

# TWO-COMMODITY PERISHABLE INVENTORY SYSTEM WITH PARTIAL BACKLOG DEMANDS

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## ABSTRACT

*This article examines a two-commodity continuous review perishable inventory system. The demands are arrived at for each product by independent Markovian arrival processes (MAP). Lifetimes follow an exponential distribution. The commodities are assumed to be substitutable. If both commodities have reached zero, demand is backlogged up to predetermined levels. This article's novelty has been a local purchase, which is made to clear the backlog instantaneously when demand reaches a predetermined level. In the steady-state, the joint probability distribution of inventory levels of both commodities is obtained. Several metrics of system performance in steady-state are derived and also provided as numerical examples to explain the optimum values of the system's parameters.*

## KEYWORDS

*Two-commodity Inventory system, Substitutable items, Joint ordering Policy, Markov arrival demands, Partial backlog.*

# 1 INTRODUCTION

Over the last few decades, researchers have been fascinated with the study of a two-commodity inventory system. It has more importance because these systems are more sophisticated than single commodity inventory systems due to the large number of items held and their coordinated behaviors. Also, many organisations have increasingly used multi-commodity inventory systems. However, the correlation of the reorder points for each item is the major challenge in multi-product inventory systems. Unlike systems that deal with a single commodity, the reordering methods in these systems are more complicated. So, replenishment orders for groups of products must be well coordinated. Initially, research focused on inventory models with independently defined reorder points. The individual ordering policy includes calculating the best order quantity and reordering duration for each item. This ordering policy implementation provides the system with significant flexibility in picking the appropriate inventory models for each item and separately modifying the policy. However, joint ordering policies are preferred over individual ordering policies when the products share the same storage space and transportation facilities. Joint replenishment has several advantages because the joint ordering policy allows for the simultaneous replenishment of several commodities, quantity discounts, and significant savings in ordering and purchasing expenses. The joint replenishment was proposed by Balintfy and developed by Silver. More details about joint replenishment can be seen in Anbazhagan et al. (2012, 2015), Senthil Kumar, and Sivakumar. Various models with two-commodity readers can read in Anbazhagan and Arivarignan, Benny et al., Krishnamoorthy et al., and Ozkar et al.

In the earlier literature on inventory systems, it has generally been recognized that inventory models built under the presumption of a product's lifetime being indefinite until its storage, i.e., an item once placed in a storeroom stays unmodified and entirely functional for supplying future demand. However, this is not the case. When constructing inventory models, one aspect for consideration is an item's perishability, as commodities do not necessarily retain their properties when held for future use. In general, perishability is the outcome of stock depletion, which consists of obsolescence, breakage, decay, losing usefulness, and many other factors. Some examples of perishable objects are meals, evaporative fluids, chemicals, drugs, and radioactive substances. For more details about perishable product readers can refer Karthikeyan and Sudesh, Nahmias, Sivakumar et al., Smrutirekha Debataa et al., Umay and Bahar, Yadavalli et al. (2010, 2015), Zhang et al.

Several research articles examine inventory systems in which required products are directly provided from stock if the item is available. Demand that appears during stock-out times results in either lost sales or a backlog (demand satisfied immediately after the arrival of ordered items). Initially, it is believed that there is a total backlog of unfilled demand. In actuality, many customers are willing to wait until the end of the shortage period to pick up their orders, while others are not. As a result, it is presumed that any predefined quantity of demand (partial backlog) that appeared during the stock-out time is satisfied. For more details about backlog concept readers can refer Adak Sudip and Mahapatra, Cárdenas-Barrón Leopoldo et al., Khan et al., Kurt et al., San José et al., Stanley et al. and Tai et al. Generally, customer satisfaction generates a lot of profit for the system. So the shopkeeper does the maximum amount of work to satisfy the customers. In a practical situation, the local purchase is made by the shopkeeper when the shop runs out of stock and that item's replenishment has been delayed. We can see this act in clothing stores, supermarkets, and all the retailers' shops.

In this article we assume that demands during the stock-out periods are backlogged. We further assume that when the number of backlogged demands reaches a prefixed level a local purchase is made to clear the backlog instantaneously so that the inventory level of the corresponding commodity becomes zero. In the following sections, We have obtained the joint probability distribution for the inventory levels of both commodities in the steady state case in section 3. Various system performance measures in the steady state are derived in section 4 and the cost analysis and the results are illustrated numerically in section 5 and 6.

## 2 THE MODEL

We consider a two-commodity inventory system with the maximum capacity  $S_i$  units for  $i$ -th commodity ( $i = 1, 2$ ). The demands for  $i$ -th commodity is of unit size. The demands for commodity-1 arrive according to a Markovian arrival process (MAP) with representation  $(D_0, D_1)$  where  $D$ 's are of order  $m_1 \times m_1$ . The underlying Markov chain  $J_1(t)$  of the MAP has the generator  $D (= D_0 + D_1)$  and a stationary row vector  $\lambda_1$  of length  $m_1$ . Independently of this process, demands for commodity-2 arrive according to a MAP with representation  $(F_0, F_1)$  where  $F$ 's are of order  $m_2 \times m_2$ . The underlying Markov chain  $J_2(t)$  of this MAP has the generator  $F (= F_0 + F_1)$  and a stationary row vector  $\lambda_2$  of length  $m_2$ . The items are perishable in nature. The life time of each commodity is assumed to be distributed as exponential with parameter  $\gamma_i$ , ( $i = 1, 2$ ). The two-commodities serve as substitute for each other, that is, a demand for a commodity that is sold out, is satisfied with the other commodity when still in stock. If both the commodities are out of stock, any arriving demands are backlogged. The backlog is allowed up to the level  $N_i (< \infty)$  for the  $i$ -th commodity ( $i = 1, 2$ ). Whenever the backlog level reaches  $N_i$ , ( $i = 1, 2$ ) an order for  $N_i$  items is placed which is replenished instantaneously. The reorder level for the  $i$ -th commodity is fixed at  $s_i (1 \leq s_i \leq S_i)$  with an ordering quantity for the  $i$ -th commodity is  $Q_i (= S_i - s_i > s_i + N_i + 1)$  items when both inventory levels are less than or equal to their respective reorder levels. The requirement  $S_i - s_i > s_i + N_i + 1$  ensures that after the replenishment the inventory levels of both commodities will be always above the respective reorder levels; otherwise it may not be possible to place reorder (according to this policy) which leads to perpetual shortage. More explicitly if  $L_i(t)$  represents inventory level of  $i$ -th commodity at time  $t$ , then a reorder for both commodities is made when  $L_1(t) \leq s_1$  and  $L_2(t) \leq s_2$ . The lead time is assumed to be distributed as negative exponential with parameter  $\beta (> 0)$ .

