TENSOR PRODUCTS OF WEAK HYPERRIGID SETS

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Reception: 17/10/2022 **Acceptance:** 01/11/2022 **Publication:** 29/12/2022

Suggested citation:

V. A. Anjali, Athul Augustine and P. Shankar (2022). Tensor products of weak hyperrigid sets *3C Empresa. Investigación y pensamiento crítico*, *11*(2), 164-171. https://doi.org/10.17993/3cemp.2022.110250.164-171

ABSTRACT

In this article, we show that concerning the spatial tensor product of W^*-algebras, the tensor product of two weak hyperrigid operator systems is weak hyperrigid. We prove this result by demonstrating unital completely positive maps have unique extension property for operator systems if and only if the tensor product of two unital completely positive maps has unique extension property for the tensor product of operator systems. Consequently, we prove as a corollary that the tensor product of two boundary representations for operator systems is boundary representation for the tensor product of operator systems. The corollary is an analogue result of Hopenwasser's [9] in the setting of W^*-algebras.

KEYWORDS

Operator system, W^*-algebra, Weak Korovkin set, Boundary representation.

2020 Mathematics Subject Classification: Primary 46L07; Secondary 46L06, 46L89.

https://doi.org/10.17993/3cemp.2022.110250.164-171

1 INTRODUCTION

Positive approximation processes play a fundamental role in approximation theory and appear naturally in many problems. In 1953, Korovkin [11] discovered the most powerful and, at the same time, the simplest criterion to decide whether a given sequence $\{\phi_n\}_{n\in\mathbb{N}}$ of positive linear operators on the space of complex-valued continuous functions $C(X)$, where X is a compact Hausdorff space is an approximation process. That is, $\phi_n(f) \to f$ uniformly on X for every $f \in C(X)$. In fact it is sufficient to verify that $\phi_n(f) \to f$ uniformly on X only for $f \in \{1, x, x^2\}$. This set is called a Korovkin set. Starting with this result, during the last thirty years, many mathematicians have extended Korovkin's theorem to other function spaces or, more generally, to abstract spaces such as Banach algebras, Banach spaces, C^*-algebras and so on. At the same time, strong and fruitful connections of this theory have been revealed with classical approximation theory and other fields such as Choquet boundaries, convexity theory, uniqueness of extensions of positive linear maps, and so on.

Here, we provide an expository review of the non-commutative analogue of Korovkin's theorems with weak operator convergence and norm convergence. The notion of *boundary representation* of a C^*-algebra for an *operator system* introduced by Arveson [2] greatly influenced the theory of noncommutative approximation theory and other related areas such as Korovkin type properties for completely positive maps, peaking phenomena for operator systems and noncommutative convexity, etc. Arveson [4] introduced the notion of *hyperrigid set* as a noncommutative analogue of classical Korovkin set and studied the relation between hyperrigid operator systems and boundary representations extensively.

In 1984, Limaye and Namboodiri [13] studied the non-commutative Korovkin sets on $B(H)$ using weak operator convergence, which they named weak Korovkin sets. Limaye and Namboodiri [13] proved an exciting result using a famous boundary theorem of Arveson [3] that is as follows: An irreducible subset of $B(H)$ containing identity and a nonzero compact operator is weak Korovkin in $B(H)$ if and only if the identity representation of the C^*-algebra generated by the irreducible set has a unique completely positive linear extension to the C^*-algebra when restricted to the irreducible set. Limaye and Namboodiri gave many examples to establish these notions and theorems.

Namboodiri, inspired by Arveson's paper [4] on hyperrigidity, modified [15] the notion of weak Korovkin set on $B(H)$ to weak hyperrigid set in the context of W^*-algebras using weak operator convergence. He generalized the theorem in [13], characterizing the weak Korovkin set without assuming the presence of compact operators, and explored all nondegenerate representations. The result is as follows: An operator system is weak hyperrigid in the W^*-algebra generated by it if and only if every nondegenerate representation has a unique completely positive linear extension to the W^*-algebra when restricted to the operator system. Using this theorem, he established the partial answer to the non-commutative analogue of Saskin's theorem [18] relating weak hyperrigidity and Choquet boundary. Namboodiri gave a brief survey of the developments in the 'non-commutative Korovkin-type theory' in [14]. Namboodiri, Pramod, Shankar, and Vijayarajan [16] studied the non-commutative analogue of Saskin's theorem using the notions quasi hyperrigidity and weak boundary representations. Shankar and Vijayarajan [21] proved that the tensor product of two hyperrigid operator systems is hyperrigid in the spatial tensor product of C^*-algebras. Arunkumar and Vijayarajan [1] studied the tensor products of quasi hyperrigid operator systems introduced in [16]. Shankar [19] established hyperrigid generators for certain C^*-algebras.

