EDVCATIO PHYSICORVM
QVO NON ASCENDAM?
ISSN 1870-9095

Euler–Bernoulli beam theory in the kitchen: The fridge-violin

Luis Darmendrail[1,2], Andreas Müller[1,2]

[1]*Faculty of Sciences/Physics Section, University of Geneva, Quai Ernest-Ansermet 30, 1211 Geneva, Switzerland.*

[2]*Example: Institute of Teacher Education, University of Geneva, Pavillon d'Uni Mail (IUFE), Boulevard du Pont d'Arve 40, 1211 Geneva, Switzerland.*

E-mail: Luis.Darmendrail@etu.unige.ch

(Recibido el 12 de diciembre de 2020, aceptado el 15 de febrero de 2021)

Abstract

An application of the Euler–Bernoulli beam theory (a classic in vibration and acoustics) to a curious sound phenomenon occurring in a daily life situation is presented. A smartphone is used as an experimental tool to measure and analyse the acoustic signal. The experimental data are compared to a prediction from the Euler-Bernoulli model, yielding satisfactory agreement. Limitations of the analysis are discussed. The low-cost nature and the investigation of a curious phenomenon make the example a good candidate *e.g.*, for undergraduate lab-work courses or research projects, which can spark students interest in acoustics, and more generally develop their interest and skill in applying physics to phenomena of everyday life**.**

Keywords: acoustics, smartphones, education.

Resumen

Se presenta una aplicación de la teoría del haz de Euler-Bernoulli (un clásico en vibración y acústica) a un curioso fenómeno sonoro que ocurre en una situación de la vida diaria. Se utiliza un teléfono inteligente como herramienta experimental para medir y analizar la señal acústica. Los datos experimentales se comparan con una predicción del modelo de Euler-Bernoulli, obteniendo un acuerdo satisfactorio. Se discuten las limitaciones del análisis. La naturaleza de bajo costo y la investigación de un fenómeno curioso hacen que el ejemplo sea un buen candidato, p. para cursos de trabajo de laboratorio de pregrado o proyectos de investigación, que pueden despertar el interés de los estudiantes en la acústica y, en general, desarrollar su interés y habilidad en la aplicación de la física a los fenómenos de la vida cotidiana.

Palabras clave: acústica, teléfonos inteligentes, educación.

I. THE PHENOMENON

While doing homework one of the authors recently encountered a puzzling phenomenon: When rubbing – quite strongly – the ventilation grill of a fridge with a slightly humid cleaning sponge (a bit like a bow and violin, see Fig. 1) a strange, high frequency and well audible sound was emitted (see link for a video file in the web material). Where does it come from? In the following, we present measurements of this acoustic phenomenon by a smartphone and a simplified, but satisfactorily accurate model. The contribution shows another example of the potential of mobile communication devices (smartphones, tablets) as experimental tools for physics education [1, 2, 3, 4, 5, 6].

II. ACUSTICAL MEASUREMENT

The sound produced as described above was recorded using the built-in microphone of an iPhone 7 plus. The acoustic measurements from the microphone are directly transformed to a spectrum (Fig. 2) by the application SpectrumView [7] using a sample rate of 48 kHz with a 13th order Fast Fourier Transform and Hamming windows function. The frequency spectrum data can be analysed directly in the smartphone application and/or transfer as a csv file to a computer for further analysis.

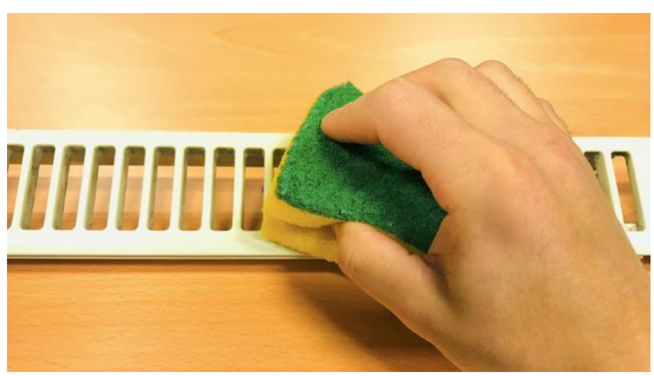

FIGURE 1. The experiment showing how the sound from the fridge grill when rubbing the transverse elements (“bridges”) with a slightly humid sponge.

