FIXED POINT THEOREMS FOR SUZUKI NONEXPANSIVE MAPPINGS IN BANACH SPACES

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ABSTRACT

In this paper, we investigate the existence of fixed points for Suzuki nonexpansive mappings in the setting of Banach spaces using the asymptotic center technique. We also establish the convergence of regular approximate fixed point sequence to the fixed points of Suzuki nonexpansive mappings. Examples are also given to illustrate the results. Our theorems generalize several results in the literature.

KEYWORDS

Banach space, Nonexpansive mapping, Suzuki nonexpansive mapping, Fixed point, Approximate fixed point.

1 INTRODUCTION

Fixed point results for nonexpansive mappings in Banach spaces are of great importance in the development of fixed point theory and are widely used to solve problems in diverse fields such as differential equations, game theory, engineering, medicine and many more (see [3, 14, 16]. The possibility of using the theory in a wide range of applications has attracted many researchers and has consequently resulted in a rapid growth of research in this field. Several authors have introduced extensions of nonexpansive mappings such as generalized nonexpansive mappings, relatively nonexpansive mappings, α- nonexpansive mappings, etc. (see [1, 6, 15]) and proved fixed point results in Banach spaces and various other spaces as well. In 2008, Suzuki introduced a new condition called condition (C) [17]. The mapping which satisfies condition (C) is now known as Suzuki nonexpansive mapping. Suzuki proved that all nonexpansive mappings satisfy the condition (C). Unlike nonexpansive mappings, the Suzuki nonexpansive mappings need not be always continuous. We can find a couple of examples for mappings which are not continuous but satisfying condition (C) in [17].

There are several techniques for finding the fixed points of nonexpansive mappings. One of the most widely used techniques, introduced by Edelstein in 1972 [5], uses the concept of asymptotic radius and asymptotic center of a sequence relative to a set K. Many researchers have used the properties and geometric behavior of the asymptotic center of sequences under consideration, to prove several fixed point results for nonexpansive mappings (see [5, 9]). For any sequence, asymptotic center can be considered as the intersection of some closed balls [7]. Therefore, the asymptotic center is always closed. But it need not be nonempty. Researchers proved that if a set K is nonempty, weakly compact and convex, then the asymptotic center of any sequence in K has the same properties as K [7]. These results boosted the usefulness of asymptotic center technique as a tool to find fixed points for Suzuki nonexpansive mapping. Dhompongsa [4] used this technique in Suzuki nonexpansive mapping and proved that if K is a Banach space and T is a self mapping of K satisfying condition (C), then for any bounded approximate fixed point sequence in K, the asymptotic center relative to K is invariant under T. Another equally important method to find fixed points in Banach spaces is the Chebyshev center technique, which uses the concept of Chebyshev radius and Chebyshev center to analyze the geometric structure of a set. A good amount of research work is reported in literature which makes use of these two techniques to find fixed points (see [4, 5, 7–10, 13].

In this paper, we primarily focus on the asymptotic center technique and show that it is possible to derive an interesting relation between the asymptotic radius and Chebyshev radius under certain conditions. In [8], Kirk proved a fixed point result for nonexpansive mapping in reflexive Banach spaces having normal structure. We extended this result for Suzuki nonexpansive mapping in weakly compact Banach spaces. Also, we investigated some sufficient conditions for the existence of fixed points for Suzuki nonexpansive mapping in a closed, bounded and convex subset of a Banach space. Apart from these, we also developed certain sufficient conditions for the convergence of regular approximate fixed point sequences to the fixed points of Suzuki nonexpansive mappings.

1.1 PRELIMINARIES

Definition 1. [7] *A mapping T on a subset K of a Banach space X is called a nonexpansive mapping if $||Tx - Ty|| \leq ||x - y||$ for all $x, y \in K$.*

Definition 2. [17] *Let T be a mapping on a subset K of a Banach space X. Then T is said to satisfy condition (C) if for all $x, y \in K$,*

$$\frac{1}{2}||x - Tx|| \leq ||x - y|| \implies ||Tx - Ty|| \leq ||x - y||.$$

The mapping satisfying condition (C) is called Suzuki nonexpansive mapping.

