

INNOVATION STRATEGY IN INDUSTRY: CASE OF THE SCHEDULING PROBLEM ON PARALLEL IDENTICAL MACHINES

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ABSTRACT

In this paper, we propose an innovation strategy in the industry (case of the scheduling problem on two parallel identical machines), with the objective of minimizing the weighted sum of the end dates of jobs, this problem is NP-hard. In this frame, we suggested a novel heuristics: (H1), (H2), (H3), with two types of neighborhood (neighborhood by SWAP and neighborhood by INSERT). Next, we analyze the incorporation of three diversification times (T1), (T2), and (T3) with the aim of exploring unvisited regions of the solution space. It must be noted that job movement can be within one zone or between different zones. Computational tests are performed on 6 problems with up to 2 machines and 500 jobs.

Keywords: innovation; Scheduling; parallel identical machines

1. INTRODUCTION

A scheduling problem consists of organizing jobs realization time with consideration of time constraints (time limits, tasks series character) and constraints related to using and availability of required resources.

The case of scheduling problems on parallel identical machines is studied by many authors like (Schmidt, 1984; Zribi & al, 2005; Chang & al, 2011; Adamu & Adewunmi, 2012, 2013; Selt & zitouni, 2016).

In (1984) Schmidt has studied the scheduling problem of parallel identical machines with different unavailability intervals and different job deadlines. He used the method of Branch and Bound based on two procedures: the first is the generation by decomposition and cut approach and the second is the hybridization of procedures of generation by cut.

Zribi and al (2005) have studied the problem $P_m // N - C // \sum_{j=1}^n w_j C_j$ and have compared two exact methods, the Branch and Bound method and the integer programming one. They have concluded that the Branch and Bound method has better performance and it allows resolving instances of more than 1000 tasks.

Chang and al (2011) proposed a genetic algorithm (GA) enhanced by dominance properties for single machine scheduling problems to minimize the sum of the job's setups and the cost of tardy or early jobs related to the common due date.

Adamu and Adewunmi (2012, 2013) have studied the problem $P_m // \sum_{j=1}^n w_j (U_j + V_j)$, they proposed some metaheuristics for scheduling problem on parallel identical machines to minimize a weighted number of early and tardy jobs.

In (2013), they carried out a comparative study of different (a genetic algorithm, particle swarm optimization and simulated annealing with their hybrids) metaheuristics for identical machines

Zitouni and Selt (2016) have studied the problem $P_m // N - C // \sum_{j=1}^n w_j C_j$ they proposed a novel heuristic for scheduling problems on parallel identical machines to minimize the weighted sum of the end dates of tasks.

In this paper, the results of Zitouni and Selt research works are exploited to develop a different new approach to solve job scheduling problems on parallel identical machines under different constraints.

2. PROBLEM DESCRIPTION

This problem consists in scheduling n jobs for m parallel identical machines $\{M_1, M_2, \dots, M_m\}$ where $n \gg m \geq 2$ with unavailability zones.

We assume that the jobs $\{j_1, j_2, \dots, j_n\}$ are all available at $t = 0$ and their operation times are independent of the choice of machines performing these jobs.

In the generic case of the problem, each one of the m machines can show some unavailability zones during scheduling horizon and each job must be executed on time.

This problem noted by $P_m // N - C // \sum_{j=1}^n w_j C_j$ consists in assigning n jobs to m machines over availability zones in a manner to enforce the weighted sum of the end dates of tasks referred to as $\sum_{j=1}^n w_j C_j$ to be minimal.

It must be noted that there is $(n!)m$ possibility to assign n jobs to m machines (Sakarovitch, 1984).

3. PROPOSED METHOD

3.1. Tabu Search (TS)

Tabu Search is a metaheuristic originally developed by (Glover, 1986).

This method combines local search procedures with some rules and mechanisms to surmount local optima obstacles avoiding the cycling trap.

Tabu search is the metaheuristic that keeps track of the regions of the solution space that have already been searched in order to avoid repeating the search near these areas (Glover & Hanafi, 2002).