In this article, we study the weak hyperrigidity of operator systems in W^*-algebras in the context of tensor products of W^*-algebras. It is interesting to investigate whether the tensor product of weak hyperrigid operator systems is weak hyperrigid. As a result of Hopenwasser [9], the tensor product of boundary representations of C^*-algebras for operator systems is a boundary representation if one of the constituent C^*-algebras is a GCR algebra. Since weak hyperrigidity implies that all irreducible representations are boundary representations for W^*-algebra, we will be able to deduce Hopenwasser's result for W^*-algebras as a spacial case. We achieve this by establishing first that unique extension property for unital completely positive maps on operator systems carry over to the tensor product of those maps defined on the tensor product of operator systems in the spatial tensor product of W^*-algebras.

2 PRELIMINARIES

To fix our notation and terminology, we recall the fundamental notions. Let H be a complex Hilbert space and let $B(H)$ be the bounded linear operators on H. An *operator system* S in a W^*-algebra $\mathcal{M}$ is a self-adjoint linear subspace of $\mathcal{M}$ containing the identity of $\mathcal{M}$. An *operator algebra* A in a W^*-algebra $\mathcal{M}$ is a subalgebra of $\mathcal{M}$ containing the identity of $\mathcal{M}$.

Let ϕ be a linear map from a W^*-algebra $\mathcal{M}$ into a W^*-algebra $\mathcal{N}$, we can define a family of maps $\phi_n : M_n(\mathcal{M}) \to M_n(\mathcal{N})$ given by $\phi_n([a_{ij}]) = [\phi(a_{ij})]$. We say that ϕ is *completely bounded* (CB) if $||\phi||_{\text{CB}} = \sup_{n\geq 1} ||\phi_n|| < \infty$. We say that ϕ is *completely contractive* (CC) if $||\phi||_{\text{CB}} \leq 1$ and that ϕ is *completely isometric* if ϕ_n is isometric for all $n \geq 1$. We say that ϕ is *completely positive* (CP) if ϕ_n is positive for all $n \geq 1$, and that ϕ is *unital completely positive* (UCP) if in addition $\phi(1) = 1$.

Definition 1. *[2] Let S be an operator system in a W^*-algebra $\mathcal{M}$. A nondegenerate representation $\pi : \mathcal{M} \to B(H)$ has a unique extension property (UEP) for S if $\pi_{|_S}$ has a unique completely positive extension, namely π itself to $\mathcal{M}$. If π is an irreducible representation, then π is said to be a boundary representation for S.*

Definition 2. *[15] A set G of generators of a W^*-algebra $\mathcal{M}$ containing the identity $1_{\mathcal{M}}$ is said to be weak hyperrigid if for every faithful representation $\mathcal{M} \subseteq B(H)$ of $\mathcal{M}$ on a separable Hilbert space H and every net $\{\phi_\alpha\}_{\alpha\in I}$ of contractive completely positive maps from $B(H)$ to itself.*

$$\lim_{\alpha} \phi_\alpha(g) = g \textit{ weakly } \forall\ g \in G \Longrightarrow \lim_{\alpha} \phi_\alpha(a) = a \textit{ weakly } \forall\ a \in \mathcal{M}.$$

Theorem 1. *[15] For every separable operator system S, that generates a W^*-algebra $\mathcal{M}$, the following are equivalent.*

(i) S is weak hyperrigid.

(ii) For every nondegenerate representation $\pi : \mathcal{M} \to B(H)$, on a separable Hilbert space H and every net $\{\phi_\alpha\}_{\alpha\in I}$ of contractive completely positive maps from $\mathcal{M}$ to $B(H)$.
$\lim_\alpha \phi_\alpha(s) = \pi(s)$ weakly $\forall\ s \in S \Longrightarrow \lim_\alpha \phi_\alpha(a) = \pi(a)$ weakly $\forall\ a \in \mathcal{M}$.

(iii) For every nondegenerate representation $\pi : \mathcal{M} \to B(H)$ on a separable Hilbert space H, $\pi_{|_S}$ has a unique extension property.