The recorded spectrum for two excitation conditions with the sponge is shown in Fig. 2. A pronounced maximum at a frequency of

$$f_e = 6385\ Hz, \tag{1}$$

occurred (red curve), compatible also with the frequencies of the first and second harmonics (see Tab. 1). In order to check for consistency, another measurement was carried out (Fig. 2, case b, yellow curve) with the sponge rubbed against only one "bridge" element. The audible the sound and measured fundamental frequency are close to those of the main experiment. The measurement in Fig. 2 yields a frequency of 6217 Hz, which is about 2.6% below the value of the first measurement. Regarding the two methods of measurement shown in the Fig. 2, it is clear that the first alternative of rubbing the sponge on the ventilation grill as a whole (the way the phenomenon was found) is best to obtain a clear signal and data in a home environment. For the second method rubbing an individual bridge element is manually more difficult due to its smallness, and a less loud and sustained sound is obtained.

TABLE I. Measured frequencies from the "fridge violin".

mode number n	frequency f (Hz)	f/n (Hz)
1 (fundamental)	6385	6385
2 (1st harmonic)	12790	6395
3 (2nd harmonic)	19163	6387

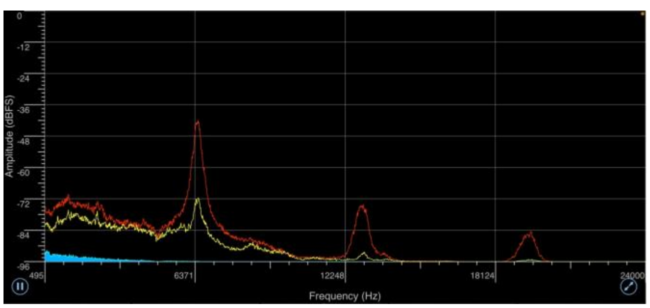

FIGURE 2. Frequency spectra of the sponge experiment for different excitation conditions: a) when rubbing the sponge against the ventilation grill (red), b) against one beam/bridge element (yellow).

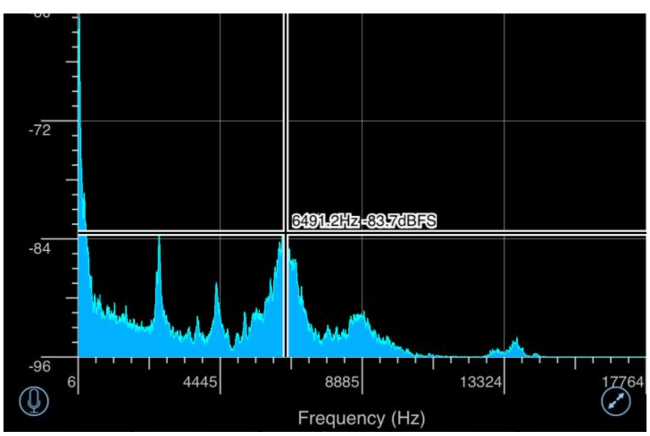

FIGURE 3. Frequency spectrum of the fingertip experiment.

Additionally, we carried out the experiment rubbing a fingertip instead of the sponge against a bridge element. The sound is still produced, but less loud, and when rubbing much more gently than in the sponge experiment. The frequency measurement (Fig. 3) reveals the following main resonance frequencies: $f_{e1}^* = 2540\ Hz$, $f_{e2}^* = 4340\ Hz$, $f_{e3}^* = 6491\ Hz$, $f_{e4}^* = 8800\ Hz$, $f_{e5}^* = 13670 Hz$. Note that f_{e3}^* is quite close to f_e (deviation < 2%), and that $f_{e4}^*/2$ and $f_{e5}^*/3$ are very close to f_{e2}^*, (deviation < 4%) analogously to the occurrence of harmonics in the excitation with the sponge (see above).

