

## **Simple theoretical proposal of the dependence of the deGennes extrapolation parameter with the surface temperature of a superconducting sample**

### **Simple propuesta teórica de la dependencia del parámetro de extrapolación de deGennes con la temperatura en la superficie de una muestra superconductora**

José José Barba-Ortega  <sup>1</sup>, Jesús D. González , y Miryam Rincón-Joya <sup>3</sup>

Recibido: 18 de mayo de 2018  
Aceptado: 06 de febrero de 2019

---

#### Cómo citar / How to cite

J. Barba-Ortega, J. D. González y M. Rincón-Joya, "Simple theoretical proposal of the dependence of the deGennes extrapolation parameter with the surface temperature of a superconducting sample". *TecnoLógicas*, vol. 22, no. 45, pp. 1-7, 2019. <https://doi.org/10.22430/22565337.1301>



---

<sup>1</sup> PhD en Física, Departamento de Física, Universidad Nacional de Colombia, Bogotá-Colombia, [jjbarbao@unal.edu.co](mailto:jjbarbao@unal.edu.co)  
<sup>2</sup> PhD en Física, Grupo en Teoría de la Materia Condensada, Universidad del Magdalena, Santa Marta-Colombia, [jgonzaleza@unimagdalena.edu.co](mailto:jgonzaleza@unimagdalena.edu.co)  
<sup>3</sup> PhD en Física, Departamento de Física, Universidad Nacional de Colombia, Bogotá-Colombia, [mrinconj@unal.edu.co](mailto:mrinconj@unal.edu.co)

## Abstract

The Time-dependent Ginzburg–Landau model (TDGLM) is a robust tool widely used to analyze the magnetization of the single-vortex state of a mesoscopic superconducting sample in presence of a magnetic field. The algorithm implemented in this work is applied to a square geometry surrounded by different kinds of materials simulated by deGennes extrapolation length  $b$ . The inside of the sample remains at constant temperature  $T_i$ , while its boundary remains at temperature  $T_s$ . This temperature variation in the sample can be generated by a continuous laser wave injected into all the internal points, except for a thin surface layer in the boundary of the material. We found that the  $b$  value at  $T_s = T_i = 0.0$ , which mimics the magnetization curve for a corresponding  $T_s$ , presents a linear dependence with the temperature. Therefore, although within the domain of validity TDGLM the parameter  $b$  is to be considered temperature-independent in the vicinity of the bulk critical temperature and that  $b$  depends on the density of states near the surface, we propose a simple dependence of  $b$  using a TDGLM.

## Keywords

Time-dependent Ginzburg–Landau equations, deGennes parameter, Superconductor, Mesoscopic, Magnetization.

## Resumen

El modelo de Ginzburg - Landau (TDGLM) es una fuerte herramienta ampliamente utilizada para analizar la magnetización de un estado de vórtice simple en una muestra superconductor mesoscópica en presencia de un campo magnético. El algoritmo implementado es aplicado a una geometría cuadrada rodeada de diferentes tipos de materiales (simulados por la longitud de extrapolación de deGennes  $b$ ). El interior de la muestra se mantiene a una temperatura constante  $T_i$ , mientras su frontera permanece a una temperatura  $T_s$ . Esta variación de temperatura en la muestra puede ser generada por una onda laser continua inyectada en todos los puntos internos, excepto en una delgada capa en la superficie del material. Encontramos que, el valor de  $b$  en  $T_s = T_i = 0.0$ , cual imita la curva de magnetización para un respectivo  $T_s$ , presenta una dependencia lineal con la temperatura. Por lo tanto, aunque dentro del dominio de validez de la TDGLM el parámetro  $b$  es considerado independiente de la temperatura en la vecindad de la temperatura crítica volumétrica y que  $b$  depende de la densidad de estados cercal a la superficie, proponemos una dependencia simple de  $b$  usando TDGLM.

## Palabras Claves

Ecuaciones Ginzburg-Landau dependientes del tiempo, Parámetro de deGennes, Superconductor, Mesoscopicos, Magnetización.