(iv) For every W^*-algebra $\mathcal{N}$, every homomorphism $\theta : \mathcal{M} \to \mathcal{N}$ such that $\theta(1_{\mathcal{M}}) = 1_{\mathcal{N}}$ and every contractive completely positive map $\phi : \mathcal{N} \to \mathcal{N}$,
$\phi(x) = x\ \forall\ x \in \theta(S) \Longrightarrow \phi(x) = x\ \forall\ x \in \theta(\mathcal{M})$.

In this context, mentioning the 'hyperrigidity conjecture' posed by Arveson [4] is relevant. The hyperrigidity conjecture states that if every irreducible representation of a C^*-algebra A is a boundary representation for a separable operator system $S \subseteq A$ and $A = C^*(S)$, then S is hyperrigid. Arveson [4] proved the conjecture for C^*-algebras having a countable spectrum, while Kleski [10] established the conjecture for all type-I C^*-algebras with some additional assumptions. Recently Davidson and Kennedy [6] proved the conjecture for function systems.

Using the apparent correspondence between representations and modules, one can translate many aspects of the above notions into Hilbert modules. Muhly and Solel [12] gave an algebraic characterization of boundary representations in terms of Hilbert modules. Following Muhly and Solel, Shankar and Vijayarajan [20, 22] established a Hilbert module characterization for hyperrigidity (weak hyperrigidity) of specific operator systems in a C^*-algebra (W^*-algebras).

We need to consider tensor products of W^*-algebras in this article. Let $A_1 \otimes A_2$ denote the algebraic tensor product of A_1 and A_2. Let $A_1 \otimes_s A_2$ denote the closure of $A_1 \otimes A_2$ provided with the spatial norm, which is the minimal C^*-norm on the tensor product of W^*-algebras. In what follows, we will consider the spatial norm for the tensor product of W^*-algebras. We know that if representations π_1 is nondegenerate on A_1 and π_2 is nondegenerate on A_2, then the representation $\pi_1 \otimes \pi_2$ is nondegenerate

https://doi.org/10.17993/3cemp.2022.110250.164-171

on $A_1 \otimes A_2$. Conversely, from [5, Theorem II.9.2.1] and [17, Proposition 1.22.11] we can see that if π is a nondegenerate representation of $A_1 \otimes A_2$, then there are unique nondegenerate representations π_1 of A_1 and π_2 of A_2 such that $\pi = \pi_1 \otimes \pi_2$.

Tensor products of operator spaces (linear subspaces) of C^*-algebras and operator spaces of tensor product of C^*-algebras were explored by Hopenwasser earlier in [8], and [9] to study boundary representations. In [8], it was shown that boundary representations of an operator subspace of a C^*-algebra $A \otimes M_n(\mathbb{C})$ under certain conditions are parameterized by the boundary representations of an operator subspace of the C^*-algebra A which is given by the operator subspace in $A \otimes M_n(\mathbb{C})$. In [9], it was proved that if one of the C^*-algebras of the tensor product is a GCR algebra, then the boundary representations of the tensor product of C^*-algebras correspond to products of boundary representations.

3 MAIN RESULTS

In our main result, we investigate the relationship between the weak hyperrigidity of the tensor product of two operator systems in the tensor product W^*-algebra and the weak hyperrigidity of the individual operator systems in the respective W^*-algebras. The following result shows that the unique extension property of completely positive maps on operator systems carries over to the tensor product of those maps defined on the tensor product of operator systems.

Theorem 2. *Let S_1 and S_2 be operator systems generating W^*-algebras A_1 and A_2 respectively. Let $\pi_i : S_i \to B(H_i), i = 1, 2$ be unital completely positive maps. Then π_1 and π_2 have unique extension property if and only if the unital completely positive map $\pi_1 \otimes \pi_2 : S_1 \otimes S_2 \to B(H_1 \otimes H_2)$ has unique extension property for $S_1 \otimes S_2 \subseteq A_1 \otimes_s A_2$.*

Proof. Assume that $\pi_1 \otimes \pi_2$ has unique extension property, that is $\pi_1 \otimes \pi_2$ has unique completely positive extension $\widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2 : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$ which is a representation of $A_1 \otimes_s A_2$. We will show that π_1 and π_2 have unique extension property. On the contrary, assume that one of the factors, say π_1 does not have unique extension property. This means that there exist at least two extensions of π_1, a completely positive map $\phi_1 : A_1 \to B(H_1)$ and the representation $\widetilde{\pi}_1 : A_1 \to B(H_1)$ such that $\phi_1 \neq \widetilde{\pi}_1$ on A_1, but $\phi_1 = \widetilde{\pi}_1 = \pi_1$ on S_1. Using [5, II.9.7], we can see that the tensor product of two completely positive maps is completely positive. We have $\phi_1 \otimes_s \widetilde{\pi}_2$ is a completely positive extension of $\pi_1 \otimes \pi_2$ on $S_1 \otimes S_2$, where $\widetilde{\pi}_2$ is a unique completely positive extension of π_2 on S_2. Hence $\phi_1 \otimes_s \widetilde{\pi}_2 \neq \widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2$ on $A_1 \otimes_s A_2$. This contradicts our assumption.

