Nonlinear Analysis of Cable Net Structures

Análisis No-lineal de estructuras compuestas por cables

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Resumen

Objective: Present a methodology implemented in Mathcad based on force densities for analytical form finding of three-dimensional cable net structures subjected to static loading. The applied methodology should present a solution algorithm that is stable and of fast convergence. **Methodology:** Theoretical background of cable net structures and the Force Density Method (FDM) are first introduced. Then, a series of Mathcad funtions are created to perform the nonlinear analysis of cable net structures. Then, a well-documented cable net structure example found in the literature is used to validate the results provided by the created Mathcad script. **Results:** The results show that the methodology implemented in Mathcad can accurately estimate the free node displacements and cable forces of 3D cable net structures subjected to static loading. **Conclusions:** This paper demonstrates that the proposed methodology produces the same results obtained in the reference paper and the Cabnet software. The stability of the Mathcad script is controlled and no numerical issues were found in the analyzed examples. The Mathcad script provided a clear way to follow the procedure, methods, and critical data obtained in the geometric analysis of cable net structures helping to overcome the inherit limitations of commercial packages whose focus solely on outputs generated in response to a set of input information, without any information about its internal calculations and procedures.

Keywords: Cable net structures, force density method, Mathcad.

Abstract

Objetivo: VPresentar una metodología basada en basada en el Método de Densidad de Fuerza (MDF) para realizar el análisis no-lineal de estructuras tridimensionales conformadas por cables sometidas a cargas estáticas, e implementada en el software Mathcad. Dicha metodología no debe presentar problemas en la obtención de la solución numérica y por tanto debe obtener una convergencia numérica eficiente y estable. Metodología: Primero se presenta el marco teórico referente a las estructuras conformadas por cables y al método de análisis basado en el MDF. Seguidamente, el código programado en Mathcad es creado para realizar el análisis no-lineal de estructuras compuestas por cables. Finalmente, los resultados obtenidos por las rutinas son validados a través de su comparación con los resultados presentados en el artículo de investigación tomado como referencia y los resultados provistos por un software de estructuras conformadas por cables. Resultados: Se demostró que el código creado provee resultados adecuados y confiables del análisis no-lineal de estructuras compuestas por cables sometidos a cargas estáticas. Conclusiones: El código implementado en Mathcad provee resultados consistentes con los hallados en el artículo tomado como referencia. En los ejemplos analizados en este artículo de investigación, no se encontraron problemas numéricos que pudieran generar inestabilidad. El código desarrollado provee una forma sencilla de visualizar las consideraciones, procedimiento y resultados obtenidos en el proceso de realizar análisis que involucre no-linealidad geométrica de estructuras compuestas por cables.

Palabras clave: DMathcad, sistema de cables, método de densidad de fuerza.

Introduction

Cable net structures are commonly encountered in facade systems, buildings lobbies, stadiums, and other structures. These type of structures are comprised by tension only hinge-connected members. When comparing with traditional structures like buildings and bridges, cable net structures provide a major advantage in terms of light weight, high ultimate strength, appealing architectural shapes, and extraordinary versatility, which contributes an efficient utilization of structural materials, economy, and aesthetics appearance.

The analysis of cable structures represents a significant challenge because of factors such as: (i) the definition of the analytical model to find the initial shape of the cable net structure and the cable forces associated to that state, (ii) in cable net analysis is rather common to find divergence in the calculations as a result of the selected iterative scheme, (iii) cable net structures are generally more flexible than conventional structural systems, but the system can exhibit a significant increase of rigidity when cable pretension is applied, (iv) the relation between a change in the loading and the corresponding displacement is nonlinear, (v) cable structures cannot transfer external loads through shear, axial compression or bending moment, and (vi) the possibility of wind-induced oscillations to affect cable net structures. Analytical approaches dealing with cable structures are mainly classified into two categories; the first one includes the models where truss or catenary elements are used to represent each cable, and the refined formulations based on finite element procedures, which includes a variational formulation in curvilinear coordinates with finite deformations [1]. Finite element methods in conjunction with the co-rotational formulation are employed to account for large displacements. Commonly, the displacement is decomposed into a rigid body motion in the global coordinate system and a pure small deformation in the local coordinate system [2].

Before 1970's, the only way to find the deformed shape of cable structures under external loads was to use photogrammetric tool. The importance of accurately determining the final deformed shape of cable net structures has to do not only with the structural, aesthetical and economical point of view, but in some cases with the operating requirements. An example is the case of deployable space antennas, where a key aspect for the operating requirements of a reflector in orbit is the finalshape [3,4,5]. Nevertheless, since 1971, the Force Density Method (FDM) provided an analytical way to find the values of deformation and forces in cable structures [6]; later on, the FDM was expanded by Sheck in 1974 [7]. The FDM converts a system of non-linear equations into a set of linear equations that can be solved directly using Gaussian elimination, conjugate gradient method, Cholesky factorization, or any other mathematical methodology. The required input data for solving a cable structure in light of the FDM are the boundary conditions, the nodal coordinates describing its topology, the information about the element connectivity, material's modulus of elasticity, cable cross-section, and the so-called force density of each cable, which is defined as the ratio of the tension force to the respective length of each cable; the force density is assumed to be constant for each cable. The FDM involves the use of an iterative approach to accomplish equilibrium and compatibility.

