t_i)$ will contain enough information to uniquely determine future evolution, and hence define unique states for the system'.

Proof that this approach constitutes an embedding that preserves invariances was provided by Takens (1981). For a system with an attractor contained in a manifold of dimension d, Takens showed that an embedding dimension, m < 2d+1, will in general exist and will fill out a set of the same dimension as the underlying attractor. For further details, the reader is referred to Smith (1992).

For a finite sample, the quality (in terms of reconstructing the state space) of the reconstructed trajectory in phase space is a function of τ. At least two methods can be used to determine τ. Either the first minimum of the autocorrelation function or the value that minimises the mutual information function between the past and the future can be used. In this thesis, the first method is used.

See Blank (1995) for a general methodology for analysing non-linearity and Guillaume (1995) for further details on reconstructing the embedding state space.

5.2.1. Correlation Dimension

The Correlation Integral given by Grassberger and Procaccia (1983a, 1983b) is defined as:

$$y = \log\left(C^m(\tau)\right), \qquad C^m(\tau) = \frac{1}{N^2} \sum_{i,\ i=1;i\neq i}^{N} \theta\left(\tau - \left\| \hat{\mathrm{x}}_{\mathrm{i}} - \hat{\mathrm{x}}_{\mathrm{j}} \right\|\right)$$

where r is the radius of the neighbourhood around the phase space point $\hat{\mathrm{x}}\mathrm{i}$, *N* is the length of the signal (number of observations), and θ denotes an indicator function with

$$\theta(\tau) = \begin{cases} 0 \text{ if } \tau < 0 \\ 1 \text{ if } \tau \geq 0 \end{cases}$$

The Correlation Integral counts the number of points that are within a distance *r* of each other. The correlation dimension is computed as follows:

$$d^m = \lim_{\tau \to 0} \frac{\log\left(C^m\right)}{\log\left(r\right)}$$

A white-noise generating process will yield a d^m that scales with *m*, whereas if the underlying system is deterministic, d^m will converge to the actual correlation dimension, $d^{\dim} = \mathrm{d}$. In practice, the dimensionality (d) of the attractor is given by the slope of log|C(r)| versus log|r|.

For a certain range of *r* (the so-called scaling region), C(r) behaves as a function of *r* and *d* of the following form:

$$C\left(r\right) = r^d$$

It is critical that the radius be chosen as close to zero as possible given the sample size (for further details see Peters, 1995; Eldridge et al, 1995; Liu et al, 1995; Brooks, 1996).

5.2.2. Pointwise Correlation Dimension Graph (PD2)

The correlation dimension, the Pointwise Correlation dimension is related to the Pointwise Correlation integral. The latter is a variant of the correlation integral that yields a value for each sample of the time series. It is defined as:

$$C_2\left(r,i\right) = \frac{1}{N-1} \sum_{i=O;\, j \neq i}^{N-1} \theta\left(r - \left\|\hat{x}_i - \hat{x}_j\right\|\right)$$

where r denotes the radius of the phase space neighbourhood, $\hat{x}_i$ and $\hat{x}_j$ are the phase-space vectors of delay coordinates, N is the length of the signal and θ is the indicator function

$$\theta\left(\tau\right) = \begin{cases} 0 \text{ if } \tau < 0 \\ 1 \text{ if } \tau \geq 0 \end{cases}$$

Skinner's (1993) PD2 algorithm to compute the Pointwise Correlation Dimension:

$$D_2\left(i\right) := \frac{d}{d\left[\log \mathrm{r}\right]} \log\left(\frac{1}{N-1} \sum_{i=O;\, j \neq i}^{N-1} \theta\left(r - \left\|\hat{x}_i - \hat{x}_j\right\|\right) \right)$$

Variations in a system's complexity dynamics are translated as sudden phase transitions in stationary time series of dynamical systems. As the Pointwise Correlation dimension detects such phase transitions, abrupt changes of the statistic reflect an increase in complexity.

5.2.3. Lyapunov Exponents

For a discrete dynamical system of the form

$$\mathrm{x}_{\mathrm{i}+1} = \mathrm{f}^{n}(\mathrm{x}_{\mathrm{i}}), \mathrm{f} \in C^{1}(R^{m} \to R^{m})$$

the Lyapunov exponents are defined as the logarithmic eigenvalues of the matrix

$$\lim_{n \to \infty} \left[J\mathrm{f}^{n}(\mathrm{x}) \right]^{\frac{1}{n}}$$

where J is the Jacobian matrix.

The exponents can be interpreted as the average exponential growth rates of initially close points under the flow generated by f. Lyapunov (1907) showed that, if one of the eigenvalues of the determinant equation has a positive real part, the 'troubled' movement is unstable. Sensitive dependence to initial conditions (SDIC) can be used as a definition of chaotic behaviour although others are also possible.

The concept of SDIC can be formalised using the largest Lyapunov exponent: an infinitesimal change in the initial conditions, δx(0), yields an iterated change in the system that grows exponentially as a function of t and λ, the largest Lyapunov exponent:

$$|\delta \mathrm{x}(\mathrm{t})| \approx |\delta \mathrm{x}(0)| \mathrm{e}^{\lambda \mathrm{t}}$$

Lyapunov exponents are normally quoted in units of base 2, and can therefore be interpreted as the information lost in bits per iteration. Brooks (1996) claims that a positive Lyapunov exponent is evidence of chaotic dynamics and it is impossible to predict the behaviour of the system as any useful information will be lost in a few time steps with a high probability. For a discussion on Lyapunov exponents and economic series, the reader is referred to Brooks (1996) and to Peters (1995).

By convention, Lyapunov exponents are numbered from the largest to the smallest and the resulting set is called the Lyapunov spectrum.

By defining a moving window and computing the local logarithmic eigenvalues for the sub-sample a 'Momentary Largest' Lyapunov exponent can be computed.

5.2.4. BDS Test

Brock, Dechert and Sckeinkman (1986) developed a diagnostic methodology to test for the presence of dynamic, possibly stochastic, non-linear structure based on the properties of the correlation integral. According to Brock and Potter (1992), this methodology should be seen as a non-linear analogue of the Ljung-Box “portmanteau’ test on estimated residuals of fitted models.

Following Brooks (1996) and Willey (1995), the BDS statistic is defined as follows:

$$W^m(r,T) = T^{\frac{1}{2}} \frac{[C^m(r,T) - C^1(r,T)^m]}{\sigma_{m,\tau}(r)}$$

where

$$\sigma_{m,\tau} = 4\left[K^m + 2\left(\sum_{j=1}^{m-1} K^{m-j} C(r)^{2j}\right) + (m-1)^2 C(r)^{2m} - m^2 K(r) C(r)^{2m-2}\right]$$

and K(r) is defined to be:

$$K(r) = \frac{6\sum_{t,s,v} h_r\left(x_t^m, x_s^m, x_v^m\right)}{\left[Tm(Tm-1)(Tm-2)\right]}$$

and

$$h_r(i,j,k) = \left[I_r(i,j) I_r(j,k)\right]$$

The BDS statistic is asymptotically distributed as a standard normal variable under the null hypothesis of an independent and identical distribution (i.i.d.) against an unspecified alternative. As the test has power against many forms of deviation from i.i.d., many possibilities may account for the rejection of the null: deterministic chaos, non-linear stochastic processes, linear processes[8] and non-stationarity. Testing the estimated residuals of a linear or GARCH model can reject some of these. The radius, r, must be chosen by the user. A value between 1 and 1.5 of the standard deviation of the data should be used in order to maximise the power of the test. More details can be found in Liu et al. (1995).

