Theoretical investigation of upper critical magnetic field (H_{c2}) of the heavy fermion superconductor $CeRhIn_5$

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Abstract

This research work focuses on the theoretical investigation of the upper critical magnetic field of superconductor$CeRhIn_5$. By using the well known Ginzburg-Landau (GL) phenomenological equation, we found the direct relationship between the GL coherence length (ξ_{GL}) and penetration depth (λ_{GL}) with temperature. From the GL equations and the results obtained for the GL coherence length, the expression for the upper critical magnetic field (H_{C2}) is obtained for the superconductor$CeRhIn_5$. The result is plotted as a function of temperature. The phase diagram shows the linear dependence of upper critical magnetic field (H_{C2}) with temperature (T). The current finding is in agreement with experimental observations.

Keywords: Ginzburg-Landau Equation, upper critical magnetic field, $CeRhIn_5$.

Resumen

Este trabajo de investigación se centra en la investigación teórica del campo magnético crítico superior del superconductor $CeRhIn_5$. Utilizando la conocida ecuación fenomenológica de Ginzburg-Landau (GL), encontramos la relación directa entre la longitud de coherencia GL (ξ_{GL}) y la profundidad de penetración (λ_{GL}) con la temperatura. A partir de las ecuaciones GL y los resultados obtenidos para la longitud de coherencia GL, se obtiene la expresión para el campo magnético crítico superior (H_{C2}) para el superconductor$CeRhIn_5$. El resultado se representa en función de la temperatura. El diagrama de fase muestra la dependencia lineal del campo magnético crítico superior (H_{C2}) con la temperatura (T). El hallazgo actual está de acuerdo con las observaciones experimentales.

Palabras clave: Palabras clave: Ecuación de Ginzburg-Landau, campo magnético crítico superior, $CeRhIn_5$.

I. INTRODUCTION

Superconductivity (SC) is a phenomenon of zero resistance. It was discovered by Heike Kamerlingh Onnes in 1911 [1]. The phenomenon was discovered when Onnes observed an enormous drop in the DC resistance of pure mercury metal at T_C = 4.2 K. The transition of a normal metal into the superconducting state is revealed by the total disappearance of the electrical resistance at low temperature. Indeed, the current in a closed superconducting circuit can circulate for a long period of time without attenuation. Magnetic field plays an important role in the field of superconductivity. Based on the way superconductors behave or withstand to an applied magnetic field, they can be classified as type I (soft) or type II (hard)superconductors. The application of strong magnetic field can destroy the superconducting state of a material. For a type I superconductor [2], there is one small critical applied magnetic field above which the superconductor becomes a normal metal. They expel the applied magnetic field if it is less than the critical field. Hence type I superconductors exhibit complete Meissner effect (perfect diamagnetism) for $H<H_C$. This is the characteristic of many pure elemental superconductors. The critical temperature, T_C in the materials decreases with increasing of applied magnetic field and the magnitude of the critical magnetic field varies with temperature according to the expression [3].

$$H_C(T) = H_C(0)[1 - (\frac{T}{T_c})^2]. \qquad (1)$$

Where $H_C(0)$ is the maximum value of the applied magnetic field above which superconductivity is destroyed.

On the other hand, type II superconductors are characterized by two critical magnetic fields, designated by H_{C1} and H_{C2}. If the applied magnetic field is less than the lower critical field H_{C1} , the superconductor behaves as type I and there will be perfect diamagnetism. When the applied magnetic field is between the two critical fields ($H_{C1}<H<H_{C2}$), the materials superconduct but there will be flux penetration and results in a mixed (vortex) state. Hence, Meissner effect (perfect diamagnetism) is not complete. If the applied magnetic field exceeds the upper critical field, they will be in a normal state.

In type II superconductors, H_{C2} can be expressed in terms of H_C as,

$$H_{C2} = \frac{H_C^2}{H_{C1}}, \qquad (2)$$

where $H_C = H_C[1 - (\frac{T}{T_C})^2]$.