### Notations

$[A]_{ij}$	: The element/submatrix at $(i, j)$ -th position of $A$ .
$\mathbf{0}$	: Zero matrix.
$I$	: An identity matrix.
$I_k$	: An identity matrix of order $k$ .
$A \otimes B$	: Kronecker product of matrices $A$ and $B$ .
$A \oplus B$	: Kronecker sum of matrices $A$ and $B$ .
$e$	: A column vector of 1's of appropriate dimension.

## 3 ANALYSIS

From the assumptions made on the input and output processes it can be shown that the quadruple  $(L_1, L_2, J_1, J_2) = \{(L_1(t), L_2(t), J_1(t), J_2(t)), t \geq 0\}$  is a Markov process with state space given by

$$\begin{aligned}
 E &= \{(i, k, j_1, j_2) | i = 1, 2, \dots, S_1, k = 0, 1, \dots, S_2, j_1 = 1, 2, \dots, m_1, j_2 = 1, 2, \dots, m_2\} \\
 &\cup \{(i, k, j_1, j_2) | i = 0, k = -(N_2 - 1), -(N_2 - 2), \dots, S_2, j_1 = 1, 2, \dots, m_1, j_2 = 1, 2, \dots, m_2\} \\
 &\cup \{(i, k, j_1, j_2) | i = -(N_1 - 1), -(N_1 - 2), \dots, -1, k = -(N_2 - 1), -(N_2 - 2), \dots, 0, \\
 &\quad j_1 = 1, 2, \dots, m_1, j_2 = 1, 2, \dots, m_2\}.
 \end{aligned}$$

Define the following ordered sets :

$$\begin{aligned}
 \mathbf{i} &= ( (i, 0), (i, 1), \dots, (i, S_2) ), \\
 \langle i \rangle &= ( (i, -N_2 + 1), (i, -N_2 + 2), \dots, (i, S_2) ), \\
 [i] &= ( (i, -N_2 + 1), (i, -N_2 + 2), \dots, (i, 0) ), \\
 (i, j) &= ( (i, j, 1), (i, j, 2), \dots, (i, j, m_1) ), \\
 (i, j, k) &= ( (i, j, k, 1), (i, j, k, 2), \dots, (i, j, k, m_2) ),
 \end{aligned}$$

Then the state space is ordered as  $([-N_1 + 1], [-N_1 + 2], \dots, [-1], < 0 >, \mathbf{1}, \mathbf{2}, \dots, \mathbf{S}_1)$ . The infinitesimal generator of  $P$  of the Markov process  $(L_1, L_2, J_1, J_2)$  has the following block partitioned form :

$$[P]_{ij} = \begin{cases} B_i, & j = i - 1, & i = 0, 1, \dots, S_1, \\ \widehat{B}, & j = i - 1, & i = -(N_1 - 2), -(N_1 - 3), \dots, -1, \\ \widetilde{B}, & j = i + (N_1 - 1), & i = -(N_1 - 1), \\ C, & j = i + Q_1, & i = 1, 2, \dots, s_1, \\ \widehat{C}, & j = i + Q_1, & i = 0, \\ \widetilde{C}, & j = i + Q_1, & i = -(N_1 - 1), -(N_1 - 2), \dots, -1, \\ A_i, & j = i, & i = 0, 1, \dots, S, \\ \widehat{A}, & j = i, & i = -(N_1 - 1), -(N_1 - 2), \dots, -1, \\ \mathbf{0}, & \text{otherwise,} \end{cases}$$

where

$$[C]_{kl} = \begin{cases} \beta I_{m_1} \otimes I_{m_2}, & l = k + Q_2, \quad k = 0, 1, \dots, s_2, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[\widehat{C}]_{kl} = \begin{cases} \beta I_{m_1} \otimes I_{m_2} & l = k + Q_2, \quad k = -(N_2 - 1), -(N_2 - 2), \dots, s_2, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[\widetilde{C}]_{kl} = \begin{cases} \beta I_{m_1} \otimes I_{m_2}, & l = k + Q_2, \quad k = -(N_2 - 1), -(N_2 - 2), \dots, 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For  $i = 2, 3, \dots, S_1$ ,

$$[B_i]_{kl} = \begin{cases} D_1 \otimes I_{m_2} + i\gamma_1 I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 1, 2, \dots, S_2, \\ D_1 \oplus F_1 + i\gamma_1 I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For  $i = 1$ ,

$$[B_i]_{kl} = \begin{cases} D_1 \otimes I_{m_2} + i\gamma_1 I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 1, 2, \dots, S_2, \\ D_1 \oplus F_1 + i\gamma_1 I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For  $i = 0$ ,

$$[B_i]_{kl} = \begin{cases} D_1 \otimes I_{m_2}, & l = k, \quad k = -(N_2 - 1), (N_2 - 2), \dots, 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[\widehat{B}]_{kl} = \begin{cases} D_1 \otimes I_{m_2}, & l = k, \quad k = -(N_2 - 1), (N_2 - 2), \dots, 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[\widetilde{B}]_{kl} = \begin{cases} D_1 \otimes I_{m_2}, & l = k, \quad k = -(N_2 - 1), (N_2 - 2), \dots, 0, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For  $i = 1, 2, \dots, s_1$ ,

$$[A_i]_{kl} = \begin{cases} I_{m_1} \otimes F_1 + k\gamma_2 I_{m_1} \otimes I_{m_2}, & l = k - 1, \quad k = 1, 2, \dots, S_2, \\ D_0 \oplus F_0 - (i\gamma_1 + \beta) I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 0, \\ D_0 \oplus F_0 - (i\gamma_1 + \beta + k\gamma_2) I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 1, 2, \dots, s_2 \\ D_0 \oplus F_0 - (i\gamma_1 + k\gamma_2) I_{m_1} \otimes I_{m_2}, & l = k, \quad k = s_2 + 1, s_2 + 2, \dots, S_2 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For  $i = s_1 + 1, s_1 + 2, \dots, S_1,$