## 1. INTRODUCTION

One way to modify the magnetic response of superconducting samples at low temperature is controlling their boundary conditions. Several theoretical physics models enable to simulate different kinds of material in contact with the sample by varying the deGennes extrapolation length  $b$  in the boundary conditions, thus allowing controlling the superconducting critical parameters [1]-[2]. In a recent work, we studied the magnetic response of a superconducting film in absence of a magnetic field, under a continuous electric current, and subjected to different boundary conditions; we found that the critical current for which the first vortex-anti-vortex penetrates the sample, depends strongly with the material in contact with the superconductor [3]. Flux trapping and boundary effects in superconducting slabs and circular geometries have been studied by several authors who found that a superconductor-dielectric interface leads to Bean-Livingston surface barrier, which is responsible for the hysteresis in the magnetization curve. Additionally, a superconductor in contact with another superconductor at different critical temperature  $T_C$  increases the second critical field [4]-[7]. Fink et al. studied the effect of the presence of another material in contact with the superconducting sample (simulated by  $b$ ) [3,5] on  $T_C$  for various geometries. They found that the deGennes parameter is useful to analyze the variation of  $T_C$  in mesoscopic superconductors [8]. Other authors found that the  $T_C$  of the sample is very sensitive to electrical fields in a reversible way [9]-[11]. In order to study the  $b(T)$ , we used a TDGLM in a mesoscopic square where temperature  $T$  is locally modified.  $T_i$  is the temperature in the internal points of the sample, while its lateral surface remains at  $T_S$ . We changed the boundary conditions until we found a  $b$  value that mimicked the

magnetization curve at  $T_S$  and  $T_i = 0$  in a single vortex state.

This paper is organized as follows. Section 2 introduces the dimensionless time-dependent Ginzburg–Landau equations [12]-[15] taking into account the general boundary conditions. Section 3 presents the results of the computational simulations for certain temperatures and the deGennes parameter of a superconducting square at  $T_S$ .

## 2. THEORETICAL FORMALISM

In this work, a mesoscopic superconducting square immersed in an external magnetic field  $\mathbf{H} = H\mathbf{z}$  is simulated with the TDGLM for the order parameter  $\psi(x, y)$  and the potential vector  $\mathbf{A}(x, y)$ , where  $\mathbf{B} = \nabla \times \mathbf{A}$  is the magnetic induction. The time-dependent Ginzburg–Landau equations are written as [16]:

$$\frac{\partial \psi}{\partial t} = -(i\nabla + \mathbf{A})^2 \psi + (1 - T)(1 - |\psi|^2)\psi \quad (1)$$

$$\frac{\partial \mathbf{A}}{\partial t} = (1 - T)Re[\bar{\psi}(-i\nabla - \mathbf{A})\psi] - \kappa^2 \nabla \times \mathbf{B} \quad (2)$$

The equations are complemented with specific boundary conditions for the order parameter  $\mathbf{n} \cdot (i\nabla + \mathbf{A})\psi = -i\psi/b$ ,  $b$  is the deGennes parameter and it is characteristic of a superconducting interface with any kind of material.  $\kappa$  is the Ginzburg–Landau parameter, which is material dependent. For convenience, we selected the parameter  $\gamma = 1 - \delta/b$ , where  $\delta$  is the size of the grid used to solve Eqs. 1 and 2 computationally. (i)  $\gamma = 0$  simulates an interface at the normal state, ( $b = 0$ , where  $\psi = 0$ ); (ii)  $0 < \gamma < 1$ , a superconductor/metal interface ( $b > \delta$ ); (iii)  $\gamma = 1$ , a superconductor/vacuum interface ( $b \rightarrow \infty$ ); and (iv) a superconductor/superconductor interface is described by  $\gamma > 1$  ( $b < 0$ ) [12]-[17].

We chose a square with dimensions  $L_x = L_y = 3\xi(0)$ ,  $\delta = 0.1$ ,  $\kappa = \lambda/\xi = 5.0$ . (typical values for an Pb-In alloy) [18]-[22].  $\xi(0)$  is the coherence length at zero temperature. In this study, we do not consider the heat equation (thermal dissipation), which is associated with the relaxation of the Cooper pair density and it is very important in measurements of magnetization as a function of time [23].