It starts from a random initial solution and successively moves to one of the neighbors of the current solution.

The difference between tabu search and other Meta-heuristic approaches is based on the notion of the tabu list, which is a special short-term memory, storing of previously visited solutions including prohibited moves. In fact, short-term memory stores only some of the attributes of solutions instead of whole solutions. So, it gives no permission to revisit solutions, and then, avoids cycling and being stuck in local optima.

During the local search, only those moves that are not tabu will be examined, if the tabu move does not satisfy the predefined aspiration criteria. These aspiration criteria are used, because the attributes in the tabu list may also be shared by unvisited good quality solutions. A common aspiration criterion is better fitness, i.e. the tabu status of a move in the tabu list is overridden if the move produces a better solution.

3.2. Algorithm (TS)

Table 1: The process of (TS) can be represented as follows:

Initialization: $\mathbf{X} = \text{initial solution}, f_{\min} = f(\mathbf{x}), \mathbf{X}: = \mathbf{x}$
Step 1: generate a neighborhood $N(\mathbf{x})$
Step 2: $f(\mathbf{x}') = [f(\mathbf{x}_i)]$
Steps 3: add $(\mathbf{x}', \text{TABU})$
Step 4: $\mathbf{x}: = \mathbf{x}'$
Step 5: If $f(\mathbf{x}) < f_{\min}$
Step 6: $f_{\min}: = f(\mathbf{x})$
Step 7: $\mathbf{X}_{\min}: = \mathbf{x}$
Step 8: End if

4. NEIGHBORHOODS

4.1. Neighborhood by (swap)

Formal statement 1. Consider a sequence σ , the set's cardinal of $N_1(\sigma)$ is $\frac{n(n-1)}{2}$.

Example

Consider a sequence $\sigma = 1234$,

Table 2: the neighborhood $N(\sigma)$ is: $N(\sigma) = \{2134, 3214, 4231, 1324, 1432, 1243\}$.

job i	job j	Sequence
1	2	2134
	3	3214
	4	4231
2	3	1324
	4	1432
3	4	1243

4.2. Neighborhood by (insert)

Formal statement 2. Consider a sequence σ , the set's cardinal of $N_2(\sigma)$ is $(n-1)^2$.

Example

Consider a sequence $\sigma = 1234$,

Table 3: the neighborhood $N(\sigma)$ is:
 $N(\sigma) = \{2134, 2314, 2341, 1324, 1342, 3124, 1243, 4123, 1423\}$

Position job	1	2	3	4
1		2134	2314	2341
2	2134		1324	1342
3	3124	1324		1243
4	4123	1423	1243	

5. PROPOSED HEURISTICS

An initial solution is always necessary. For this reason, we suggest in this part the following heuristics:

Assign the (best) job h where $(P_i = \frac{p_j}{w_j})$ and $(P_i = \max P_i)$ to the best machine

(Selt and Zitouni, 2014); based on two principles justified by the two following propositions:

Proposition 1. In optimal scheduling, it is necessary to schedule the tasks in each availability zone of the machine according to the order SWPT.

Proof. It results directly by adjacent job exchange like used by (Smith,1956) for the corresponding zones.

Proposition 2. It is not useful to let the machine (idle) if a job can be assigned to this machine.

Notations:

We denote by:

$J = \{1, 2, \dots, n\}$: The set of jobs.

p_h : Execution time of the job h .

$I = \{1, 2, \dots, m\}$: The set of machines

$Z = \{1, 2, \dots, \alpha\}$: Availability Zones.

$S_z^{(i)} (z \in Z)$: The beginning of the unavailability time of the machine $i \in I$.

$T_z^{(i)} (z \in Z)$: The end of the unavailability time of the machine $i \in I$.

$C_z^{(i)} (z \in Z)$: Execution time of the job $j \in J_z^{(i)}$.

w_j : the weight of the job h .

5.1. Heuristic (H1)

Initialization

$J = \{1, 2, \dots, n\}$, $\sigma = \emptyset$, $F=0$; $C_i = 0$; $P_i = \text{random}(1.99)$; $W_i = \text{random}(1.10)$;
 $z=1$.