Finally, the purpose of the paper is to present a FDM methodology implemented in Mathcad. Programming has been recognized as an important strategy when it comes to educational purposes. It is necessary for the successful realization of teachers and students activity, likewise it helps to develop the professional competences [8]. Despite the fact that there are several computational programs for cable net structures, most of them are proprietary "black boxes" with no adequate information with regard the limitations, simplifications and methods of analysis [9]. With this regard, a method based on force densities is implemented in Mathcad to perform simulations of multi-degree-of-freedom 3D cable net structures

subjected to static loads. This software was chosen after thoroughly reviewing the capabilities of a series of programming environments commonly used for academic and professional purposes such as Mathematica, Matlab, and Microsoft Excel (Visual Basic for Applications). Mathcad is a widely used software for the structural engineering community (i.e. students, professionals and structural engineering firms), it is easy to learn, and its interface is highly visual which constitutes a convenient tool to be applied to solve cable structure systems. The convenience of using Mathcad for creating user-applications lies in the fact that one can focus on the fundamental concepts rather than on the mathematical solution [10].

Methodology

The goal of this paper is to implement Mathcad as a tool to develop a script able to perform the nonlinear analysis of 3D cable structures based on the FDM.. While many software packages for structural analysis have addressed this topic in last decades, most of them are proprietary "black boxes" with no comprehensive information applied to these particular structures [9]. To overcome this limitation, a customized version of the FDM is first introduced.

Then, a two-step process is applied in the creation of the Mathcad script. First, the nonlinear analysis of cable structures is decomposed into smaller well-defined Mathcad programming functions, and in a second step, the combination of these programing functions comprises the way to perform the structural analysis of cable structures.

Finally, a well-documented cable net structure example found in the literature is used to validate the output results provided by the Mathcad script.

Force Density Method (FDM)

The FDM was originally proposed by Linkwitz and Schek [6]. This approach is considered as one of the most powerful techniques to analyze cable net structures [11]. The key feature of this method is that the nonlinear equilibrium equations for unknown locations of the nodes are transformed to a set of linear equations.. This system is constructed using the force-to-length ratios of each cable, also called "force densities", as the network description parameters.

The assumptions are:

- Members are connected by rotationally unrestrained joints.
- Connectivity between the nodes and members of the structure is known.
- The geometrical configuration of the structure can be described in terms of nodal coordinates only.
- Self-weight is neglected.

In the last decades, the FDM has been widely used in the analysis and design of a variety of structures where form-finding is involved.   The FDM has been employed to find the deformed configuration of tensegrity structures (i.e. self-equilibrated structure comprised by pre-stressed cable and struts members connected by pins or hinges) subjected to external loads [12,13,14,15]. In cable membrane structures, the FDM has been used as a shape-finding tool [16]. In addition, the equilibrium of funicular networks can be conveniently linearized through the FDM [17,18,19]. In beam shell structures, nonlinear finite element

procedures in conjunction with the FDM have been successfully utilized to perform the structural analysis of these structures [20]. The above facts indicate the power and versatility of the FDM.

In the next sections, the fundamentals of the FDM are presented. Before continuing, it should be indicated the nomenclature used in this paper. The subscript "s" refers to restrained nodes (i.e. nodes which cannot have any displacement), whereas the subscript "f" refers to nodes that are not constrained (i.e. free nodes). The system of linear equations that defines the force density method is the following:

$$[D][C] = \{F\} \tag{1}$$

where [D] is the force density matrix, [C] is the coordinates of the system, and {F} is the applied force vector. Arranging this into free and restrained nodes the coordinates of the free ones may be written using partitioned matrices as indicated in equation (2). A multiplication of the partitioned system in equation (2) yields to equation (3) and (4). Solving equation (3) for the vector of free nodal coordinates C_f, results in equation (5).