Definition 3. [10] *Let $T : K \to K$ be any mapping. A sequence $\{x_n\}$ in K is called approximate fixed point sequence if $||Tx_n - x_n|| \to 0$ as $n \to \infty$.*

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Definition 4. [4] *Let K be a nonempty closed and convex subset of a Banach space X and $\{x_n\}$, a bounded sequence in X. For $x \in X$, the asymptotic radius of $\{x_n\}$ at x is defined as*

$$r(x, \{x_n\}) = \limsup\{||x_n - x||\}.$$

The asymptotic radius and asymptotic center of $\{x_n\}$ relative to K are defined as follows:

$$r \equiv r(K, \{x_n\}) = \inf\{r(x, \{x_n\}) : x \in K\}$$

$$A \equiv A(K, \{x_n\}) = \{x \in K : r(x, \{x_n\}) = r\}.$$

Definition 5. [9] *A bounded sequence is said to be regular if each of its subsequence has the same asymptotic radius.*

Definition 6. [9] *A bounded sequence is said to be uniform if each of its subsequence has the same asymptotic center.*

Definition 7. [7] *For any subset K of X, the radius of K relative to x, Chebyshev radius of K, Chebyshev center of K and the diameter of K are defined as follows:*
For any $x \in X$, $r_x(K) = \sup\{||x - y|| : y \in K\}$

$$r(K) = \inf\{r_x(K) : x \in K\}$$

$$C(K) = \{x \in K : r_x(K) = r(K)\}$$

$$diam(K) = \sup\{r_x(K) : x \in K\}.$$

Definition 8. *A nonempty, closed, convex subset D of a given set K is said to be a minimal invariant set for a mapping $T : K \to K$ if $T(D) \subseteq D$ and D has no nonempty, closed and convex proper subsets which are T-invariant.*

Definition 9. [7,8] *A convex subset K of X is said to have normal structure if each bounded, convex subset S of K with $diamS > 0$ contains a nondiametral point.*

Definition 10. [2] *A convex set K of X is said to have asymptotic normal structure if, given any bounded convex subset S of K which contains more than one point and given any decreasing net of nonempty subsets $\{s_\alpha; \alpha \in A\}$ of S, the asymptotic center of $\{s_\alpha; \alpha \in A\}$ in S is a proper subset of S.*

Definition 11. [11] *Let X be a Banach space. X is said to have Opial property if for each weakly convergent sequence $\{x_n\}$ in X with weak limit z and for all $x \in X$ with $x \neq z$,*

$$\limsup ||x_n - z|| < \limsup ||x_n - x||.$$

Lemma 1. [17, Lemma 6] *Let T be a mapping on a bounded convex subset K of a Banach space X. Assume that T satisfies condition (C). Define a sequence $\{x_n\}$ in K by $x_1 \in K$ and*

$$x_{n+1} = \lambda T x_n + (1 - \lambda) x_n$$

for $n \in \mathbb{N}$, where λ is a real number belonging to $[\frac{1}{2}, 1)$. Then

$$\lim_{n\to\infty} ||Tx_n - x_n|| = 0$$

holds.

Lemma 2. [7, Lemma 9.1] *Let $\{x_n\}$ be a sequence in a Banach space X and K a nonempty subset of X.*
(a) If K is weakly compact, then $A(K, \{x_n\}) \neq \phi$.
(b) If K is convex, then $A(K, \{x_n\})$ is convex.

Lemma 3. [4, Lemma 3.1] *Let K be a subset of a Banach space X, and $T : K \to K$ be a mapping satisfying condition (C). Suppose $\{x_n\}$ is a bounded approximate fixed point sequence for T. Then $A(K, \{x_n\})$ is invariant under T.*

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Proposition 1. [9, Preposition 1] *Every bounded sequence has a regular subsequence.*

Theorem 1. [7, Theorem 3.2] *Suppose K is a nonempty, weakly compact, convex subset of a Banach space. Then for any mapping $T : K \to K$ there exists a closed convex subset of K which is minimal T-invariant.*

Theorem 2. [12, Theorem 1] *A convex subset K of X has normal structure if and only if K has asymptotic normal structure.*