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[8] See Brock (1992) for a rigourous definition of linearity in this context.

6. Hinich Bispectrum Tests

Following Swami, Mendel and Nikias (1998), the autocorrelation function of a stationary process x(n) is defined as Rxx(m)=E{x*(n)x(n+m)}. As an autocorrelation sequence cannot provide any evidence of non-linearity, high-order cumulants must be estimated from the high-order moments. These are a generalisation of the autocorrelation, of which cumulants are specific combinations. The cumulants up to order three are defined as C_{1x}= E{x(n)}, C_{2x}(k)= E{ x*(n)x(n+k)} and C_{3x}(k,l)= E{ x*(n)x(n+k)x(n+l)}. The first-order cumulant is the mean of the process and the second-order cumulant is the autocovariance function.

The Kth-order polyspectrum is defined as the Fourier transform of the corresponding sequence of cumulants. As we are interested in checking for the assumption that the time series is linear and even Gaussian, in which case the second-order covariances (and spectra) would contain all of the useful information, we must compute the third-order spectra to study departures from linearity and Gaussianity (Rao, 1992). The bispectrum, or third-order polyspectrum, is defined as:

$$S_{3x}\left(f_1,f_2\right)=\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}C_{3x}(k,l)e^{-j2\pi f_1 k}\,e^{-j2\pi f_2 l}$$

The cross-bicoherence is defined as:[9]

$$bic_{xyz}\left(f_1,f_2\right)=\frac{S_{xyz}\left(f_1,f_2\right)}{\sqrt{S_{2x}\left(f_1+f_2\right)S_{2y}\left(f_1\right)S_{2z}\left(f_2\right)}}$$

Hinich (1982) developed algorithms to test for Gaussianity and linearity based on the idea that, if the third-order cumulants of a process are zero, then its bi-spectrum is flat (zero), and hence its bi-coherence is zero. If the bi-spectrum is not zero, then the process is non-Gaussian. The bi-coherence is a non-zero constant if the process is linear and non-Gaussian. The Gaussianity test has the null (H_0) of a zero bi-spectrum against the alternative (H_1) of a non-zero bi-spectrum. If the alternative hypothesis (H1) holds we can test for linearity with a null (H_0') of a constant bi-coherence against an alternative (H_1') of non-constant bi-coherence (see Swami, Mendel and Nikias, 1998).

An estimate of the bi-coherence is given by the following formula:

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[9] The autobicoherence is obtained when x=y=z.

$$\left|\hat{bic}_{xxx}\left(f_1,f_2\right)\right|^2=\frac{\left|\hat{S}_{xxx}\left(f_1,f_2\right)\right|^2}{S_{2x}\left(f_1+f_2\right)S_{2x}\left(f_1\right)S_{2x}\left(f_2\right)}$$

If the bi-spectrum is equal to zero, the statistic is distributed as a χ^2 random variable with two degrees of freedom. The actual test statistic is defined as follows (Brooks, 1996; Brooks and Hinich, 1998):

$$S=2\sum_m\sum_n\left|\hat{bicxxx}\left(f_m,f_n\right)\right|^2$$

The *S* statistic has a χ^2 distribution with 2p degrees of freedom when the null hypothesis is one of non-linearity. A statistical test to determine whether the observed S is consistent with a central χ^2 distribution can be performed. If we have evidence that the data is not Gaussian, then if it is linear, the squared bi-coherence will be constant for all f_1 and f_2. In practice, this constant value can be estimated by computing the mean value over all of the points in the non-redundant region and is called l, by convention. In this case, the squared bi-coherence has a χ^2 distribution with 2 degrees of freedom. The user must set the frame-size parameter; as recommended in the literature, the square root of the number of observations is used in this thesis.

In the next chapter, we study the Colombian foreign exchange market from the perspective of a policymaker – monetary policy design and implementation – as well as its microstructure and statistical properties, using the tests described in this section.

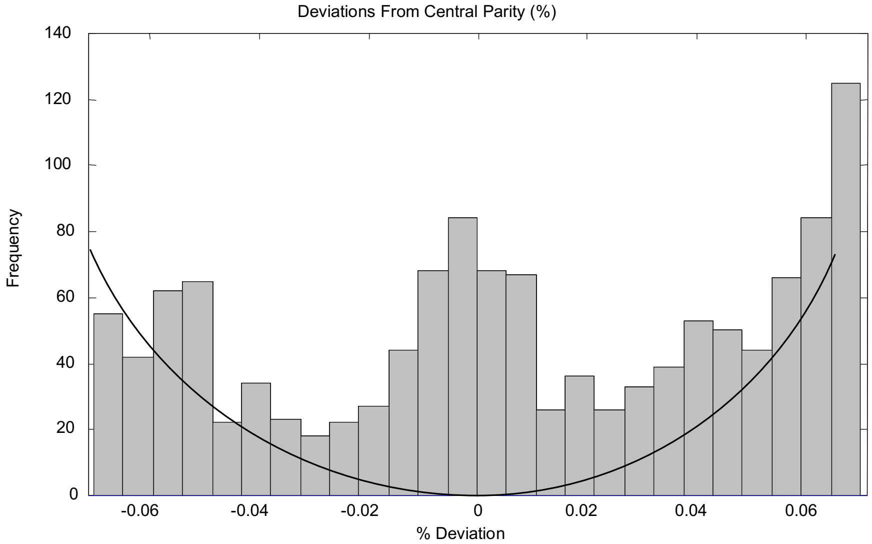
V. COLOMBIAN PESO MARKET DATA ANALYSIS

1. Preliminary Analysis

During the period 1994-Aug 1999, the nominal exchange rate was close to the limits of its band (± 7% from the central parity). Thus, the histogram of the deviations from central parity presented in Figure 8 shows a concentration on the upper side of the band and more observations near the central parity than expected by currency bands model. These predict a U-shaped distribution if no intra-marginal intervention is implemented by the Authorities to drive the exchange rate towards the middle of the band.

Figure 8

Histogram for COP deviations from central parity
(Jan-14-1994/Aug-31-1999)



The excess observations near the central parity result from a revaluation of the Central Parity by 7% at the end of 1994 that *de facto*, as the rate was hitting the bottom, left the currency in the middle of the new band until mid 1995, and the realignment of the band in September 1998.

The high concentration on the weak side reflects the political crisis from August 1995 to June 1996 (alleged financing of the presidential campaign by the drug cartels) and expectations of a realignment from the first half of 1998 onwards. The time spent in each of the regimes defined in section III.1 for the COP model was 38%, 40% and 22% for regimes 1, 2 and 3, respectively.

In common with the majority of financial return series, the distribution of COP returns has fat tails and a peak at the mean. Skewness and excess kurtosis are both significantly different from zero at the 1% level, the latter confirming the hypothesis of a leptokurtic distribution – see Figure 9.

The normality plot (Figure 10) illustrates whether returns are normally distributed. If the data are normal, the plot will be linear while other distributions will introduce curvature. It is worth noting that most of the non-linearity arises from extreme positive outliers.

These findings are consistent across the three returns measures: logarithmic daily spot market returns (spot returns), logarithmic daily returns computed using 2 consecutive days' average prices (daily returns), and logarithmic hourly returns (hourly returns). Serial correlation of returns is not very strong in the daily spot market, whereas correlation in daily returns seems to be more important for the reasons discussed in the previous section. Hourly returns are slightly correlated from lag three onwards but with no definite sign bias and returns vary abruptly as shown in Figures 11 and 12, respectively.