$$\begin{bmatrix} D_{ff} & D_{fs} \\ D_{sf} & D_{ss} \end{bmatrix} \begin{Bmatrix} C_f \\ C_s \end{Bmatrix} = \begin{Bmatrix} F_f \\ F_s \end{Bmatrix} \tag{2}$$

$$[D_{ff}][C_f] + [D_{fs}][C_s] = \{F_f\} \tag{3}$$

$$[D_{sf}][C_f] + [D_{ss}][C_s] = \{F_s\} \tag{4}$$

$$\{C_f\} = [D_{ff}]^{-1}(\{F_f\} - [D_{fs}][C_s]) \tag{5}$$

Before getting into a description of the Mathcad script developed to perform the nonlinear analysis of cable-net structures, it is useful to give a broad overview on how a cable element structurally works. In essence, when a cable is subjected to tension, its lateral stiffness is increased. The geometric stiffness is a function of the applied pretension load and the cable length, but since the cable force is unknown prior to the analysis, the geometric stiffness must be determined through an iterative procedure; the elastic stiffness is based on the undeformed configuration thus it does not change during the analysis. In order to obtain the exact solution in a cable net structure, it is necessary to use the geometric stiffness in the calculation. Omitting either the elastic or the geometric stiffness can lead to erroneous results. In a cable net structure, a correct solution must satisfy equilibrium and compatibility. In fact, it is important to highlight the implications of the change in geometry in the structural analysis. First, it is required to find the cable net shape under pretensioning forces only, also referred as "initial shape", subsequently, the so called "final shape" is determined as external loads are applied. For these structures, geometric nonlinearities arise because the external loads are balanced by large changes of shape but small strains [21]. The importance of the geometric stiffness will decrease as long the cable is gaining drape, but for small drapes, geometric stiffness will significantly contribute to the transverse stiffness of the system since the geometric stiffness depends on the level of pretensioning forces.

The implementation of the FDM is based on the flowchart presented in Figure 1. Firstly, the input information pertaining to the topology of the cable structure, material properties, cables cross-section, pretension and external forces is required.

Secondly, since the FDM involves the use of an iterative approach in order to accomplish equilibrium and compatibility (i.e. by means of iteration process), by nature, the computational model requires that the

numerical solution accomplishes the prescribed convergence tolerance "Tol", defined for this study as 0.0001. The tolerance can be treated as a user-defined input to meet the user needs. In essence, when the difference between the two most recent force iterations is less than the specified tolerance, the Mathcad functions arrive at a solution.

Thirdly, the force density matrix and its sub-assemblages are created, as indicated in the left hand-side in equation (3). Also the structural system is assembled. The cable system is idealized by linear elements connecting nodes in the system, and the behavior of the material is assumed linearly-elastic, homogeneous and isotropic. The cross-sectional areas are assumed to be constant, and external forces are applied only on nodal points. For the development of equilibrium equations that satisfy equilibrium and compatibility, the tangent stiffness matrix is used. It consists of the sum of the elastic stiffness, [Ke], and the geometric stiffness, [Kg], as indicated in equations (6-8). The matrices [Ke] and [Kg] are transformed from the element stiffness matrix based on the element's local coordinate system to the global coordinate system before combining in order to create the structural stiffness matrix. In equations (6-8), "E" is the modulus of elasticity, "A" is cross sectional area, "L" is the cable length, "f" is the cable force, and "lx", "mx", "nx" are direction cosine values of the angles between local "x" axes and global "X", "Y" and "Z" axis.

Finally, if a solution is found within the specified tolerance limits, the updated free-node coordinates and the change of pretension are the output variables.