III. THEORY

A. Vibration theory in beams (Euler-Bernoulli Model)

From a physical point of view the sound produced in this experiment is caused by structural vibrations of the grill [8] [9]. In this case, the rubbing motion of the sponge works as an external exciting force, a bit like a bow on a violin string, and such an external force can excite some specific modes of structural vibration with its corresponding resonance frequency. Specifically, we propose that each bridge (transverse element) of the grill has vibration modes in the audible spectrum (20Hz to 20kHz) which are the source of the "fridge violin" sound. Euler-Bernoulli beam theory [10] is used to calculate the vibration frequencies of the bridges in order to compare the theoretical results with those obtained experimentally with the smartphone.

In the following, a derivation of the resonance frequencies of a beam according to the Euler-Bernoulli theory [10] in seven steps is provided.

1. The differential equation for a beam is given by

$$\frac{\partial^4 \Psi}{\partial x^4} + \frac{\rho A}{YI} \cdot \frac{\partial^2 \Psi}{\partial x^2} = 0, \tag{2}$$

where Ψ is the transverse displacement of a point x of the beam at time t, where ρ (density) and Y (Young's modulus) the material constants of the beam, A (cross section) and I (area moment of inertia of cross section) are its geometrical parameters; L is the length, to be used below.

2. With a factorisation ansatz:

$$\Psi(x,t) = y(x) \cdot \sin(\omega t),$$

the in the differential equation becomes

$$\Rightarrow \frac{d^4 y}{dx^4} + \frac{\rho A}{YI} \cdot \omega^2 y = 0,$$

3. It has the characteristic equation

$$r^4 - q^4 = 0,$$

with

$$q = \left(\frac{\rho A}{YI}\cdot \omega^2\right)^{1/4}. \qquad (3)$$

There are two real roots and two imaginary roots, leading to a solution of the form:

$$y(x) = C_1 e^{qx} + C_2 e^{-qx} + C_3 e^{iqx} + C_4 e^{-iqx},$$

or equivalently

$$y(x) = A_1 \sinh(qx) + A_2 \cosh(qx) + A_3\, sin(qx) + A_4\, cos(qx)$$

The slope is:

$$\frac{dy}{dx} = q(A_1 \cosh(qx) + A_2 \sinh(qx) + A_3 \cos(qx) - A_4 \sin(qx)).$$

4. Now, the boundary conditions for the clamped-clamped situation are applied

$$x = 0,\ \frac{dy(0)}{dx} = 0 \ \Rightarrow \begin{cases} 0 = A_2 + A_4 \\ 0 = A_1 + A_3 \end{cases},$$

$$x = L,\ \frac{dy(L)}{dx} = 0\,, \Rightarrow \begin{cases} 0 = A_1(\sinh(qL) - \sin(qL)) + A_2(\cosh(qL) - \cos(qL)), \\ 0 = A_1(\cosh(qL) - \cos(qL)) + A_2(\sinh(qL) + \sin(qL)). \end{cases}$$

5. Elimination of A_1 and A_2 allows to obtain an equation for qL:

$$\begin{cases} A_1 = -A_2 \dfrac{\cosh(qL) - \cos(qL)}{\sinh(qL) - \sin(qL)}, \\ A_1 = -A_2 \dfrac{\sinh(qL) + \sin(qL)}{\cosh(qL) - \cos(qL)}, \end{cases}$$

$$\Rightarrow \frac{\cosh(qL) - \cos(qL)}{\sinh(qL) - \sin(qL)} = \frac{\sinh(qL) + \sin(qL)}{\cosh(qL) - \cos(qL)},$$

$$\Rightarrow (\sinh(qL) - \sin(qL))\cdot(\sinh(qL) + \sin(qL)) - (\cosh(qL) - \cos(qL))^2 = 0.$$

6. Then we set $q_n L = r_n$ and arrive the following "roots" of the above equation:

$q_n L = r_n$ with r_n = 4.73, 7.85, 11.00, 14.14, 17.27 ... obtained numerically.