### 3. RESULTS AND DISCUSSION

We analyzed the magnetization curves of the deGennes parameter in the single-vortex state using several values. Fig. 1 shows magnetization  $-4\pi M$  as a function of applied magnetic field  $H$  for a superconducting/vacuum interface in a single vortex state at  $T = 0.0$  (Fig. 1a) and Meissner state at  $T_i = 0.5T_C$  (Fig. 1b). It is widely known that magnetization decreases as  $T_i$  increases and the sample becomes less diamagnetic. It can be seen from the plots that the lower critical field  $H \sim 1.726H_{c2}$  is practically independent of  $T_S$ , and the upper field depends on  $T_S$  (see Fig. 2). The Meissner state is presented for the case with  $T_i = 0.5T_C$ , and a slow superconductor/normal state occurs due to

the temperature dependence with vortex size [2]. In Fig. 3, we plot magnetization as a function of  $H$  for several values of  $\gamma$  and  $T_S$ . Fig. 3(a):  $T_i = 0.0$  and  $\gamma = 0.9985$ ,  $T_S = 0.1T_C$  (red and blue lines),  $\gamma = 0.9975$ ,  $T_S = 0.2T_C$  (green and black lines) and  $\gamma = 0.9965$ ,  $T_S = 0.3T_C$  (dark cyan and navy blue lines). Fig. 3(b):  $T_i = 0.5T_C$  and  $\gamma = 0.998$ ,  $T_S = 0.7T_C$  (red and blue lines),  $\gamma = 0.9965$ ,  $T_S = 0.8T_C$  (green and black lines).

We calculated the error in the maximum difference of the magnetization curves by comparing all cases with  $\gamma = 1.0$  and  $T_i = T_S = 0.0$ . We thus found a percentual error  $E \sim 1\%$  and, based on this criterion, we can say that the curves are very similar. We also established that  $\gamma$  presents an analytical dependence with  $T = T_S$ , as  $\gamma(T) = 0.9980 - 0.0118T$  (see Fig. 3, left). In Fig. 4, we plot  $\gamma$  as a function of  $T_S$  when the magnetization curves are similar for Fig. 4(a)  $T_i = 0.0$  and Fig. 4(b)  $T_i = 0.5$ . As we can see in Fig. 4(b), there is a deflection in the slope of  $\gamma(T)$  when  $T_S > T_i$  from  $\gamma(T) = 1.0062 - 0.0125T$  for  $\gamma > 1.0$  to  $\gamma(T) = 1.0085 - 0.0150T$  for  $\gamma < 1.0$ . This is due to a superconductor-superconductor (superconductor-metal) interface. The surface barrier is