Figure 1. Flowchart for implementation of the FDM Mathcad
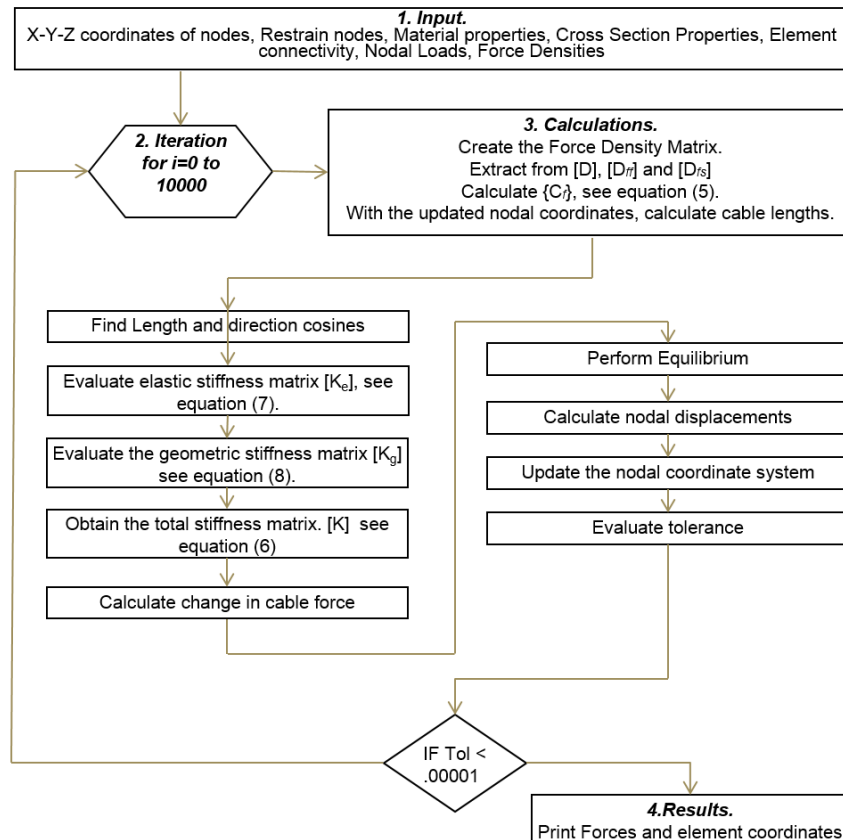
Source: Own elaboration
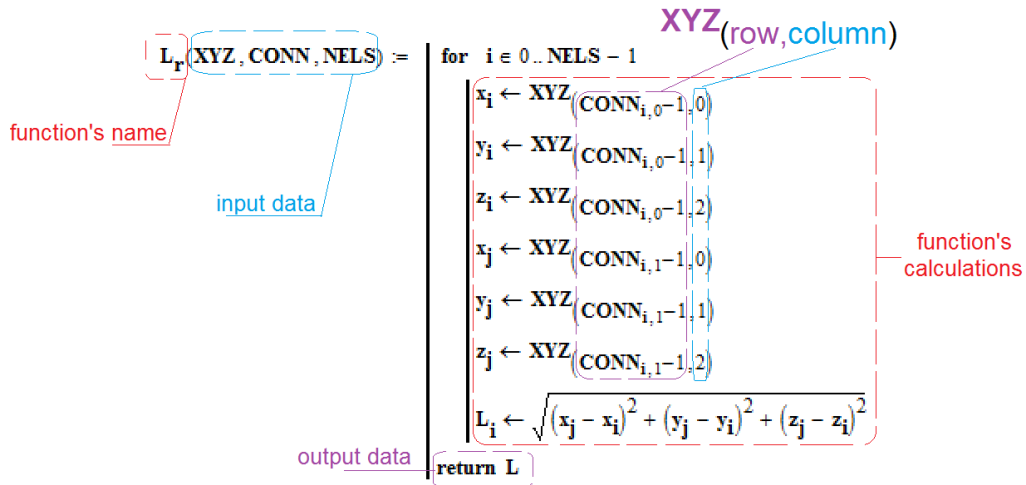
$$[K] = [K_e] + [K_g] \tag{6}$$

$$[K_e] = \frac{EA}{L} \begin{bmatrix} l_x^2 & l_x m_x & l_x n_x & -l_x^2 & -l_x m_x & -l_x n_x \\ l_x m_x & m_x^2 & m_x n_x & -l_x m_x & -m_x^2 & -m_x n_x \\ l_x n_x & m_x n_x & n_x^2 & -l_x n_x & -m_x n_x & -n_x^2 \\ -l_x^2 & -l_x m_x & -l_x n_x & l_x^2 & l_x m_x & l_x n_x \\ -l_x m_x & -m_x^2 & -m_x n_x & l_x m_x & m_x^2 & m_x n_x \\ -l_x n_x & -m_x n_x & -n_x^2 & l_x n_x & m_x n_x & n_x^2 \end{bmatrix} \tag{7}$$

$$[K_g] = \frac{f}{L} \begin{bmatrix} m_x^2 + n_x^2 & -l_x m_x & -l_x n_x & -(m_x^2 + n_x^2) & l_x m_x & l_x n_x \\ -l_x m_x & l_x^2 + n_x^2 & -m_x n_x & l_x m_x & -(l_x^2 + n_x^2) & m_x n_x \\ -l_x n_x & -m_x n_x & l_x^2 + m_x^2 & l_x n_x & m_x n_x & -(l_x^2 + m_x^2) \\ -(m_x^2 + n_x^2) & l_x m_x & l_x n_x & m_x^2 + n_x^2 & l_x m_x & l_x n_x \\ l_x m_x & -(l_x^2 + n_x^2) & m_x n_x & l_x m_x & l_x^2 + n_x^2 & m_x n_x \\ l_x n_x & m_x n_x & -(l_x^2 + m_x^2) & l_x n_x & m_x n_x & l_x^2 + m_x^2 \end{bmatrix} \tag{8}$$
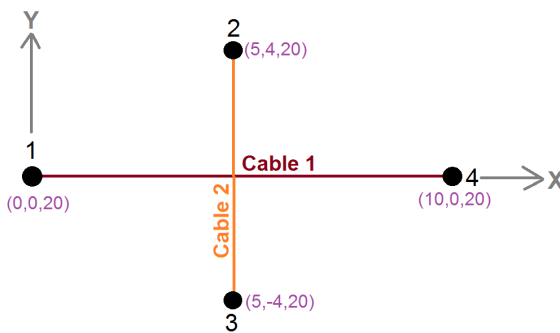
Mathcad Overview

