FIXED POINT THEOREMS IN THE GENERALIZED RATIONAL TYPE OF C-CLASS FUNCTIONS IN B-METRIC SPACES WITH APPLICATION TO INTEGRAL EQUATION

Mehdi Asadi
Department of Mathematics, Zanjan Branch, Islamic Azad University, Zanjan, (Iran)
E-mail:masadi.azu@gmail.com; masadi@iau.ac.ir
ORCID: https://orcid.org/0000-0003-2170-9919
Corresponding author

Mehdi Afshar
Department of Mathematics, Zanjan Branch, Islamic Azad University, Zanjan, (Iran)
E-mail:afshar1979@yahoo.com
ORCID: https://orcid.org/0000-0003-3764-2704

Reception: 14/08/2022 **Acceptance:** 29/08/2022 **Publication:** 29/12/2022

Suggested citation:

Medhi Asadi and Medhi Afshar (2022). Fixed point theorems in the generalized rational type of C-class functions in b-metric spaces with Application to Integral Equation. *3C Empresa. Investigación y pensamiento crítico*, *11*(2), 64-74. https://doi.org/10.17993/3cemp.2022.110250.64-74

ABSTRACT

In this paper, we study some results of existence and uniqueness of fixed points for a C-class of mappings satisfying an inequality of rational type in b-metric spaces. After definition of C-class functions covering a large class of contractive conditions by Ansari [2]. Our results extend very recent results in the literature; as well as Khan in [14] and later Fisher in [9] gave a revised improved version of Khan's result and Piri in [17] a new generalization of Khan's Theorem. At the end, we present an example of finding solutions for an integral equation.

KEYWORDS

Metric space; fixed point, C-function.

1 INTRODUCTION

In 1989, Bakhtin [4] introduced b-metric spaces as a generalization of metric spaces. Since then, many articles have been published in the field of fixed point theory and Banach generalization. The contraction principle in such spaces, known as Banach's contraction principle, states that every self-contracting mapping in a complete metric space has a unique fixed point. This principle has been generalized and expanded in several ways.

In this paper, we study certain results of existence and uniqueness of fixed points for a C-class of mappings satisfying an inequality of rational type in b-metric spaces. The C-class functions cover a large class of contractive conditions which applyed by Ansari [2]. Our results develop very recent results in the literature; as well as Khan in [14] and later Fisher in [9] gave a revised improved version of Khan's result and Piri in [17] a new generalization of Khan's Theorem. At the end of the paper, we present examples of finding solutions for integral equations. For more detail and recent papers refer to [3, 7, 8, 11, 16, 18].

Definition 1 ([10]). *Let X be a (nonempty) set and $s \geq 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is called a b-metric on X if the following conditions hold for all $x, y, z \in X$:*

(i) $d(x, y) = 0$ *if and only if* $x = y$,

(ii) $d(x, y) = d(y, x)$,

(iii) $d(x, y) \leq s[d(x, z) + d(z, y)]$ *(b-triangular inequality).*

Then, the pair (X, d) is called a b-metric space with parameter s.

Example 1 ([15]). *Let (X, d) be a metric space and let $\beta > 1, \lambda \geq 0$ and $\mu > 0$. For $x, y \in X$, set $\rho(x, y) = \lambda d(x, y) + \mu d(x, y)^\beta$. Then (X, ρ) is a b-metric space with the parameter $s = 2^{\beta-1}$ and not a metric space on X.*

Definition 2 ([5]). *Let (X, d) be a $b-$metric space. Then a sequence $\{x_n\}$ in X is called:*

(i) $b-$convergent if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$. In this case, we write $\lim_{n\to\infty} x_n = x$.

(ii) A $b-$Cauchy sequence if $d(x_n, x_m) \to 0$ as $n, m \to \infty$.