Figure 9

COP Returns Histogram

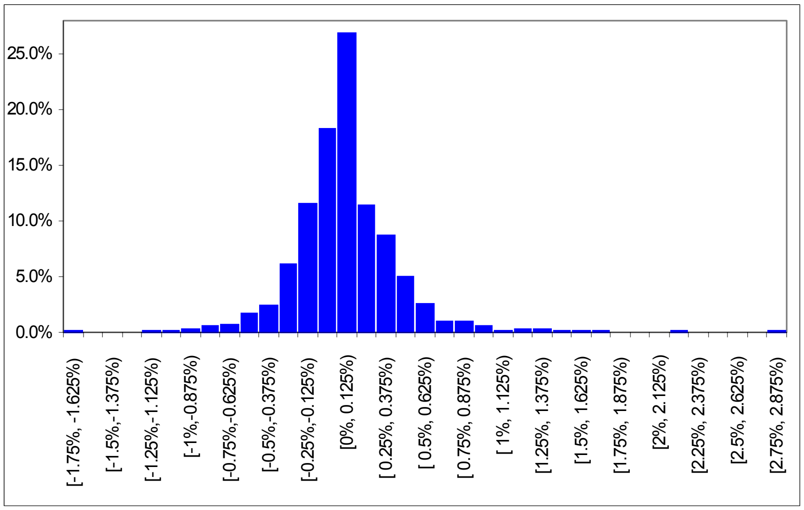


Figure 10

Normality plot for COP returns

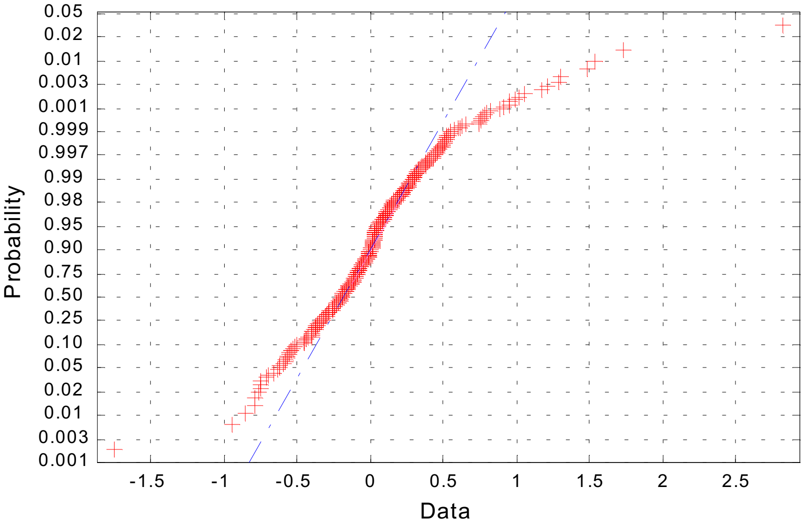


Figure 11

Autocorrelation function of Hourly COP returns

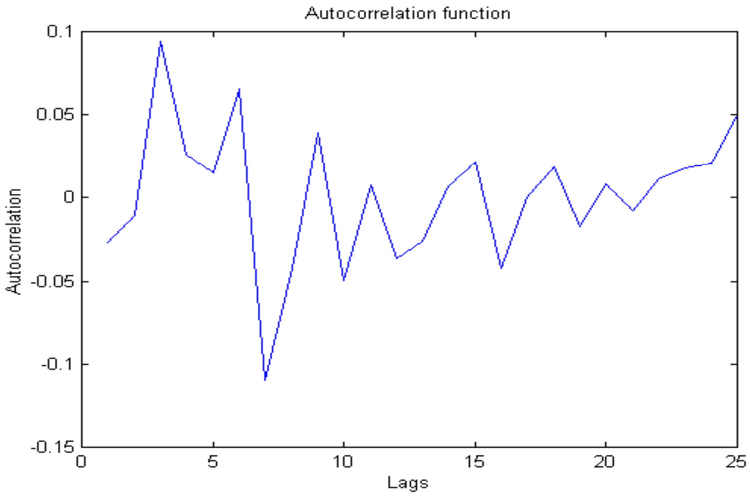
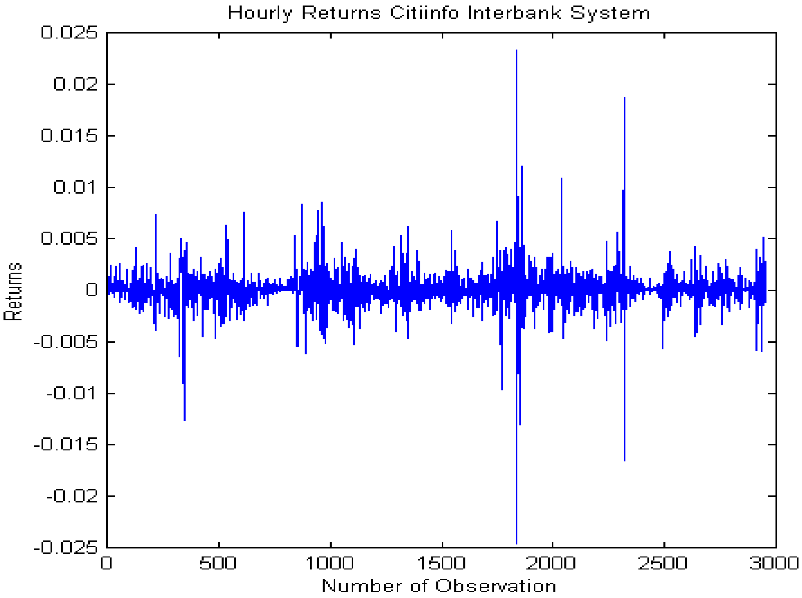


Figure 12

Hourly COP returns (Citiinfo Interbank System)



A Hurst Exponent around 0.35 indicates that the COP returns time series is mean reverting. This is consistent for all three measures of returns- see table 5.

Table 5
COP Returns Hurst Exponent

Returns	Hurst Exponent
Daily Returns	0.401
Spot Market Returns	0.315
Hourly Returns	0.354

However, a daily trend of 0.06% seems to be present in the returns data. This could result from the drift implicitly imposed by the authorities in the crawling rate of the band (11%, 13%, 15% and 13% for 1994, 1995 and 1996, 1997 and 1998, respectively) as the annualised trend arising from the data for the whole period is equivalent to 15.3%. The series of returns computed using the daily opening and closing prices confirms the result with a smaller daily trend of 0.035%.

A slowly-decaying auto-correlation function combined with a rapidly-decaying and oscillating partial autocorrelation function confirms this result. The Dickey-Fuller test indicates that there is a unit root in the price series. As expected, prices seem to follow a random walk. The unit-root hypothesis is rejected for each of the three returns series at the 1% significance. Squared returns statistically differ from zero for all lags tested using the Ljung Box statistic. This is true for each of the returns measures and rejection is strongest for hourly returns.

The auto-correlation function for the daily range of the exchange rate is also high and confirms these findings. The daily range (max-min), presented in Figure 13, shows that high volatility follows high volatility and low volatility follows low volatility.[10]

Within the context of a regression analysis, a model of returns as a function of Central-Bank intervention, real-sector supply and demand and changes in financial sector assets and liabilities was constructed. The results are presented in Annex 2. Statistically significant, real-sector demand and supply do not seem to (linearly) account for changes in the exchange rate. This is in agreement with currency bands theory, in which the exchange rate is not determined exclusively by the fundamentals and non-linearity does not necessarily cancel out in the long run. Following Bertola and Caballero (1992), one could assert that the density function of the reserves of the country were not well defined as a consequence of the multiple objectives simultaneously followed by the Monetary Authorities.