Mathcad is a computer software used for calculating and graphing mathematical equations. Mathcad offers the flexibility to produce self-explanatory documents, performs a series of high-level calculations, generates quality graphs integrated directly to the output document, and incorporates units as part of a computation. This application is programmer friendly, the script are easy to write, visualize, edit and update. Programming takes advantages of the active learning techniques to users better understand applications, to de-mystify and to facilitate the learning process [22]. Mathcad it is highly visual, intuitive, and it requires a minimal amount of programming knowledge to create simple user-defined functions. In order to provide an overview of a Mathcad script, Figure 2 is presented. This figure shows a code for calculating the cable lengths in a cable net structures. Three input data is required, "XYZ" refers the spatial coordinates of the nodes, "CONN" provides information with regard element connectivity (i.e. what nodes comprise the cable element), "NELS" refers to the number of cable elements. For a single cable, the element length "L" is calculated as the square root of the sum of the squares of the "X", "Y", and "Z" lengths. The output is a vector that presents all cable lengths. Notice that the first row of the CONN array is identified by the index 0.
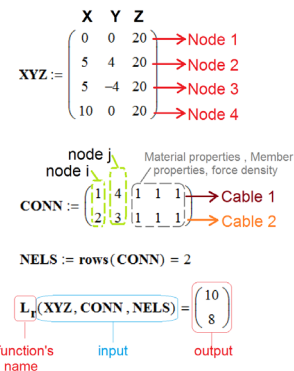
Figure 2. Sample of Mathcad function to obtain an array Lr with the lengths of each cable,

$$L_r(XYZ, CONN, NELS) :=$$

for $i \in 0 .. NELS - 1$

$$x_i \leftarrow XYZ_{(CONN_{i,0}-1, 0)}$$
$$y_i \leftarrow XYZ_{(CONN_{i,0}-1, 1)}$$
$$z_i \leftarrow XYZ_{(CONN_{i,0}-1, 2)}$$
$$x_j \leftarrow XYZ_{(CONN_{i,1}-1, 0)}$$
$$y_j \leftarrow XYZ_{(CONN_{i,1}-1, 1)}$$
$$z_j \leftarrow XYZ_{(CONN_{i,1}-1, 2)}$$
$$L_i \leftarrow \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$

return L

$XYZ_{(row, column)}$

function's name

input data

function's calculations

output data

(A)   Mathcad function

(B) Cable-net example

Y

2 (5,4,20)

1 (0,0,20)   Cable 1   4 → X
                         (10,0,20)

Cable 2

3 (5,-4,20)

(C) Input and output information

$$XYZ := \begin{pmatrix} X & Y & Z \\ 0 & 0 & 20 \\ 5 & 4 & 20 \\ 5 & -4 & 20 \\ 10 & 0 & 20 \end{pmatrix} \rightarrow \begin{matrix} \text{Node 1} \\ \text{Node 2} \\ \text{Node 3} \\ \text{Node 4} \end{matrix}$$

node j
node i
Material properties, Member properties, force density

$$CONN := \begin{pmatrix} 1 & 4 & 1 & 1 & 1 \\ 2 & 3 & 1 & 1 & 1 \end{pmatrix} \rightarrow \begin{matrix} \text{Cable 1} \\ \text{Cable 2} \end{matrix}$$

$$NELS := rows(CONN) = 2$$

$$L_r(XYZ, CONN, NELS) = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

function's name   input   output

Source: Own elaboration

Mathcad functions implementation

A series of Mathcad functions are created to implement the procedure indicated in Figure 1. The Mathcad functions are based on the information provided and the results of user-defined functions. Regarding the input information, Table 1 presents the required data to perform the analysis of a given cable net 3D structure. Notice that for input information pertaining to nodes and cable elements, the number of rows indicates the number of nodes and elements describing the topology of the cable net structure. The Mathcad functions required to perform the analysis are listed in Table 2. As may be observed, twenty functions are needed. The parenthesis of these routines presents the arguments that represent input data, (as indicated in column 1 of Table 2), or information that is created and presented as output information (i.e. as indicated in column 2 of Table 2).

Table 1. Required Input for Mathcad Analysis of Cable Structures

Function name	Number of rows	Number of columns	Description
XYZ	number of nodes	3	Global X-Y-Z coordinates of each node
R	number of nodes	3	Nodal restraints: 1=fixed, 0=free.
MAT	number of materials	1	Material Properties: Modulus of elasticity (E)
PROP	number of properties	1	Member Properties: Cross-sectional area (A)
CONN	number of elements	5	Member connectivity and properties: I end, J end, Material properties , Member properties, force density vector.
F	3 x (number of free nodes)	1	Nodal loads (Fx,Fy,Fz)

Source: Own elaboration.

The proposed Mathcad functions follow a similar nomenclature of those indicated in the direct stiffness method of matrix structural analysis used in the so-called transformational approach [23]. Most of the Mathcad functions listed in Table 2 are self-explanatory, but Figure 3 presents the arguments of the most relevant functions.