Lemma 1 ([1]). *Let (x, d) be a b-metric space with s. If sequences $\{x_n\}$ and $\{y_n\}$ are b-convergent to x and y in X. Then*

$$\frac{1}{s^2} d(x, y) \leq \liminf_{n\to\infty} d(x_n, y_n) \leq \limsup_{n\to\infty} d(x_n, y_n) \leq s^2 d(x, y).$$

Specially if $x = y$, then $\lim_{n\to\infty} d(x_n, y_n) = 0$. Further if $z \in X$ we have

$$\frac{1}{s} d(x, z) \leq \liminf_{n\to\infty} d(x_n, z) \leq \limsup_{n\to\infty} d(x_n, z) \leq s d(x, z).$$

Definition 3 ([6]). *Let Ψ denote all functions $\psi : [0, \infty) \to [0, \infty)$ satisfied:*

(i) ψ *is strictly increasing and continuous,*

(ii) $\psi(t) = 0$ *if and only if* $t = 0$.

We let Ψ denote the class of the altering distance functions.

Definition 4 ([2]). *An ultra altering distance function is a continuous, nondecreasing mapping $\varphi : [0, \infty) \to [0, \infty)$ such that $\varphi(t) > 0$ for $t > 0$.*

https://doi.org/10.17993/3cemp.2022.110250.64-74

We let Φ denote the class of the ultra altering distance functions.

Zoran Kadelburg and et al. in [19], and Imdad and Ali [12, 13] defined and studied implicit functions and utilized same to prove several fixed point results for rational type condition. Also, in 2014 Ansari in [2] introduced C-type functions as follows:

Definition 5 ([2]). *A mapping $F : [0,\infty)^2 \to \mathbb{R}$ is called C-class function if it is continuous and satisfies following axioms:*

1. $F(s,t) \leq s$;
2. $F(s,t) = s$ *implies that either* $s = 0$ *or* $t = 0$.

It's clear that $F(0,0) = 0$. We denote C-class functions by $\mathcal{C}$.

Let

$$\Omega := \{\varphi \in C([0,\infty),[0,\infty)) : \varphi^{-1}(0) = 0\}.$$

Example 2 ([2]). *The following functions $F : [0,\infty)^2 \to \mathbb{R}$ are elements of $\mathcal{C}$, for all $s,t \in [0,\infty)$:*

1. $F(s,t) = s - t$.
2. $F(s,t) = \frac{s}{(1+t)^r}$; $r \in (0,\infty)$.
3. $F(s,t) = s - \varphi(s)$ *for* φ *where* $\varphi \in \Omega$

2 RESULTS

We improve the recent results by C-class functions to generalization of contractions in the cases of fractional contraction.

Theorem 1. *Let (X,d) a b-metric space with s and $T : X \to X$ a self mapping satisfying*

$$\psi(d(Tx,Ty)) \leq \begin{cases} g(x,y), & A_{xy} \neq 0; \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

for $x,y \in X$ where $x \neq y$ and

$$g(x,y) := F(\psi(m(x,y)), \varphi(m(x,y)))$$

$$m(x,y) := \frac{d(x,Tx)d(x,Ty) + d(y,Ty)d(y,Tx)}{d(x,Ty) + d(Tx,y)},$$

$$A_{xy} := \max\{d(x,Ty), d(Tx,y)\}$$

and $F \in \mathcal{C}, \varphi \in \Phi, \psi \in \Psi$. Then T has a unique fixed point.

Proof.

Let $x_0 \in X$. For $n \in \mathbb{N}$ put $x_n := Tx_{n-1}$. It's clear when for some $n \in \mathbb{N}$ we have $x_n = x_{n+1}$. So we assume that

$$x_n \neq x_{n+1} \quad \forall n \in \mathbb{N}.$$

Put

$$A_{n,n-1} = \max\{d(x_n, Tx_{n-1}), d(Tx_n, x_{n-1})\} \neq 0.$$

 https://doi.org/10.17993/3cemp.2022.110250.64-74

by (1) we get

$$\begin{aligned}
&\psi(d(x_n, x_{n+1})) \\
&= \psi(d(Tx_{n-1}, Tx_n)) \\
&\leq F(\psi(m(x_{n-1}, x_n)), \varphi(m(x_{n-1}, x_n))) \\
&\leq F(\psi(d(x_{n-1}, x_n)), \varphi(d(x_{n-1}, x_n))) \\
&\leq \psi(d(x_{n-1}, x_n)).
\end{aligned} \tag{2}$$