Remark 1. [7] *It is clear that if for any sequence $\{x_n\}$ in K and $x \in X$, $r(x, \{x_n\}) = 0$ if and only if $\lim_{n\to\infty} x_n = x$.*

Remark 2. [9] *If $\{x_{n_k}\}$ is a subset of $\{x_n\}$, then $r(K, \{x_{n_k}\}) \leq r(K, \{x_n\})$ and if $r(K, \{x_{n_k}\}) = r(K, \{x_n\})$, then $A(K, \{x_n\}) \subseteq A(K, \{x_{n_k}\})$.*

2 RESULTS

Theorem 3. *Let K be a weakly compact convex subset of a Banach space X and $T : K \to K$ satisfies condition (C). Assume that K is minimal T-invariant and $\{x_n\}$ is an approximate fixed point sequence in K. Then*
(i) $A(K, \{x_n\}) = K$
(ii) $r(K, \{x_n\}) = r(K)$.

Proof. Let K be a weakly compact and convex subset of a Banach space X.
Suppose $diam(K) = 0$. Then there is nothing to prove.
Now, suppose $diam(K) > 0$.
Let $\{x_n\}$ be any bounded approximate fixed point sequence in K. Then $A(K, \{x_n\})$ is closed.
Also by Lemma 2, $A(K, \{x_n\})$ is nonempty and convex. Thus $A(K, \{x_n\})$ is weakly compact.
By Lemma 3, $A(K, \{x_n\})$ is T-invariant and by the minimality of K, we have $A(K, \{x_n\}) = K$.
Since K is weakly compact, there exist a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and $z \in K$ such that $x_{n_k} \to z$ weakly.
Clearly $\{x_{n_k}\}$ is an approximate fixed point sequence in K.
Since $A(K, \{x_{n_k}\}) = K$, $\limsup ||x_{n_k} - x|| = r(K, \{x_{n_k}\})$ for all $x \in K$.
Also, we have for any $x \in K$, $\limsup ||x_{n_k} - x|| \leq \sup\{||x - y|| : y \in K\}$.
Therefore,

$$r(K, \{x_{n_k}\}) \leq r_x(K) \implies r(K, \{x_{n_k}\}) \leq r(K). \tag{1}$$

Now, for any $y \in K$, $x_{n_k} - y \to z - y$ weakly and hence we have

$$||z - y|| \leq \limsup ||x_{n_k} - y|| = r(K, \{x_{n_k}\}).$$

Hence for all $y \in K$,

$$\begin{aligned} ||z - y|| \leq r(K, \{x_{n_k}\}) &\implies r_z(K) \leq r(K, \{x_{n_k}\}) \\ &\implies r(K) \leq r(K, \{x_{n_k}\}). \end{aligned} \tag{2}$$

Thus from (1) and (2) we get

$$r(K, \{x_{n_k}\}) = r(K). \tag{3}$$

We know that for any subsequence $\{x_{n_k}\}$ of $\{x_n\}$,

$$r(K, \{x_{n_k}\}) \leq r(K, \{x_n\}). \tag{4}$$

Since $r(K, \{x_{n_k}\}) = r(K)$, if $x \in C(K)$, then for all $y \in K$, $||x - y|| \leq r(K, \{x_{n_k}\})$.
Therefore, for all x_n,

$$||x_n - x|| \leq r(K, \{x_{n_k}\}) \implies r(K, \{x_n\}) \leq r(K, \{x_{n_k}\}). \tag{5}$$

Hence from (4) and (5), $r(K, \{x_n\}) = r(K, \{x_{n_k}\})$. Thus from (3) we get
$r(K, \{x_n\}) = r(K)$.

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Corollary 1. *Let K be a weakly compact convex subset of a Banach space X and $T : K \to K$ satisfies condition (C). If K is minimal T-invariant, then every approximate fixed point sequence in K are regular and uniform.*

Proof. Let $\{x_n\}$ be an approximate fixed point sequence in K. Then every subsequence $\{x_{n_k}\}$ of $\{x_n\}$ is also an approximate fixed point sequence.
Hence by (ii) in Theorem 3, $r(K, \{x_n\}) = r(K, \{x_{n_k}\})$.
Therefore, $\{x_n\}$ is regular.
By (i) in Theorem 3, $A(K, \{x_n\}) = A(K, \{x_{n_k}\}) = K$.
Hence $\{x_n\}$ is uniform.