Heavy Fermions are probably the first and best documented cases that evidence a strong relationship between the appearance of magnetic quantum criticality and superconductivity [4, 5]. Amongst the heavy fermion superconductors, the 115 family ($CeMIn_5$, M being a transition metal), is of paramount importance which is often presented as bridging the gap between the low T_C heavy fermions and the high-T_C cup rates, owing to its 2D character, d-wave superconducting states, and pronounced "non Fermi-liquid" features [6, 7, 8].

$CeRhIn_5$ is an antiferromagnet at ambient pressure ($T_N \approx 3.8$ K) which evolves toa superconducting state for $P > P_C \approx 16$ kbar and $T_C \approx 2$ K and reaches 2.4 K at a pressure of 2.3 GPa [9]. $CeRhIn_5$ is a member of the $Ce_mM_nIn_{3m+2n}$ family. (M: transition metals). $CePt_2In_7$ is also a member of the same family and possesses two-dimensional tetragonal crystal structure and cylindrical Fermi surfaces elongated along c-axis [10]. In such compounds with 2D crystal structure with long lattice parameter along the c-axis, H_{C2} is usually anisotropic, namely, H_{C2} along the magnetic field H⫴ c-axis is smaller than that along $H \perp$c-axis due to orbital limiting and the anisotropy of the effective mass of the conduction electron.

Amongst the $Ce_mM_nIn_{3m+2n}$ families, $CeCoIn_5$ and $CeIrIn_5$ show such anisotropy in H_{C2} with quasi-two dimensional Fermi surfaces [11, 12, 13, 14]. On the other hand, pressure induced superconductor $CeRhIn_5$ shows opposite behavior in spite of the similar topology of the Fermi surfaces except the volume of the Fermi surfaces. Namely, H_{C2} (0) = 16.9 T along H ⫴[001] (c-axis) is larger than H_{C2} (0) = 9.7 T along H ⫴[100] at P_C = 2.45GPa [15]. In $CeRhIn_5$, antiferromagnetic state is induced by magnetic field just above P_C, where the antiferromagnetic state disappears by pressure at zero magnetic field. Pauli paramagnetic pair-breaking effect seems to be anisotropic in the field-induced antiferromagnetic state in $CeRhIn_5$, while isotropic in $CeCoIn_5$ [15].

II. MATHEMATICAL FORMULATIONS TO FIND THE UPPER CRITICAL MAGNETIC FIELD OF SUPERCONDUCTOR $CeRhIn_5$

A. The Basic Ginzburg-Landau Theory

Ginzburg-Landau (GL) theory is a mathematical theory used to describe superconductivity. It deals with Type-I and Type-II superconductors and enables compute H_{C1} and H_{C2} [16]. The Ginzburg-Landau theory was derived based on the BCS microscopic theory by Lev Gorkov.

The basic postulate of the GL is that if ψ is small and varies slowly in space, the free-energy density (F_s *(r))* can be expanded in a series of the form,

$$F_s = F_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\nabla - \frac{e^*}{c}A\right)\psi\right|^2 + \frac{|H|^2}{8\pi}. \qquad (3)$$

Where α and β are phenomenological parameters ($\beta > 0$ and the sign of α is temperature dependent), $m^* = 2m$ is an effective mass, $e^* = q^* = 2e$ is the charge of an electron, $\boldsymbol{A}$ is the magnetic vector potential and $\boldsymbol{B} = \nabla X \boldsymbol{A}$ [16, 17].

If $\psi = 0$, equation (3) reduces to the free energy of the normal state and becomes, $F_s = F_n + \frac{|H|^2}{8\pi}$.

Now, by minimizing the free energy with respect to fluctuations in the order parameter and the vector potential, one arrives at the Ginzburg-Landau equations given by,

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}A\right)^2\psi = 0. \qquad (4)$$

The current density from the Hamilton's energy of particles is given by $H = \frac{1}{2}m^*v_d^2 = \frac{1}{2m^*}\left[p - \frac{e^*}{c}A\right]^2$. From which we get, $v_d = \frac{1}{m^*}\left[p - \frac{e^*}{c}A\right]$. But $p = -i\hbar\nabla$. Thus, we get

$$v_d = \frac{1}{m^*}\left[-i\hbar\nabla - \frac{e^*}{c}A\right]$$

But, $J_s = e^* n_s v_d$, where J_s is the supercurrent density and $n_s = \psi(r)^*\psi(r)$