Now by increasing ψ and relation (2)

$$d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n).$$

Since the $\{d(x_{n-1}, x_n)\}$ in decreasing therefore for some r

$$\lim_{n\to\infty} d(x_{n-1}, x_n) = r. \tag{3}$$

Again by (2)

$$\psi(r) \leq F(\psi(r), \varphi(r)) \leq \psi(r).$$

So $\psi(r) = 0$ or $\varphi(r) = 0$ hence

$$\lim_{n\to\infty} d(x_{n-1}, x_n) = r = 0. \tag{4}$$

It's time to show that $\{x_n\}$ is a Cauchy sequence. If not, then for $\varepsilon > 0$ there exit $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ in $\{x_n\}$ and $m(k) > n(k) > k$ such that

$$d(x_{m(k)}, x_{n(k)}) \geq \varepsilon, d(x_{m(k)}, x_{n(k)-1}) < \varepsilon$$

and

$$\lim_{n\to\infty} d(x_{m(k)}, x_{n(k)+1}) = \lim_{n\to\infty} d(x_{m(k)+1}, x_{n(k)}) = \varepsilon. \tag{5}$$

$$\exists N \in \mathbb{N}\ \forall k \geq N \quad \max\{d(x_{m(k)}, x_{n(k)+1}), d(x_{m(k)+1}, x_{n(k)}) \geq \frac{\varepsilon}{2} > 0.$$

By take a look at (1), for each $k \geq N$

$$\begin{aligned}
\psi(d(x_{m(k)+1}, x_{n(k)+1})) &= \psi(d(Tx_{m(k)}, Tx_{n(k)})) \\
&\leq F(\psi(m(x_{m(k)}, x_{n(k)})), \varphi(m(x_{m(k)}, x_{n(k)}))) \\
&\leq \psi(d(x_{n-1}, x_n))
\end{aligned} \tag{6}$$

even by the relations (3) and (5)

$$\psi(\varepsilon) \leq F(\psi(\varepsilon), \varphi(\varepsilon)) \leq \psi(\varepsilon),$$

so $\varepsilon = 0$ which is not. Thus $\{x_n\}$ is a Cauchy sequence in complete b-metric space. Therefore it is convergent to some $x^* \in X$.

$$\lim_{n\to\infty} d(x_n, x^*) = 0. \tag{7}$$

If $x_{n+1} = Tx^*$ for some n, then

$$x^* = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} Tx^* = Tx^*.$$

So

$$\exists N \in \mathbb{N}\ \forall n \geq N \quad d(x_n, Tx^*) > 0,$$

and also

$$\max\{d(x_n, Tx^*), d(x^*, Tx_n) > 0.$$

From (1)

$$\begin{aligned} \psi(d(x_{n+1}, Tx^*)) &= \psi(d(Tx_n, Tx^*)) \\ &\leq F(\psi(m(x_n, x^*)).\varphi(m(x_n, x^*))) \\ &\leq \psi(d(x_{n-1}, x_n)) \end{aligned} \quad (8)$$

where

$$m(x_n, x^*) = \frac{d(x_n, Tx_n)d(x_n, Tx^*) + d(x^*, Tx^*)d(x^*, Tx_n)}{d(x_n, Tx^*) + d(Tx_n, x^*)}$$

and or

$$m(x_n, x^*) = \frac{d(x_n, x_{n+1})d(x_n, Tx^*) + d(x^*, Tx^*)d(x^*, x_{n+1})}{d(x_n, Tx^*) + d(x_{n+1}, x^*)}$$

by taking lim inf and lim sup from (8) and relation (4) and (7) we have:

$$\limsup_{n\to\infty} m(x_n, x^*) \leq 0 \times sd(x^*, Tx^*) + sd(x^*, Tx^*) \times 0 = 0$$

