MIXED SYMMETRY STATES IN ^{96}MO AND ^{98}RU ISOTONES IN THE FRAMEWORK OF INTERACTING BOSON MODEL

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ABSTRACT

The software package IBM code for interacting boson model-1 and Neutron Proton Boson $NPBOS$ code for interacting boson model-2 have been used to calculate energy levels for, for *by estimating a set of parameters which are used to predict the behavior of even-even isotones within the current scope of work there is clear competition between the two parameters (ε and* a_2*) in* isotones, *as an inverse relationship. This means that vibrational qualities are continuous mixed with the rotational properties*. In interacting boson model-2 parameters(ε, κ, χ_π and χ_ν) have been shown similarity with interacting boson model-1 expected. *The Majorana parameter effect (ζ_2) on the calculated excitation energy level* for isotones has been accomplished by vary the ζ_2 around the optimum-matches to practical data. The effect of increasing ζ_2 on mixing symmetry states is the same in all isotones but different from state to another, we find the state $J^+ = 2_3{}^+$ was the lowest mixing symmetry states still approximately constant in the all. In that time,isotones have $1^+, 3^+{}_1, 5^+{}_1$ mixed symmetry states, rapidly increasing with increasing ζ_2. The results of the calculated energy levels were in acceptable agreement with the experimental data. There is no pure vibrational property of these isotones

KEYWORDS

Interacting boson model-1&2, MSS's, IBM code and $NPBOS$ code

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INTRODUCTION

Even-even isotones considered as medium nuclei mass number, which are always referred to be as vibrational nuclei, due to the small bosons number outside the closed N and Z shells, probably what appears from the sequence of energy levels in the modern experimental decay schemes are stay away from their values of typical harmonic oscillator (pure vibration), which indicates to energy levels distortion such as 0_2^+, 2_2^+ , $4_1^+$and 6_1^+. For this reason, isotones have been re-examined in modern experimental decay schemes. The interacting boson model, suggests that the collective behavior rises from the coupling, through the interaction of the nucleon-nucleon of the isolated low-lying systems of valence protons and neutrons that is definite in accordance to the respect of the major shell closure. It is capable of describing nuclear characteristics such as energies and spins of the levels, decay probabilities for the emission of gamma quanta, probabilities of electromagnetic transitions and their reduced matrix elements for different transitions, multipole moments, and mixing ratios[1-3]. IBM special cases are existed named "dynamic symmetries"[4-8]. They correspond to the well-known "limits", vibrational, rotational, and gamma unsteady nuclei. The concept of dynamic symmetry is of a basic significance in the IBM, because it permits the exact and analytic solution of the associated eigenvalue problem for a restricted class of boson Hamiltonians.The interacting boson model is suitable for describing the low-lying collective states in even (N,Z) nuclei by a system of interacting s and d-bosons carrying angular momentum's 0 and 2 respectively [9].The structure of medium mass nuclei are a focus of nuclear structure research [10-13]. The structure of nuclei with proton numbers 42,44 and 46 and the number of neutrons greater than 50 was for many years a challenge to theoretical explanations because of the fluctuating transition of nuclear properties between the vibrational features and weak-rotational features within$IBM1$ and $IBM2$[14,16].

INTERACTING BOSON MODEL-1

Realizing that there is only one body and two body parts in the interacting boson paradigm, formation $(s^\dagger, d_m^\dagger)$ and destruction (s,dm) actions are introduced with the index m=0,±1,±2. When considering on-boson terms in the boson-boson contact, the greatest Hamiltonian is [1-4].

$$H = \varepsilon_s\left(s^\dagger s\right) + \varepsilon_d \sum_m d_m^\dagger d_m + V\ldots.(1)$$

The boson-boson interacting energy is introduced by $\varepsilon_s, \varepsilon_d$are the s and d boson dynamisms and V. The greatest extensively recycled method of $IBM1$ Hamiltonian is [2-5].

$$H = \varepsilon n_d + a_0 P^\dagger P + a_1 L\,.\,L + a_2 Q\,.\,Q + a_3 T_3 T_3 + a_4 T_4 T_4 \ldots(2)$$

In order to simplify it, sometimes the boson $\varepsilon = \varepsilon_d - \varepsilon_s$, $\varepsilon_s$vigor value is set to zero. In addition, the forte of the quadruple points only $\varepsilon = \varepsilon_d$ appears

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$a_0, a_1, a_2, a_3, and a_4$ is determined by the angular momentum that results from the interaction of the bosons. Thus, the five and six bosons are lengthy by their solitary constituent. In cases where the number of bosons is fixed, it is signified by the group $U(6)$ (As for the round oscillator. Correspondingly, it is shaped by the three apertures, which are $U(5)$, $SU(3)$ and $O(6)$ [14-16], these symmetries are related to the geometrical idea of the spherical vibrator, deformed rotor and a symmetric ($\gamma-$soft) deformed rotor, respectively.