7. Inserting the values of the "roots" q_n in eq. (3), the vibration frequencies $\omega_n = 2\pi f_n$ are obtained as

$$f_n = \frac{r_n^2}{2\pi}\sqrt{\frac{Y\cdot I}{\rho\cdot A\cdot L^4}} = C_n\sqrt{\frac{Y\cdot I}{\rho\cdot A\cdot L^4}}, \qquad (4)$$

with $C_n = r_n^2/2\pi$ = 3.56, 9.82, 19.2, 31.8, 47.5 …. for short.

B. Frequency calculation for the case of the fridge grill

Each bridge element is considered as a U-shaped beam clamped at both ends (Fig. 4). The parameters necessary for eq. (1) are as follows:

The geometric parameters are given in eq. (4). The linear dimensions are obtained by measurement with a caliper. The cross-section $A_\sqcap$ of the U-shaped beam profile (see Fig. 4) is calculated as $A_\sqcap = A_e - A_i$, where A_e and A_v are the areas of the exterior (4x8 mm^2) and the void (2x6 mm^2) rectangular sections of Fig. 4. The area moments if inertia of the U-shaped beam profile are $I_{\sqcap l}$ and $I_{\sqcap s}$ with respect to the long (b) and short side (a) of the cross-section, respectively.

geometric parameters
$$L = 39.5\ \text{mm},\quad A_\sqcap = 20\ \text{mm}^2,\quad I_{\sqcap l} = 1.155\cdot 10^{-10},\ I_{\sqcap s} = 3.87\cdot 10^{-11}\ \text{m}^4. \qquad (5)$$

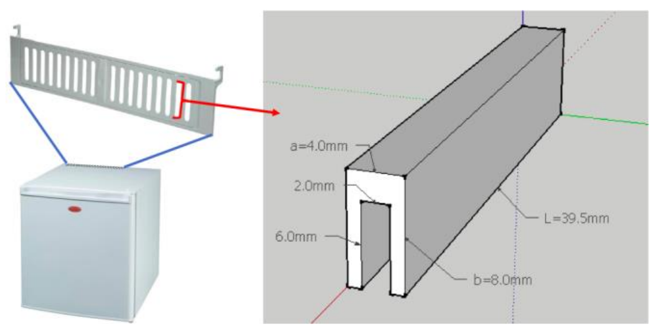

FIGURE 4. U-shaped beam profile of the bridge elements of the fridge grill.

The material constants are considered for Polypropylene (PP, a typical material for elements like ventilation grills). For its density and elastic modulus, a range of values is found in the literature, see Shackelford & Doremus (2008) [11] and eq. (6).

material constants
Y = 1325 MPa, ρ = 900 kg/m^3, (data set 1, [12])
Y =1500 MPa, ρ = 941 kg/m^3 (data set 2, [13]). (6)

Inserting these values in eq. 4 yields the fundamental frequencies (n = 1) for the two vibration directions and the 1st and 2nd data set, respectively,

a) f_{1l} = 6652 Hz (1st data set [12]),
f_{1l} = 6921 Hz (2nd data set [13]), (7)
b) f_{1s} = 3851 (1st data set [12]);

f_{1s} = 4007 Hz (2nd data set [13]).

IV. DISCUSSION

For the original experiment with the sponge a frequency of 6385 *Hz* is found which agrees quite well with the calculated frequencies f_{1l} of the Euler-Bernoulli model according to eq. (7) a) (relative differences of 4% for the first and 8% for the second data set, respectively). For the fingertip experiment, frequency f_{e3}^* = 6491 *Hz* also agrees well with this prediction (relative differences 2.5% and 7% for the two datasets). Moreover, the fingertip experiment yields another resonance at f_{e2}^* = 4340 *Hz*, which agrees reasonably well with the calculated frequencies f_{1s} for the other mode according to eq. (7) b) (relative differences for the first/second data are 11%/8%. Thus, the frequencies of both lowest vibration modes of the bridge elements according to the Euler-Bernoulli-theory are indeed present in the experimental spectra (depending on the excitation procedure); the accuracy of the predictions (some %) appears as acceptable for the simplified approach proposed here. The presence of different modes and harmonics is also quite common in structural analysis, however requires a more detailed understanding of the excitation mechanism (see below) and how it influences the vibratory behaviour of the structure [14].