thus

$$\psi(\frac{1}{s}d(x^*, Tx^*)) \leq F(\psi(0), \varphi(0)) \leq \psi(0) = 0$$

and consequently $x^* = Tx^*$.
For an other condition may be to happen: for some $n \in \mathbb{N}$

$$A_{n,n-1} = \max\{d(x_n, Tx_{n-1}), d(Tx_n, x_{n-1})\} = 0.$$

So (1) thus $d(Tx_{n-1}, Tx_n) = 0$ $Tx_{n-1} = Tx_n$ means that $x_n = Tx_n$.
For uniqueness, let x^* and y^* be two fixed point of T such that $d(x^*, y^*) > 0$. By (1)

$$\begin{aligned} \psi(d(x^*, y^*)) &= \psi(d(Tx^*, Ty^*)) \\ &\leq F(\psi(m(x^*, y^*)), \varphi(m(x^*, y^*))) \\ &\leq F(\psi(0), \varphi(0)) \leq \psi(0) = 0, \end{aligned}$$

it should be $d(x^*, y^*) = 0$.

For the next result, it's enough $F(s,t) = \frac{s}{2+t}$.

Corollary 1. *Let (X, d) a b-metric space with s and $T : X \to X$ a selfmapping satisfying*

$$\psi(d(Tx, Ty)) \leq \begin{cases} g(x, y), & A_{xy} \neq 0; \\ 0, & otherwise \end{cases}$$

for $x, y \in X$ where $x \neq y$ and

$$g(x, y) := \frac{\psi(m(x, y))}{+\varphi(m(x, y))}$$

$$m(x, y) := \frac{d(x, Tx)d(x, Ty) + d(y, Ty)d(y, Tx)}{d(x, Ty) + d(Tx, y)},$$

$$A_{xy} := \max\{d(x, Ty), d(Tx, y)\}$$

and $\varphi \in \Phi, \psi \in \Psi$. Then T has a unique fixed point.

And if we put $F(s,t) = s - t$:

 https://doi.org/10.17993/3cemp.2022.110250.64-74

Corollary 2. *Let (X,d) a b-metric space with s and $T:X\to X$ a selfmapping satisfying*

$$\psi(d(Tx,Ty)) \leq \begin{cases} g(x,y), & A_{xy}\neq 0\ ; \\ 0, & otherwise \end{cases}$$

for $x,y\in X$ where $x\neq y$ and

$$g(x,y) := \psi(m(x,y)) - \varphi(m(x,y))$$

$$m(x,y) := \frac{d(x,Tx)d(x,Ty)+d(y,Ty)d(y,Tx)}{d(x,Ty)+d(Tx,y)},$$

$$A_{xy} := \max\{d(x,Ty),d(Tx,y)\}$$

and $\varphi\in\Phi,\psi\in\Psi$. Then T has a unique fixed point.

And also we put $F(s,t)=ks$, $\psi(t)=t$ and $s=1$, hence:

Corollary 3 ([14]). *Let (X,d) a b-metric space with s and $T:X\to X$ a selfmapping satisfying*

$$\psi(d(Tx,Ty)) \leq \begin{cases} g(x,y), & A_{xy}\neq 0; \\ 0, & otherwise \end{cases} \tag{9}$$

for $x,y\in X$ where $x\neq y$ and

$$g(x,y) := km(x,y)$$

$$m(x,y) := \frac{d(x,Tx)d(x,Ty)+d(y,Ty)d(y,Tx)}{d(x,Ty)+d(Tx,y)},$$

$$A_{xy} := \max\{d(x,Ty),d(Tx,y)\}.$$

Then T has a unique fixed point.

Our results improve the following Corollary in [17] which it is extension of the Khan's Theorem with $s=1$.