Conversely, assume that π_1 and π_2 have the unique extension property, that is π_1 and π_2 have unique completely positive extensions $\widetilde{\pi}_1 : A_1 \to B(H_1)$ and $\widetilde{\pi}_2 : A_2 \to B(H_2)$ respectively where $\widetilde{\pi}_1$ and $\widetilde{\pi}_2$ are representations of A_1 and A_2 respectively. We will show that $\pi_1 \otimes \pi_2$ has the unique extension property. We have $\widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2 : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$ is a representation and an extension of $\pi_1 \otimes \pi_2$ on $S_1 \otimes S_2$. It is enough to show that if $\phi : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$ is a completely positive extension of $\pi_1 \otimes \pi_2$ on $S_1 \otimes S_2$ then $\phi = \widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2$ on $A_1 \otimes A_2$.

Let P be any rank one projection in $B(H_2)$. The map $a \to (1 \otimes P)\phi(a \otimes 1)(1 \otimes P)$ is completely positive on A_1, since the map is a composition of three completely positive maps. Let v be a unit vector in the range of P and let K be the range of $1 \otimes P$. Define $U : H_1 \to K$ by $U(x) = x \otimes v,\ x \in H_1$, U is a unitary map. Let $\hat{\pi} = U\widetilde{\pi}_1(a)U^*,\ a \in A_1$ and $\hat{\pi}(a)$ is the restriction to K of $\widetilde{\pi}_1(a) \otimes P = (1 \otimes P)(\widetilde{\pi}_1(a) \otimes 1)(1 \otimes P)$. Since $\hat{\pi}$ is unitarily equivalent to $\widetilde{\pi}_1$, the representation $\hat{\pi}_{|S_1}$ has unique extension property. Let $\psi(a)$ be the restriction to K of $(1 \otimes P)\phi(a \otimes 1)(1 \otimes P)$ which implies that ψ is a completely positive map that agrees with $\hat{\pi}$ on S_1, hence on all of A_1.

Let $x, y \in H_1$ and $r \in H_2$. From the above paragraph we have, for any $a \in A_1$, $\langle \phi(a \otimes 1)(x \otimes r), y \otimes r \rangle = \langle (\widetilde{\pi}_1(a) \otimes 1)(x \otimes r), y \otimes r \rangle$. (Letting P be the rank one projection on the subspace spanned by r.) Let $D = \phi(a \otimes 1) - \widetilde{\pi}_1 \otimes 1$. Then we have $\langle D(x \otimes r), y \otimes r \rangle = 0$, for all $x, y \in H_1$, $r \in H_2$. Using polarization formula

https://doi.org/10.17993/3cemp.2022.110250.164-171

$$\begin{aligned} 4\langle D(x\otimes r), y\otimes s\rangle &= \langle D(x\otimes(r+s)), y\otimes(r+s)\rangle \\ &\quad -\langle D(x\otimes(r-s)), y\otimes(r-s)\rangle \\ &\quad +i\langle D(x\otimes(r+is)), y\otimes(r+is)\rangle \\ &\quad -i\langle D(x\otimes(r-is)), y\otimes(r-is)\rangle . \end{aligned}$$

We have $\langle D(x\otimes r), y\otimes s\rangle = 0$, for all $x, y \in H_1$ and for all $r, s \in H_2$. Consequently, if $z_1 = \sum_{i=1}^{n} x_i \otimes r_i$ and $z_2 = \sum_{i=1}^{m} y_i \otimes s_i$, then $\langle Dz_1, z_2\rangle = 0$. Since z_1, z_2 run through a dense subset of $H_1 \otimes H_2$ and D is bounded, $D = 0$. Therefore $\phi(a\otimes 1) = \widetilde{\pi}_1(a)\otimes 1$, for all $a \in A_1$. In the same way we can obtain $\phi(1\otimes b) = 1\otimes\widetilde{\pi}_2(b)$, for all $b \in A_2$. Since ϕ is a completely positive map on $A_1\otimes A_2$ and $\phi(1\otimes b) = 1\otimes\widetilde{\pi}_2(b)$, for all $b \in A_2$, using a multiplicative domain argument, e.g., see [9, Lemma 2] we have