The answer to the main question – where does the sound effect when cleaning the fridge grill come from, see sect. **¡Error! Marcador no definido.** – thus can be answered: from an oscillation of the bridge elements, as described by the Euler-Bernoulli-theory of beam oscillations.

It is a limitation of the present treatment that it does not allow to analyse the excitation process, *i.e.*, which mode is excited with which intensity by a given excitation mechanism, and whether harmonics of a mode are excited.

For instance, it is not implausible that the vibration perpendicular to the grill is excited more strongly in the sponge experiment, when pressure on the elements is much larger than the rather gentle soft touch in the fingertip experiment, and also that it is less damped by the friction with the sponge than the vibration parallel to the grill: however, more advanced methods are necessary to provide evidence about the precise influence of the excitation mechanism [15].

Still, while acknowledging this limitation, one can say that the treatment presented here fulfils its educational purpose, *i.e.* to explain a curious acoustical phenomenon met in everyday life with experimental and theoretical means on the undergraduate level. Some further perspectives are discussed in the next section

V. CONCLUSIONS AND PERSPECTIVES

In this experiment we have shown a classic example of acoustics [8], viz. vibrations of beams lit to a sound signal occurring in a daily life situation. Data are obtained with the help of a common smartphone and an application capable of frequency analysis showing a clear resonance frequency at 6385 Hz. The Euler-Bernoulli model provides a specific prediction for the beam resonance frequencies. For typical values of the material constants, the calculated values are quite close to the empirical value (relative deviation a few %).

In a more general perspective, we present an exploratory experiment trying to broaden the scope from the laboratory to physics in everyday life, and thus to provide an enrichment to the traditional role of experiments in physics education.

The contribution is intended for undergraduate students (perhaps also student projects at supper secondary level), and lecturers with an interest in connections of physics to everyday life. Within that scope, the contribution shows another example of the potential of mobile communication devices (smartphones, tablets) as experimental tools, here for analysis of oscillations and sound phenomena (for related topics, see cracking knuckles [16], tunnel pressure waves [17], elevator oscillations [18], or use as hydrophones [19].

For further exploration by the interested reader, similar work in this and other journals for many areas of physics education has appeared in the last years, from other work on oscillations and waves [3, 20, 21], through electromagnetism [22, 23, 24] and optics [25] to explorations of radioactivity [26]. Moreover, Darmendrail *et al*. (2019) [27] discuss links and integrated approaches between science and technology education based on smartphones as experimental devices, and several journals run series about the topic since several years [1, 28].

REFERENCES

Url for an example of an audio and video record of the phenomenon, and its spectral analysis: https://www.youtube.com/watch?v=G-40zrynPaE

[1] Kuhn, J. and Vogt, P., *iPhysicsLabs (Series)*, Column Editors' Note, The Physics Teacher **50**, 372 (2012).

[2] Vieyra, R., Vieyra, C., Jeanjacquot, P., Marti, A. and Monteiro, M., *Turn your smartphone into a science laboratory*, The science teacher **82**, 32 (2015).

[3] González, M. Á. and González, M. Á., *Smartphones as experimental tools to measure acoustical and mechanical properties of vibrating rods*, European Journal of Physics **37,** 045701 (2016).

[4] Gil, S. and Di Laccio, J., *Smartphone una herramienta de laboratorio y aprendizaje: laboratorios de bajo costo para el aprendizaje de las ciencias*, Latin-American Journal of Physics Education **11**, 5 (2017).

[5] Staacks, S., Hütz, S., Heinke, H. and Stampfer, C. *Advanced tools for smartphone-based experiments: phyphox*, Physics Education **53**, 045009 (2018).

[6] Montealegre, J. S. C., Romero, D. D. P. J. and Muñoz, J. H., *App's como herramientas pedagógicas para el proceso de Enseñanza-Aprendizaje de la Física*, Revista Científica, 160-168 (2019).

[7] Oxford Wave Research, SpectrumView – Frequency Analysis Software, Retrieved from https://www.oxfordwaveresearch.com/products/spectrumviewapp

[8] Fletcher, N. H. and Rossing, T. D., *The Physics of Musical Instruments*, (Springer, New York, 1998).