Corollary 4 ([17]). *Let (X,d) a b-metric space with s and $T:X\to X$ a self mapping satisfying*

$$\psi(d(Tx,Ty)) \leq \begin{cases} g(x,y), & A_{xy}\neq 0; \\ 0, & otherwise \end{cases}$$

for $x,y\in X$ where $x\neq y$ and

$$g(x,y) := m(x,y) - \varphi(m(x,y))$$

$$m(x,y) := \frac{d(x,Tx)d(x,Ty)+d(y,Ty)d(y,Tx)}{d(x,Ty)+d(Tx,y)},$$

$$A_{xy} := \max\{d(x,Ty),d(Tx,y)\}.$$

where $\varphi\in\Phi$. Then T has a unique fixed point.

3 Application to integral equation

In this section, we looking for the solutions of the following integral equation:

$$u(t) = g(t,u(t)) + \int_0^t G(t,s,u(s))ds, \quad t\in[0,\infty), \tag{10}$$

where $G:[0,\infty)\times[0,\infty)\times\mathbb{R}\to\mathbb{R}$ $g:[0,\infty)\times\mathbb{R}\to\mathbb{R}$ are continuous.

Let X be complete space $BC([0,\infty))$ containing of bounded, continuous and real valued functions on $[0,\infty)$ such that

$$\|u\| = \sup\{|u(t)| : t\in[0,\infty)\}.$$

 https://doi.org/10.17993/3cemp.2022.110250.64-74

Example 3. *We show that the following integral equation:*

$$(Tu)(t) = g(t, u(t)) + \int_0^t G(t, s, u(s))ds, \quad t \in [0, \infty), \tag{11}$$

has unique solution, if we have the under following conditions.

Let $d(u, v) = \|u - v\|^2$. So d is a b-metric with $s = 2$.

1. $\psi(t) = \varphi(t) = t$
2. $F(s, t) = s$
3. $\|u - v\| \leq \min\{\|u - Tu\|, \|v - Tv\|\}$
4. $|g(t, u(t)) - g(t, v(t))| \leq \frac{|u(t)-v(t)|}{a} \quad t \in [0, \infty)$
5. for $t \in [0, \infty)$
$$|\int_0^t \big(G(t, s, u(s)) - G(t, s, v(s))\big)ds| \leq \frac{\|u - v\|}{b}$$
6. $a, b > 0$, $c > 2$, $c = \min\{a, b\}$ and $k = \frac{4}{c^2}$.

At first we observe that

$$\begin{aligned} \|u - v\|^2(\|u - Tv\|^2 + \|Tu - v\|^2) &= \|u - v\|^2\|u - Tv\|^2 + \|u - v\|^2\|Tu - v\|^2 \\ &\leq \|u - Tu\|^2\|u - Tv\|^2 + \|Tv - v\|^2\|Tu - v\|^2, \end{aligned}$$

so

$$d(u, v) = \|u - v\|^2 \leq \frac{\|u - Tu\|^2\|u - Tv\|^2 + \|Tv - v\|^2\|Tu - v\|^2}{\|u - Tv\|^2 + \|Tu - v\|^2} = m(u, v).$$

And by $(a + b)^2 \leq 2(a^2 + b^2)$

$$|(Tu)(t) - (Tv)(t)| \leq |g(t, u(t)) - g(t, v(t))| + \int_0^t |G(t, s, u(s)) - G(t, s, v(s))|ds$$

$$\begin{aligned} |(Tu)(t) - (Tv)(t)|^2 &\leq (|g(t, u(t)) - g(t, v(t))| + \int_0^t |G(t, s, u(s)) - G(t, s, v(s))|ds)^2 \\ &\leq 2(|g(t, u(t)) - g(t, v(t))|^2 + (\int_0^t |G(t, s, u(s)) - G(t, s, v(s))|ds)^2) \\ &\leq 2\left(\frac{|u(t) - v(t)|}{a})^2 + (\frac{\|u - v\|}{b})^2\right) \leq 4\left(\frac{\|u - v\|}{c}\right)^2. \end{aligned}$$

So

$$\begin{aligned} d(Tu, Tv) &= \|Tu - Tv\|^2 \\ &\leq 4\left(\frac{\|u - v\|}{c}\right)^2 = kd(u, v) \leq km(u, v) \\ &\leq F(\psi(m(u, v)), \varphi(m(u, v))). \end{aligned}$$

Therefore integral equation (11) by the Theorem 1 has a unique answer.

