Investigaciones Europeas de Dirección y Economía de la Empresa
Vol. 5, Nº 2, 1999, pp. 93-102

CORRELATION DYNAMICS BETWEEN THE BENELUX AND THE UK STOCK MARKETS: IMPLICATIONS FOR RISK MANAGEMENT

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ABSTRACT

We present a model for the correlation dynamics between the BeNeLux and UK stock markets. Using daily data we estimate the Correlated GARCH model of Christodoulakis and Satchell (1998) to uncover the dynamics of the pair-wise correlations between markets. We then propose how such a model can be used as risk management tool. The empirical results suggest that a constant correlation model would cause significant misallocations in any process that attempted to jointly allocate investment or capital between BeNeLux, UK and other markets.

KEYWORDS: Risk Management; Stock Markets.

INTRODUCTION[1]

There is substantial evidence that correlations vary over time, especially between high-frequency financial asset returns. Such time variation implies that a pair of markets, asset classes or individual assets may experience common innovation shocks but the degree of commonality is non-constant over time. Such a relationship has important implications for the management of financial risks, as correlations are key inputs for optimal financial decisions such as asset allocation, pricing and the design of derivatives.

The evidence on the time variation of correlations goes back to the work of Kaplanis (1988), Fustenberg and Jeon(1989), Bertero and Mayer(1990) Koch and Koch(1991), King, Sentana and Wadhwani(1994) and Longin and Solnik (1995) among others. They use various kinds of statistical procedures to test for the stability of a correlation matrix and present substantial evidence against such a hypothesis. Further, correlations were found to increase immediately after or during commonly high volatile periods in the markets, be related with observable factors such proxies for the business cycle as well as latent factors. Christodoulakis and Satchell (1998), henceforth CS, model correlation as a stationary discrete-time stochastic process and show explicitly how persistent common (unobservable) innovations for a pair of asset returns lead to a stationary model for correlations, generating correlation clustering and implying predictability. In this paper we use daily data on european stock markets to estimate their joint correlation structure based on the CS (1998) model, and discuss how such a methodology can be used for more efficient risk management.

Our data set consists of daily returns[2] over the period of July 1988 to July 1998 from the UK, Belgium, Netherlands and Luxemburg stock markets as well as a joint BeNeLux market index. We use the DataStream-calculated indexes in US dollars for these markets and consider the pair-wise correlations between the UK and each of the BeNeLux markets; in each case we eliminate from the sample the data points corresponding to the non-common trading days for the two markets. A preliminary statistical analysis of the data set uncovered no significant serial correlation for the levels of returns, but highly significant ARCH-type effects as expected. We believe that the joint correlation structure of BeNeLux and the UK is likely to vary over time and this time variation can be captured by our methodology which in turn has important risk implications. Section 2 discusses how our methodology can address issues in risk management, section 3 describes the mathematics of the Correlated ARCH model for which we present our empirical results and conclusions in section 4.

CORRELATION AND RISK MANAGEMENT ISSUES

There are many areas of risk management where variable correlation influences risk management decisions, we shall discuss just three. The first is *asset allocation* whilst the second is what we might term *correlation products*, the third is conventional *hedging*. In a standard Global Asset Allocation model, the fund manager elects to build a global equity/bond product by holding appropriate indices in a set of countries loosely coincident with the OECD; the manager may choose to include currency as well. Thus he would have a portfolio of the order of 80 to 90 components, we shall call this number N. His model would consist of forecasts of expected returns for each index together with time varying volatility. Usually, the manager would assume a constant correlation between each pair of N indices. Since he would work with a rolling window of 60 to 80 monthly data points, the correlation would change through time but only very slowly. The advantage of our model is that we could now capture large correlation clusters, so if the UK and the Belgian equity market both moved substantially last period, our model would forecast a high co-movement next period. Using this higher correlation would assign a higher risk to the UK and Belgium in our global portfolio and our mean-variance optimizer would accordingly reduce our position in these two markets, assuming that our forecasts of expected returns and volatility were unchanged.