[9] Gerges, S. N. Y. and Arenas, J. P., *Fundamentos y control del ruido y vibraciones*, (NR, Florianópolis, Brasil, 2004).

[10] Han, S. M., Benaroya, H. and Wei, T., *Dynamics of transversely vibrating beams using four engineering theories*, Journal of Sound and vibration **225**, 935-988 (1999).

[11] Shackelford, J. F. and Doremus, R. H., *Ceramic and glass materials*, (Structure, Properties and Processing, Springer, 2008).

[12] Designer Data, *Polypropylene (PP)*, Retrieved from https://www.designerdata.nl/plastics/thermo+plastics/PP (visited on September 31, 2020)

[13] The Engineering Toolbox, *Young's Modulus – Tensile and Yield Strength for common Materials*. Retrieved from https://www.engineeringtoolbox.com/young-modulus-d_417.html (visited on September 31, 2020)

[14] Modak, S. V., *Separation of structural modes and harmonic frequencies in Operational Modal Analysis using random decrement*, Mechanical Systems and Signal Processing **41**, 366-379 (2013).

[15] Brincker, R. and Ventura, C. Introduction to operational modal analysis, (Wiley, Chichester (UK), 2015).

[16] Müller, A., Vogt, P., Kuhn, J. and Müller, M., *Cracking knuckles—A smartphone inquiry on bioacoustics*, The Physics Teacher **53**, 307-308 (2015).

[17] Müller, A., Hirth, M. and Kuhn, J., *Tunnel pressure waves–A smartphone inquiry on rail travel*, The Physics Teacher **54**, 118-119 (2016).

[18] Kuhn, J., Vogt, P. and Müller, A., Analyzing elevator oscillation with the smartphone acceleration sensors, The Physics Teacher **52**, 55-56 (2014).

[19] Monteiro, M. and Martí, A. C., *Using smartphones as hydrophones: two experiments in underwater acoustics*, Physics Education **55**, 033013 (2020).

[20] Osorio, M., Pereyra, C. J., Gau, D. L. and Laguarda, A., *Measuring and characterizing beat phenomena with a smartphone*, European Journal of Physics **39**, 025708 (2018).

[21] López, V. J. and Arias, V., *Física y aplicaciones móviles en la escuela: un estado del arte enfocado en la enseñanza de movimientos oscilatorios*, Latin-American Journal of Physics Education **13**, 8 (2019).

[22] Arribas, E., Escobar, I., Suarez, C. P., Najera, A. and Beléndez, A., *Measurement of the magnetic field of small magnets with a smartphone: a very economical laboratory practice for introductory physics courses*, European Journal of Physics **36**, 065002 (2015).

[23] Monteiro, M., Stari, C., Cabeza, C. and Marti, A. C., *Magnetic field 'flyby'measurement using a smartphone's magnetometer and accelerometer simultaneously*, The Physics Teacher **55**, 580-581 (2017).

[24] Monteiro, M., Organotin, G. and Marti, A. C., *Magnetic fields produced by electric railways*, arXiv preprint arXiv:2006.06481 (2020).

[25] Sans, J. A., Gea-Pinal, J., Gimenez, M. H., Esteve, A. R., Solbes, J. and Monsoriu, J. A., *Determining the efficiency of optical sources using a smartphone's ambient light sensor*, European Journal of Physics **38**, 025301 (2016).

[26] Keller, O., Benoit, M., Müller, A. and Schmeling, S., *Smartphone and Tablet-Based Sensing of Environmental Radioactivity: Mobile Low-Cost Measurements for Monitoring*, Citizen Science, and Educational Purposes, sensors **19**, 4264 (2019).

[27] Darmendrail, L., Keller, O. and Müller, A., *Data, Data Everywhere, and Quite a Bit (e) to Learn*. In Zur Bedeutung der Technischen Bildung in Fächerverbünden., (Springer Spektrum, Wiesbaden 29 – 42, (2019).

[28] Kuhn, J., Wilhelm, T. and Lück, S., *Physik mit Smartphones und Tablet-PCs*, Physik in unserer Zeit **44**, 44-45 (2013).