Corollary 2. [10, Preposition 6.3] *Let K be a weakly compact convex subset of a Banach space X, and $T : K \to K$ be a nonexpansive mapping. Assume that K is minimal for T, that is, no closed convex bounded proper subset of K is invariant for T. If $\{x_n\}$ is an approximate fixed point sequence in K, then $A(K, \{x_n\}) = K$.*

Proof. Since every nonexpansive mapping satisfies condition (C), by above theorem, $A(K, \{x_n\}) = K$.

Theorem 4. *Let K be a nonempty, weakly compact and convex subset of a Banach space X and suppose K has normal structure. Then every mapping $T : K \to K$ satisfying condition (C) has a fixed point.*

Proof. By Theorem 1, we can consider K as closed, convex minimal T- invariant subset.
Suppose $diamK > 0$.
Consider an approximate fixed point sequence $\{x_n\} \subseteq K$.
By (i) in Theorem 3, $A(K, \{x_n\}) = K$.
Now, define $W_n := \{x_m : m \geq n\},\ n \in \mathbb{N}$.
Clearly, $\{W_n, n \in \mathbb{N}\}$ is a decreasing chain of nonempty bounded subsets of K.
We can easily prove that Asymptotic center of $\{W_n, n \in \mathbb{N}\} = A(K, \{x_n\}) = K$.
Since K has normal structure, by Theorem 2, K has asymptotic normal structure.
Thus $A(K, \{x_n\}) = K$, which is a contradiction.
Therefore, $diamK = 0$ and hence K has only one element x (say).
Thus $T(x) = x$.

We obtain the result of Kirk [8] and Theorem 4.1 in [7] as corollaries of our result.

Corollary 3. [8, Theorem] *Let K be a nonempty, bounded, closed and convex subset of a reflexive Banach space X, and suppose that K has normal structure. If $T : K \to K$ is nonexpansive, then T has a fixed point.*

Proof. Since a bounded, closed and convex subset of a reflexive Banach space is weakly compact and every nonexpansive mapping is Suzuki nonexpansive, by the above theorem, T has a fixed point.

Corollary 4. [7, Theorem 4.1] *Let K be a nonempty, weakly compact, convex subset of a Banach space, and suppose K has normal structure. Then every nonexpansive mapping $T : K \to K$ has a fixed point.*

Proof. Since every nonexpansive mapping is Suzuki nonexpansive, by the above theorem, T has a fixed point.

The following theorem gives sufficient conditions for the existence of fixed points for Suzuki nonexpansive mapping in Banach spaces.

Theorem 5. *Let K be a closed, bounded and convex subset of a Banach space X and $T : K \to K$ satisfies condition (C). If $T(K)$ is contained in a compact subset of K, then T has a fixed point in K.*

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Proof. Let $\{x_n\}$ be an approximate fixed point sequence in K.
Therefore, $||Tx_n - x_n|| \to 0$.
Consider the sequence $\{Tx_n\}$ in $T(K)$.
Since $T(K)$ is a subset of a compact set of K, there exist a subsequence $\{Tx_{n_k}\}$ of $\{Tx_n\}$ and $z \in K$ such that $Tx_{n_k} \to z$.
Therefore, $\lim_{k\to\infty} ||x_{n_k} - z|| = \lim_{k\to\infty} ||x_{n_k} - Tx_{n_k}|| = 0$.
Hence $x_{n_k} \to z$.
Clearly, $\{x_{n_k}\}$ is a bounded approximate fixed point sequence and $A(K, \{x_{n_k}\}) = \{z\}$.
Also by Lemma 3, $A(K, \{x_{n_k}\})$ is T-invariant and hence $T(z) = z$.

The following is an example to illustrate this theorem.