$$\phi(a\otimes b) = \phi(a\otimes 1)(1\otimes\widetilde{\pi}_2(b)) = (1\otimes\widetilde{\pi}_2(b))\phi(a\otimes 1)$$

for all $a \in A_1$, $b \in A_2$. Also $\phi(a\otimes 1) = \widetilde{\pi}_1(a)\otimes 1$, for all $a \in A_1$. Hence $\phi = \widetilde{\pi}_1 \otimes_s \widetilde{\pi}_2$ on $A_1 \otimes_s A_2$.

Corollary 1. *Let S_1 and S_2 be separable operator systems generating W^*-algebras A_1 and A_2 respectively. Assume that either A_1 or A_2 is a GCR algebra. Then S_1 and S_2 are weak hyperrigid in A_1 and A_2 respectively if and only if $S_1 \otimes S_2$ is weak hyperrigid in $A_1 \otimes_s A_2$.*

Proof. Assume that $S_1 \otimes S_2$ is weak hyperrigid in the W^*-algebra $A_1 \otimes_s A_2$. By theorem 1, every unital representation $\pi : A_1 \otimes_s A_2 \to B(H_1 \otimes H_2)$, $\pi_{|S_1\otimes S_2}$ has unique extension property. We have if π is a unital representation of $A_1 \otimes_s A_2$, since one of the W^*-algebras is GCR then by [7, Proposition 2] there are unique unital representations π_1 of A_1 and π_2 of A_2 such that $\pi = \pi_1 \otimes_s \pi_2$. Using theorem 2, we can see that $\pi_{1|S_1}$ and $\pi_{2|S_2}$ have unique extension property. This implies that S_1 and S_2 are weak hyperrigid in A_1 and A_2 respectively again by theorem 1.

Conversely, assume that S_1 is weak hyperrigid in A_1 and S_2 is weak hyperrigid in A_2. By theorem 1, for every unital representations $\pi_1 : A_1 \to B(H_1)$ and $\pi_2 : A_2 \to B(H_2)$, $\pi_{1|S_1}$ and $\pi_{2|S_2}$ have unique extension property. We have, if π_1 and π_2 are unital representations of A_1 and A_2 respectively, then $\pi_1 \otimes_s \pi_2$ is an unital representation of $A_1 \otimes_s A_2$. Using theorem 2, we can see that $\pi_1 \otimes_s \pi_{2|S_1\otimes S_2}$ has unique extension property. Now, by theorem 1 $S_1 \otimes S_2$ is weak hyperrigid in $A_1 \otimes_s A_2$.

Let $A_1 \otimes_m A_2$ denote the closure of $A_1 \otimes A_2$ provided with maximal C^*-norm. There are C^*-algebras A_1 for which the minimal and the maximal norm on $A_1 \otimes A_2$ coincide for all C^*-algebras A_2 and consequently the C^*-norm on $A_1 \otimes A_2$ is unique. Such C^*-algebras are called nuclear. The spatial norm assumption in the above results is redundant if the C^*-algebras are nuclear. But general C^*-algebras with the lack of injectivity associated with other C^*-norms, including the maximal one, will require additional assumptions.

Let A_1 and A_2 be W^*-algebras and γ is any C^*-cross norm on $A_1 \otimes A_2$. If π_1 and π_2 are irreducible representations of A_1 and A_2 respectively, then $\pi_1 \otimes_\gamma \pi_2$ is an irreducible representation of $A_1 \otimes_\gamma A_2$. Conversely, every irreducible representation π on $A_1 \otimes_\gamma A_2$ need not factor as a product $\pi_1 \otimes_\gamma \pi_2$ of irreducible representations. If we assume, one of the W^*-algebra is a GCR algebra, and then by [7, Proposition 2], every irreducible representation does factor. Since GCR algebras are nuclear, there is a unique C^*-cross norm on $A_1 \otimes A_2$, which we denote by $A_1 \otimes_\gamma A_2$.

Using the above facts, the result by Hopenwasser [9] relating boundary representations of tensor products of C^*-algebras will become a corollary to our theorem 2.