$$[A_i]_{kl} = \begin{cases} I_{m_1} \otimes F_1 + k\gamma_2 I_{m_1} \otimes I_{m_2}, & l = k - 1, \quad k = 1, 2, \dots, S_2, \\ D_0 \oplus F_0 - i\gamma_1 I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 0, \\ D_0 \oplus F_0 - (i\gamma_1 + k\gamma_2) I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 1, 2, \dots, S_2 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For  $i = 0,$

$$[A_i]_{kl} = \begin{cases} D_1 \oplus F_1 + k\gamma_2 I_{m_1} \otimes I_{m_2}, & l = k - 1, \quad k = 1, 2, \dots, S_2, \\ I_{m_1} \otimes F_1, & l = k - 1, \quad k = -(N_2 - 2), -(N_2 - 3), \dots, -1, 0, \\ & (or) \\ D_0 \oplus F_0 - \beta I_{m_1} \otimes I_{m_2}, & l = k + N_2 - 1, \quad k = -(N_2 - 1), \\ D_0 \oplus F_0 - (\beta + k\gamma_2) I_{m_1} \otimes I_{m_2}, & l = k, \quad k = -(N_2 - 1), -(N_2 - 2), \dots, 0, \\ D_0 \oplus F_0 - (k\gamma_2) I_{m_1} \otimes I_{m_2}, & l = k, \quad k = 1, 2, \dots, s_2, \\ D_0 \oplus F_0 - k\gamma_2 I_{m_1} \otimes I_{m_2}, & l = k, \quad k = s_2 + 1, s_2 + 2, \dots, S_2, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[\hat{A}]_{kl} = \begin{cases} I_{m_1} \otimes F_1, & l = k - 1, \quad k = -(N_2 - 2), -(N_2 - 3), \dots, -1, 0, \\ & (or) \\ D_0 \oplus F_0 - \beta I_{m_1} \otimes I_{m_2}, & l = k + N_2 - 1, \quad k = -(N_2 - 1), \\ \mathbf{0}, & l = k, \quad k = -(N_2 - 1), -(N_2 - 2), \dots, 0, \\ & \text{otherwise.} \end{cases}$$

It may be noted that the matrices  $A_i, i = 1, 2, \dots, S_1, B_i, i = 2, 3, \dots, S_1$  and  $C$  are of size  $(S_2 + 1)m_1m_2 \times (S_2 + 1)m_1m_2, B_1$  is of size  $(S_2 + 1)m_1m_2 \times (S_1 + N_2)m_1m_2, B_0$  is of size  $(S_2 + N_2)m_1m_2 \times N_2m_1m_2, \hat{B}$  is of size  $N_2m_1m_2 \times N_2m_1m_2, \tilde{B}$  is of size  $N_2m_1m_2 \times (S_2 + N_2)m_1m_2, \hat{C}$  is of size  $(S_2 + N_2)m_1m_2 \times (S_2 + 1)m_1m_2, \tilde{C}$  is of size  $N_2m_1m_2 \times (S_2 + 1)m_1m_2, A_0$  is of size  $(S_2 + N_2)m_1m_2 \times (S_2 + N_2)m_1m_2$  and  $\hat{A}$  is of size  $N_2m_1m_2 \times N_2m_1m_2.$

### 3.1 STEADY STATE ANALYSIS

It can be seen from the structure of  $P$  that the homogeneous Markov process  $\{(L_1(t), L_2(t), J_1(t), J_2(t)), t \geq 0\}$  on the finite state space  $E$  is irreducible. Hence the limiting distribution  $\phi_{(i,k,j_1,j_2)} =$

$$\lim_{t \rightarrow \infty} Pr [L_1(t) = i, L_2(t) = k, J_1(t) = j_1, J_2(t) = j_2 | L_1(0), L_2(0), J_1(0), J_2(0)]$$

exists. Let

$$\begin{aligned} \phi_{(i,k,j_1)} &= (\phi_{(i,k,j_1,1)}, \phi_{(i,k,j_1,2)}, \dots, \phi_{(i,k,j_1,m_2)}), j_1 = 1, 2, \dots, m_1, \\ \phi_{(i,k)} &= (\phi_{(i,k,1)}, \phi_{(i,k,2)}, \dots, \phi_{(i,k,m_1)}), k = -N_2 + 1, -N_2 + 2, \dots, S_2, \\ \phi^{(i)} &= \begin{cases} (\phi_{(i,0)}, \phi_{(i,1)}, \dots, \phi_{(i,S_2)}), & \text{if } i = 1, 2, \dots, S_1, \\ (\phi_{(i,-N_2+1)}, \phi_{(i,-N_2+2)}, \dots, \phi_{(i,S_2)}), & \text{if } i = 0, \\ (\phi_{(i,-N_2+1)}, \phi_{(i,-N_2+2)}, \dots, \phi_{(i,0)}), & \text{if } i = -N_1 + 1, -N_1 + 2, \dots, -1. \end{cases} \end{aligned}$$

and

$$\Phi = (\phi^{(-N_1+1)}, \phi^{(-N_1+2)}, \dots, \phi^{(S_1-1)}, \phi^{(S_1)}).$$

