Young Children's Use of Manipulatives to Represent Addition Concept

Uso de manipuladores por parte de niños pequeños para representar el concepto de adición

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ABSTRACT

One of the first fundamental arithmetic concepts that young children learn in school is addition. This article explores how young children use manipulatives while working on tasks relating to number and addition. The study employed case study research design and involved six children (aged six years) in one preschool center. Observation of children's work during mathematical tasks revealed children's early understanding of the addition concept. Hence, it is important that educators value and support the early development of children's mathematical representations to facilitate their successful use in learning and understanding mathematics concepts.

Keywords: Young children, Addition concept, Manipulatives, Representation.

RESUMEN

Uno de los primeros conceptos aritméticos fundamentales que los niños pequeños aprenden en la escuela es la suma. Este artículo explora cómo los niños pequeños usan manipulativos mientras trabajan en tareas relacionadas con el número y la suma. El estudio empleó un diseño de investigación de estudio de caso e involucró a seis niños (de seis años) en un centro preescolar. La observación del trabajo de los niños durante las tareas matemáticas reveló la comprensión temprana de los niños del concepto de suma. Por lo tanto, es importante que los educadores valoren y apoyen el desarrollo temprano de las representaciones matemáticas de los niños para facilitar su uso exitoso en el aprendizaje y la comprensión de los conceptos matemáticos.

Palabras clave: niños pequeños, concepto de adición, manipulativos, representación.

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1. INTRODUCTION

Representations are inevitable in mathematics classrooms worldwide. A variety of representations are commonly used across all grade levels. Research provides substantial evidence that the use of representations has a positive influence in the teaching and learning of mathematics. Representations can support the communication of mathematical ideas, develop conceptual understanding and facilitate problem solution (Bakar & Karim, 2019; Bakar, 2017; Rosli, Goldsby & Capraro, 2015; Ainsworth, 1999; Ayub, Ghazali, & Othman, 2013). The National Council of Teachers of Mathematics highlights the use of representation in mathematics classrooms to support the understanding of abstract concepts (NCTM, 2000). As gaining understanding is critical in mathematics (Hiebert, 1997), representation can play a major function as there is a strong connection between representation and conceptual understanding (Yuanita, Zulnaidi, & Zakaria, 2018; Abdullah, Halim & Zakaria, 2014; Abdullah, Zakaria & Halim, 2012; Ainsworth, 1999).

Research on representation use indicates a positive link between representation use and students' performances in problem solving. Researchers have asserted that students who created representations managed to solve the problems successfully (e.g. Bakar, 2018; Edens & Potter, 2007; Uesaka, Manalo & Ichikawa, 2007). Through the processes of internalization and externalization during representation-creation and through making connections between the two (Goldin & Stheingold, 2001), problem context can clearly be 'seen' which in turn facilitates problem solutions.

Although researchers have been investigating students' performances as a result of representational usage and revealed useful information about the positive contribution of representation in mathematical learning, still there is a limited picture about children's understanding of the concept being explored. Furthermore, studies on young children and their use of representations are limited (Johns, 2015) specifically relating to the use of manipulatives in facilitating young children's concept formation and understanding of abstract mathematics concepts.

2. RESEARCH BACHGROUND

Researchers have confirmed that manipulatives are a powerful aid in teaching and learning mathematics. This type of representation is commonly used in mathematics classrooms worldwide and across all grade levels. Teachers included manipulatives in instruction as they believed children profited from their use (Howard & Perry, 1997). As for children, they enjoyed learning with manipulatives (Howard & Perry, 1997).

An advantage of that approach, as suggested by Manches and O'Malley (2016), is that actions on manipulatives aided problem-solving by leading children to particular procedures. Actions on manipulatives, such as grouping objects into two equal sets, swapping over groups and moving objects, corresponded to part-whole relationships leading children to produce more solutions when solving additive composition tasks compared with using pictorial representation in the absence of manipulatives.

Furthermore, manipulatives are widely used to help students to bridge concrete objects with their abstractions. Concrete materials such as counters and blocks are useful in developing students' mental images. Teachers often expect that manipulatives facilitate students' creation of an internal representation of the external concepts being taught. Students' constructions of mental imagery are either rigidly or flexibly connected to the materials used (Thomas & Tabor, 2012). Unfortunately, there is no promise that students automatically create the intended internal representation in their mind and some students are likely to 'see' different concepts in the same manipulatives (Puchner, Taylor, O'Donnell, & Fick, 2008). Indeed, the external representation is useful only after one has internally made sense of it (Goldin & Shteingold, 2001).