Example 4. *Let*

$$Tu(t) = \frac{u(t)}{2} + \int_0^t e^{-(t-s)}\frac{u(s)}{3}ds. \tag{12}$$

 https://doi.org/10.17993/3cemp.2022.110250.64-74

So

$$Tu(t) = \frac{u(t)}{2} + e^{-t} * \frac{u(t)}{3} = u * \left(\frac{\delta(t)}{2} + \frac{e^{-t}}{3}\right), \quad g(t, u(t)) = \frac{u(t)}{2},$$

$$G(t, s, u(s)) = e^{-(t-s)}\frac{u(s)}{3}, \quad a = 2, \quad b = 3,$$

where $*$ *is convolution of* u *and* v*; i.e.,*

$$u(t) * v(t) = \int_0^t u(t-s)v(s)ds.$$

We see that

$$u(t) - Tu(t) = u(t) * \left(\frac{\delta(t)}{2} - \frac{e^{-t}}{3}\right), \quad v(t) - Tv(t) = v(t) * \left(\frac{\delta(t)}{2} - \frac{e^{-t}}{3}\right)$$

and

$$\begin{aligned}
\left|\int_0^t \big(G(t,s,u(s)) - G(t,s,v(s))\big)ds\right| &= \left|\int_0^t \left(e^{-(t-s)}\frac{u(s)}{3} - e^{-(t-s)}\frac{v(s)}{3}\right)ds\right| \\
&\le \int_0^t e^{-(t-s)}\frac{\|u-v\|}{3}ds \\
&\le \frac{\|u-v\|}{3}(1-e^{-t}) \\
&\le \frac{\|u-v\|}{3}
\end{aligned}$$

where $\delta(t)$ *is Dirichlet function with Laplace transformation* $L(\delta) = 1$.

$$\begin{aligned}
|Tu(t) - Tv(t)| &= \left|u(t) * \left(\frac{\delta(t)}{2} + \frac{e^{-t}}{3}\right) - v(t) * \left(\frac{\delta(t)}{2} + \frac{e^{-t}}{3}\right)\right| \\
&= \left|(u(t) - v(t)) * \left(\frac{\delta(t)}{2} + \frac{e^{-t}}{3}\right)\right| \\
&\le \frac{5}{6}|u(t) - v(t)|,
\end{aligned}$$

without of loss of generality, let $|u(t)| \le |v(t)|$ *for all* $t \in [0, \infty)$.

$$\left|u(t) * \left(\frac{\delta(t)}{2} - \frac{e^{-t}}{3}\right)\right| \le \left|v(t) * \left(\frac{\delta(t)}{2} - \frac{e^{-t}}{3}\right)\right|$$

therefore $|Tu(t) - u(t)| \le |Tv(t) - v(t)|$.

$$\begin{aligned}
|u(t) - v(t)| &\le |u(t) - Tu(t)| + |Tu(t) - Tv(t)| + |Tv(t) - v(t)| \\
&\le |u(t) - Tu(t)| + \frac{5}{6}|u(t) - v(t)| + |Tv(t) - v(t)| \\
\frac{1}{6}|u(t) - v(t)| &\le |u(t) - Tu(t)| + |Tv(t) - v(t)|
\end{aligned}$$

so for some positive number l *we have*

$$\|u - v\| \le 4l \min\{\|u - Tu\|, \|v - Tv\|\}.$$

All conditions of Example (3) *with* $F(r, s) = 4ls$ *hold, and integral equation* (12) *has a unique solution* $u = 0$.

Availability of data and material

Not applicable.

https://doi.org/10.17993/3cemp.2022.110250.64-74

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have read and approved the final manuscript.

ACKNOWLEDGMENT

We would like to thank the referees for their comments and suggestions on the manuscript.

REFERENCES

[1] **Aghajani, A.**,**Abbas, M.**, and **Roshan, J.**, (2014). "Common fixed point of generalized weak contractive mappings in partially ordered b-metric spaces." *Mathematica Slovaca, 64, 2*), 941–960.