Turning to the use of correlation products in risk management we first need to define what they are. Broadly speaking, they consist of that class of derivative products whose pay-off's are non-linear and depend upon more that one asset. These two features guarantee under fairly general conditions that the fair price of the product will depend upon the correlation between individual assets. These products have captured the imagination of financial engineers and financial products purveyors but they are not used to the same extent as conventional derivatives. Products that fall under this category include *diff swaps*, *quanto swaps*, options that give you the minimum or the maximum price of a number of assets et c. Mahoney (1995) provides definitions of these types of contracts and includes a discussion on the risk management issues associated with correlation products. He notes that conventional contracts have risk aspects that are additive so that risk can be enumerated at the trading level and global risk issues require only aggregation with respect to separate risk factors. With correlation products however, the global risk manager requires a more active and quantitative role as he has to consider correlation risk as well as the individual risks of the two or more correlated assets.

The last issue, hedging, can be dealt with straightforwardly. It is natural to think of an asset, call it X, that an institution is obliged to hold and a second asset Y, called *the hedge,* which the institution will buy/sell to reduce the risk of his fixed position in X. For many hedging schemes, and in particular regression-based ones, the optimal hedging coefficient will depend upon, inter alia, the correlation coefficient between X and Y. Thus if our model were, ceteris paribus, to forecast an increase in correlation, we would need to hold less units of Y to achieve the degree of hedge that we had previously.

Concluding, we see that a time-varying correlation model will have immediate and important implications for a wide range of risk management problems.

CORRELATED ARCH MODELLING

Following the notation of CS (1998), let y_t be a 2×1 vector of asset returns with conditional mean equation

$$y_t = \mu + v_t \qquad (1)$$

where μ is a 2×1 vector that can have a general structure and v_t is a vector of error terms such that

$$v_t = \begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix} = \begin{pmatrix} \sigma_{1,t}\xi_{1,t} \\ \sigma_{2,t}\xi_{2,t} \end{pmatrix}$$

with $\sigma_{i,t}$ representing the conditional standard deviation of asset *i* returns and $\xi_{i,t}$ represents an innovation process. The joint generating process of the two innovations is assumed to follow

$$\begin{pmatrix} \xi_{1,t} \\ \xi_{2,t} \end{pmatrix} \sim D\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & \rho(z_{12,t}) \\ \rho(z_{12,t}) & 1 \end{pmatrix}\right) \qquad (2)$$

which is an independent but non-identically distributed sequence, the distribution *D* of which will be specified accordingly later in the text. Under this framework, asset returns experience common innovation shocks through the covariance term $\rho(z_{12,t})$ which is allowed to vary over time. Further we now see that

$$v_t | I_{t-1} \sim D(0, H_t)$$

where I_{t-1} is the sigma field generated by the available information set and H_t is the time-varying covariance matrix, such that $H_t = C_t R_t C_t$ or

$$H_t = \begin{pmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{pmatrix}\begin{pmatrix} 1 & \rho(z_{12,t}) \\ \rho(z_{12,t}) & 1 \end{pmatrix}\begin{pmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{pmatrix}$$

where conditional variances $\sigma_{i,t}^2$ are assumed to follow any type of GARCH process[3], e.g. a GARCH(1,1) would be $E(v_{i,t}^2) = \sigma_{i,t}^2 = \omega_i + \alpha_i v_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2$. Provided that variances are positive through the GARCH parameter constraints, H_t is guaranteed to be positive definite for every t if $\rho_{12,t}$ is less than one in absolute value. By the definition of conditional correlation we have

$$E\left(\frac{v_{1,t}v_{2,t}}{\sigma_{1,t}\sigma_{2,t}}\middle| I_{t-1}\right)=\rho_{12,t} \qquad (3)$$

and the sequence of estimated innovations $\frac{v_{1,t-1}v_{2,t-1}}{\sigma_{1,t-1}\sigma_{2,t-1}},\frac{v_{1,t-2}v_{2,t-2}}{\sigma_{1,t-2}\sigma_{2,t-2}},\cdots$ will also be available in the information set I_{t-1} as it is generated by the joint modelling of the conditional means and variances. This forms a real-valued serially uncorrelated sequence and provides a minimal information set driving the evolution of correlation. To ensure that $|\rho_{12,t}|<1$ for all t, we adopt the *Fisher's-z* transformation[4] of the correlation coefficient

$$z(\rho_{12,t})=\frac{1}{2}\ln\left(\frac{1+\rho_{12,t}}{1-\rho_{12,t}}\right) \qquad (3a)$$

which is a one-to-one functio mapping (-1,1) onto the real line.