Thus, we obtain,

$$J_s = \frac{e^*}{m^*}\left[-i\hbar\nabla - \frac{e^*}{c}A\right]\psi(r)^*\psi(r),$$

$$J_s = \frac{e^*}{m^*}\left[\psi^*\left(-i\hbar\nabla - \frac{e^*}{c}A\right)\psi + \psi\left(-i\hbar\nabla - \frac{e^*}{c}A\right)\psi^*\right], \qquad (5)$$

or

$$J_s = \frac{e^*}{m^*}\left[\psi^*\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}A\right)\psi + \psi\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}A\right)\psi^*\right]. \qquad (6)$$

Equation (4) determines the order parameter ψ based on the applied magnetic field and equation (6) yields the superconducting current density. The Ginzburg-Landau equation provides a complete information about the superconducting state ψ (r) that gives the spatial distribution of the Cooper pair density taking into account a possible variation in their concentration, whereas $\boldsymbol{A}$(r) describes the local distribution of the magnetic field in the superconductor.

In the absence of external magnetic field (at free surface), there will not be superconducting current and the equation for ψ becomes,

$$F_s - F_n = \alpha\psi + \beta|\psi|^2\psi. \qquad (7)$$

This equation has a trivial solution, $\psi = 0$ and it corresponds to the normal state of $T > T_C$. Below the superconducting transition temperature (T_C), equation (7) is expected to have a non-trivial solution (*i.e.* $\psi \neq 0$) and the equation can be rearranged as,

$$|\psi|^2 = -\frac{\alpha}{\beta}. \quad (8)$$

If the second part of equation (4) is positive, then there is a non zero solution for ψ and this can be achieved by assuming the temperature dependence of α such that $\alpha(T) = \alpha_o(T - T_C)$ with $\frac{\alpha_o}{\beta} > 0$ and $n_s \propto (T_c - T)$.

For $T > T_C$, the expression $\frac{\alpha(T)}{\beta}$ is positive and the second part of equation (4) is negative and only $\psi = 0$ solves the Ginzburg-Landau Equation. For $T < T_C$, the second part of equation (4) is positive and there is a non-trivial solution for ψ. Thus equation (8) can be expressed as,

$$|\psi| = \left(\frac{\alpha_o(T_c - T)}{\beta}\right)^{\frac{1}{2}}. \quad (9)$$

Equation (9) yields the Ginzburg Landau order parameter [17, 18].

B. Calculation of Ginzburg-Landau Coherence Length

The Ginzbrug-Landau coherence length (ξ_{GL}) is a measure of the distance in the superconducting electron concentration that cannot change drastically in a spatially-varying magnetic field. The Ginzbrug-Landau coherence length (ξ_{GL}) is a temperature-dependent as well as a material dependent quantity. In the case of the absence of the magnetic vector potential, equation (4) reduces to,

$$\alpha\psi + \beta|\psi|^3 + \frac{1}{2m^*}(-i\hbar\nabla)^2\psi = 0. \quad (10)$$

Now, if we consider a wave function that varies only in the z-direction with zero applied magnetic field, then the first GL equation becomes one dimensional. That is,

$$\alpha\psi + \beta|\psi|^3 - \frac{\hbar^2}{2m^*}\frac{d^2\psi}{dx^2} = 0. \quad (11)$$

Assuming ψ is real and neglecting the term $\beta|\psi|^3$ in comparison with α, equation (11) becomes,

$$\alpha\psi = \frac{\hbar^2}{2m^*}\frac{d^2\psi}{dx^2}. \quad (12)$$

For the plane wave function, the solution of equation (12) is in the form of,

$$\psi(x) = e^{\left(\frac{ix}{\xi_{GL}}\right)} = exp\left(\frac{ix}{\xi_{GL}}\right). \quad (13)$$