Begin

Sort jobs $h \in J$ in increasing order according to the criterion $\frac{P_j}{w_j}$ in L_1

While ($L_1 \neq \emptyset$) **do**

if ($Z_2^1 > P_0$) and ($Z_2^1 \geq P_0$)

Determine the machine M such that $Z_2^1 - C_{22}^S \leq \min(P_{h1}, P_{h0})$

Assigned the job h to the machine M ;

Compute C_j ; $F = f_{\sigma_1} + f_{\sigma_2} = f_{\sigma_1} + f_{\sigma_2} + C_i W_i$

Delete the job h from L_1

Else

Set $Z = Z + 1$;

End if

$\sigma = \sigma_1 \cup \sigma_2$; //obtained sequence

End

Example1

Table 4: Consider the problem P1 with the following data:

job	1	2	3	4	5	6
P_j	72	97	17	18	44	97
W_j	7	9	1	10	3	7
P_j/W_j	10.2	10.7	17	1.8	14.6	19.5

Results of heuristic (H_1) are: $f = 3576$.

Execution time = 0.034sec.

Results of tabu (swapping) are: $f = 3110$.

Execution time = 0.006 sec.

Results of tabu (insertion) are: $f = 2431$.

Execution time = 0.008 sec.

The best results are obtained by using tabu by swapping for $f = 3310$.

5.2. Heuristic (H2)

Initialization

$J = \{1, 2, \dots, n\}$, $\sigma = \emptyset$, $F = 0$; $C_i = 0$; $P_i = \text{random}(1.99)$; $W_i = \text{random}(1.10)$;

$z = 1$.

Begin

Sort jobs $h \in J$ in increasing order according to the criterion $\frac{p_j}{w_j}$ in a list L_1

Sort jobs $h \in J$ in decreasing order according to the criterion p_j in a list L_2

While ($L_1 \neq \emptyset$) **do**

If ($Z_a^2 > P_0$) and ($Z_a^1 \geq P_0$)

Determine the machine M such that $Z_a^1 - C_{i2}^a \leq \min(P_{h1}, P_{h0})$

Assigned the job h_0 to the machine M

Compute C_j ; $f_{\sigma_1} = f_{\sigma_1} + C_i W_i$

Delete the job h_0 from L_1 and L_2

Else

If ($Z_a^2 > P_1$) and ($Z_a^1 \geq P_1$)

Determine the machine M such that $Z_a^2 - C_{i2}^a \leq \min(P_{h1}, P_{h0})$

Assigned the job h_1 to the machine M

Compute C_j ; $F = f_{\sigma_1} + f_{\sigma_2} = f_{\sigma_1} + f_{\sigma_2} + C_i W_i$

Delete the job h_1 from L_1 and L_2

Else

Set $Z = Z + 1$;

End if

$\sigma = \sigma_1 \cup \sigma_2$; //obtained sequence

End

Example 2

Table 5: Consider the problem P2 with the following data:

job	1	2	3	4	5	6
P_j	82	56	52	19	19	85
W_j	5	3	6	4	1	9
P_j/W_j	16.4	18.6	8.6	4.7	19	9.4

Results of heuristic (H₂) are: $f = 3536$. Execution time = 0.039 sec.

Results of tabu (swapping) are: $f = 3110$. Execution time = 0.005 sec.

Results of tabu (insertion) are: $f = 3120$. Execution time = 0.006 sec.

The best results are obtained by using tabu by swapping for $f = 3110$.