We now make $z_{12,t}$ evolve as a linear function of the available information, that is

$$z_{12,t}=a_0+\varphi_1\tilde{v}_{t-1}+\ldots+\varphi_q\tilde{v}_{t-q} \qquad (4)$$

where

$$\tilde{v}_t=\frac{v_{1,t}v_{2,t}}{\sigma_{1,t}\sigma_{2,t}}-\rho$$

and $\rho=E\left(\frac{v_{1,t}v_{2,t}}{\sigma_{1,t}\sigma_{2,t}}\right)$ is the first joint unconditional moment of a process as in (2). We define (1), (2) and (4) as a Correlated ARCH (CorrARCH) process of order q. The order of lag q determines the length of time for wich a shock persists in conditioning the correlation of subsequent return errors.

As q increases, the memory of shocks is longer and a very long lag structure of (4) will eventually call for a more parsimonious representation, as usual in time series analysis. Under the usual stability conditions, the process can be represented as

$$z_{12,t} = \lambda + \zeta_1 z_{t-1} + \ldots + \zeta_p z_{t-p} + \phi_1 \widetilde{v}_{t-1} + \ldots + \phi_q \widetilde{v}_{t-q} \qquad (5)$$

where $\lambda = a_0(1 - \zeta_1 - \ldots - \zeta_p)$. We define (1), (2) and (5) as a Correlated GARCH (CorGARCH) process of order (p,q). This specification will allow for longer memory and a more flexible lag structure. For p=0 the CorGARCH(p,q) process reduces to the CorrARCH(q) process (4), and for p=q=0 the model reduces to the constant conditional correlation model of Bollerslev (1990).For further technical details on correlation processes as well as their autocorrelation structure see CS (1998).

In the case that ξ_t is normally distributed and independent of y_t and any exogenous variables that may contribute to the information set, the conditional density will be bivariate Gaussian which in log form will be written as:

$$\ln\left(f\left(y_t | I_{t-1}\right)\right) = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln|H_t| - \frac{1}{2}\left(v_t' H_t^{-1} v_t\right)$$

where N=2 is the number of rows in y_t. For a sample of T observations the conditional log-likelihood function will be the sum of the conditionally normal log-probabilities

$$\ln L = -T\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln|H_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' H_t^{-1} v_t$$

Recalling that $H_t = C_t R_t C_t$ we can write more analytically

$$\begin{aligned}\ln L = & -T\ln(2\pi) - \sum_{t=1}^{T}\ln\left(\sigma_{1,t}^2\right) - \sum_{t=1}^{T}\ln\left(\sigma_{2,t}^2\right) - \frac{1}{2}\sum_{t=1}^{T}\ln\left(1-\rho\left(z_{12,t}\right)\right) \\ & -\frac{1}{2}\sum_{t=1}^{T}\begin{pmatrix} v_{1,t} \\ v_{2,t}\end{pmatrix}'\begin{pmatrix}\sigma_{1,t}^2 & \sigma_{1,t}\sigma_{2,t}\rho\left(z_{12,t}\right) \\ \sigma_{1,t}\sigma_{2,t}\rho\left(z_{12,t}\right) & \sigma_{2,t}^2\end{pmatrix}^{-1}\begin{pmatrix} v_{1,t} \\ v_{2,t}\end{pmatrix}\end{aligned} \qquad (6)$$

Now our log-likelihood function is expressed in terms of $v_{i,t}$'s which are functions of the mean parameters μ_i, of $\sigma_{i,t}^2$ which evolve as univariate GARCH processes of any type and of $z_{12,t}$ which evolves as a Correlated ARCH or GARCH process of any order. Our purpose is to estimate the values of the unknown parameters involved in the conditional mean, conditional variance and the conditional correlation equations that will give the globally maximum value of (6). We thus need to evaluate its first and second order derivatives with respect to the vector of the unknown parameters. For an explicit derivation of the score and the Hessian see CS (1998). As the log-likelihood function is highly non-linear and a closed-form solution of the first order conditions is not available, (6) can be numerically maximized through the

Newton-Raphson algorithm which we find more stable compared to others. For non-normal innovations, such as t-distributed, the log-likelihood in (6) is based on the conditional t distribution involving further unknown shape-related parameters to be estimated form the data.