Despite its advantages, researchers have expressed concerns about the use of manipulatives. This is because students have used manipulatives in a rote manner without attaching meaning to the manipulatives (Puchner et al., 2008). Furthermore, even if they use manipulatives, actions encouraged by the physical materials do not always support learning. Lesh, Post & Behr (1987) propose that students use various forms of representations flexibly, instead of depending only on one type of representation (such as relying only on symbols).

Researchers emphasize the importance of developing children's arithmetic skills in the early years of instruction (Gelman & Gallistel, 1978; Patel & Canobi, 2010; Resnick, 1992). Skills learned in the early years of school are important knowledge for use in many aspects of everyday life as well as for use in future learning and life. Children's early counting experiences provide an important base towards understanding addition concepts (Gelman & Gallistel, 1978). Research relates the strong relationship between children's quantitative knowledge in the early years with success in later years of schooling (Krajewski & Schneider, 2009).

In Malaysia, preschool children (aged four to six years old) are expected to understand the addition concepts and solve problems using the operation (Ministry of Education Malaysia, 2010). This basic skill is one of the important foundations needed for later mathematical learning. For example, knowledge and understanding of addition may support learning of subtraction and multiplication (as repeated addition).

Since teachers reported that their pre-schools' children faced difficulties understanding the addition concept and continued struggling with this basic operation in Year One (Tyng, Zaman, & Ahmad, 2011), this study explored the use of manipulatives in learning addition concept. More specifically, it is important to discover if manipulatives are beneficial in facilitating the learning and understanding of the addition topic.

3. THE STUDY

The purpose of this study is to explore the use of manipulatives by young children to portray their understanding of addition concept. Specifically, this research will investigate how manipulatives support the students' understanding of mathematics concepts involving the concept of number and addition.

The study involved six children from the same 'pre-school' classroom in Malacca, Malaysia. The study was conducted during the first term of school, hence their mathematical achievement was solely based on a pre-test that focused on counting abilities and the application of counting skills to basic addition and subtraction tasks. It is important to note that despite fact that the post-test was not being presented in this paper, the children's performance in basic number skills was found improved.

The study engaged children in one-to-one tasks with no explicit teaching of addition undertaken as part of the data collection process. The researcher, who acted as the teacher with this group of children over a period of five weeks, introduced the addition process through a series of developmental steps beginning from modelling with concrete materials and finally producing the number sentence. Together, the researcher and the children explored various addition scenarios, with the use of a variety of representations actively encouraged. The children were required to represent various quantities and addition situation.

The children's creation and use of manipulatives during the tasks were examined to obtain insight into the children's understanding of the concept. Specifically, the strategies, procedures, behaviours, and discussions as they manipulated the concrete materials were analyzed in detail, as this provided evidence of their understanding of the concepts.

4. DATA SOURCES AND ANALYSIS

Several sources of data were collected during the study including observations, field notes, conversations with children, artefacts, audio and video recording. Initially, children's work was analysed. Video analysis of children working exhibited how children created their representations. Conversation with children helped to clarify the representation they created and explain what they have in mind during their engagement with the tasks. Each child's pre-test score, representation creations and associated talk, as well as events and behaviours that informed their mathematical thinking were summarised in a table. This allowed analysis of various representation forms from different children. Additionally, the table allowed for comparison of representation creations and thinking among different children. The representation created by the children, in combination with the thinking involved (identified through observations and conversations with children) revealed the children's understanding of the concept of number and addition.

5. DATA SOURCES AND ANALYSIS

The findings presented in this paper involved only a small part of a larger project investigating the variety and use of mathematical representation by children (aged 6 years). The way children created and used the representation is first described. We then report the children's understanding of the concepts, as revealed through the procedures and strategies observed during the creation and use of representations. Next, we summarize the key findings related to children's representation creation and use, as well as their understanding of the concepts.

Representations of Number and Addition

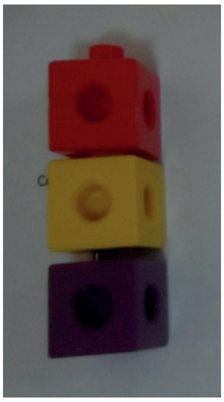
Representing Quantity

In this study, the researcher introduced the children to the addition concepts using a variety of manipulatives. Examples of concrete materials included linking cubes, coins, pegs and marbles. All these concrete materials were familiar to the children and are commonly found either in their learning or home environments where they have experienced using them or observed others manipulating the objects. The initial task involved the children's creation and representation of numbers using various concrete materials (i.e. including cubes) with assistance from the researcher. In later tasks, children were required to employ cubes independently to demonstrate their understanding of addition.