Example 1. *In the space l_2 consider the closed unit ball*
$K = \{x = (x_1, x_2, ...) \in l_2 : ||x||_2 \leq 1\}$.
Define $T : K \to K$ as

$$T(x) = \begin{cases} (\frac{1}{3}, 0, 0, ..), & if\ x = (1, 0, 0, ...) \\ 0, & otherwise. \end{cases}$$

For all $x, y \neq (1, 0, 0, ...)$ in K, $||Tx - Ty||_2 = 0 \leq ||x - y||_2$.
If $x = (1, 0, 0, ...)$ then $\frac{1}{2}||x - Tx||_2 = \frac{1}{2}||(\frac{2}{3}, 0, 0, ...)||_2 = \frac{1}{3}$.
Therefore, if $\frac{1}{2}||x - Tx||_2 \leq ||x - y||_2$ for any $y \in K$, then $\frac{1}{3} \leq ||x - y||_2$.
Thus $||Tx - Ty||_2 = \frac{1}{3} \leq ||x - y||_2$.
Hence T satisfies condition (C).
$T(K) = \{(0, 0, 0, ...), (\frac{1}{3}, 0, 0, ...)\}$ is a subset of a compact set of K.
Thus all conditions in the above theorem are satisfied.
Since $T(0) = 0$, T has a fixed point.

Theorem 6. *Let X be a Banach space with Opial property and K be a closed, bounded and convex subset of X. Let $T : K \to K$ satisfies condition (C) and if $T(K)$ is contained in a weakly compact subset of K, then T has a fixed point in K.*

Proof. Let $\{x_n\}$ be an approximate fixed point sequence in K. Then we have, $||Tx_n - x_n|| \to 0$.
Since $\frac{1}{2}||Tx_n - x_n|| \leq ||Tx_n - x_n||$ and T satisfies condition (C), we have $||T^2x_n - Tx_n|| \leq ||Tx_n - x_n||$ for all $n \in \mathbb{N}$.
Therefore, $\lim_{n\to\infty} ||T^2x_n - Tx_n|| \leq \lim_{n\to\infty} ||Tx_n - x_n|| = 0$.
Hence $\{T(x_n)\}$ is an approximate fixed point sequence.
Since $T(K)$ is a subset of a weakly compact set of K, there exist a subsequence $\{Tx_{n_k}\}$ of $\{Tx_n\}$ and $z \in K$ such that $Tx_{n_k} \to z$ weakly as $k \to \infty$.
By Opial property, for any $x \neq z$, $\limsup ||Tx_{n_k} - z|| < \limsup ||Tx_{n_k} - x||$.
Therefore, for any $x \neq z, r(z, \{Tx_{n_k}\}) < r(x, \{Tx_{n_k}\}) \implies r(K, \{Tx_{n_k}\}) = r(z, \{Tx_{n_k}\})$ and $A(K, \{Tx_{n_k}\}) = \{z\}$.
By Lemma 3, $A(K, \{x_{n_k}\})$ is T-invariant and hence $T(z) = z$.

Corollary 5. [17, Theorem 4] *Let T be a mapping on a convex subset K of a Banach space X. Assume that T satisfies condition (C). Assume also that either of the following holds:*
(i) K is compact;
(ii) K is weakly compact and X has the Opial property.
Then T has a fixed point.

Proof. Suppose K is compact and convex. Therefore, K is closed, bounded and convex.
Also, since $T : K \to K$, we have $T(K) \subseteq K$. Thus $T(K)$ is a subset of a compact set.
Hence by Theorem 5, T has a fixed point.
Now suppose K is weakly compact, convex and X has the Opial property.
Therefore, K is closed, bounded and convex. Also, since $T : K \to K$, we have $T(K) \subseteq K$.

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Thus $T(K)$ is a subset of a weakly compact set.
Hence by Theorem 6, T has a fixed point.