The parameters estimated for the Central Bank intervention support the statement of the Central Bank that any intra-marginal participation in the market is aimed at reducing volatility as the sings the coefficients imply that when the exchange rate

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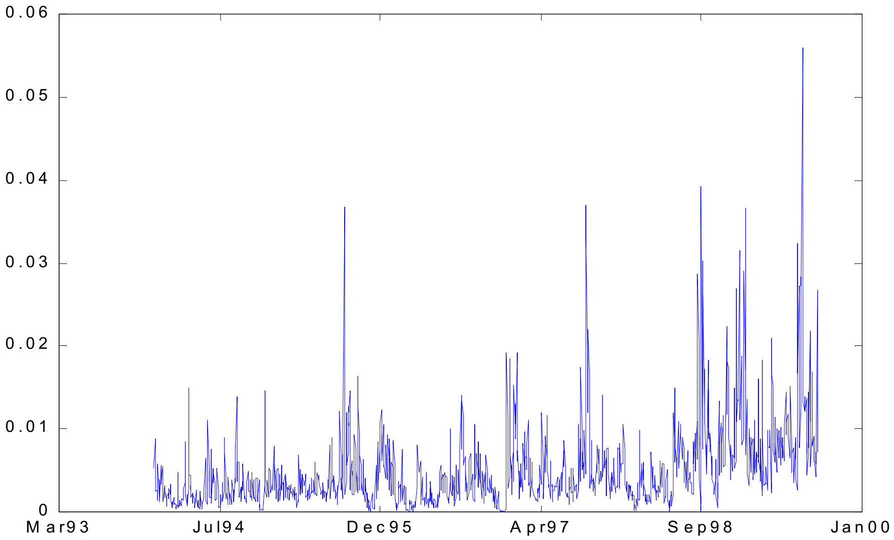
[10] The unit root hypothesis is rejected at the 1% significance level.

increases (decreases) the Authorities sell (buy). The evidence strongly suggests that the exchange rate causes the intervention, and not vice versa. Moreover, the coefficients seem to be small but this can result from the fact that the intervention is not performed consistently.

Figure 13
Daily Range COP Citiinfo Inter-bank System



Financial sector position changes do not seem to be statistically significant. This could be for any one of three reasons: cash positions and derivative contract positions change not only from participating in the market but also from liquidating contracts; real sector trades are not necessarily directed at the market immediately as a result of traders' inventory management; and next-day market operations and derivatives positions may be reported either on the transaction day or settlement day as the regulation did not seem to be precise enough and a proper (and correct) convention was established only in 1997.

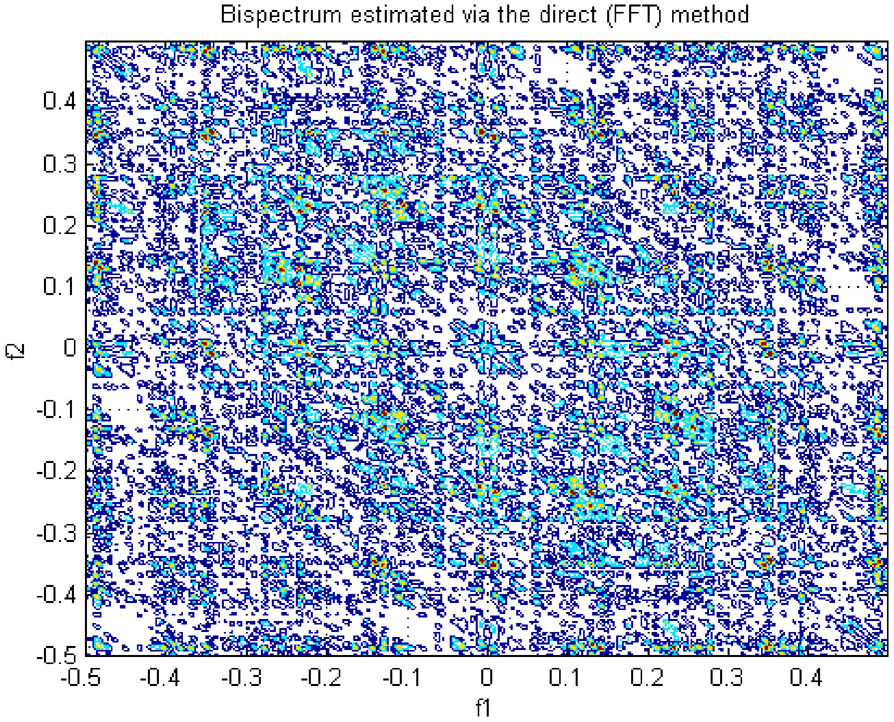
Lags were included in the model but the results did not change significantly. Also, non-linearity may be present in the price formation process as implied by currency models. In the next section, non-linearity tests are applied to the return series.

2. Non-linearity Tests

Gaussianity is rejected for each of the returns series when Hinich's test is applied. This result is supported by the bi-spectrum plots (Figure 14 below). In these plots, for time series that follow a Gaussian distribution, the plot should be empty as the third order cumulants are inexistent.

Also, we cannot reject the null hypothesis of linearity since the estimated interquantile range is not very different from the theoretical value for each of the series. The bi-spectrum plot for the Citiinfo spot market is presented in Figure 14.

Figure 14
Bi-spectrum Plot for COP Returns Spot Market



The null hypothesis is strongly rejected for all definitions and frequencies of returns when the BDS test is applied to raw data. The linear null and GARCH hypotheses were tested by fitting linear and GARCH(1,1) models, respectively and by testing the estimated residuals using BDS. In the first case, the pattern of rejection did not change. When the residuals from a GARCH (1,1) model are tested,[11] although for daily returns the null is still rejected (less strongly), for hourly returns the null is not rejected for low embedded dimensions. It is not surprising that some non-linearity is still present as the solution for the exchange-rate behaviour under crawling bands includes a non-linear expression. Other sources of non-linearity (see Brock and Potter, 1992) can include non-stationarity, deterministic chaos and other non-linear stochastic sources. The presence of a unit root was rejected by the Augmented Dickey Fuller test for all frequencies and lags up to 10, including one-hour returns.

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[11] In accordance with squared-return autocorrelations, the ARCH tests support the hypothesis of ARCH effects for each of the returns series.

The hypothesis of deterministic chaos does not seem plausible, since the correlation dimension does not converge (see Figure 15 and Table 6) in accordance with the results of the white-noise test, which in turn is not rejected in the case of spot and hourly returns at the 1% and 10% significance levels respectively.