Substituting the value of plane wave function into equation (12) (in terms of (ψ (x))), we get,

$$\alpha\left[exp\left(\frac{ix}{\xi_{GL}}\right)\right] = \frac{\hbar^2}{2m^*}\left(\frac{i}{\xi_{GL}}\right)^2 exp\left(\frac{ix}{\xi_{GL}}\right).$$

From which we get,

$$-\frac{\hbar^2}{2m^*}\left(\frac{1}{\xi_{GL}^2(T)}\right) = \alpha. \quad (14)$$

Solving for ξ_{GL} at superconducting state, that is, when α is negative yields,

$$\xi_{GL}(T) = -\sqrt{\frac{\hbar^2}{2m^*|\alpha|}} = \sqrt{\frac{\hbar^2}{2m^*|\alpha_o(T_C - T)|}}, \quad (15)$$

where $\alpha = \alpha_o(T - T_C)$, $\Rightarrow -\alpha = \alpha_o(T_C - T)$ and $\xi_{GL}(0) = \sqrt{\frac{\hbar^2}{2m^*|\alpha_o|T_C}}$ and is the zero temperature GL coherence length.

Equation (15) yields the expression for the GL coherence length [19]. Since α depends on temperature, as $\alpha \propto (T - T_C)$, then wecan conclude that, the GL coherence length is temperature dependent.

Now let us consider cases:

CaseI. For the superconducting state ($T < T_C$), we have,

$$\xi_{GL}(T) = \xi_{GL}(0)\left(1 - \frac{T}{T_c}\right)^{-\frac{1}{2}}, \quad (16)$$

and

CaseII. For the normal state ($T > T_C$), we have,

$$\xi_{GL}(T) = \xi_{GL}(0)\left(\frac{T}{T_C} - 1\right)^{-\frac{1}{2}}. \quad (17)$$

Case III. At $T = T_C$, the GL theory becomes invalid.

C. Calculation of Ginzburg-Landau Penetration Depth

The surface current flows in a very thin layer of thickness (λ_{GL}) which is called the Ginzburg-Landau penetration depth [19]. The temperature and magnetic field dependence of the penetration depth appear quite naturally in Ginzburg-Landau (GL) theory. Like the London model, the GL model is independent of the underlying mechanism for superconductivity. Ginzburg-Landau theory is strictly valid only in superconducting phase boundary and is thus not generally applicable at low temperatures [19]. In the Ginzburg-Landau theory, a complex order parameter(ψ) is a function of temperature, magnetic field and the spatial coordinates [16, 17]. The total free energy per unit volume of the superconducting state in the presence of a magnetic field is minimizing this expression with respect to the first

GL equation and with respect to the current density equation.

$$F_{GL} = \frac{1}{2m^*}\left|-i\hbar\nabla - \frac{e^*}{c}\vec{A}\right|^2 \psi + \alpha\psi + \beta|\psi|^2\psi = 0, \quad (18)$$

where $m^* = 2m$ and $e^* = 2e$.

$$|\psi| = |\psi_o| = \sqrt{\frac{-\alpha}{\beta}}. \quad (19)$$

Using equation (18), we get the expression for current density as follows,

$$J_s = -\frac{e^*\hbar i}{m^*}[\psi^*\nabla\psi + \psi\nabla\psi^*] - \frac{{e^*}^2}{m^*c}A|\psi|^2. \quad (20)$$

Neglecting $\nabla\psi$and $\nabla\psi^*$ equation (20), becomes

$$J_s = -\frac{{e^*}^2}{m^*c}|\psi|^2\vec{A}. \quad (21)$$

Using Maxwell's equation:

$$\nabla X\vec{B} = \frac{4\pi}{c}\vec{J}_s. \quad (22)$$

Taking the curl on both sides of equation (22), we get,

$$\nabla X\nabla X\vec{B} = \frac{4\pi}{c}(\nabla XJ_s). \quad (23)$$

$$\nabla(\nabla.\vec{B}) - \nabla^2\vec{B} = \frac{4\pi}{c}(\nabla X\vec{J}_s). \quad (24)$$

where$\nabla.\vec{B} = 0$, $\nabla X\vec{J}_s = -\frac{{e^*}^2}{m^*c}|\psi|^2\nabla X\vec{A}$ and $\vec{B} = \nabla X\vec{A}$.