Then the vector of limiting probabilities  $\Phi$  satisfies

$$\Phi P = \mathbf{0} \quad \text{and} \quad \Phi \mathbf{e} = 1. \tag{1}$$

The first equation of the above yields the following set of equations:

$$\begin{aligned}
\phi^{(i+1)}\widehat{B} + \phi^{(i)}\widehat{A} &= \mathbf{0}, & i = -N_1 + 1, -N_1 + 2, \dots, -2, \\
\phi^{(i+1)}B_{i+1} + \phi^{(i)}\widehat{A} &= \mathbf{0}, & i = -1, \\
\phi^{(i+1)}B_{i+1} + \phi^{(i)}A_i + \phi^{(i-N_1+1)}\widetilde{B} &= \mathbf{0}, & i = 0, \\
\phi^{(i+1)}B_{i+1} + \phi^{(i)}A_i &= \mathbf{0}, & i = 1, 2, \dots, Q_1 - N_1, \\
\phi^{(i+1)}B_{i+1} + \phi^{(i)}A_i + \phi^{(i-Q_1)}\widetilde{C} &= \mathbf{0}, & i = Q_1 - N_1 + 1, Q_1 - N_1 + 2, \dots, Q_1 - 1, \\
\phi^{(i+1)}B_{i+1} + \phi^{(i)}A_i + \phi^{(i-Q_1)}\widehat{C} &= \mathbf{0}, & i = Q_1, \\
\phi^{(i+1)}B_{i+1} + \phi^{(i)}A_i + \phi^{(i-Q_1)}C &= \mathbf{0}, & i = Q_1 + 1, Q_1 + 2, \dots, S_1 - 1, \\
\phi^{(i)}A_i + \phi^{(i-Q_1)}C &= \mathbf{0}, & i = S_1.
\end{aligned} \tag{2}$$

The equations (except (2)) can be recursively solved to get

$$\phi^{(i)} = \phi^{(Q_1)}\theta_i, \quad i = -N_1 + 1, -N_1 + 2, \dots, S_1,$$

where

$$\theta_i = \begin{cases} -\theta_{i+1}\widehat{B}\widehat{A}^{-1}, & i = -(N_1 - 1), -(N_1 - 2), \dots, -2, \\ -\theta_{i+1}B_0\widehat{A}^{-1}, & i = -1, \\ -(\theta_{i+1}B_{i+1} + \theta_{i-N_1+1}\widetilde{B})A_i^{-1}, & i = 0, \\ -\theta_{i+1}B_{i+1}A_i^{-1}, & i = 1, 2, \dots, Q_1 - N_1, \\ -(\theta_{i+1}B_{i+1} + \theta_{i-Q_1}\widetilde{C})A_i^{-1}, & i = Q_1 - N_1 + 1, Q_1 - N_1 + 2, \dots, Q_1 - 1, \\ I, & i = Q_1, \\ -(\theta_{i+1}B_{i+1} + \theta_{i-Q_1}C)A_i^{-1}, & i = Q_1 + 1, Q_1 + 2, \dots, S_1 - 1, \\ -\theta_{i-Q_1}CA_i^{-1}, & i = S_1. \end{cases}$$

Substituting the values of  $\theta_i$  in equation (2) and in the normalizing condition we get the value of  $\phi^{(Q_1)}$ .

## 4 SYSTEM PERFORMANCE MEASURES

In this section we derive some stationary performance measures of the system. Using these measures, we can construct the total expected cost per unit time.

### 4.1 MEAN INVENTORY LEVEL

Let  $\eta_{I_i}$  denote the mean inventory level of  $i$ -th commodity in the steady state ( $i = 1, 2$ ). Since  $\phi_{(i,j)}$  is the steady state probability vector for inventory level of first commodity is  $i$  and the second commodity is  $j$ , we have

$$\eta_{I_1} = \sum_{i=1}^{S_1} \sum_{k=0}^{S_2} i \phi_{(i,k)} \mathbf{e}.$$

and

$$\eta_{I_2} = \sum_{i=0}^{S_1} \sum_{k=1}^{S_2} k \phi_{(i,k)} \mathbf{e}.$$

### 4.2 MEAN REORDER RATE

A reorder for both commodities is made when the joint inventory level, drops to either  $(s_1, s_2)$  or  $(s_1, j), j < s_2$  or  $(i, s_2), i < s_1$ . Let  $\zeta_R$  denote the mean joint reorder rate for both commodities in the

steady state and it is given by

$$\begin{aligned} \eta_R &= \frac{1}{\lambda_1} \sum_{k=0}^{s_2} \phi_{(s_1+1,k)} (D_1 \otimes I_{m_2}) \mathbf{e} + \frac{1}{\lambda_2} \sum_{i=0}^{s_1} \phi_{(i,s_2+1)} (I_{m_1} \otimes F_1) \mathbf{e} \\ &+ \frac{1}{\lambda_1} \phi_{(0,s_2+1)} (I_{m_1} \otimes F_1) \mathbf{e} + \frac{1}{\lambda_2} \phi_{(s_1+1,0)} (D_1 \otimes I_{m_2}) \mathbf{e} \\ &+ (s_1 + 1)\gamma_1 \sum_{k=0}^{s_2} \phi_{(s_1+1,k)} \mathbf{e} + (s_2 + 1)\gamma_2 \sum_{i=0}^{s_1} \phi_{(i,s_2+1)} \mathbf{e}. \end{aligned}$$

Let  $\eta_{R_i}$  denote the mean individual reorder rate for commodity- $i$  in the steady state ( $i = 1, 2$ ). When the inventory level of commodity-1 is  $-(N_1 - 1)$ , a demand for commodity-1 will trigger the individual reorder for commodity-1. Hence we get

$$\eta_{R_1} = \frac{1}{\lambda_1} \sum_{k=-N_2+1}^0 \phi_{(-N_1+1,k)} (D_1 \otimes I_{m_2}) \mathbf{e}.$$

Similar arguments lead to

$$\eta_{R_2} = \frac{1}{\lambda_2} \sum_{i=-N_1+1}^0 \phi_{(i,-N_2+1)} (I_{m_1} \otimes F_1) \mathbf{e}.$$

### 4.3 AVERAGE BACKLOG

Let  $\eta_{B_i}$  denote the mean backlog of commodity- $i$  in the steady state ( $i = 1, 2$ ). Then we have

$$\eta_{B_1} = \sum_{i=-N_1+1}^{-1} \sum_{k=-N_2+1}^0 |i| \phi_{(i,k)} \mathbf{e}.$$

and

$$\eta_{B_2} = \sum_{i=-N_1+1}^0 \sum_{k=-N_2+1}^{-1} |k| \phi_{(i,k)} \mathbf{e}.$$