INTERACTING BOSON MODEL-2

The interacting boson model-2, is a further step in the development of the interacting boson model. It is an approach which gives the collective nuclear states as described by interacting boson -1,a microscopic foundation, since the interacting boson model-2 can in principle be derived from the shell model. This development, which is based on the concept of the generalized fermion seniority [1,2], has been introduced by Arima et. al. [3-8]. The model has given the bosons a direct physical interpretation as correlated pairs of particles with $J^\pi = 0$ and $J^\pi = 2$.The Hamiltonian operator in interacting boson model-2 will have three parts: one part for each of proton and neutron bosons and a third part for describing the proton-neutron interaction[17-19].

$$\mathrm{H} = \mathrm{H}_\pi + \mathrm{H}_\nu + \mathrm{V}_{\pi\nu}\ldots\ldots(3)$$

A simple schematic Hamiltonian guided by microscopic consideration is given by[17-19]:-

$$\mathrm{H} = \varepsilon\left(\mathrm{n}_{\mathrm{d}\pi} + \mathrm{n}_{\mathrm{d}\nu}\right) + \kappa \mathrm{Q}_\pi \,.\, \mathrm{Q}_\nu + \mathrm{V}_{\pi\pi} + \mathrm{V}_{\nu\nu} + \mathrm{M}_{\pi\nu}\ldots(4)$$

where

$$Q_\rho = (d_\rho^\dagger s_\rho + s_\rho^\dagger d_\rho)_\rho^2 + \chi_\rho (d_\rho^\dagger d_\rho)_\rho^2 \;\; \rho = \pi, \nu\ldots(5)$$

$$V_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L^\rho \left[\left(d_\rho^\dagger d_\rho^\dagger\right)^{(L)} . \left(d_\rho d_\rho\right)^{(L)}\right]^{(0)}\ldots(6)$$

$\varepsilon_\pi, \varepsilon_\nu$ are proton and neutron energy respectively, they are assumed equal $\varepsilon_\pi = \varepsilon_\nu = \varepsilon$. The last term in Eq.(4) contains the Majorana operator $\mathrm{M}_{\pi\nu}$ and it is usually added in order to remove states of mixed proton neutron symmetry. This term can be written as[18-21]:-

$$\mathrm{M}_{\pi\nu} = \zeta_2 (\mathrm{s}_\nu^\dagger \mathrm{d}_\pi^\dagger - \mathrm{d}_\nu^\dagger \mathrm{s}_\pi^\dagger)^{(2)} . \left(\mathrm{s}_\nu \mathrm{d}_\pi - \mathrm{d}_\nu \mathrm{s}_\pi\right)^{(2)} + \sum_{\mathrm{k}=1,3} \zeta_\mathrm{k} (\mathrm{d}_\nu^\dagger \mathrm{d}_\pi^\dagger)^{(\mathrm{k})} - \left(\mathrm{d}_\nu \mathrm{d}_\pi\right)^{(\mathrm{k})}\ldots(7)$$

If there is empirical proof of the existence of the so-called "mixing symmetrical condition," the Majorana factor is changed in order to adjust the placement of these levels in the continuum. Due to this approximation, a system of neutron and proton bosons is taken into consideration. In the interacting boson model2, the microscopic interpretation of the boson number $\mathrm{N} = \mathrm{N}_\pi + \mathrm{N}_\nu$fixes the total number of bosons, N, which was previously treated as a parameter in the interacting boson model 1. It is

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possible to determine the levels of energy by diagonalizing Hamiltonian Eq. (4) and experimenting with the parameters $\varepsilon, \kappa, \chi_\pi, \chi_\nu$ and C_L to find the best match to the observed spectrum. One form of boson can be used to produce spectra that resemble those of the interacting boson model1 [20,21]. When $(\varepsilon \gg \kappa)$, $(\varepsilon \ll \kappa$ and $\chi_\pi = \chi_\nu = -\sqrt{7}/2)$, and($\varepsilon \ll \kappa$ and $\chi_\nu = -\chi_\pi$)are present, respectively. The U(5) limit, SU(3) limit, and O(6) limits are present. Most nuclei fall halfway between two of these three limiting instances rather than strictly falling under one of them. The interactive boson framework enables for a streamlined process between the restrictive conditions for different isotopes. The systematics of energy ratios of successive levels of collective bands in even –even medium and heavy mass nuclei were studied for vibrational and rotational limits for a given band for each I the following ratio were constructed to define the symmetry of excited band[22].

$$r\left(\frac{(J+2)}{J}\right) = \frac{R\left(\frac{(J+2)}{J}\right)exp. - \left(\frac{(J+2)}{J}\right)vib.}{R\left(\frac{(J+2)}{J}\right)rot. - \left(\frac{(J+2)}{J}\right)vib.}$$

$$= \frac{R\left(\frac{(J+2)}{J}\right)exp. - \left(\frac{(J+2)}{J}\right)}{2(J+2)/J(J+1)} \ldots .(8)$$

where $R((J+2)/J)exp.$ denotes the ratio's experimental value. For vibrational nuclei, the value of energy ratios, r, has vibrates to (0.1r0.35); for transitional nuclei, (0.4r0.6) and for rotating nuclei, (0.6r1).