From equations (22) and (24), we get,

$$\nabla X\vec{J}_s = -\frac{{e^*}^2}{m^*c}|\psi|^2\nabla X\vec{A} = -\frac{4e^2}{m^*c}|\psi|^2\vec{B} \quad (25)$$

$$\nabla^2\vec{B} = -\frac{4\pi}{c}(\nabla X\vec{J}_s) = -\frac{16\pi e^2}{m^*c}n_s\vec{B}. \quad (26)$$

Since $|\psi|^2 = n_s = \frac{-\alpha}{\beta}$ and $\lambda_{GL}^2(T) = \frac{m^*c^2}{16\pi e^2 n_s}$, we get,

$$\nabla^2\vec{B} = -\frac{4\pi}{c}(\nabla X\vec{J}_s) = \frac{\vec{B}}{\lambda_{GL}^2(T)}. \quad (27)$$

Hence, we get,

$$\lambda_{GL}^2(T) = \frac{m^*c^2\beta}{16\pi e^2\alpha}. \quad (28)$$

$$\lambda_{GL}(T) = \sqrt{\frac{m^*c^2\beta}{16\pi e^2|\alpha|}} = \sqrt{\frac{m^*c^2}{16\pi e^2 n_s}} = \sqrt{\frac{mc^2}{8\pi e^2 n_s}}, \quad (29)$$

where $m^* = 2m$ and $n_s = -\frac{\alpha}{\beta} = \frac{\alpha_o(T_C - T)}{\beta}$. Therefore, we obtain,

$$\lambda_{GL}(T) = \sqrt{\frac{mc^2\beta}{8\pi e^2\alpha_o(T_C - T)}}. \quad (30)$$

Assuming that,

$$\lambda_L(0) = \sqrt{\frac{mc^2\beta}{8\pi e^2\alpha_o T_C}}. \quad (31)$$

The Ginzburg-Landau penetration depth ($\lambda_{GL}(T)$) varies as a function of temperature as,

$$\lambda_{GL}(T) \propto \left[1 - \left(\frac{T}{T_C}\right)\right]^{-\frac{1}{2}}. \quad (32)$$

From which we obtain,

$$\lambda_{GL}(T) = \lambda_L(0)\left[1 - \left(\frac{T}{T_C}\right)^4\right]^{-\frac{1}{2}}, \quad (33)$$

where$\lambda_L(0)$ is the London penetration depth at absolute zero temperature [20].

D. Calculation of the Upper Critical Magnetic Field Using Ginzburg-Landau Theory

The upper critical magnetic field (UCMF) is the magnetic field which completely suppresses superconductivity in type-II superconductors. More properly, the UCMF is a function of temperature (and pressure). Superconducting region nucleates spontaneously within anormal conductor when the applied magnetic field is decreased below a value denoted by H_{C2} [21]. At the onset of superconductivity, ψ is small and we linearize the GL equations and becomes,

$$H\psi = \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}A\right)^2\psi = -\alpha\psi = E\psi. \quad (34)$$

Since $m^* = 2m$, $e^* = 2e$, $\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$ and $E = E_x + E_y + E_z$.

The upper critical magnetic field (H_{C2}) can be calculated by linearizing equation (34) and substituting the value of ∇ as,

$$\frac{1}{2m^*}\left[-i\hbar\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) - \frac{e^*}{c}A\right]^2\psi = -\alpha\psi. \quad (35)$$

The magnetic field in a superconducting region at the onset of superconductivity is just the applied field, so that ***A*** = ***B***(0, *x*,0) = ***B****x* and equation (35) becomes,

$$\frac{1}{2m^*}\left[-i\hbar\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) - \frac{e^*}{c}Bx\right]^2\psi = -\alpha\psi, \quad (36)$$

where the eigen value of the crystal momentum is $\hbar k$. Hence, we have,

$$\frac{1}{2m^*}\left[-i\hbar\left(\frac{\partial}{\partial x} + k_y + k_z\right) - \frac{e^*}{c}Bx\right]^2\psi = -\alpha\psi. \quad (37)$$