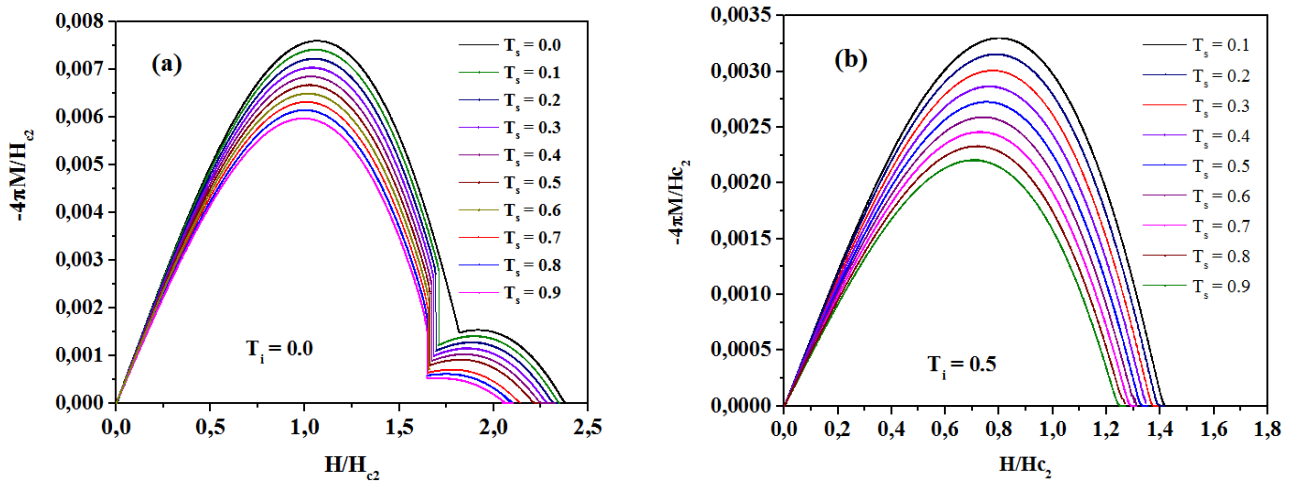


Fig. 1. (Color online) Magnetization curves for  $T_S/T_C = 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$  and  $\gamma = 1.0$ , for (a) the single vortex state at  $T_i = 0.0$  and (b) the Meissner state at  $T_i = 0.5T_C$ . Source: Authors' own work.

enhanced (suppressed) and, consequently, diamagnetism increases (decreases) and an interface with  $\gamma > 1.0$  ( $0 < \gamma < 1.0$ ) must be chosen. Our results show dependence for the critical temperature with the boundary simulated by the deGennes parameter  $b$ .

#### 4. CONCLUSIONS

In summary, we analyzed the numerical results of the magnetization curves of a mesoscopic superconducting square sample embedded in a thermal bath at different temperatures  $T$  and boundary conditions  $\gamma$ . A metallic (superconducting) material in contact with the sample suppressed (enhanced) the surface superconductivity. Therefore, we could find a temperature of the surface for which  $T_s \neq T_i$  when  $\gamma = 1$ , mimicking the result for  $T_s = T_i$  when  $\gamma \neq 1$ . We found a highly possible value for  $\gamma$  at  $T_i = T_s = 0.0$  at which the magnetization curve is similar to the one simulated with each  $T_s$ . We found that, for a square sample surrounded by different kinds of materials, in the Ginzburg–Landau model, the critical temperature of these materials for the studied samples at  $T = 0.0$  exhibits a dependence  $\gamma(T) = 0.998 - 0.0118T$ . In terms of the deGennes parameter, we have  $b(T) = (0.085 - 0.15T)^{-1} \xi(0)$ .

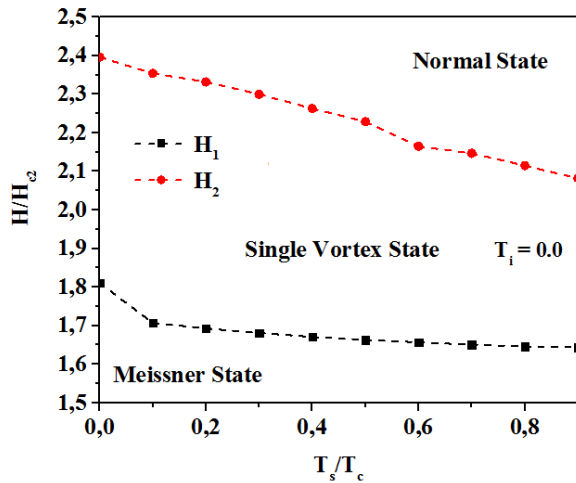


Fig. 2. (Color online) Phase diagram  $H - T$  showing the first vortex penetration field  $H_1$  and superconducting-normal transition field  $H_2$  at  $T_i = 0.0$  and  $\gamma = 1.0$ . Source: Authors' own work.