Table 2. Custom Mathcad Functions needed for Cable Net Analysis

name	Input	Purpose	Output
Nr	(NODES,R)	Obtain number of free and restrained degrees of freedom	(N_f, N_s, m, n)
Ffr	(m)	Create force matrix	(F_f)
Dr	(NODES,NELS,D,TC,CONN)	Create force density matrix	(D)
Dffr	(m,D,Nf)	Extract the upper right submatrix of D	(D_{ff})
Dfsr	(m,n,D,Nf,Ns)	Extract the upper left submatrix of D	(D_{fs})
Cfr	(Dff,Ff,Dfs,Cs)	Create the X-Y-Z coordinates of the free nodes	(C_f)
fo	(Lo,Tc,CONN,NELS)	Create the initial cable forces vector	(f)
IDr	(NODES,R)	Create an equation number array	(ID)
DIRr	(FN,NELS,CONN)	Calculate unbalance force	$(DIRR)$
Lr	(XYZ,CONN,NELS)	Calculate cable lengths	(L)
lxr	(NELS,XYZ,L,CONN)	Create an array of element direction cosine X,Y,Y component	(l_x, m_x, n_x)
Br	(NELS,NEQ,CONN,ID)	Create the transformation matrix, local to global	(BB)
kr	(PROP,MAT,CONN,lX,mx,nx,NELS,Lo)	Create the 6x6 element stiffness matrix in global coordinates	(k)
Kr	(NELS,B,k,K)	Create the global structure stiffness matrix	(K_r)
Δfr	(L,Lo,PROP,MAT,NELS,CONN)	Create an array containing the change in element forces	(Δ_f)
Kgr	(f,Lo,lx,B,Kg,mx,nx,NELS)	Create the global geometric matrix	(K_g)
δr	(KT,U,δ,NEQ)	Create a vector with nodal displacement	(δ)
Ur	(NEQ,NELS,U,DIR,f,DC,FN,h)	Create a vector with the unbalanced forces	(U)
TOLr	(U,NEQ)	Calculate the scalar value of the absolute maximum unbalanced force	(TOL)
XYZr	(CORD,R,δ,NODES)	Create the new X-Y-Z coordinates of the cable system	(XYZ)

Source: Own elaboration

Figure 3. Relevant Mathcad functions

Source: Own elaboration

Pre-stressed hyperbolic paraboloid example

The structure analyzed is shown in Figure 4; this example was originally presented by Thornton and Birnstiel [24], and it has been taken as reference for many researchers in the last decades [24,25,26,27,29]. The same example is taken as the reference in this study 252526, and 2924.The analytical response from the Mathcad functions is compared with the results from the software CABNET [29].

The cable net system studied here is a hyperbolic paraboloid formed by 14 intersecting cables. The idealized system is constituted by 41 nodes, and as indicated in Figure 4, nodes 1 to 25 are unrestrained nodes (f),

and the remaining nodes are constrained nodes (s). Nodes 26 and 34 are at grade, node 13 is 120 in. above the grade. Nodes 30 and 38 are 240 in. above grade, and the origin of the coordinate system is 120 in. below node 13. The cable net system is comprised by 64 elements where the bearing cables are oriented along the "X" direction and the stabilizing cables are aligned along the "Y" direction. The system is geometrically symmetric about "X" and "Y" axes.

Figure 4. Pre-stress Hyperbolic Paraboloid Cable Net (Plan View)
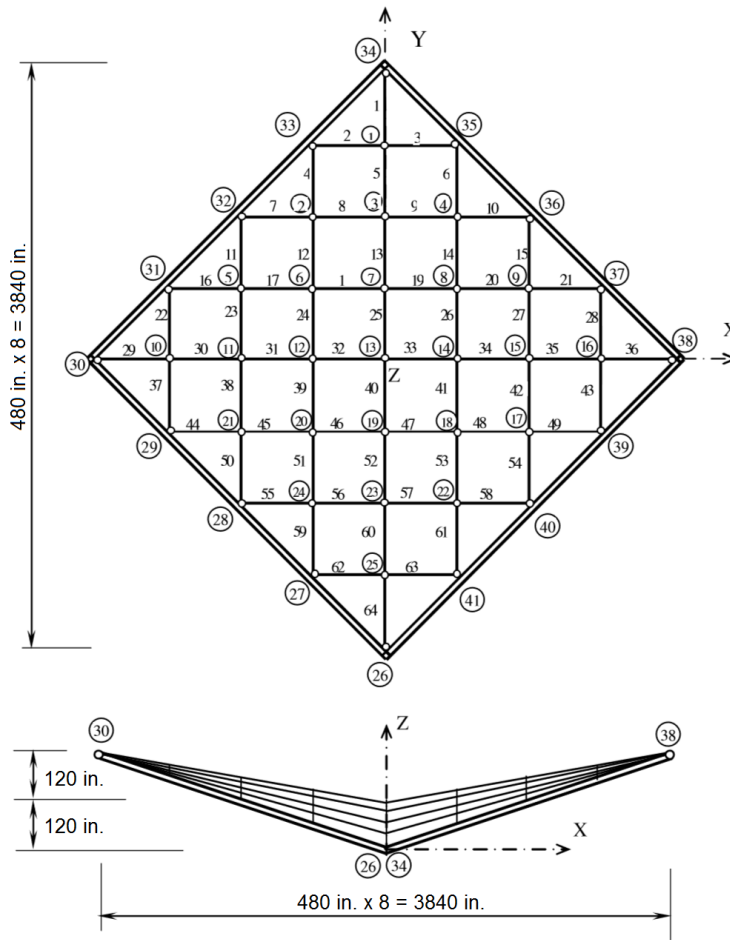
Source: Adapted from [26].