Since the expression of the Hamiltonian's energy given in equation (37) does not depend on coordinates (y and z) the corresponding momentum components (k_y , k_z) are conserved.

$$E_x\psi = -\alpha\psi - \frac{\hbar^2}{2m^*}\left(k_y^2 + k_z^2\right)\psi, \qquad (38)$$

$$\Rightarrow E_x\psi = -\left[\frac{1}{2m^*}\left[-i\hbar\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) - \frac{e^*}{c}Bx\right]^2\psi\right] - \frac{\hbar^2}{2m^*}\left(k_y^2 + k_z^2\right)\psi.$$

From which we get,

$$E_x\psi = \frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2}\right)\psi + \frac{m^*}{2}\left(\frac{e^*Bx}{m^*c}\right)^2\psi. \qquad (39)$$

The largest value of the magnetic field (***B***) for which the solution of equation (39) of the lowest eigenvalue is given by,

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c = \left(n + \frac{1}{2}\right)\frac{\hbar e^* B_{max}}{m^* c\frac{\hbar^2 k_z^2}{2m^*}}. \qquad (40)$$

Let us take the smallest eigenvalues n = 0 and K_z= 0 corresponding to the highest field in which superconductivity can nucleate in the interior of a bulk sample which occurs with the upper critical magnetic field in the coefficients change of sign.

From equation (40), we have,

$$\frac{1}{2}\hbar\omega_c = \frac{\hbar e^* B_{max}}{2m^* c}, \qquad (41)$$

where ω_c is the cyclotron frequency and is given by,

$$\omega_c = \frac{e^* B_{max}}{m^* c\frac{2\alpha}{\hbar}}. \qquad (42)$$

Since $BC2_{max}$, solving for H_{C2}, we get,

$$H_{C2} = -\frac{2m^* c}{\hbar e^*}|\alpha| \;. \qquad (43)$$

From the relation $\alpha = \alpha_o(T - T_C)$, we get,

$$-\alpha = \alpha_o(T_C - T) = \frac{\hbar^2}{2m^*}\left(\frac{1}{\xi_{GL}^2(T)}\right),$$

$$\xi_{GL}(T) = \xi_{GL}(0)\left[\left(1 - \frac{T}{T_C}\right)\right]^{-\frac{1}{2}},$$

and

$$\xi_{GL}^2(T) = \frac{\xi_{GL}^2(0)}{\left(1 - \frac{T}{T_C}\right)}.$$

Thus, we obtain the expression for the temperature dependent upper critical magnetic field (H_{C2}) as,

$$H_{C2} = \left(\frac{2m^* c}{\hbar e^*}\right)\left(\frac{\hbar^2}{2m^*\xi_{GL}^2(0)}\right)\left[\left(1 - \frac{T}{T_{C2}}\right)\right]$$

$$\Rightarrow H_{C2} = \frac{\hbar c}{e^*\xi_{GL}^2(0)}\left(1 - \frac{T}{T_C}\right).$$

Hence, we obtain,

$$H_{C2} = \frac{\varphi_o}{2\pi\xi_{GL}^2(T)} = \frac{\varphi_o}{2\pi\xi_{GL}^2(0)}\left(1 - \frac{T}{T_C}\right), \qquad (44)$$

where $\varphi_o = \frac{2\pi\hbar c}{e^*}$.

E. Anisotropic Mass Tensor Model

Now, let us consider anisotropy in mass, by introducing the effective mass tensor to the kinetic energy term of equation (4), where m^* is an effective mass tensor which is given by,

$$m^* = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & m_z \end{bmatrix}.$$

Since the coherence length $\xi_{GL}(0)$ depends on the effective mass as $\xi_{GL}(T) \propto \frac{1}{\sqrt{m^*}}$, the resulting equation is formally identical with the Schrödinger equation of a particle with charge e^*, an isotropic mass tensor m^*in a uniform magnetic field H and the energy levels that have the harmonic oscillator are given by,

$$-\alpha = \left(n + \frac{1}{2}\right)\hbar\omega_C(\theta) \;. \qquad (45)$$

Let us consider Newton's law of motion under the influence of Lorentz force (F_L), *i.e.*

$$F_L = m^*.\dot{v} = \frac{e^*}{c}vXH, \qquad (46)$$

where v is the velocity.