CALCULATIONS AND RESULTS

The isotones have neutron number $N = 54$ which equivalent(two particles bosons) and atomic number $(Z = 42\ and\ 44)$ respectively, which equivalent $(4{,}3\ and\ 2)$ hole proton boson number. By calculating a number of variables specified in the formulas for the Hamiltonian component equations(2)&(3), the energy levels for have been calculated using the software packages $IBM - code$ computer code for interacting boson model-1 and Neutron Proton Boson $NPBOS - code$ for interacting boson model-2 [5]. Table (1) and Figure (1) provide the anticipated results for the computations of the stimulated energy state for three isotones that are conducted low-lying.

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Table 1. The parameters have been used in the interacting boson model1&2 Hamiltonian for even-even isotones (in MeV).

IBM1-Parameters in MeV, except χ		
Isotopes / **The parameters**	^{96}**Mo**	^{98}**Ru**
N	6	5
ε	0.44	0.57
a_0	0.0	0.0
a_1	0.02	0.008
a_2	-0.04	-0.018
a_3	0.001	0.001
a_4	0.001	0.001
χ	-0.8	-0.8
$IBM2$**-Parameters in MeV, except** χ		
Isotopes / **The parameters**	^{96}**Mo**	^{98}**Ru**
N_π	4	3
N_ν	2	2
ε_d	0.9	0.86
κ	-0.08	-0.08
χ_ν	-0. 8	-0. 8
χ_ν	-0. 8	-0. 8
ζ_2	0.01	0.01
$\zeta_{1,3}$	0.001	0.001
C_ν^L	(-0.6,0.8,0.1)	(-0.4,0.8,0.1)
C_π^L	(-0.56,0.8,-0.08)	(0.6,0.8,-0.08)

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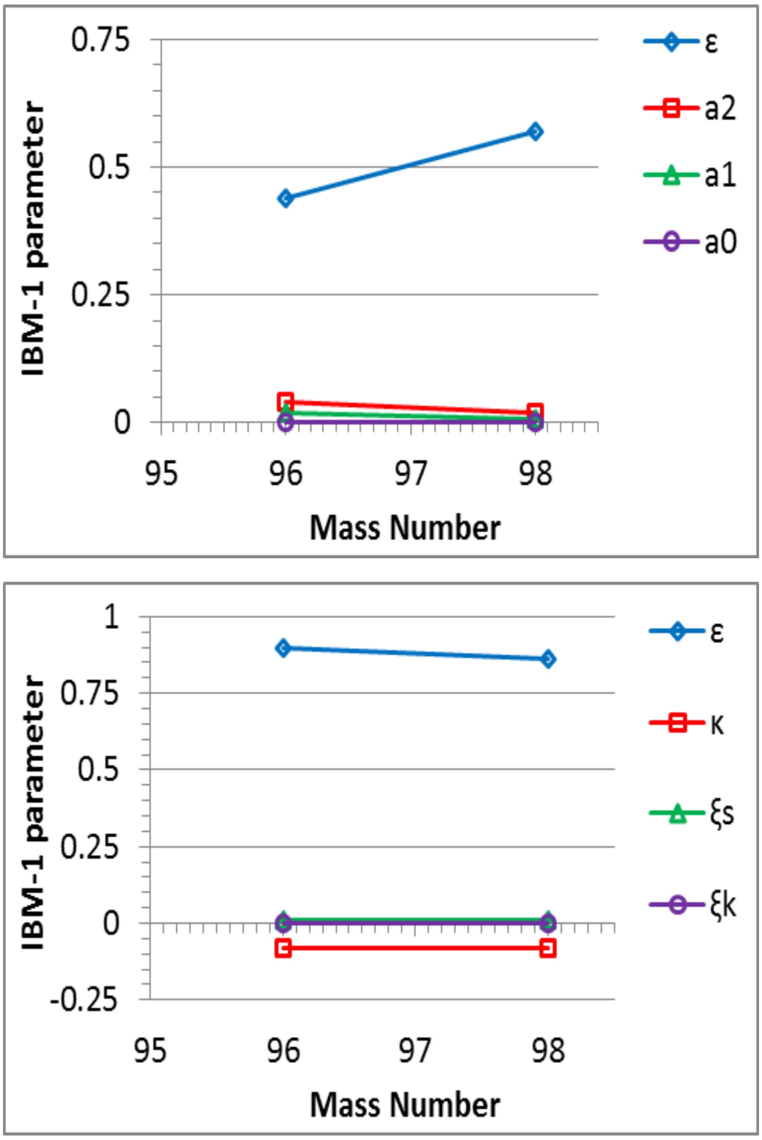