Figure 15

Correlation Dimension for COP Returns (Spot Market)

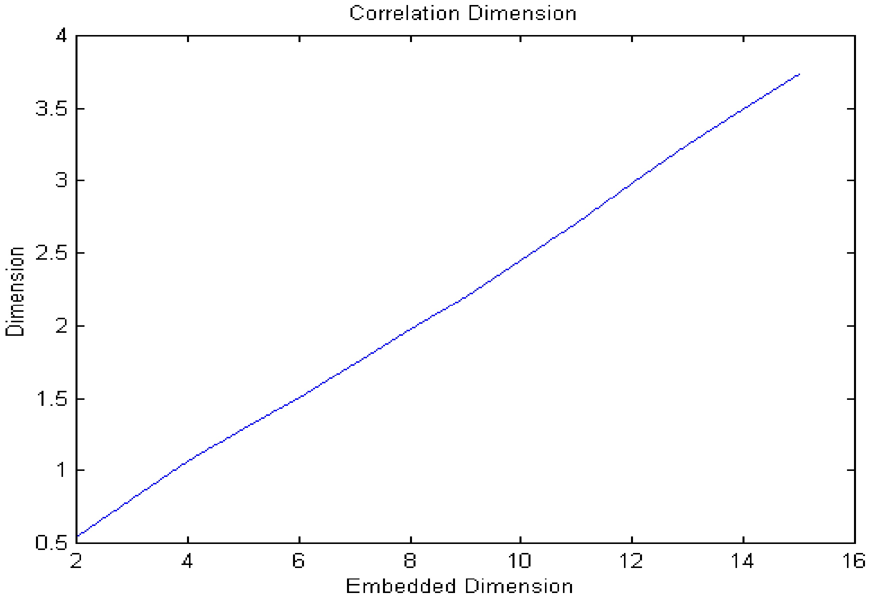

Table 6

Correlation Dimension Parameters

Tau (min acf[12])	Min dimension	Max Dimension	Min rad	Max rad
17	2	15	0.11	0.13

Interestingly, in performing non-linearity tests Brooks (1996) found that the degree of saturation is often around 5-6 (embedded dimensions) for exchange rates without government intervention. This seems to support the fact that the effective complexity of the currency band arrangement is greater than a free-float regime. Moreover, as Figure 16 shows, the complexity of the system, as measured by changes in the Pointwise Correlation Dimension, varies significantly.[13] Coincidentally, the

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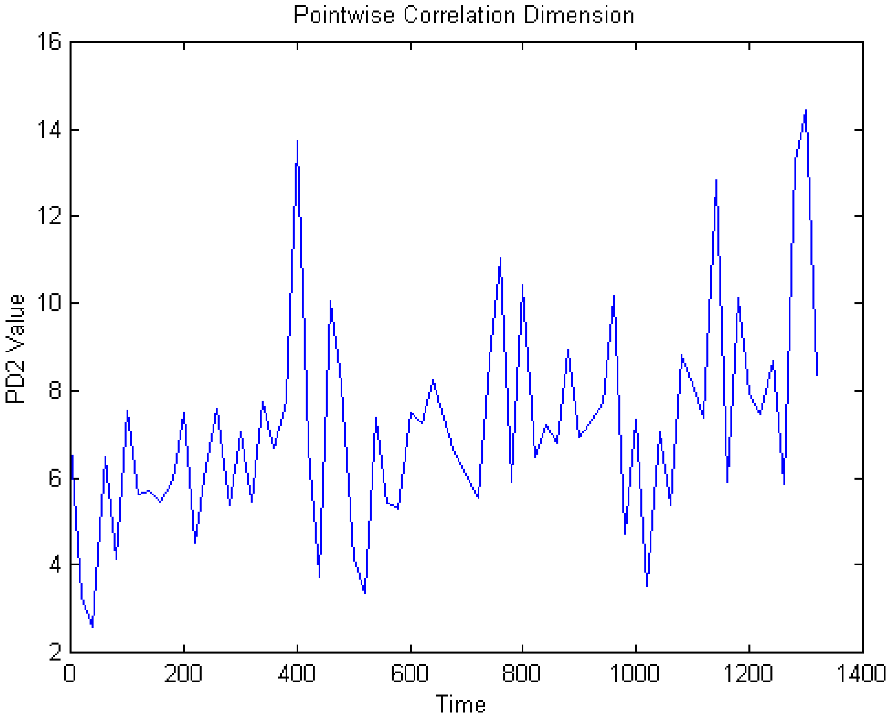
[12] Autocorrelation function

[13] Parameters used are Tau=1, Mininimum Dimension=2 and Maximum Dimension=15.

first great variation (around observation 400) occurred in August 1995, at the same time as the President was accused of financing his campaign with drug cartel resources. It also increased markedly before the currency band's move in 1998 and the elimination of the currency regime in 1999.

Figure 16

Pointwise Correlation Dimension for COP Returns (Spot Market)



As expected from the correlation dimension results, Lyapunov exponents do not have a definite sign for changes in the estimation parameters and the largest temporal Lyapunov exponents (for each of the returns series) vary dramatically.

Using the same variables as in the linear regression, a neural network with two layers of 5 and 1 neurons, respectively (sigmoid and linear functions) was constructed and trained with a validation set. It was then tested on a final set. The data was divided up into training, validation and test subsets using one-half, one-quarter and one-quarter of the data respectively, by taking equally-spaced points throughout the original data.[14] Annex 3 describes the network architecture and the training, validation and test sets used.

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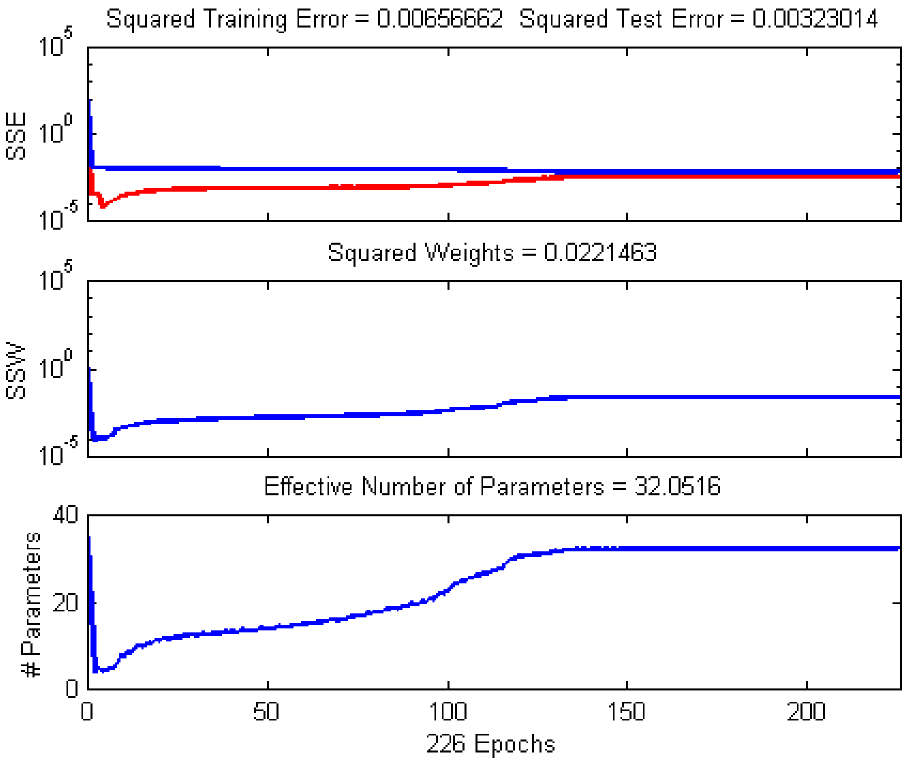
[14] Every 4 points, starting at points 1 and 3 for the two training subsets and at points 2 and 4 for the validation and testing subsets, respectively.

To improve generalisation, the Automated Bayesian Regularisation algorithm was implemented, as it does not allow the network to overfit the data and eliminates the guesswork in determining the required network size. See Hagan et al. (1996) and Demuth and Beale (1998) for details. The results obtained seem reasonable since the test-set errors and the validation-set errors have similar characteristics as they converge - see Figure 17.