The upper critical magnetic field can be expressed by using cyclotron frequency with the lowest free energy given by,

$$-\alpha = \frac{1}{2}\hbar\omega_c(\theta).$$

The solution of upper critical magnetic field by applying elliptical orbits traversed with cyclotron frequency is given by,

$$\omega_c(\theta) = \frac{|e^*|B_{max}}{c\left(\frac{\sin^2\theta}{m_x m_z} + \frac{\cos^2\theta}{m_x}\right)^{\frac{1}{2}}}\,. \qquad (47)$$

The solution of the lowest free energy corresponds to $n = 0$ and by using equation (47), we get

$$\frac{1}{2}\hbar\omega_C(\theta) = -\alpha = \frac{1}{2}\hbar\left[\frac{|e^*|B_{max}}{c\left(\frac{sin^2\theta}{m_x m_z}+\frac{cos^2\theta}{m_x}\right)^{\frac{1}{2}}}[]\right].$$

Where θ is the angle the magnetic field makes with the z-axis.

$$-\alpha = -\alpha_o(T - T_C) = \alpha_o(T_C - T)$$

$$H_{C2} = \frac{2c\alpha_o(T_C - T)}{\hbar e^*\left[\frac{sin^2\theta}{m_x m_z}+\frac{cos^2\theta}{m_x}\right]^{\frac{1}{2}}}. \quad (48)$$

Using the general expression of coherence length and equation (44) we have,

$$\xi_x = \left[\frac{\hbar^2}{2m_x\alpha_o(T_c - T)}\right]^{\frac{1}{2}}, \quad (49)$$

and

$$\xi_z = \left[\frac{\hbar^2}{2m_z\alpha_o(T_c - T)}\right]^{\frac{1}{2}}. \quad (50)$$

Using the expression for the flux quantization, $\varphi_o = \frac{hc}{|e^*|}$ and equation (44), H_{C2} can be expressed as,

$$H_{C2} = \frac{\varphi_o}{2\pi\left[\frac{sin^2\theta}{\xi_x^2\xi_z^2}+\frac{cos^2\theta}{\xi_x^4}\right]^{\frac{1}{2}}}. \quad (51)$$

For fields parallel and perpendicular to the symmetry plane we can write Equation (51) as:

$$H_{C2} \parallel = \frac{\varphi_o}{2\pi\xi_z^2}, \quad (52)$$

and

$$H_{C2} \perp = \frac{\varphi_o}{2\pi\xi_x^2}. \quad (53)$$

Equations (52) and (53) are the mathematical expressions of the upper critical magnetic field (H_{C2}) for fields parallel and perpendicular to the symmetry axis [22].

III. RESULTS AND DISCUSSION

From equation (16) and using plausible experimental values of coherence length for parallel and perpendicular to the symmetry axis of superconducting $CeRhIn_5$ [15], we obtained the phase diagram which demonstrates the relationship between the GL coherence length and temperature (T) as indicated in Fig. 1. As can be seen from the figure, the GL coherence length increases with temperature and diverges as $T \rightarrow T_C$. $\xi_{GL}(0)$ has the same value as that of the conventional BCS coherence length, ξ_o [19]. Furthermore, equation (33) yields the expression for the GL penetration depth and the relationship it has with temperature(T) as shown in Fig. 2. From the figure, we observe the increase of the GL penetration depth with temperature (T) and generally, we see that, the penetration depth rises asymptotically as the temperature approaches T_C. Thus, the penetrationof the magnetic field increases as the temperature approaches T_C.

Finally, we have determined the expression for the upper critical magnetic field for superconducting $CeRhIn_5$ using the GL equation and by considering some experimental data for upper critical magnetic fields for parallel and perpendicular and plotted the upper critical magnetic fields for parallel and perpendicular to the symmetry axis as shown in Fig. 3 [9, 15, 23]. From the figure, we can see that, the upper critical magnetic field decreases as temperature increases and reaches to zero at the critical temperature of superconducting $CeRhIn_5$, which agrees with the experimental observations [15]. We also observe that, the upper critical magnetic field (H_{C2}) parallel and perpendicular to the symmetry axis of superconducting $CeRhIn_5$ is inversely proportional to the GL coherence length.