EMPIRICAL RESULTS AND CONCLUSIONS

We present our empirical results in Table 1. For each pair of markets we first estimate a bivariate uncorrelated GARCH process. Then we relax the no-correlation assumption and allow initially for constant correlation and eventually for time varying correlations. Our model selection procedure is based on Likelihood Ratio statistics as well as Bayesian Information Criteria such as Akaike (AIC) and Schwartz (SIC). In selecting the particular models presented in our tables, we estimated several different specifications within each category and then performed model selection procedures as described above.

We found convergent estimates for the UK versus Belgium, Netherlands as well as the BeNeLux index. For Luxemburg versus UK we failed to obtain convergent results; there may be some institutional explanation for this since even simple models such as the uncorrelated bivariate GARCH could not be calculated. The t statistics for the estimated parameters in all cases suggests that all of them are statistically significant. An inspection of the log likelihood values in each table uncovers that, in all cases, relaxing the no-correlation assumption improves dramatically the log likelihood value which can be seen from the value of the likelihood ratio test statistics and the information criteria such as the Akaike and Schwartz. A similar picture is revealed when we relax the constant correlation assumption to allow for CorGARCH effects which further increases the log likelihood values. Thus, our results strongly suggest that a model for the joint distribution of the BeNeLux and UK stock markets is best described with a Correlated GARCH process.

Furthermore we see that there is persistence in common innovation shocks between the markets implying strong correlation time variation and predictability, see the CorGARCH block of Table 1 where ζ_1 takes values 0.94, 0.97 and 0.96 for the UK versus Belgium, Netherlands and the BeNeLux respectively. These high values of the autoregressive parameter - always greater than 0.9- suggest strong persistence of shocks on $z_{12,t}$. We can gain some intuition about the degree of persistence by looking the on average *half life* of a shock associated to ζ_1, that is the number of periods s such that $\zeta_1^s = \frac{1}{2}$. Table 2 presents these estimates from which we can see that a common shock may need on average to exhaust its half life from eleven to twenty three days.

Table 1. UK versus Belgium, Netherlands and BeNeLux.

Parameters	**No Correlation**			**Constant Correlation**			**CorGARCH**		
	UK-Bel	UK-Ne	UK-BNL	UK-Bel	UK-Ne	UK-BNL	UK-Bel	UK-Ne	UK-BNL
μ_1	0.05 (3.16)	0.05 (3,13)	0.05 (3.15)	0.05 (3.14)	0.05 (2.99)	0.05 (3.00)	0.05 (3.11)	0.05 (3,10)	0.05 (3.22)
μ_2	0.06 (3.89)	0.07 (4.58)	0.06 (4.57)	0.06 (3.72)	0.06 (4.50)	0.06 (4.23)	0.06 (3.66)	0.06 (4.41)	0.06 (4.56)
ω_1	0.02 (1.66)	0.02 (1.40)	0.02 (1.80)	0.03 (3.12)	0.04 (3.91)	0.03 (3.98)	0.03 (3.16)	0.03 (3.81)	0.03 (4.00)
α_1	0.06 (3.27)	0.06 (2.75)	0.06 (3.51)	0.07 (5.53)	0.08 (6.19)	0.08 (6.39)	0.07 (5.96)	0.08 (6.60)	0.08 (7.15)
β_1	0.91 (28.3)	0.91 (25.1)	0.91 (30.0)	0.90 (39.9)	0.86 (34.8)	0.87 (40.2)	0.89 (40.6)	0.87 (41.5)	0.87 (47.0)
ω_2	0.02 (3.05)	0.02 (2.62)	0.02 (3.20)	0.03 (3.27)	0.03 (4.17)	0.03 (4.23)	0.03 (3.38)	0.03 (4.10)	0.02 (4.31)
α_2	0.08 (5.63)	0.06 (4.60)	0.07 (5.73)	0.08 (5.36)	0.08 (6.17)	0.07 (6.37)	0.08 (6.12)	0.07 (6.60)	0.08 (7.22)
β_2	0.88 (38.7)	0.91 (41.9)	0.90 (48.8)	0.87 (31.6)	0.89 (47.0)	0.89 (45.2)	0.88 (39.5)	0.89 (53.3)	0.89 (54.1)
λ		-		0.56 (28.3)	0.77 (39.1)	0.76 (38.6)	0.02 (2.19)	0.01 (2.37)	0.02 (2.38)
ϕ_1		-			-		0.02 (4.05)	0.02 (3.65)	0.02 (4.04)
ζ_1		-			-		0.94 (40.2)	0.97 (98.6)	0.96 (67.9)
$\ln L$	3030.3	3092.5	3175.4	3412.9	3790.5	3857.8	3422.9	3806.0	3870.9
LR		-		765.1 [3.84]	1396.0 [3.84]	1364.8 [3.84]		-	
AIC	3022.3	3084.5	3164.4	3403.9	3781.5	3848.8	3411.96	3797.00	3859.9
SIC	2998.9	3061.1	3144.0	3377.6	3755.2	3822.5	3379.79	3762.83	3827.7