5.3. Heuristic (H3)

Initialization

$J = \{1, 2, \dots, n\}$, $\sigma = \emptyset$, $F = 0$; $C_i = 0$; $P_i = \text{random}(1.99)$; $W_i = \text{random}(1.10)$; $z = 1$

Begin

Sort jobs $h \in J$ in increasing order according to the criterion $\frac{P_j}{W_j}$ in L_1

Sort jobs $h \in J$ increasing order according to the criterion p_j in L_2

While ($L_1 \neq \emptyset$) do

If ($Z_a^z > P_0$) and ($Z_a^1 \geq P_0$)

Determine the machine M such that $Z_a^z - C_{i_2}^a \leq \min(P_{h1}, P_{h0})$

Assigned the job h_0 to the machine M; Compute C_j ;

$F = f_{\sigma 1} + f_{\sigma 2} = f_{\sigma 1} + f_{\sigma 2} + C_i W_i$

Delete the job h_1 from L_1 and L_2

Else

If ($Z_a^z > P_1$) and ($Z_a^1 \geq P_1$)

Determine the machine M such that $Z_a^z - C_{i_2}^a \leq \min(P_{h1}, P_{h0})$

Assigned the job h_1 to the machine M

Compute C_j ;

$$F = f_{\sigma_1} + f_{\sigma_2} = f_{\sigma_1} + f_{\sigma_2} + C_i W_i$$

Delete the job h_1 from L_1 and L_2

Else

Set $Z=Z+1$;

End if

$\sigma = \sigma_1 \cup \sigma_2$; // obtained sequence

End

Example 3

Table 6: Consider the problem P 3 with the following data

job	1	2	3	4	5	6
P_j	82	56	52	19	19	85
W_j	5	3	6	4	1	9
P_j/W_j	16.4	18.6	8.6	4.7	19	9.4

Results of (H_3) are: $f = 3530$.

Execution time = 0.016sec.

Results of tabu (swapping) are: $f = 3091$.

Execution time = 0.007 sec.

Results of tabu (insertion) are: $f = 3095$.

Execution time = 0.008 sec.

The best results are obtained by using tabu by swapping for $f = 3091$.

6. DATA GENERATION

The heuristics were tested on problems generated with 500 jobs similar to that used in previous studies : (M'Hallah & Bulfin, 2005; Lee, 1996, 1997; Schmidt, 2000) for each task j an integer processing time P_j was randomly generated in the interval (1.99), with a weight randomly w_j chosen in the interval (1.10).

7. DISCUSSION OF RESULTTS

We have chosen MATLAB as our programming and testing tool. In this part we illustrate a comparison between heuristics (H1), (H2), (H3) and metaheuristic TS, from our testing, we noticed the following: If the number of jobs n is less than 150, then the proposed heuristics present good results. If the number of jobs n is between 150 and 250, the Tabu method by Swapping gives better results (Figures 1, 2 and 3). If the number of jobs exceeds

250, in this case, the tabu method by swapping whose complexity is $O(n^3)$ becomes practically useless (results of tables 3,4 and 5).

Tables 7, 8 and 9 below presents: BC: The best costs, AC: Average costs, AT: Average time.

Table 7: Heuristic (H1) cost amelioration based on (TS).

JOBS	Results of heuristic (H ₁)	AT (sec)	(TS)Swap		(TS)Insert		BC
			AC	AT (sec)	AC	AT (sec)	
N=30	38432	0.013	29562	0.85	30659	1.22	29562
	48056	0.012	37335	1.02	37258	1.95	37238
	34420	0.01	26967	0.93	27265	159	26967
N=50	113123	0.04	96256	5.50	97945	8.56	96256
	105562	0.08	93625	6.60	93959	9.62	93625
	102225	0.07	94215	5.30	93165	8.23	93165
N=150	931265	0.17	869856	52.35	911025	60.10	869856
	926921	0.15	912694	60.25	908223	62.68	908223
	882230	0.19	858541	58.12	859624	63.31	858541
N=250	2655846	0.26	2630354	135.53	2641873	137.36	2630354
	2559125	0.21	2549623	165.36	2545280	164.23	2545280
	2478415	0.22	2459225	123.68	2465968	124.65	2459225
N=350	4965280	0.26	4962171	265.25	4964382	276.95	4962171
	4771183	0.31	4767183	296.32	4768245	300.34	4767183
	4896954	0.24	4889864	240.36	4887262	268.21	4887262
N=500	9213434	0.55	9107596	436.6	9110652	435.24	9107596
	9126543	0.6	9122261	370.65	9123621	381.23	9122261
	9506951	0.7	9499251	395.12	9498926	397.15	9498926

Table 8: Heuristic (H2) cost amelioration based on (TS).