### 4.4 MEAN PERISHABLE RATE

Let the mean perishable rate of commodity- $i$  in the steady state be denoted by  $\zeta_{F_i}$ , ( $i = 1, 2$ ). Then we have

$$\eta_{F_1} = \sum_{i=1}^{S_1} \sum_{k=0}^{S_2} i \gamma_1 \phi_{(i,k)} \mathbf{e}.$$

and

$$\eta_{F_2} = \sum_{i=0}^{S_1} \sum_{k=1}^{S_2} k \gamma_2 \phi_{(i,k)} \mathbf{e}.$$

## 5 COST ANALYSIS

The expected total cost per unit time (expected total cost rate) in the steady state for this model is defined to be

$$\begin{aligned} TC(S_1, S_2, s_1, s_2, N_1, N_2) &= c_{h_1} \eta_{I_1} + c_{h_2} \eta_{I_2} + c_r \eta_R + c_{r_1} \eta_{R_1} + c_{r_2} \eta_{R_2} \\ &+ c_{b_1} \eta_{B_1} + c_{b_2} \eta_{B_2} + c_{p_1} \eta_{F_1} + c_{p_2} \eta_{F_2}, \end{aligned}$$

where



- $c_r$  : Setup cost per order.
- $c_{r_i}$  : Setup cost for the  $i$ -th commodity under local purchase ( $i=1,2$ ).
- $c_{h_i}$  : Holding cost for the  $i$ -th commodity per unit time,  $i = 1, 2$ .
- $c_{p_i}$  : Perishable cost per unit item per unit time of  $i$ -th commodity ( $i=1,2$ ).
- $c_{b_i}$  : Cost per unit backlog for the  $i$ -th commodity per unit time,  $i = 1, 2$ .

By substituting the values for  $\eta$ 's we can compute the value of  $TC(S_1, S_2, s_1, s_2, N_1, N_2)$ .

Since the evaluation of the  $\phi$ 's involve recursive computations, it is quite difficult to show the convexity of the total expected cost rate. However we present the following example to demonstrate the computability of the results derived in our work, and to illustrate the existence of local optima when the total cost function is treated as a function of only two variables.

## 6 NUMERICAL ILLUSTRATION

We consider the following numerical example : The demand for first commodity is given by  $(D_0, D_1)$  where

$$D_0 = \begin{pmatrix} -50 & 0 \\ 0 & -5 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 39 & 11 \\ 3.9 & 1.1 \end{pmatrix}.$$

The demand for second commodity is given by  $(F_0, F_1)$  where

$$F_0 = \begin{pmatrix} -20 & 0 \\ 0 & -2 \end{pmatrix}, \quad F_1 = \begin{pmatrix} 19 & 1 \\ 1.9 & 0.1 \end{pmatrix}.$$

In the following tables, the optimal cost for each row is shown in underlined and the optimal cost for each column is shown in bold.

Let  $\gamma_1 = 1, \gamma_2 = 1, \beta = 25, s_1 = 2, s_2 = 2, N_1 = 3, N_2 = 3, c_{h_1} = 0.01, c_{h_2} = 0.01, c_r = 75, c_{r_1} = 2, c_{r_2} = 2, c_{b_1} = 1, c_{b_2} = 1, c_{p_1} = 2, c_{p_2} = 1$ .

$$\text{Let } \overline{TC}(S_1, S_2) = TC(S_1, S_2, 2, 2, 3, 3).$$

From table 1, the numerical values shows that  $\overline{TC}(S_1, S_2)$  is a convex function in  $(S_1, S_2)$  and the

Table 1 – Total Expected Cost Rate of  $S_1$  and  $S_2$

$S_2$	10	11	12	13	14
$S_1$					
13	9.872429	9.630808	<u>9.596855</u>	9.743616	9.775500
14	9.709404	9.520833	<u>9.501561</u>	9.633814	9.684700
15	9.594168	9.451205	<u>9.446881</u>	9.569767	9.634135
16	9.517610	<u>9.413976</u>	<b>9.424364</b>	<b>9.541746</b>	<b>9.616449</b>
17	9.472896	<b>9.403231</b>	9.427757	9.542487	9.625701
18	<b>9.454763</b>	<u>9.414471</u>	9.452342	9.566473	9.657093
19	9.459070	<u>9.444205</u>	9.494506	9.609474	9.706772

(possibly local) optimum occurs at  $(S_1, S_2) = (17, 11)$ .

Let  $\gamma_1 = 0.01, \gamma_2 = 0.8, \beta = 18, S_2 = 20, s_2 = 3, N_1 = 3, N_2 = 3, c_{h_1} = 0.01, c_{h_2} = 0.01, c_r = 0.55, c_{r_1} = 0.45, c_{r_2} = 0.5, c_{b_1} = 0.1, c_{b_2} = 0.1, c_{p_1} = 0.1, c_{p_2} = 0.4$ .

$$\text{Let } \overline{TC}(S_1, s_1) = TC(S_1, 20, s_1, 3, 3, 3).$$

From table 2, the numerical values shows that  $\overline{TC}(S_1, s_1)$  is a convex function in  $(S_1, s_1)$  and the

Table 2 – Total Expected Cost Rate of  $S_1$  and  $s_1$

$s_1$	4	5	6	7	8
$S_1$					
49	5.076500	<u>5.075467</u>	5.080127	5.088899	5.100611
50	<b>5.076459</b>	<u>5.075080</u>	5.079444	5.087956	5.099435
51	5.076589	<u>5.074872</u>	5.078945	5.087203	5.098452
52	5.076888	<b>5.074837</b>	5.078626	5.086635	5.097661
53	5.077351	<u>5.074974</u>	<b>5.078484</b>	5.086248	5.097055
54	5.077976	<u>5.075278</u>	5.078514	5.086039	5.096631
55	5.078760	<u>5.075747</u>	5.078714	<b>5.086004</b>	5.096385
56	5.079700	<u>5.076377</u>	5.079080	5.086140	<b>5.096314</b>
57	5.080793	<u>5.077165</u>	5.079609	5.086443	5.096414

(possibly local) optimum occurs at  $(S_1, s_1) = (52, 5)$ .