Notes: Bel, Ne and BNL stand for Belgium, Netherlands and BeNeLux respectively. AIC=ln L-k, SIC=ln L-0.5k ln T, t-statistics in brackets and chi square critical values in square brackets, T=2560 observations. Parameter notation follows section 3.

Table 2. Half-Life Estimates.

	UK-Belgium	UK-Netherlands	UK-BeNeLux
ζ_1	0.94	0.97	0.96
$s_d = -\dfrac{\ln 2}{\ln \zeta_1}$	11.2	22.8	17

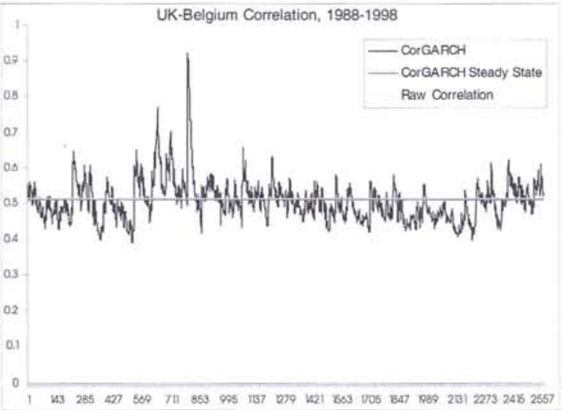
Table 3. Steady-State Correlations.

	UK-Belgium		UK-Netherlands		UK-BeNeLux	
	ρ	z	ρ	z	ρ	z
Raw Data	.5137	.5678	.6439	.7650	.6415	.7607
Constant Correlation	.5083	.5612	.6493	.7742	.6437	.7646
CorGARCH(1,1)	.5110	.5641	.6486	.7730	.6450	.7667

As a final check we calculate the "steady-state" value of ρ_t and *Fisher's-z*$_t$ as defined in equations (3) and (3a) respectively. These results are presented in Table 3 and show a pleasing consistency between the three methods i.e. the raw sample correlation, the bivariate constant-correlation GARCH and the steady-state correlation implied by our model.

To obtain some intuition of the difference of our results, the reader should inspect Figure 1 where we plot, as an example, the UK vs Belgium correlation as estimated by CorGARCH, Constant Correlation GARCH and raw correlation methods for the whole sample period. It is evident that our model demonstrates substantial fluctuations in the correlation, correlation clustering and shows how poor approximation constant correlation is.

Figure 1. UK-Belgium correlation (1988-1998).



Overall, we present empirical evidence for the correlation dynamics between the UK and BeNeLux stock markets, applying the Correlated ARCH model of Christodoulakis and Satchell (1998). Our results strongly support arguments for time-varying joint correlations and correlation clustering. The joint evolution of the markets is shown to be best described by the CorGARCH model implying an explicit autocorrelation structure for the correlation coefficient and predictability. Based on these results, we discuss important implications for risk management. We have not focussed in our application on risk management calculations, e.g. plotting some varying hedge ratios or values-at-risk. We hope to do this in future research; we also hope to apply evolutionary models to this problem to capture the non-stationary shift from a pre-Euro to post-Euro world.

NOTES

(1) We are grateful to participants of the "Risk management in Finance" session of the EURO XVI conference, Brussels July 1998 and to Prof. Constantin Zopounidis for useful comments.

(2) We use the daily percentage change of the price index as a measure of the daily return. For small changes, the difference of the natural logarithm of price could also be adopted.

(3) For more details on ARCH and GARCH processes see Engle (1982), Bollerslev (1986) as well as in excellent survey paper by Bera and Higgins (1993)

(4) The range of the Fisher's-z will be $(-\infty,+\infty)$. As a standard result, it will approach normality much more rapidly than the correlation coefficient, see Muirhead (1982).

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