[2] **Ansari, A.H.**, (2014). "Note on $\varphi - \phi$-contractive type mappings and related fixed point. *The 2nd regional conference on mathematics and applications", Payame Noor University*, 377–380.

[3] **Asadi, M.**, **Moeini, B.**, **Mukheimer, A.**, and **Hassen, A.**, (2019). Complex valued $M-$metric spaces and related fixed point results via complex C-class function. *Journal of Inequalities and Special Functions, 10, 1*, 101–110.

[4] **Bakhtin, I. A.**, (1989). The contraction mapping principle in almost metric space. *Funct. Anal. Gos. Ped. Inst. Unianowsk, 30*, 26–37.

[5] **Boriceanu, M.**, **Bota, M.**, and **Petrusel, A.**, (2010). "Mutivalued fractals in b-metric spaces". *Cent. Eur. J. Math.*, 367–377.

[6] **Choudhury, B. S.**, (2005). "A common unique fixed point result in metric spaces involving generalised altering distances." *Mathematical Communications, 10, 2*, 105–110.

[7] **Eshraghisamani, M.**, **Vaezpour,S.M.**, and **Asadi, M.**, (2-18). New fixed point results with $\alpha_{qs^p}-$admissible contractions on b–Branciari metric spaces. *Journal of Inequalities and Special Functions, 9, 3*, 38–46.

[8] **Eshraghisamani, M.**, **Vaezpour, S.M.**, and **Asadi, M.**, (2017). New fixed point results on Branciari metric spaces. *Journal of Mathematical Analysis, 8, 6*, 132–141.

[9] **B. Fisher**, (1978). "On a theorem of Khan". *Riv. Math. Univ. Parma. 4*, 35–137.

[10] **Faraji, H.**, **Savic, D.**, and **Radenovic, S.**, (2019). "Fixed Point Theorems for Geraghty Contraction Type Mappings in b-Metric Spaces and Applications". *Axioms, 8*, 484–490.

[11] **Ghezellou, S.**, **Azhini,M.**, and **Asadi, M.**, (2021). Best proximity point theorems by k, c and mt types in b–metric spaces with an application. *Int. J. Nonlinear Anal. Appl., 12, 2*, 1317-1329.

[12] **Ali, J.**, and **Imdad, M.**, (2008). An implicit function implies several contraction conditions. *Sarajevo J. Math. 4, 2*, 269–285.

[13] **Ali, J.** and **Imdad, M.**, (2009). Common fixed point theorems in symmetric spaces employing a new implicit function and common property (E.A), Bull. Belg. Math. Soc. Simon Stevin **16**(3), 421–433.

[14] **Khan, M.S.** , **Swaleh, M.** and **Sessa, S.**, (1984). "Fixed point theorems by altering distances between the points". *Bull. Austral. Math. Soc. 30, 1*, 1–9.

 https://doi.org/10.17993/3cemp.2022.110250.64-74

[15] **Kirk, W.** and **Shahzad, N.**, (2014). "Fixed point Theory in Distance Spaces". *Springer: Berlin, Germany.*

[16] **Monfared, H.**, **Azhini, M.**, and **Asadi, M.**, (2017). C-class and $F(\psi,\varphi)$-contractions on M–metric spaces. *Int. J. Nonlinear Anal. Appl., 8, 1*, 209-224.

[17] **H. Piri**, **S. Rahrovi** and **P. Kumam**, (2017). "Generalization of Khan fixed point theorem". *J. Math. Computer Sci., 17*, 76–83.

[18] **Younis, M.**, **Singh, D.**, **Asadi, M.**, and **Joshi, V.**, (2019).. Results on contractions of Reich type in graphical b–metric spaces with applications. *FILOMAT, 33, 17*, 5723–5735.

[19] **Kadelburg, Z.**, **Mohammad Imdad**, and **Sunny Chauhanm**, (1999). Unified common fixed point theorems under weak reciprocal continuity or without continuity. *Demonstratio Math., 32, 1*, 157-163.