Figure 17

Network Training Statistics



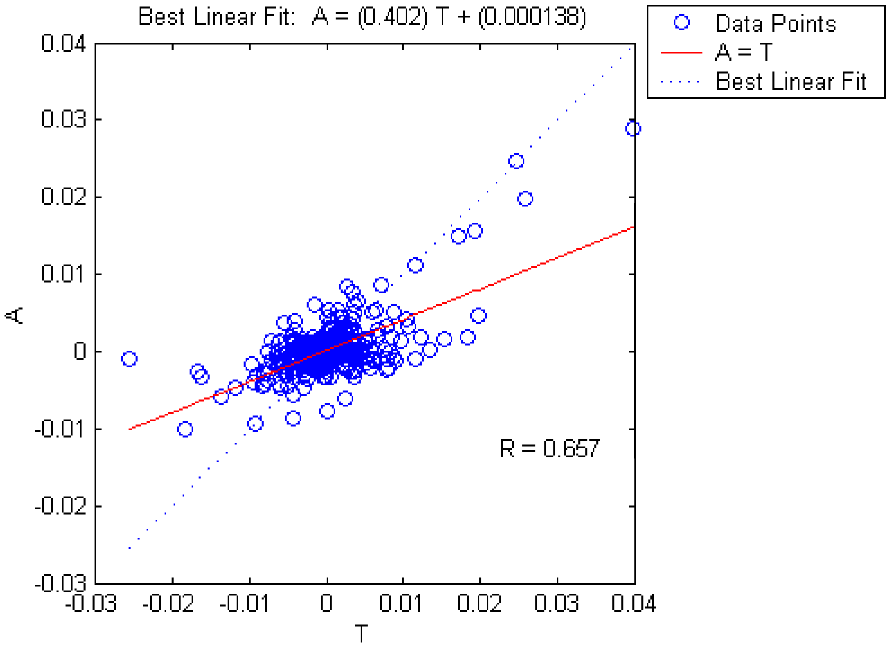
A regression of the output vector obtained from inputting the entire data set into the estimated network, against the corresponding target (exchange-rate returns), yields an R-squared of 65.7% as can be seen from Figure 18.

A plot of the actual returns and the network output shows a high correlation between the two series. As the third panel of Figure 5-21 indicates, thirty-two parameters were effectively used out of a possible 35.

The resulting weights are reported in Annex 3. Weights analysis in neural networks is difficult but in general, in the first layer, Central Bank intervention usually has opposite signs from the financial sector and for three out of five weights for the real sector. Moreover, Central Bank intervention weights are all positive whereas financial sectors weights are mostly negative. The Central Bank can be seen as either a liquidity provider or as a compensating counterparty for the financial sector. In general, in accordance with the results of the linear regression, Central Bank intervention and real-sector participation weights are greater in absolute value. Results from the neural network exercise support the hypothesis of non-linearity since a very simple network greatly improves the fit against a linear alternative. As before, lags were included without improving the results.

Figure 18

Regression of Network Output Versus Actual Returns



3. Volatility Analysis

In a currency band system, possible future exchange-rate paths are limited. Thus, even when the Authorities are not defending the limits and the exchange-rate lies within the band, exchange-rate dynamics differ from a free-float mechanism. In fact, when the Central Bank's ability to defend the bands is credible, the exchange-rate is determined not only by the evolution of the fundamentals, but also by the impact on agents' expectations of the Authorities' resolve to intervene in the boundaries of the band.

Given that agents' expectations may depend on the level of the exchange rate relative to both the central parity and the limits of the band, the volatility of the returns and the proportion of time elapsed is analysed under each of the three regimes defined for the stochastic Colombian Peso Model. The evolution of the Colombian Peso in each of those regimes from January 24, 1994 to August 31, 1999 can be approximately divided into 16 sub-intervals in which the rate is confined mostly to one of these regimes.

Summary statistics for these sub-intervals are presented in Table 10, and a number of features are worth noting. From July 1996 onwards, the exchange rate tends to remain in regime 1 or regime 3, while passage through regime 2 is either very fast or as a result of a realignment of the band. In general terms, the annualised volatility of 7.2%, for the whole sample, is low compared to actively-traded currencies such as the Deutsche Mark or the Japanese Yen, which have annualised volatility around 12% or 14%.[15] From Table 7, a gradual structural increase in volatility seems apparent, as the market was developing during the period.[16]

Table 7
Annual volatility and participation by regime

Period	Ann. Volatility	Days	Regime	% time
1: 24/01/94 - 26/06/94	2.7%	104	2	7.6%
2: 27/06/94 - 09/12/94	3.0%	113	3	8.3%
3: 12/12/94 - 15/08/95	4.2%	164	2	12.0%
4: 16/08/95 - 26/07/96	5.3%	231	1	17.0%
5: 27/07/96 - 07/10/96	6.1%	49	2	3.6%
6: 08/10/96 - 28/01/97	6.0%	73	3	5.4%
7: 29/01/97 - 04/03/97	7.8%	25	2	1.8%
8: 05/03/97 - 20/08/97	4.1%	111	3	8.2%
9: 21/08/97 - 12/09/97	15.7%	17	2	1.2%
10: 15/09/97 - 24/06/98	5.6%	187	1	13.7%
11: 25/06/98 -19/08/98	6.7%	36	2	2.6%
12: 20/08/98 -18/11/98	13.9%	62	1	4.6%
13: 19/11/98 -20/05/99	10.2%	120	2	8.8%
14: 21/05/99 -25/06/99	9.5%	24	1	1.8%
15: 26/06/99 -17/08/99	18.4%	34	2	2.5%
16: 18/08/99 -31/08/99	10.5%	11	1	0.8%
All Sample	7.2%	1361	-	100.0%

---

[15] Flood and Garber (1992) have shown that the reduction in the conditional variance of the exchange rate works to the detriment of the interest rate's volatility. In Colombia's case, the Monetary Authorities tried to reduce this phenomenon by introducing an interest rate range. However, it seems that this was done at the expense of increasing monetary base dispersion.

[16] Trading increased from an average daily turnover of US$48 million in 1994 to US$185 million in the first quarter of 1998.

Note that, in general, the COP crossed or "touched" temporarily regime 2 for a few days with very high volatility. During the months of August and September 1997, the COP depreciated by more than 12% and the major part of the move took place over 17 days that accounted for a volatility of 15.7%. Since August 1998, volatility has increased permanently with a maximum value of 18.4% in July 1999.

As mentioned earlier, Garch(1,1) models were fitted to different measures and frequencies of returns. The results suggest that disturbances are not homoskedastic. Further, the autocorrelation function for the daily range indicates that it could depend on the previous day's range. Also ARCH tests up to lag 10 confirm the hypothesis of ARCH effects. In each case, the sum of the coefficients is close to 1 and an IGARCH process cannot be ruled out. The latter could be the result of discrete regime shifts in the variance, or unparameterised regime shifts in the conditional mean that in turn could manifest themselves as variance regime shifts. Both are clearly plausible under crawling bands with realignments and seem to be supported by the results related to volatility above, as well as the fact that Central Bank intervention seems to be directed at lower volatility and any change in the intervention mechanism would result in regime shifts in the variance. For the standarised residuals, results are not consistent for the different measures of returns and frequencies, implying that a GARCH(1,1) may not be an adequate specification, although the BDS test was not strongly rejected when GARCH residuals were tested.