When asked to represent a quantity of a spoken number with concrete materials, the ability to associate the spoken number name with the correct quantity is essential. Given the task to represent 'three' using cubes, all children had no difficulty in getting the correct quantity of cubes from the box placed in front of the classroom. They then displayed the quantity in various forms (refer Figure 1). Some children linked the cubes together while others grouped them. They constructed the quantity easily and quickly. Using the three cubes they had on hand, Norman and Aimy linked the cubes together to form the construction in Figure 1a. Similarly, Qaisya and Deliyana connected one cube with another and continued until she reached the total (refer Figure 1b). Unlike the previous children, Nadia grabbed 3 cubes and grouped them together (refer Figure 1c). As can be seen in Figure 1d, Ali and Rozy constructed the quantity similar to the way Nadia did. None of the children commented on the quantity despite the fact that they were aware of the differences in the construction of the cubes; showing that they knew the quantity remained as 'three', despite having them displayed in various ways. This showed that the children had a stable understanding of quantity.

A stack of 3 cubes to represent the quantity '3'

a) 3 cubes Linked (by Norman and Aimy)



3 cubes Linked (by Qaisya and Deliyana)

A group of 3 cubes to represent the quantity '3'

c) 3 cubes Grouped (Nadia)

d) 3 cubes Grouped (Ali and Rozy)

Figure 1: Examples of children's construction of the quantity 'three' using cubes

Representing Addition

Since the students had only recently been introduced to the addition concept, the researcher began with a simple addition situation to initiate children's manipulation of concrete materials. The children were then given different addition scenarios involving different quantities and were encouraged to use a variety of concrete materials to help them grasp the concept more firmly.

Representing the Addends

The initial task involved small numbers for the addends (3 and 1). The researcher asked the children to model the addition situation with the cubes and to also obtain the total. The majority of the children were able to represent the quantity and the addition situation easily (refer Figure 2). Whilst manipulating the cubes, Aimy and Rozy counted aloud. "1, 2, 3….4". "4" both said confidently and smiled. Similarly, Qaisya was observed to count softly and slowly as she constructed the first addend "1, 2, 3". "And here 1" as she simultaneously included the second addend. She then counted them all "1, 2, 3 and 4". "4' she said aloud. Note that in exhibiting the first addend, the children demonstrated different ways of presenting the quantity of the addends (i.e. linked them horizontally, grouped them together and linked then vertically).

3 (in a line) and 1 by Qaisya

3 (in a group) and 1 by Rozy

3 (in a pile) and 1 by Aimy

Figure 2: Examples of children's construction of addition using cubes, showing different ways of representing the first addend

As in Figure 2a, Norman, Nadia, and Qaisya formed a horizontal line with the cubes. As seen in Figure 2b, Rozy constructed a group of three cubes, while Aimy stacked the cubes vertically (refer Figure 2c). They continued modelling the second addend by putting one cube separate from the previous construction. They then counted the cubes one-by-one to obtain the total. By doing so, they identified addition as comprising of two groups of objects and counting all the objects in both groups to obtain the total.

Representing the Total

Given the addition situation "3+1", Ali lined all four cubes, that were very close to each other, in a straight line (refer Figure 3).

Figure 3: Ali's construction of addition using cubes (the total)

By contrast with other children who represented both the addends, Ali's construction contained no gap between the three cubes and one cube. Presumably he was not yet aware that addition comprises two separate quantities. By constructing the total (four cubes), he demonstrated that he knew how to perform addition. However, it was not clear if he understood the underlying principles in addition. He might have only followed the procedure of counting all to reach the solution rather than understanding the meaning of addition.

6. DISCUSSION OF FINDINGS

The findings from the data indicated that the young children's use of representation provided insights into their understanding of numbers and addition concepts. The exploration of numbers and addition using concrete materials had facilitated their understanding of numbers and addition.

Understanding Numbers

The children's manipulation of concrete objects during the construction of various quantities showed evidence of their understanding of quantity. When constructing groups of items using the cubes provided to them, the children showed the one-to-one principle as they touched/ pointed to each manipulative and simultaneously verbalized the given number names. Also, the children's concrete construction of quantities reflected their understanding of cardinality, in which the last number represents the total quantity (Batchelor, Keeble, & Gilmore, 2015). By presenting the quantities in various forms including as a set of objects as well as in a line (i.e. vertically and horizontally), the children provided evidence for their knowledge relating to the cardinality principle (i.e. knew that the final number of the set designates the total number of objects). Also, the children demonstrated a stable understanding of quantity, evidenced through the construction of quantities in various formation (objects in a line and in groups).