let  $\gamma_1 = 0.01, \gamma_2 = 0.9, \beta = 10, S_2 = 20, s_2 = 2, S_1 = 20, s_1 = 2, c_{h_1} = 0.01, c_{h_2} = 0.01, c_r = 21, c_{r_1} = 15, c_{r_2} = 18, c_{b_1} = 5, c_{b_2} = 5, c_{p_1} = 0.8, c_{p_2} = 0.75$ .

$$\text{Let } \overline{TC}(N_1, N_2) = TC(20, 20, 2, 2, N_1, N_2).$$

From table 3, the numerical values shows that  $\overline{TC}(N_1, N_2)$  is a convex function in  $(N_1, N_2)$  and the

Table 3 – Total Expected Cost Rate of  $N_1$  and  $N_2$

$N_2$	3	4	5	6	7
$N_1$					
4	10.565306	10.543407	10.533310	<u>10.531192</u>	10.535652
5	10.509154	10.491453	<u>10.486547</u>	10.490709	10.502146
6	10.476386	<u>10.465856</u>	10.468392	<b>10.481047</b>	<b>10.501967</b>
7	10.456879	<u>10.454703</u>	<b>10.465625</b>	10.487919	10.520371
8	<b>10.453162</b>	<b>10.449303</b>	10.468540	10.500486	10.545501
9	10.461411	<u>10.459465</u>	10.492293	10.541267	10.609235

(possibly local) optimum occurs at  $(N_1, N_2) = (8, 4)$ .

Let  $\gamma_1 = 0.1, \gamma_2 = 0.8, \beta = 18, S_1 = 20, s_1 = 3, s_2 = 2, N_1 = 3; c_{h_1} = 0.1, c_{h_2} = 0.1, c_r = 0.11, c_{r_1} = 0.1, c_{r_2} = 0.1, c_{b_1} = 0.1, c_{b_2} = 0.1, c_{p_1} = 0.1, c_{p_2} = 0.1$ .

$$\text{Let } \overline{TC}(S_2, N_2) = TC(20, S_2, 3, 2, 3, N_2).$$

From table 4, the numerical values shows that  $\overline{TC}(S_2, N_2)$  is a convex function in  $(S_2, N_2)$  and the

Table 4 – Total Expected Cost Rate of  $S_2$  and  $N_2$

$N_2$	5	6	7	8	9
$S_2$					
39	8.654678	<u>8.653849</u>	8.654060	8.654295	8.654545
40	<b>8.517849</b>	<b>8.517004</b>	<b>8.517197</b>	<b>8.517414</b>	<b>8.517645</b>
41	8.689863	<u>8.680004</u>	8.680181	8.680380	8.680593
42	8.843738	<u>8.842865</u>	8.843027	8.843210	8.843407
43	9.005788	<u>9.005604</u>	9.005752	9.005920	9.006102

(possibly local) optimum occurs at  $(S_2, N_2) = (40, 6)$ .

Let  $\gamma_1 = 0.1, \gamma_2 = 0.8, \beta = 18, S_1 = 20, s_1 = 3, N_2 = 3, N_1 = 3; c_{h_1} = 0.1, c_{h_2} = 0.1, c_r = 0.11, c_{r_1} = 0.1, c_{r_2} = 0.1, c_{b_1} = 0.1, c_{b_2} = 0.1, c_{p_1} = 0.1, c_{p_2} = 0.1$ .

$$\text{Let } \overline{TC}(S_2, s_2) = TC(20, S_2, 3, s_2, 3, 3).$$

From table 5, the numerical values shows that  $\overline{TC}(S_2, s_2)$  is a convex function in  $(S_2, s_2)$  and the

Table 5 – Total Expected Cost Rate of  $S_2$  and  $s_2$

$s_2$	2	3	4	5	6
$S_2$					
39	8.656617	<u>8.655266</u>	8.656616	8.657680	8.658492
40	<b>8.519803</b>	<b>8.518349</b>	<b>8.519626</b>	<b>8.520642</b>	<b>8.521425</b>
41	8.699830	<u>8.681279</u>	8.682486	8.683454	8.684207
42	8.846717	<u>8.844073</u>	8.845214	8.846135	8.846858
43	9.007478	<u>9.006747</u>	9.007824	9.008700	9.009393

(possibly local) optimum occurs at  $(S_2, s_2) = (40, 3)$ .

Let  $\gamma_1 = 0.01, \gamma_2 = 0.8, \beta = 18, S_2 = 20, s_1 = 2, s_2 = 3, N_2 = 3; c_{h_1} = 0.01, c_{h_2} = 0.01, c_r = 0.55, c_{r_1} = 0.45, c_{r_2} = 0.5, c_{b_1} = 0.1, c_{b_2} = 0.1, c_{p_1} = 0.1, c_{p_2} = 0.4$ .

$$\text{Let } \overline{TC}(S_1, N_1) = TC(S_1, 20, 2, 3, N_1, 3).$$

From table 6, the numerical values shows that  $\overline{TC}(S_1, N_1)$  is a convex function in  $(S_1, N_1)$  and the

Table 6 – Total Expected Cost Rate of  $S_1$  and  $N_1$

$N_1$	5	6	7	8	9
$S_1$					
49	5.034028	5.023524	<u>5.021157</u>	5.023724	5.029325
50	<b>5.033895</b>	5.023100	<u>5.020510</u>	5.022892	5.028327
51	5.033935	5.022856	<u>5.020048</u>	5.022249	5.027524
52	5.034146	<b>5.022789</b>	<u>5.019767</u>	5.021791	5.026910
53	5.034523	5.022894	<b>5.019662</b>	5.021515	5.026482
54	5.035065	5.023168	<u>5.019732</u>	<b>5.021415</b>	5.026235
55	5.035768	5.023608	<u>5.019971</u>	5.021490	<b>5.026165</b>
56	5.036629	5.024210	<u>5.020377</u>	5.021734	5.026268

(possibly local) optimum occurs at  $(S_1, N_1) = (53, 7)$ .

The Figure 1 grants the impact of the perishable rate  $\gamma_1$ , on the total expected cost rate TC via four curves which relate to  $\beta = 18.5, 18.6, 18.7, 18.8$ . Since figure 1, we perceive that the total cost value decreases when the perishable rate  $\gamma_1$  and the replenishment rate  $\beta$  increases.