The cross-sectional area of each cable is 1 in2, the cable material has a modulus of elasticity of 24000 ksi, and a cable pre-stress of approximately 50 kips for all loading cases. Three loading conditions were evaluated:

- Loading 1: A force of 1 kip (downwards) applied at each unconstrained nodes.
- Loading 2: Same as loading 1, but with a total of 15 kip at node 7.
- Loading 3: Same as loading 2, but with a force of 10 kip in the Y direction at node 7

The required input information is presented in Figure 5. For concreteness, only relevant information is presented. For instance, the nodal force vector {Ff} only includes information pertaining to nodes 1,3,7 and 25. Recalling, the number of rows of this vector is 75 (i.e. 25 free nodes x 3 (Fx,Fy,Fz) = 75 rows).

Figure 5. Sample of data input

Property:= {1.0 in^2} *Cross-sectional Area*

Material:= {24000 ksi} *Modulus of Elasticity*

Force Pretension:= {50 kip} *Force Pretension*

Loading 1 (kip) — $\{F_t\}:=$

Force		Node
0	FX	
0	FY	1
-1	FZ	
0	FX	
0	FY	3
-1	FZ	
0	FX	
0	FY	7
-1	FZ	
0	FX	
0	FY	13
-1	FZ	
0	FX	
0	FY	25
-1	FZ	

Loading 2 (kip) — $\{F_t\}:=$

Force		Node
0	FX	
0	FY	1
-1	FZ	
0	FX	
0	FY	3
-1	FZ	
0	FX	
0	FY	7
-15	FZ	
0	FX	
0	FY	13
-1	FZ	
0	FX	
0	FY	25
-1	FZ	

Loading 3 (kip) — $\{F_t\}:=$

Force		Node
0	FX	
0	FY	1
-1	FZ	
0	FX	
0	FY	3
-1	FZ	
0	FX	
10	FY	7
-15	FZ	
0	FX	
0	FY	13
-1	FZ	
0	FX	
0	FY	25
-1	FZ	

Boundary Conditions — R:=

Rx	Ry	Rz	Node
0	0	0	1
0	0	0	2
1	1	1	27
1	1	1	28
1	1	1	29
1	1	1	30
1	1	1	31
1	1	1	32
1	1	1	33
1	1	1	34
1	1	1	35
1	1	1	36
1	1	1	37
1	1	1	38
1	1	1	39
1	1	1	41

Connectivity — CONN:=

i-node	j-node	Material	Property	Force Density	Element
34	1	1	1	1	1
33	1	1	1	1	2
4	8	1	1	1	14
6	12	1	1	1	24
37	16	1	1	1	28
30	10	1	1	1	29
15	16	1	1	1	35
16	39	1	1	1	43
21	28	1	1	1	50
20	24	1	1	1	51
18	22	1	1	1	53
28	24	1	1	1	55
24	23	1	1	1	56
24	27	1	1	1	59
22	41	1	1	1	61
25	26	1	1	1	64

Fixed Nodes Coordinates (in.) — XYZ:=

X	Y	Z	Node
0	-1920	0	26
-480	-1440	60	27
-960	-960	120	28
-1440	-480	180	29
-1920	0	240	30
-1440	480	180	31
-960	960	120	32
-480	1440	60	33
0	1920	0	34
480	1440	60	35
960	960	120	36
1440	480	180	37
1920	0	240	38
1440	-480	180	39
960	-960	120	40
480	-1440	60	41

Source: Own elaboration

Notice that the only difference between the loading scenarios 1 and 2 is the vertical force applied at node 7, while the difference between the loading scenarios 2 and 3 is the presence of a force in the "Y" direction at node 7. The array "R" refers to the boundary conditions of each node, nodes 1 to 25 are free nodes, where the zero value "0" represents this condition; nodes 26 to 41 are restrained and this condition is represented by the one value "1". .

Results

Nodal displacements results for all loading configurations are presented in Table 3, together with the respective results obtained from the reference, and using the software CABNET [29]. Table 3 shows selected cables initial forces.

Table 3. Vertical displacements for all loading cases.