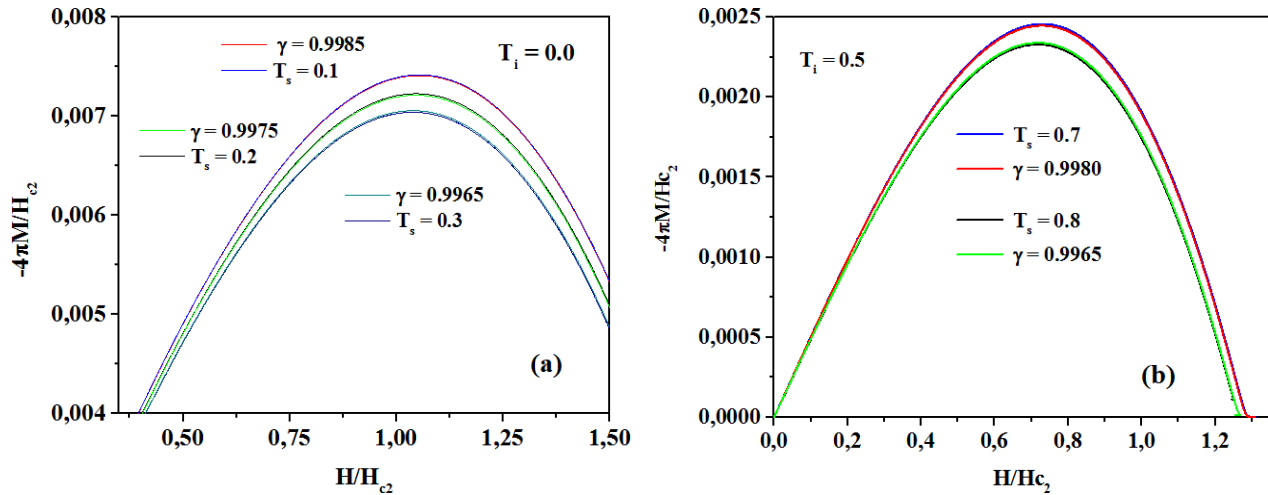


Fig. 3. (Color online) Magnetization curves for (a)  $T_i = 0.0$  and  $\gamma = 0.9985$ ,  $T_s = 0.1$  (red and blue lines),  $\gamma = 0.9975$ ,  $T_s = 0.2$  (green and black lines) and  $\gamma = 0.9965$ ,  $T_s = 0.3$  (dark cyan and navy blue lines). (b)  $T_i = 0.5$  and  $\gamma = 0.9980$ ,  $T_s = 0.7$  (red and blue lines),  $\gamma = 0.9965$ ,  $T_s = 0.8$  (green and black lines). Source: Authors' own work.

Simple theoretical proposal of the dependence of the deGennes extrapolation parameter with the surface temperature of a superconducting sample