The Figure 2 grants the impact of the perishable rate  $\gamma_2$ , on the total expected cost rate TC via three curves which relate to  $\beta = 19, 20$  and  $21$ . Since figure 2, we perceive that the total cost value decreases when the perishable rate  $\gamma_2$  and the replenishment rate  $\beta$  increases.

In tables 7 and 8, we show that the impact of the cost values on the optimal values  $(S_1^*, s_1^*)$  and the corresponding total expected cost rate. Towards this end, we first fix the parameters and cost value as  $S_2 = 20, s_2 = 3, N_1 = 3, N_2 = 3, \beta = 18, \gamma_1 = 0.01, \gamma_2 = 0.8, c_{r_1} = 0.45, c_{r_2} = 0.5$ .

Table 7 – Impact of Cost Values

$C_{h_2}$				0.01						
$C_{p_2}$				0.4			0.5			
$C_{b_2}$				0.09	0.1	0.11	0.09	0.1	0.11	
$C_r$	$C_{h_1}$	$C_{p_1}$	$C_{b_1}$							
0.4	0.01	0.10	0.09	55 6	55 6	55 6	55 6	55 6	55 6	
				5.0730	5.0733	5.0736	6.1763	6.1765	6.1768	
			0.10	55 6	55 6	55 6	55 6	55 6	55 6	
				5.0734	5.0737	5.0740	6.1767	6.1770	6.1773	
			0.11	55 6	55 6	55 6	55 6	55 6	55 6	
				5.0735	5.0738	5.0741	6.1768	6.1771	6.1774	
	0.02	0.10	0.09	55 6	55 6	55 6	55 6	55 6	55 6	
				5.1261	5.1264	5.1267	6.2202	6.2205	6.2208	
			0.10	55 6	54 6	54 6	54 6	54 6	54 6	
				5.1266	5.1269	5.1272	6.2206	6.2209	6.2212	
			0.11	54 6	54 6	54 6	54 6	54 6	54 6	
				5.1271	5.1273	5.1276	6.2211	6.2214	6.2217	
	0.5	0.01	0.10	0.09	54 6	54 6	54 6	53 6	53 6	53 6
					5.5517	5.5520	5.5523	6.6793	6.6795	6.6798
				0.10	54 6	54 6	54 6	53 6	53 6	53 6
					5.5522	5.5524	5.5527	6.6797	6.6800	6.6873
				0.11	54 6	54 5	54 5	53 5	53 5	53 5
					5.5526	5.5529	5.5532	6.6817	6.6819	6.6893
0.02	0.10	0.09	54 6	53 5	53 5	53 5	53 5	53 5		
			5.6281	5.6284	5.6287	6.6822	6.6835	6.6900		
		0.10	53 6	53 5	53 5	53 5	53 5	53 5		
			5.6296	5.6299	5.6309	6.6826	6.6837	6.6912		
		0.11	53 6	53 5	53 5	53 5	53 5	52 5		
			5.6371	5.6373	5.6376	6.6829	6.6838	6.6915		
0.5	0.01	0.10	0.09	55 6	55 6	55 6	55 6	55 6	55 6	
				5.6462	5.6468	5.6471	6.6983	6.6985	6.6988	
			0.10	55 6	55 6	55 6	55 6	55 6	55 6	
				5.6771	5.6777	5.6780	6.7067	6.7079	6.7083	
			0.11	55 6	55 6	55 6	55 6	55 6	55 6	
				5.6784	5.6787	5.6790	6.7177	6.7178	6.7179	
	0.02	0.10	0.09	55 6	55 6	55 6	55 6	55 6	55 6	
				5.6861	5.6884	5.6887	6.7202	6.7265	6.7268	
			0.10	55 6	54 6	54 6	54 6	54 6	54 6	
				5.6886	5.6889	5.6890	6.7566	6.7569	6.7570	
			0.11	54 6	54 6	54 6	54 6	54 6	54 6	
				5.6887	5.6890	5.6891	6.7571	6.7574	6.7577	
	0.02	0.10	0.09	54 6	54 6	54 6	53 6	53 6	53 6	
				5.6892	5.6894	5.6896	6.7692	6.7698	6.7791	
			0.10	54 6	54 6	54 6	53 6	53 6	53 6	
				5.6893	5.6902	5.6907	6.7857	6.7860	6.7893	
			0.11	54 6	54 5	54 5	53 5	53 5	53 5	
				5.6896	5.6909	5.6912	6.7919	6.7942	6.7958	
0.02	0.10	0.09	54 5	53 5	53 5	53 5	53 5	53 5		
			5.7281	5.7284	5.7287	6.8012	6.8015	6.8018		
		0.10	53 6	53 5	53 5	53 5	53 5	53 5		
			5.7296	5.7299	5.7309	6.8026	6.8029	6.8032		
		0.11	53 6	53 5	53 5	53 5	53 5	52 5		
			5.8371	5.8373	5.8376	6.8036	6.8044	6.8047		