4. COP Model Results

The goal of the study (Brooks and Revéiz, 2002) was to capture, in a simulated model, the stylised features of the COP exchange-rate system. For this purpose, a three-regime, volatility-switching stochastic model with parity reversion and drift was fitted to the data by minimising a loss function using a grid search algorithm. The statistical characteristics of the simulated series are similar to the statistical properties of the returns from the historical sample. In fact, they account for the kurtosis and the non-linear dependence of the exchange-rate series. Although the loss function minimises the difference between the average length of time spent by the simulation and the actual data in each regime, the resulting moments of the density function for the simulated returns fit adequately most of the characteristics of the sample returns data.

The optimal drift was slightly higher than that of the crawling rate of the band. As the inflation target for both years was approximately attained and the crawling rate is computed as the expected inflation differential between Colombia and the rest of the world, it would seem that the expected rate of devaluation implicit in the Authorities' objective was not too low. The higher drift rate could result from a worsening of the twin deficits, as a consequence of the fall in commodities prices and an increase in public expenditure, combined with local political uncertainty and external unrest in the financial markets. Another complementary factor that could generate a higher drift is asymmetry in the regulation related to net foreign currency holdings that forbids financial institutions from maintaining a negative net position for more than one day.

In relation to the parity reversion, two possibly complementary hypotheses can be formulated. Either Banco de la República intervened intra-marginally or, as currency-bands theory asserts, intervention at the limits would generate expectations that the COP will appreciate when the exchange rate is near the top of its band and expectations that the COP will depreciate near the bottom. However, as the parity reversion is around 0.0002 for a width of ±7%, the maximum daily reversion is 0.0015%, only 6.52% of the daily average change. The intervention, if performed by the Central Bank, does not seem to be directed at pushing the rate towards the parity.

The latter finding is in line with the statements made by the Central Bank in 1998 in the sense that the intra-marginal intervention, if any, is directed at minimising the volatility of the exchange rate and not at affecting any prevailing tendencies, and seems to be confirmed by the signs of the coefficients in the regression results. Also, the band's credibility seems to be low during the period under study. Cardenas and Bernal (1997) showed that interest-rate volatility is higher than that expected in a credible exchange rate band and that both interest-rate and exchange-rate volatilities are higher than those governed by the fundamentals identified in their research.

Galindo (1998) analysed the credibility of the Colombian exchange-rate zone and found that a model along the lines of Krugman's, but in which expectations of central parity depreciation are endogenous with respect to the position of the exchange rate inside its band (with the implication of increased expectations of a depreciation when the upper band is approached) properly describes the behaviour of the Colombian peso. The band's realignment in September 1998 and the further elimination of the band in September 1999 seem to support that claim.

VI. SUMMARY AND CONCLUSIONS

The COP exhibits the statistical properties expected of a financial time series and in particular of currency bands theory. The volatility of returns is greatest near the central parity and the exchange rate tends to stay close to the limits of its band. Clearly, the distribution of returns is leptokurtic and the COP is non-stationary.

Using the estimated correlation dimension and the lack of predictability as a measure of complexity, Colombian peso returns are highly complex. Although evidence of low-deterministic chaos is weak, there is, however, evidence of stochastic non-linearity. Autocorrelation functions of returns are roughly non-existent at all lags with the exception of hourly returns where negative autocorrelation of returns is present for small lags.

Persistence in variance is present in the data as detected by the Engle and BDS tests. The failure of the bi-spectrum tests - which have low power against ARCH effects - to reject the null hypothesis of linearity, indicates that we have little evidence that the data can be described by bilinear or non-linear autoregressive models. GARCH(1,1) models do not seem to accurately describe the persistence in variance of the data and the results of BDS tests on the standardized residuals suggest that not all non-linear dependency can by explained by volatility clustering.

It is plausible that instead of short-term memory, the observed non-linearity could be evidence of some sort of stochastic switching or other threshold form. This possibility seems to be supported by the results for the dynamics of the volatility of returns and from solutions to theoretical models of exchange rates under currency bands, which include a non-linear factor in order to capture the effective complexity of the system (Krugman, 1991; Froot and Obstfeld, 1992). Central Bank intervention seemed to be directed at placing bounds on volatility. This could result in variance regime shifts, whenever the authorities modify their intervention policies. Moreover, expectations are linked to the level of the exchange rate within its band and this could result in regime shifts in the conditional mean.

In this context, expectations of central parity depreciation are endogenous with respect to the position of the exchange rate inside its band and models of endogenous belief heterogeneity (Brock and LeBaron, 1995) have been shown to produce time series consistent with the ARCH effect. The implication of increased expectations that the currency will depreciate when the upper band is approached implies a positive feedback in expectations, which - without Central Bank intervention - would cause market trading to stop temporarily. However, if the Central Bank actively intervenes, becoming a liquidity provider as may be inferred by the neural network results, the situation may continue fuelled by the resulting decrease in the reserves of the country. This effect is compounded by the fact that the density function of the reserves of the country is ill defined, as a result of the multiple objectives simultaneously followed by the monetary authorities simultaneously; the prevailing regulatory framework which forbids net short positions and synthetic hedging of forward purchases; and the existence of an illiquid next-day market which seems to have allowed expectations to modify prices without actual liquid trading taking place.

We can therefore conclude that the institutional arrangements, i.e. regulation, monetary policy, trading conventions and social capital may affect both the effective and potential complexity of the market leading to the presence of non-linearity and endogenous (heterogeneous) expectations at the microscopic level that result in positive feedbacks, regularities and non-equilibrium conditions in the macroscopic level as evidenced by the realignments and subsequent elimination of the exchange-rate system.