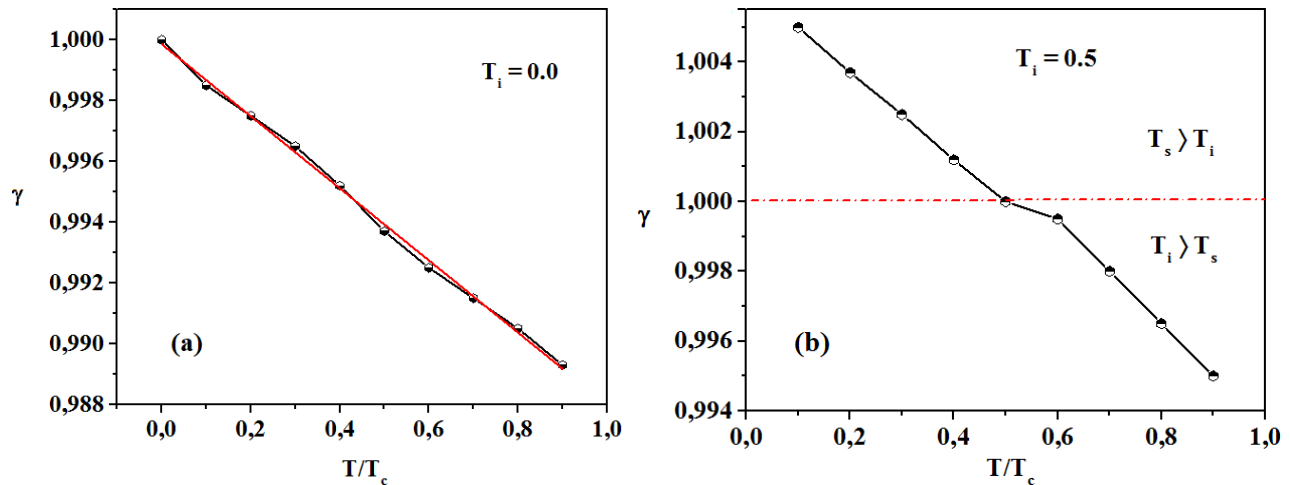


Fig. 4. Inverse of the deGennes parameter  $\gamma$  as a  $T = T_s$  function for (a)  $T_i = 0.0$  and (b)  $T_i = 0.5$ . Source: Authors' own work.

## 5. ACKNOWLEDGEMENTS

The authors would like to thank Edson Sardella for all the very useful discussions.

## 6. REFERENCES

- [1] P. G. De Gennes, *Superconductivity of Metals and Alloys*, 1st ed. New York: Advanced Books Classics, 1966.
- [2] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. New York: McGraw-Hill Book Co., 1996.
- [3] J. Barba-Ortega, E. Sardella, and R. Zadorosny, "Influence of the deGennes extrapolation parameter on the resistive state of a superconducting strip," *Phys. Lett. A*, vol. 382, no. 4, pp. 215–219, Jan. 2018. <https://doi.org/10.1016/j.physleta.2017.11.010>
- [4] J. Barba-Ortega, C. C. de S. Silva, and J. A. Aguiar, "Superconducting slab in contact with thin superconducting layer at higher critical temperature," *Phys. C Supercond.*, vol. 469, no. 14, pp. 852–856, Jul. 2009. <https://doi.org/10.1016/j.physc.2009.06.001>.
- [5] J. Barba-Ortega, E. Sardella, and J. A. Aguiar, "Superconducting boundary conditions for mesoscopic circular samples," *Supercond. Sci. Technol.*, vol. 24, no. 1, p. 015001, Jan. 2010. <https://doi.org/10.1088/0953-2048/24/1/015001>.
- [6] B. J. Baelus, B. Partoens, and F. M. Peeters, "One-dimensional modulation of the superconducting boundary condition for thin superconducting films," *Phys. Rev. B*, vol. 73, no. 21, p. 212503, Jun. 2006. <https://doi.org/10.1103/PhysRevB.73.212503>.
- [7] M. M. Doria, A. R. de C. Romaguera, and F. M. Peeters, "Effect of the boundary condition on the vortex patterns in mesoscopic three-dimensional superconductors: Disk and sphere," *Phys. Rev. B*, vol. 75, no. 6, p. 064505, Feb. 2007. <https://doi.org/10.1103/PhysRevB.75.064505>.
- [8] H. J. Fink, S. B. Haley, C. V. Giuraniuc, V. F. Kozhevnikov, and J. O. Indekeu, "Boundary conditions, dimensionality, topology and size dependence of the superconducting transition temperature," *Mol. Phys.*, vol. 103, no. 21–23, pp. 2969–2978, Nov. 2005. <https://doi.org/10.1080/00268970500226189>.
- [9] M. Salluzzo *et al.*, "Indirect Electric Field Doping of the CuO<sub>2</sub> Planes of the Cuprate NdBa<sub>2</sub> Cu<sub>3</sub> O<sub>7</sub> Superconductor," *Phys. Rev. Lett.*, vol. 100, no. 5, p. 056810, Feb. 2008. <https://doi.org/10.1103/PhysRevLett.100.056810>.
- [10] A. S. Dhoot, S. C. Wimbush, T. Benseman, J. L. MacManus-Driscoll, J. R. Cooper, and R. H. Friend, "Increased T<sub>c</sub> in Electrolyte-Gated Cuprates," *Adv. Mater.*, vol. 22, no. 23, pp. 2529–2533, May. 2010. <https://doi.org/10.1002/adma.200904024>.
- [11] X. Leng, J. Garcia-Barriocanal, S. Bose, Y. Lee, and A. M. Goldman, "Electrostatic Control of the Evolution from a Superconducting Phase to an Insulating Phase in Ultrathin YBa<sub>2</sub> Cu<sub>3</sub> O<sub>7-x</sub> Films," *Phys. Rev. Lett.*, vol. 107, no. 2, p. 027001, Jul. 2011. <https://doi.org/10.1103/PhysRevLett.107.027001>.
- [12] W. D. Gropp, H. G. Kaper, G. K. Leaf, D. M. Levine, M. Palumbo, and V. M. Vinokur, "Numerical Simulation of Vortex Dynamics