Corollary 2. *Let S_1 and S_2 be unital operator subspaces of generating W^*-algebras A_1 and A_2 respectively. Assume that either A_1 or A_2 is a GCR algebra. Then the representation $\pi_1 \otimes_\gamma \pi_2$ of $A_1 \otimes_\gamma A_2$ is a boundary representation for $S_1 \otimes S_2$ if and only if the representations π_1 of A_1 and π_2 of A_2 are boundary representations for S_1 and S_2 respectively.*

Now, we will provide some examples which illustrate the results above.

https://doi.org/10.17993/3cemp.2022.110250.164-171

Example 1. *Let $G =$ linear span(I, S, S^*), where S is the unilateral right shift in $B(H)$ and I is the identity operator. Let $A = C^*(G)$ be the C^*-algebra generated by G. We have $K(H) \subseteq A$, $A/K(H) \cong C(\mathbb{T})$ is commutative, where $\mathbb{T}$ denotes the unit circle in $\mathbb{C}$. Let Id denotes the identity representation of the C^*-algebra A. Let $S^*Id(\cdot)S$ be a completely positive map on the C^*-algebra A such that $S^*IdS_{|_G} = Id_{|_G}$, it is easy to see that $S^*IdS_{|_A} \neq Id_{|_A}$. Therefore the unital representation $Id_{|_G}$ does not have unique extension property. Using [15, Theorem 3.1], we conclude that G is not a weak hyperrigid operator system in a W^*-algebra $B(H)$.*

Let $G_1 = G$, $A_1 = A$ and Id_1 denotes the identity representation of A_1. Let $G_2 = A_2 = M_n(\mathbb{C})$ and Id_2 denotes the identity representation of the C^*-algebra A_2. The completely positive map $S^*Id_1S \otimes Id_2$ on the C^*-algebra $A_1 \otimes A_2$ is such that $S^*Id_1S \otimes Id_2 = Id_1 \otimes Id_2$ on operator system $G_1 \otimes G_2$. By the above conclusion we see that $S^*Id_1S \otimes Id_2 \neq Id_1 \otimes Id_2$ on the C^*-algebra $A_1 \otimes A_2$. Therefore the unital representation $Id_1 \otimes Id_2$ does not have unique extension property for $G_1 \otimes G_2$. Hence by theorem [15, Theorem 3.1], $G_1 \otimes G_2$ is not a weak hyperrigid operator system in a W^*-algebra $B(H) \otimes M_n(\mathbb{C})$.

Example 2. *Let the Volterra integration operator V acting on the Hilbert space $H = L^2[0,1]$ be given by*

$$Vf(x) = \int_0^x f(t)dt, \qquad f \in L^2[0,1].$$

V generates the C^*-algebra $K = K(H)$ of all compact operators. Let $S =$ linear span(V, V^*, V^2, V^{2*}) and S is weak hyperrigid [4, Theorem 1.7] and [15, Theorem 3.1] in W^*-algebra $B(H)$. Let $S_1 = S_2 = S$ and $A_1 = A_2 = B(H)$. We know that S_1 and S_2 are weak hyperrigid operator systems in the W^*-algebra A_1 and A_2 respectively. By corollary 1 we conclude that $S_1 \otimes S_2$ is weak hyperrigid operator system in the W^*-algebra $A_1 \otimes A_2$.

Example 3. *Let $G =$ linear span(I, S, S^*, SS^*), where S is the unilateral right shift in $B(H)$ and I is the identity operator. Let $A = C^*(G)$ be the C^*-algebra generated by the operator system G. We have, $K(H) \subseteq A$. $A/K(H) \cong C(\mathbb{T})$ is commutative, where $\mathbb{T}$ denotes the unit circle in $\mathbb{C}$. Since S is an isometry, G is a weak hyperrigid operator system in the W^*-algebra $B(H)$ [15, Theorem 3.1]. Let $G_1 = G$, $A_1 = B(H)$ and $G_2 = A_2 = M_n(\mathbb{C})$. It is clear that G_2 is a weak hyperrigid operator system in the W^*-algebra $A_2 = M_n(\mathbb{C})$. By corollary 1, $G \otimes M_n(\mathbb{C})$ is a weak hyperrigid operator system in $B(H) \otimes M_n(\mathbb{C})$.*

ACKNOWLEDGMENT

The Second author thanks the Council of Scientific & Industrial Research (CSIR) for providing a doctoral fellowship. The third author thanks Cochin University of Science and Technology for granting the project under Seed Money for New Research Initiatives (order No. CUSAT/PL(UGC).A1/1112/2021) dated 09.03.2021.

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