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ANNEX 1

Financial System Foreign Exchange Position

From Posicion Propia Compulsory Report to the Superintendencia Bancaria

Figure 1-1
Financial Sector Assets

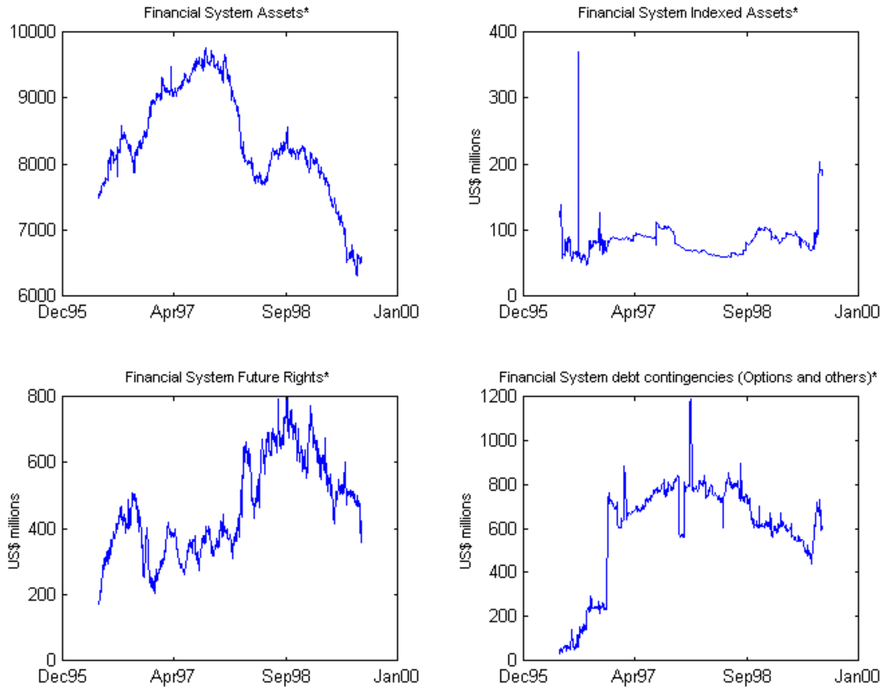


Figure 1-2
Financial Sector Liabilities

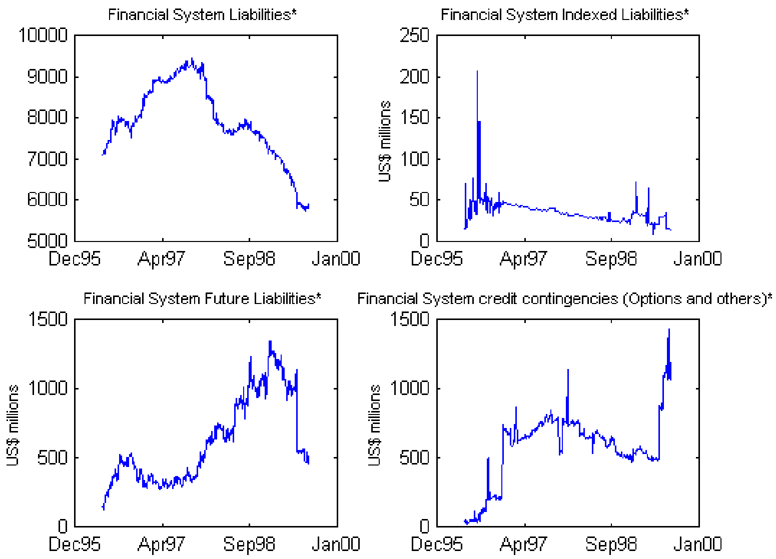


Figure 1-3
Financial Sector Total Rights, Liabilities and Net Foreign Exchange Position

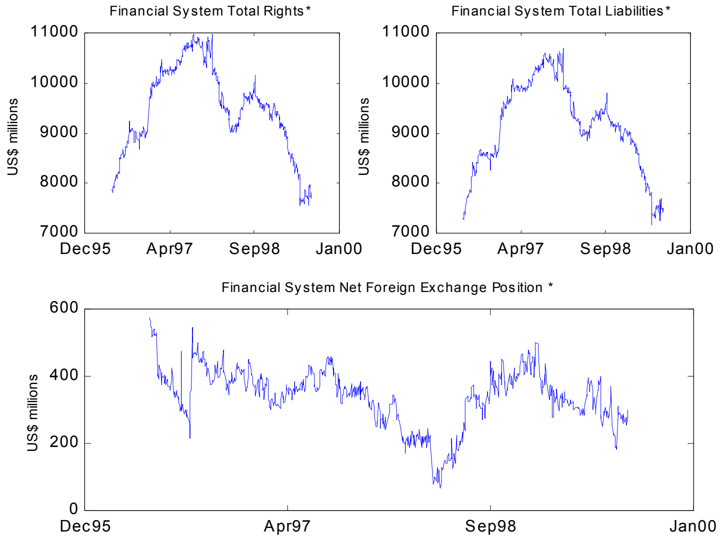
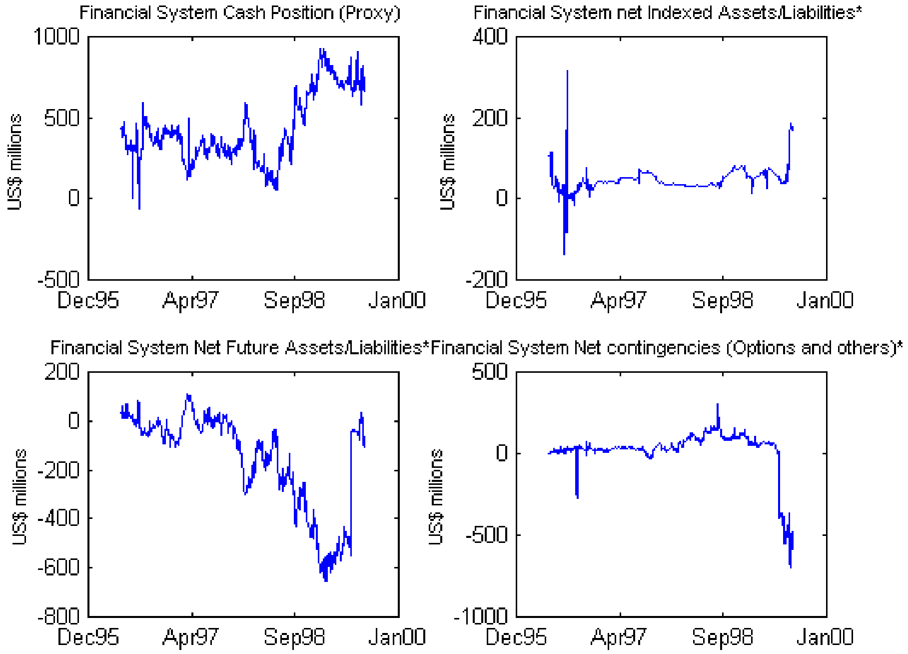


Figure 1-4
Financial Sector Net Positions



Financial System Cash Position (Proxy)

Financial System net Indexed Assets/Liabilities*

Financial System Net Future Assets/Liabilities*

Financial System Net contingencies (Options and others)*

ANNEX 2

Regression Results

Table 1

OLS Regresion Results

R-squared	Rbar-squared	sigma^2	Durbin-Watson	Number Observations	Number Variables
0.05655	0.050463	2.5637e-005	2.1741	781	6

Table 2

Estimated Coefficients

Parameters	coefficients	t-statistic	t-probability
Net Cash Pos.	7.0061e-007	0.63781	0.52379
Net Indexed Pos.	4.8379e-006	0.72963	0.46584
Net Future Pos.	8.6236e-007	0.5765	0.56445
Net Contingency Pos.	-6.442e-007	-0.41811	0.67598
Net Central Bank OO/DD.	5.7185e-005	6.6361	6.0494e-011
Net Real Sector OO/DD	3.5909e-005	4.8797	1.2901e-006

ANNEX 3

Neural Network Model for COP Returns

Table 1

Neural Network Characteristics (Matlab Conventions)

Characteristic	Description
Input Sources	1
Number of Layers	2
Bias Connections	0 0
Input Connections	1 0
Layer Connections	0 1 0 0
Output Connections	0 1
Target Connection	0 1
# Input Variables	6
Layer 1 Function	logsig
Layer 2 Function	purelin
Training Algorithm	Automated Bayesian Regularization

Table 2

Training, Validation and Test Sets Description

Sample	781 (T)
Training Set	391 (Observations:[1:4:T 3:4:T])
Validation Set	195 (Observations:[4:4:T])
Test Set	195 (Observations:[2:4:T])

Table 3

Input (6 variables) to First Layer (5 Neurons, Logsig) Weights

Neuron (Layer 1)/Input	Net Cash Pos.	Net Indexed Pos.	Net Future Pos.	Net Contingency Pos.	Net CB OO/DD.	Net Real Sector OO/DD
1	-0.010541	-0.024551	-0.0099846	-0.013832	0.008242	-0.0040188
2	-0.0049361	-0.010273	-0.006288	-0.0090758	0.022767	0.01248
3	-0.00085575	-0.0021873	-0.0021719	-0.0064023	0.010633	-0.039094
4	-0.0028455	-0.012802	-0.0023467	0.0038778	0.055763	0.017636
5	-0.0010399	-0.0040583	-0.00070996	0.00055225	0.046957	-0.027456

Table 4
First Layer (Logsig, 5 Neurons) to Second Layer (purelin, 1 Neuron) Weights

Neuron (Layer 2)/Output (Layer 1)	**1**	**2**	**3**	**4**	**5**
1	-0.04815	0.057686	-0.043328	-0.040071	0.05215