Node	Loading 1			Loading 2			Loading 3	
	Thornton and Birnstiel [24]	CABNET [28]	FDM Mathcad	Thornton and Birnstiel [24]	CABNET [29]	FDM Mathcad	Thornton and Birnstiel [24]	FDM Mathcad
34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	-3.05	-3.05	-3.05	-4.57	-4.57	-4.57	-5.34	-5.34
3	-6.64	-6.62	-6.64	-15.22	-15.22	-15.24	-16.42	-16.44
7	-9.28	-9.26	-9.28	-44.62	-44.62	-44.68	-45.00	-45.06
13	-10.34	-10.33	-10.34	-21.26	-20.66	-20.70	-19.97	-20.00
19	-9.28	-9.26	-9.28	-12.24	-12.24	-12.26	-11.56	-11.58
23	-6.64	-6.62	-6.64	-7.26	-7.26	-7.27	-6.70	-6.71
25	-3.05	-3.05	-3.05	-3.11	-3.11	-3.11	-2.74	-2.74
26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Source: Own elaboration

Results for horizontal cable forces for all loading cases can be seen in Table 4. The results obtained by the Mathcad functions are consistent with the obtained results from the software CABNET [29] and to those presented in the reference paper [24].

Table 4. Cable forces for all loading cases.

| Cable	Nodes	Loading 1		Loading 2		Loading 3	
		CABNET [28]	FDM Mathcad	CABNET [29]	FDM Mathcad	Thornton and Birnstiel [24]	FDM Mathcad
1	34-1	41.24	41.24	40.09	40.09	45.95	46.17
5	1-3	41.16	41.16	40.02	40.02	46.06	46.13
13	3-7	41.10	41.10	40.08	40.08	46.26	46.27
25	7-13	41.07	41.07	40.36	40.37	36.66	36.73
40	13-19	41.07	41.07	40.21	40.21	36.68	36.67
52	19-23	41.91	41.10	40.19	40.19	36.70	36.71
60	23-25	41.16	41.16	40.23	40.23	36.71	36.79
64	25-36	41.23	41.24	40.32	40.32	37.71	36.89

Source: Own elaboration

As may be observed from Tables 3 and 4, the proposed Mathcad functions for solving cable net structures produce accurate and reliable results as compared with the reference paper [24] and the cable net software CABNET [29].

The tangent stiffness iteration was performed for the three loading cases with a tolerance value of 0.0001. Since the developed Mathcadfunctions require less iterations to achieve convergence, they are relatively faster than other solutions indicated in the literature. For instance, the numerical solution using the so called "the point based iterative procedure" required 68 iterative steps for the same tolerance value [27]. As shown in Figure 6, the calculated tolerance values under increasing iteration number for the loading 3 is

presented. It is observed that the developed algorithm allows to obtain fast convergence with a prescribed tolerance and the analysis duration is approximately 2 seconds when a 2.6 GHz computer is used. Compared with [27], the proposed methodology is not only faster but it requires significant less iterations.

Figure 6. Calculated tolerance values under increasing iteration number.
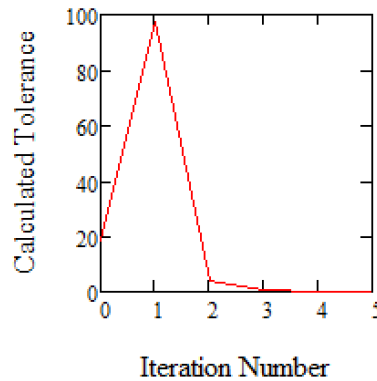
Source: Own elaboration

Conclusions

Despite the fact that today there are computational programs for cable nets structures, most of them are considered as proprietary "black boxes" with little or no guidance for the limitations, simplifications and methods involved in the analysis of these type of structures. The structural analysis of cable net structures represents a challenge due to its inherit complexity. In this paper, a methodology for implementing in Mathcad an approach based on force densities for analytical form finding of three-dimensional cable net structures subjected to static loading is presented. Programming takes advantages of the active learning techniques; it is expected that a motivated reader will be able to better understand the structural analysis of cable net structures helping to de-mystify and facilitate the analysis of this type of structures.

Tangent stiffness convergence procedure is adopted with a prescribed convergence tolerance of 0.0001 to obtain the solution for the geometrical nonlinearities presented in cable net structures. Fast convergence was achieved due to equilibrium check in every degree of freedom; less than 10 iterations were required for the three loading cases presented in this study. It was shown that the results of the Mathcad functions produce the same results given in the reference paper [24] and the software CABNET [29]. The Mathcad implemented script exhibited a negligible computational cost, since those are significantly faster than other solutions indicated in the literature (i.e. the point based iterative procedure presented in reference [27]). The stability of the algorithms is controlled, and divergence problems were not found for the analyzed examples.

Today's world demands to engineers the creation of integrated design documents that explain design process through interactive math calculations, graphs, and customizable functions in a single share-able document easy to follow, understand and update. In this sense, the present study bridges the theory related to the FDM and its step-by-step implementation in the commercial program Mathcad. The illustrative example demonstrates the capability of the Mathcad cable net functions thus simplifying the analysis of those structures under vertical and horizontal static loads.

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