Theorem 7. *Let K be a compact subset of a Banach space X. Let $T : K \to K$ satisfies condition (C). Let $\{x_n\}$ be a regular approximate fixed point sequence in K. Then $\{x_n\}$ converge strongly to a fixed point of T.*

Proof. Let $\{x_n\}$ be a regular approximate fixed point sequence in K.
Since K is compact, there exist a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and $z \in K$ such that $x_{n_k} \to z$.
Therefore, $A(K, \{x_{n_k}\}) = \{z\}$.
Since $\{x_{n_k}\}$ is an approximate fixed point sequence, by Lemma 3, $A(K, \{x_{n_k}\})$ is T-invariant. Hence $T(z) = z$.
Since $\{x_n\}$ is regular, $r(K, \{x_n\}) = r(K, \{x_{n_k}\}) = 0$.
Since K is compact and $\{x_n\}$ is an approximate fixed point sequence, $A(K, \{x_n\})$ is nonempty.
We know that if $\{x_n\}$ is regular, then for any subsequence $\{x_{n_k}\}$,
$A(K, \{x_n\}) \subseteq A(K, \{x_{n_k}\})$.
Therefore, $A(K, \{x_n\}) = \{z\}$. Hence $x_n \to z$.

The following example shows that even if T has fixed points, if $\{x_n\}$ is not regular, then $\{x_n\}$ need not converge to a fixed point.

Example 2. *Consider the compact set $K = [-1, 1]$ in $\mathbb{R}$ and define $T : K \to K$ as $T(x) = x$.*
Clearly T is a nonexpansive mapping and hence satisfies condition (C).
Consider $x_n = (-1)^n$ for all $n \in \mathbb{N}$
For all $n \in \mathbb{N}$, $||x_n - Tx_n|| = 0$. Thus $\{x_n\}$ is an approximate fixed point sequence.
But $\{x_n\}$ does not converge to a fixed point.
This will not contradict the above theorem because $\{x_n\}$ is not regular.
Asymptotic radius of $\{x_n\} = r(K, \{x_n\}) = 1$ and $A(K, \{x_n\}) = \{0\}$.
Consider the subsequence $\{x_{2n}\} = \{1, 1, 1, ...\}$. Clearly $x_{2n} \to 1$.
Therefore, $A(K, \{x_{2n}\}) = \{1\}$ and $r(K, \{x_{2n}\}) = 0$.
Hence $\{x_n\}$ is not regular.

Theorem 8. *Let K be a subset of a Banach space X and $T : K \to K$ satisfies condition (C). Suppose $T(K)$ is contained in a compact subset of K and let $\{x_n\}$ be a regular approximate fixed point sequence in K with nonempty asymptotic center. Then $\{x_n\}$ converge strongly to a fixed point of T.*

Proof. Let $\{x_n\}$ be a regular approximate fixed point sequence in K with nonempty asymptotic center.
Consider the sequence $\{Tx_n\}$ in $T(K)$. Since $T(K)$ is compact, there exist a subsequence $\{Tx_{n_k}\}$ of $\{Tx_n\}$ and $z \in T(K)$ such that $Tx_{n_k} \to z$.
Therefore, $\lim_{n\to\infty} ||x_{n_k} - z|| \leq \lim_{n\to\infty} ||x_{n_k} - Tx_{n_k}|| = 0$.
Thus $x_{n_k} \to z$ and hence $A(K, \{x_{n_k}\}) = \{z\}$.
Since $\{x_{n_k}\}$ is an approximate fixed point sequence, by Lemma 3, $A(K, \{x_{n_k}\})$ is T-invariant.
Hence $T(z) = z$.
Since $\{x_n\}$ is regular, $r(K, \{x_n\}) = r(K, \{x_{n_k}\}) = 0$.
Also if $\{x_n\}$ is regular, then for any subsequence $\{x_{n_k}\}$, we have $A(K, \{x_n\}) \subseteq A(K, \{x_{n_k}\})$.
Thus we have $A(K, \{x_n\}) = \{z\}$, which implies that $x_n \to z$.

3 CONCLUSIONS

In this paper, we have used the asymptotic center technique to establish the existence of fixed points for Suzuki nonexpansive mappings in Banach spaces. We have shown that under certain condition, the asymptotic radius and Chebyshev radius are equal. Using this result, we have established that every approximate fixed point sequence is regular as well as uniform. The convergence of regular approximate fixed point sequences to a fixed points of the Suzuki nonexpansive mapping is also established. A couple of examples are given to illustrate the results.

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