Addition Understanding Using Concrete Representations

The analysis of these data suggests that the manipulation of the concrete materials was helpful in facilitating the children's understanding. Manches and O'Malley (2016) found that the use of concrete materials (for example, by touching them) was helpful in stimulating students' senses. After constructing the cubes into two different groups, pointing to and touching the cubes while simultaneously counting, helped the children in the present study develop understanding of addition as combining the groups to obtain the total. This is particularly important for children who were introduced to the new concept who would not have been able to arrive at the total if they had not pointed to the manipulatives. Further, repeated experience manipulating concrete materials gradually assisted the children to transform their concrete knowledge into abstract knowledge. Moyer (2001) claimed that the meaning of the mathematical ideas does not lay in the physical nature of the manipulatives. This was evident in this study. After the children had reflected on their actions on the manipulatives, they were then able to make the link between manipulatives and concrete materials.

The children's ability to quickly learn the addition process through the researcher's demonstrations of addition using manipulatives supported previous research that established the contribution of such tools on students' learning (Gibbons, 2012). With the help of various manipulatives, most children succeeded in providing correct answers by counting all the quantities they constructed. Young children were more successful in performing addition problems when concrete referents were made available to them, compared with story and number fact problems in which physical referents were absent (Levine et al., 1992). As cautioned by Hiebert and Wearne (1992), students occasionally used manipulatives in a rote manner without understanding the concepts behind the procedures. Since they applied no obvious action to the concrete materials (putting the groups together), it raises concerns about the children's ability to link the manipulatives with the addition concept being taught.

Counting-all to Add

Since counting is the key foundation in performing and understanding addition (Baroody, 1987a), and since counting and adding are interrelated when performing addition, it is equally important to differentiate between the two mathematical actions in this study to be able to identify children's understanding of the concept of addition.

The children's verbal counting that accompanied the construction was analyzed as this was helpful in inferring their thinking regarding addition concept. As both counting and adding involved finding the cardinality of a set of objects, children's verbal counting would be helpful in determining whether the children were merely counting, or whether they counted to add the objects. Most children were found to be employing *counting-all*, in which they started counting from the first group, then followed by counting the second group, and finally obtained the total. This addition strategy showed some signs of addition knowledge. Evidently, the children did not merely count, but they counted to add. As identified by Siegler and Jenkins (1989), there are eight strategies used by young children to solve addition, in which five of them involve counting. Considering that the children had only recently introduced to addition, it is possible that they began with *counting-all* and used this strategy on almost all occasions, as this is one of the simplest strategies used by young children (Baroody, 1987b). Furthermore, *counting-all* is a useful and unavoidable method for addition (Fuson, 1992) especially for young children.

7. CONCLUSION, IMPLICATIONS AND RECOMMENDATIONS

The children in this study were provided with opportunities to work with quantity and addition using concrete materials. By manipulating concrete materials related to numbers and addition, they developed their understanding of the concepts. By using concrete materials (cubes) to represent quantities as well as addition, none of the children had any difficulty in completing the tasks. They could construct the required quantities easily and accurately. They even demonstrated their stable understanding of quantity as evident by representing the quantities of the addends in different arrangements, but still resulting in similar totals. All the children demonstrated addition as comprising two groups of cubes. Also, the children in this study could easily manipulate the concrete materials to represent the required addition situation.

This study has implications for the constructivist theory of learning. The children's active manipulation of the concrete materials aided the children to make sense of the addition concept rather than knowing the knowledge simply by listening to the teacher's explanation relating to the concept. Hence, curriculum developers and educators need to practice constructivist learning theory in the development of Mathematics learning content, strategies as well as assessment for young children.

This study showing children's capability of making sense of newly introduced addition concepts by exploring and manipulating concrete materials has implications for the instruction and assessment used in mathematics classrooms. Children might be in disadvantages when instructions or assessments put emphasis only on one particular representation form (i.e. symbols) as it would be detrimental to children who have not yet transitioned to the abstract level; that still require alternative representation forms (e.g. able to explore and grasp better with the help of concrete form). Hence, teachers should attend to the differences in knowledge, skills and learning styles by promoting the use of manipulative as scaffolds to learning and understanding.

Future research should include teaching intervention that supports students utilizing various forms of representation effectively including the use of concrete materials for teaching as well as learning. It is also important that both teachers and value alternative representation forms (e.g. manipulatives and visual representation forms) particularly as it is helpful when learning new mathematics concepts.

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