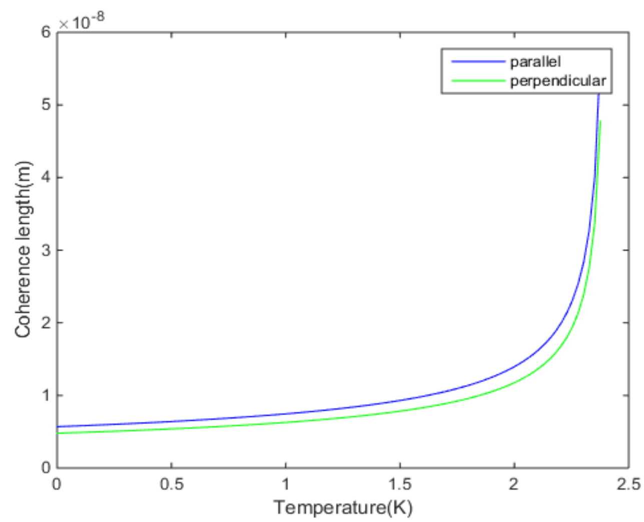

FIGURE 1. GL coherence length versus temperature (T).

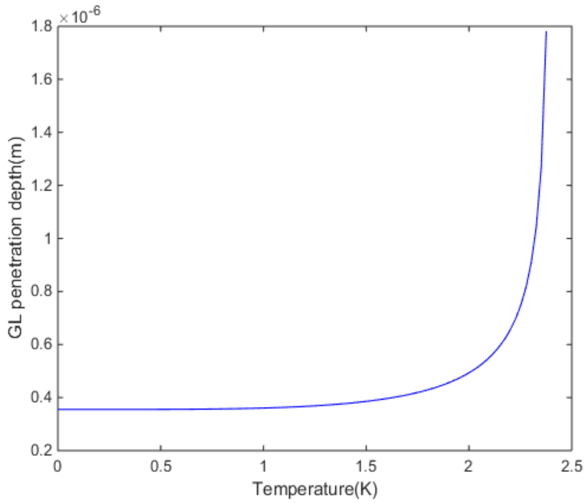

FIGURE 2. GL penetration depth versus temperature (*T*).

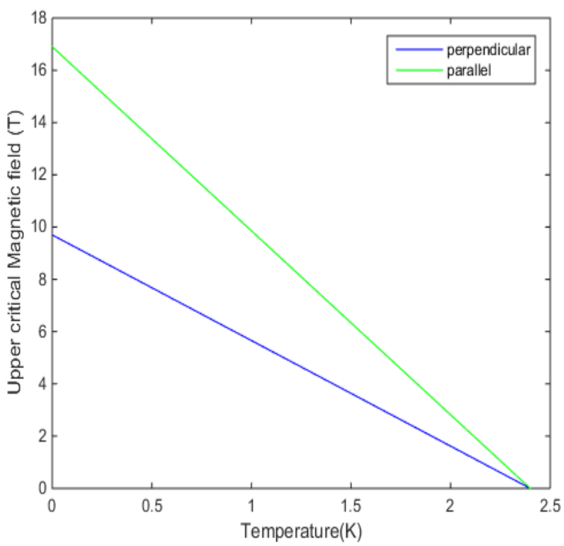

FIGURE 3. Upper critical magnetic field parallel and perpendicular to the symmetry axis versus temperature (*T*).

IV. CONCLUSION

The main purpose of this research work is to determine the upper critical magnetic field of superconducting $CeRhIn_5$ by using the Ginzburg-Landau approach. From the calculations, the effect of coherence length, penetration depth and anisotropy in mass tensor on upper critical field are considered in our model. Finally phase diagrams are plotted by using MATLAB scripts. From the phase diagrams plotted, it can be concluded that the upper critical magnetic field of superconducting $CeRhIn_5$ is inversely related to temperature which is in agreement with experimental observations [15].

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