JOBS	Results of heuristic (H ₂)	AT (sec)	(TS)Swap		(TS)Insert		BC
			AC	AT (sec)	AT	AC (sec)	
N=30	40586	0.012	31657	0.8	31057	1.18	31057
	38213	0.013	29564	0.93	30526	1.78	29564
	37592	0.015	28456	1.01	28915	1.64	28456
N=50	100172	0.022	89743	5.9	91254	8.36	89743
	111228	0.016	98625	6.2	99147	9.82	98625
	99273	0.018	89745	5.75	91450	8.11	89745
N=150	851935	0.041	803856	53.47	809256	63.23	803856
	889098	0.052	832159	62.71	820541	61.54	820541
	867296	0.039	813753	58.99	819369	60.73	813753
N=250	2324111	0.077	2299840	131.66	2293647	140.73	2293647
	2411968	0.063	2365486	164.32	2394935	174.66	2365486
	2395659	0.070	2360258	129.38	2373281	129.81	2360258
N=350	4569054	0.12	4528346	260.54	4542563	266.49	4528346
	4656261	0.129	4597750	285.97	4616542	298.67	4597750
	4448628	0.122	4425698	243.83	4416581	270.36	4416581
N=500	9031909	0.35	9016549	446.77	9029512	431.94	9016549
	9225172	0.33	9210975	365.16	9209964	388.58	9078964
	9340531	0.34	9318620	399.52	9319753	421.42	9168620

Table 9: Heuristic (H3) cost amelioration based on (TS).

JOBS	Results of heuristic (H ₃)	AT (sec)	(TS)Swap		(TS)Insert		BC
			AC	AT (sec)	AC	AT (sec)	
N=30	35910	.014	27365	0.77	27801	0.98	27365
	35708	.013	28054	0.98	28456	1.50	28054
	36114	.011	28282	1.02	28067	1.39	28067
N=50	98285	.016	90170	6.2	91843	7.90	90170
	104207	.021	96408	6.3	95290	9.58	95290
	102195	0.02	91819	5.9	94014	8.45	91819
N=150	936879	0.06	867658	51.79	880170	63.77	867658
	901542	0.063	822964	61.3	839753	65.20	822964
	910925	.067	861490	59.48	852135	66.95	852135
N=250	2530927	0.08	2489321	37.32	2500325	142.78	2490321
	2300753	0.09	2270845	59.91	2259658	174.66	2232658
	2276966	0.078	2245547	132.45	2249528	131.59	2236547
N=350	4628100	0.12	4590596	255.32	4598212	262.58	4678596
	4523330	0.2	4491627	291.75	4489365	300.12	4482365
	4559716	0.21	4530129	252.01	4533156	277.81	4518029
N=500	9363249	0.3	9339824	420.44	9321598	445.43	9236598
	9203920	0.25	9195893	383.61	9172589	398.70	9032589
	9023792	0.28	9001569	400.59	9004796	427.91	9001569

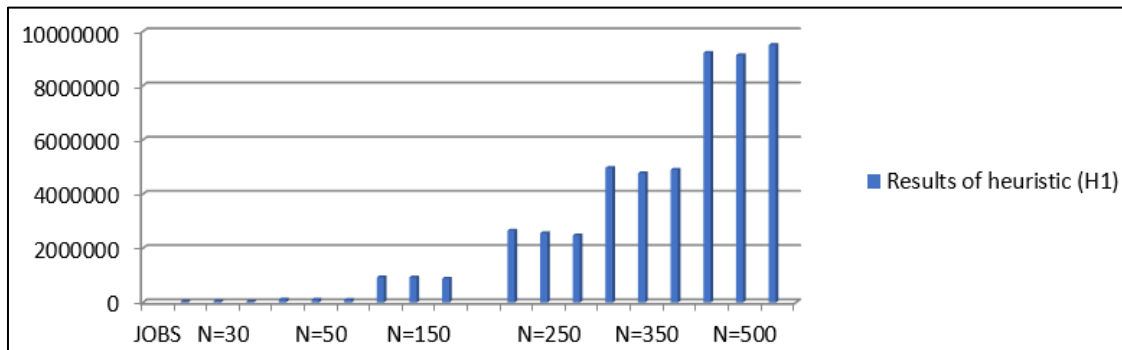


Figure1: Histogram of heuristic (H1) cost amelioration based on tabu search for different N values.