Table 8 – Impact of Cost Value

$C_{h_2}$				0.02						
$C_{p_2}$				0.4			0.5			
$C_{b_2}$				0.09	0.1	0.11	0.09	0.1	0.11	
$C_r$	$C_{h_1}$	$C_{p_1}$	$C_{b_1}$							
0.4	0.01	0.10	0.09	54 6	54 6	54 6	54 6	54 6	54 6	
				5.2189	5.2191	5.2194	6.2197	6.2213	6.2232	
			0.10	54 6	54 6	54 6	54 6	54 6	54 6	
			5.2196	5.2199	5.3101	6.2367	6.2370	6.2373		
		0.11	54 6	54 6	54 6	54 6	54 6	54 6		
			5.2234	5.2237	5.3240	6.2767	6.2770	6.2773		
		0.20	0.09	54 6	54 6	54 6	54 6	54 6	54 6	
			5.2281	5.2284	5.3287	6.3902	6.3913	6.3956		
	0.10		53 6	53 6	53 6	53 6	53 6	53 6		
			5.3296	5.3299	5.3302	6.4106	6.4109	6.4112		
	0.11		53 6	53 6	53 6	53 6	53 6	53 6		
			5.3311	5.3323	5.3346	6.4152	6.4163	6.4177		
	0.02	0.10	0.09	53 6	53 6	53 6	52 6	52 6	52 6	
			5.6517	5.6520	5.6523	6.7793	6.7795	6.7798		
0.10			53 6	53 6	53 6	52 6	52 6	52 6		
		5.6522	5.6524	5.6527	6.7797	6.7800	6.7873			
0.11		53 5	53 5	53 5	52 5	52 5	52 5			
		5.6526	5.6529	5.6532	6.7817	6.7819	6.7883			
	0.20	0.09	0.09	53 5	53 5	53 5	52 5	52 5	52 5	
			5.7281	5.7284	5.7287	6.7912	6.7915	6.7918		
0.10			52 5	52 5	52 5	52 5	52 5	52 5		
		5.7296	5.7299	5.7309	6.8026	6.8029	6.8112			
0.11		52 5	52 5	52 5	52 5	52 5	52 5			
		5.7371	5.7373	5.7376	6.8116	6.8134	6.8147			
0.5	0.01	0.10	0.09	54 6	54 6	54 6	54 6	54 6	54 6	
				5.8762	5.8768	5.8771	6.8783	6.8785	6.8788	
			0.10	54 6	54 6	54 6	54 6	54 6	54 6	
			5.8771	5.8777	5.8780	6.9767	6.9790	6.9793		
		0.11	54 6	54 6	54 6	54 6	54 6	54 6		
			5.8784	5.8787	5.8790	6.9777	6.9870	6.9873		
		0.20	0.09	0.09	54 6	54 6	54 6	54 6	54 6	54 6
				6.1761	6.1784	6.1787	7.2202	7.2565	7.2568	
	0.10			53 6	53 6	53 6	53 6	53 6	53 6	
			6.1786	6.1789	6.1792	7.2566	7.2569	7.2572		
	0.11		53 6	53 6	53 6	53 6	53 6	53 6		
			6.1791	6.1793	6.1796	7.2671	7.2674	7.2677		
	0.02	0.10	0.09	53 6	53 6	53 6	52 6	52 6	52 6	
			6.5627	6.5670	6.5823	7.6797	7.6894	7.6898		
0.10			53 6	53 6	53 6	52 6	52 6	52 6		
		6.5792	6.5802	6.5907	7.6857	7.6920	7.6923			
0.11		53 5	53 5	53 5	52 5	52 5	52 5			
		6.5906	6.5929	6.5942	7.6919	7.6942	7.6958			
	0.20	0.09	0.09	53 5	53 5	53 5	52 5	52 5	52 5	
			6.7281	6.7284	6.7287	7.7312	7.7315	7.7318		
0.10			52 5	52 5	52 5	52 5	52 5	52 5		
		6.7296	6.7299	6.7309	7.7426	7.7429	7.7512			
0.11		52 5	52 5	52 5	52 5	52 5	52 5			
		6.8371	6.8373	6.8376	7.7516	7.7534	7.7547			

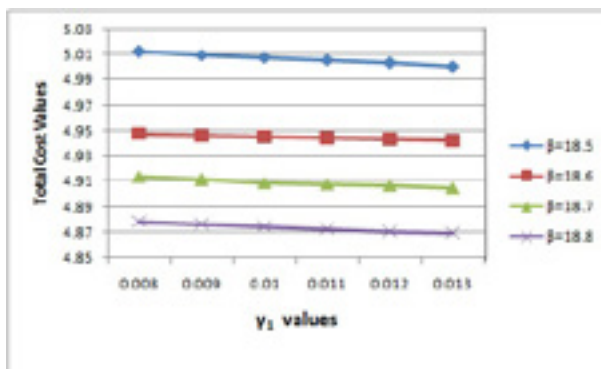


Figure 1 – TC versus  $\gamma_1$

$\gamma_2 = 0.8, S_1 = 52, s_1 = 5, S_2 = 20, s_2 = 3, N_1 = 3, N_2 = 3, c_{h_1} = 0.01, c_{h_2} = 0.01, c_r = 0.55, c_{r_1} = 0.45, c_{r_2} = 0.5, c_{b_1} = 0.1, c_{b_2} = 0.1, c_{p_1} = 0.1, c_{p_2} = 0.4.$

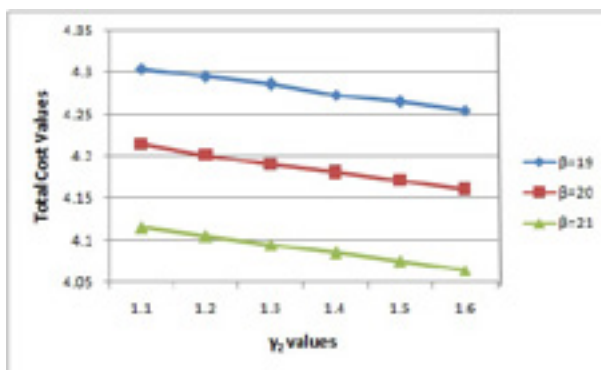


Figure 2 – TC versus  $\gamma_2$

$\gamma_1 = 0.01, S_1 = 52, s_1 = 5, S_2 = 20, s_2 = 3, N_1 = 3, N_2 = 3, c_{h_1} = 0.01, c_{h_2} = 0.01, c_r = 0.55, c_{r_1} = 0.45, c_{r_2} = 0.5, c_{b_1} = 0.1, c_{b_2} = 0.1, c_{p_1} = 0.1, c_{p_2} = 0.4.$

## 7 CONCLUSION

In this article, we examined the substitutable perishable inventory system. Specifically, we analyzed the structure of the system performance that takes place when a local purchase is made to clear the backlog instantaneously if both commodities have reached zero and demand is backlogged up to predetermined levels. Arriving customers follow a Markovian arrival process. The commodities are assumed to be substitutable. If both commodities have reached zero, demand is backlogged up to predetermined levels. Graphical results of perishable rates and replenishment rates had been presented. This shows that if the perishable and replenished rate increases then the total cost would increase. The results of the contribution were illustrated using numerical patterns to estimate the convexity of the overall cost rate of this system. The impact of cost values on total expected cost rate were shown. In the future, our proposed model can be expanded by various reordering policies and described by real data values.

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