- in Type-II Superconductors,” *J. Comput. Phys.*, vol. 123, no. 2, pp. 254–266, Feb. 1996. <https://doi.org/10.1006/jcph.1996.0022>.
- [13] G. C. Buscaglia, C. Bolech, and A. Lpez, *Connectivity and Superconductivity*, vol. 62. Berlin, Heidelberg: Springer Berlin Heidelberg, 2000. <https://doi.org/10.1007/3-540-44532-3>.
- [14] J. Berger, “Time-dependent Ginzburg–Landau equations with charged boundaries,” *J. Math. Phys.*, vol. 46, no. 9, p. 095106, Sep. 2005. <https://doi.org/10.1063/1.2010352>.
- [15] A. Crassous *et al.*, “Nanoscale Electrostatic Manipulation of Magnetic Flux Quanta in Ferroelectric/Superconductor BiFeO<sub>3</sub>/YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-δ</sub> Heterostructures,” *Phys. Rev. Lett.*, vol. 107, no. 24, p. 247002, Dec. 2011. <https://doi.org/10.1103/PhysRevLett.107.247002>.
- [16] M. V. Milošević and R. Geurts, “The Ginzburg–Landau theory in application,” *Phys. C Supercond.*, vol. 470, no. 19, pp. 791–795, Oct. 2010. <https://doi.org/10.1016/j.physc.2010.02.056>.
- [17] V. Moshchalkov, R. Woerdenweber, and W. Lang, *Nanoscience and Engineering in Superconductivity*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010. <https://doi.org/10.1007/978-3-642-15137-8>.
- [18] C. Poole, H. Farach, and R. Creswick, *Handbook of Superconductivity*, 1st ed. New York: Academic Press, 2000.
- [19] J. Barba-Ortega and M. R. Joya, “Configuración de vórtices en películas finas: Teoría Ginzburg-Landau no lineal,” *Tecnológicas*, no. 27, pp. 89–102, July 2011. Dec. 2011. <https://doi.org/10.22430/22565337.6>.
- [20] J. Barba-Ortega, E. D. Valbuena-Nino, and M. Rincón-Joya, “Transport phenomena in superconductors: kinematic vortex,” *Iteckne*, vol. 14, no. 1, pp. 11–16, Mar. 2017. <https://doi.org/10.15332/iteckne.v14i1.1625>
- [21] F. Durán-Florez, M. Rincón-Joya, and J. Barba-Ortega, “Perfil de súper-corrientes en una lámina de Al a campo magnético cero,” *Respuestas*, vol. 21, no. 2, pp. 6–12, Jan. 2016. <https://doi.org/10.22463/0122820X.769>
- [22] J. Barba-Ortega and M. Rincón-Joya, “Nucleación de vórtices y antivórtices en películas superconductoras con nanoestructuras magnéticas,” *Respuestas*, vol. 16, no. 1, pp. 45–49, Jan. 2011. <https://doi.org/doi.org/10.22463/0122820X.104>.
- [23] E. C. S. Duarte, E. Sardella, W. A. Ortiz, and R. Zadorosny, “Dynamics and heat diffusion of Abrikosov’s vortex-antivortex pairs during an annihilation process,” *Journal Phys. Condens. Matter*, vol. 29, no. 40, p. 405605, 2017. <https://doi.org/10.1088/1361-648X/aa81e6>