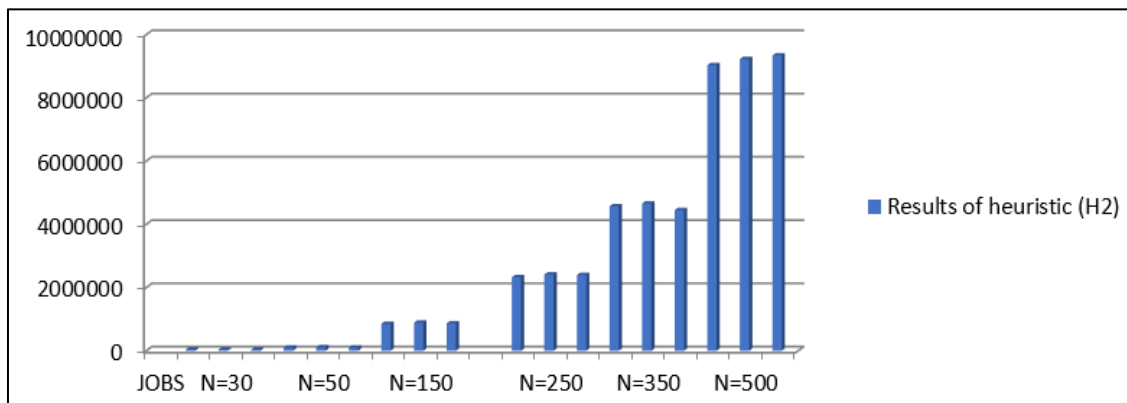


Figure2: Histogram of heuristic (H2) cost amelioration based on tabu search for different N values.

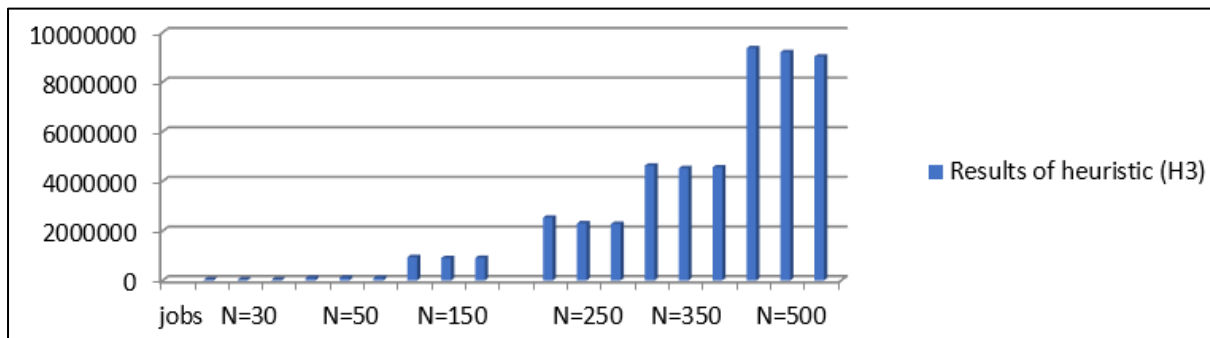


Figure3: Histogram of heuristic (H3) cost amelioration based on tabu search for different N values.

8. CONCLUSION

In this work, we propose a novel approach for scheduling problems on two parallel identical machines).

The developed approach uses a diversification technique based on search restarting from the point of the solution that was chosen among the earlier best unmaintained found solutions. According to the carried out tests, it can be concluded that the proposed heuristics ensure good results (polynomial complexity $o(n^3)$).

It must be noted that the heuristic (H2) and the neighborhood by (SWAP) present the best costs with an acceptable execution time.

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