Figure 1. For even-even isotones, the parameters (in MeV)have been employed in the interacting boson model-1&2 Hamiltonian as a function of mass numbers

Calculation of energy ratios of $(E4_1^+/E2_1^+)$, $(E6_1^+/E2_1^+)$, $(E8_1^+/E2_1^+)$ and r ratios for all examined isotones have been calculated and are shown in figure (2).Figure (3) shows the estimated energy levels for the isotones in comparison to the experimental data[23-25].

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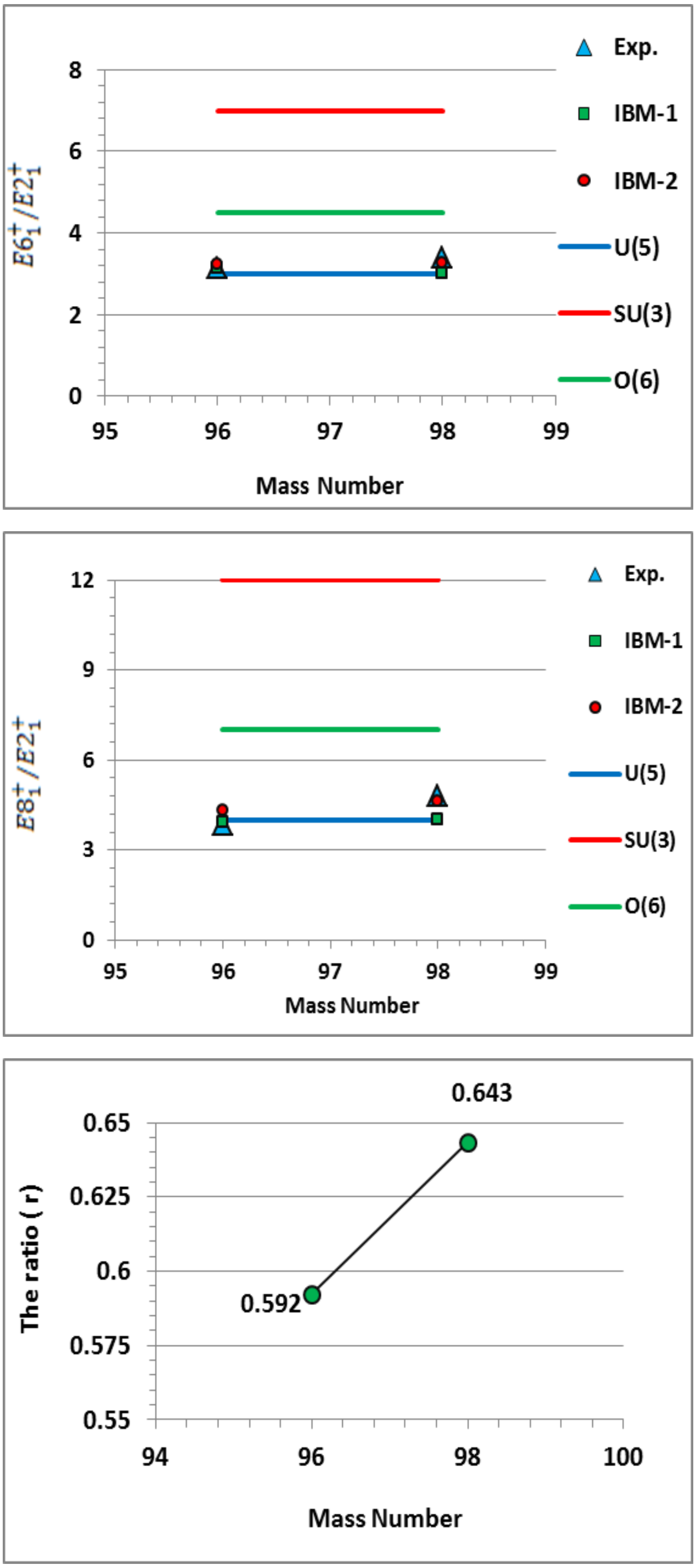

Figure 2 The experimental [23-25], theoretical and standard [1,2] energy ratios ($E4_1^+/E2_1^+$, $E6_1^+/E2_1^+$, , $E8_1^+/E2_1^+$) and r ratio[22]respectively as a function of mass numbers for even-even isotones

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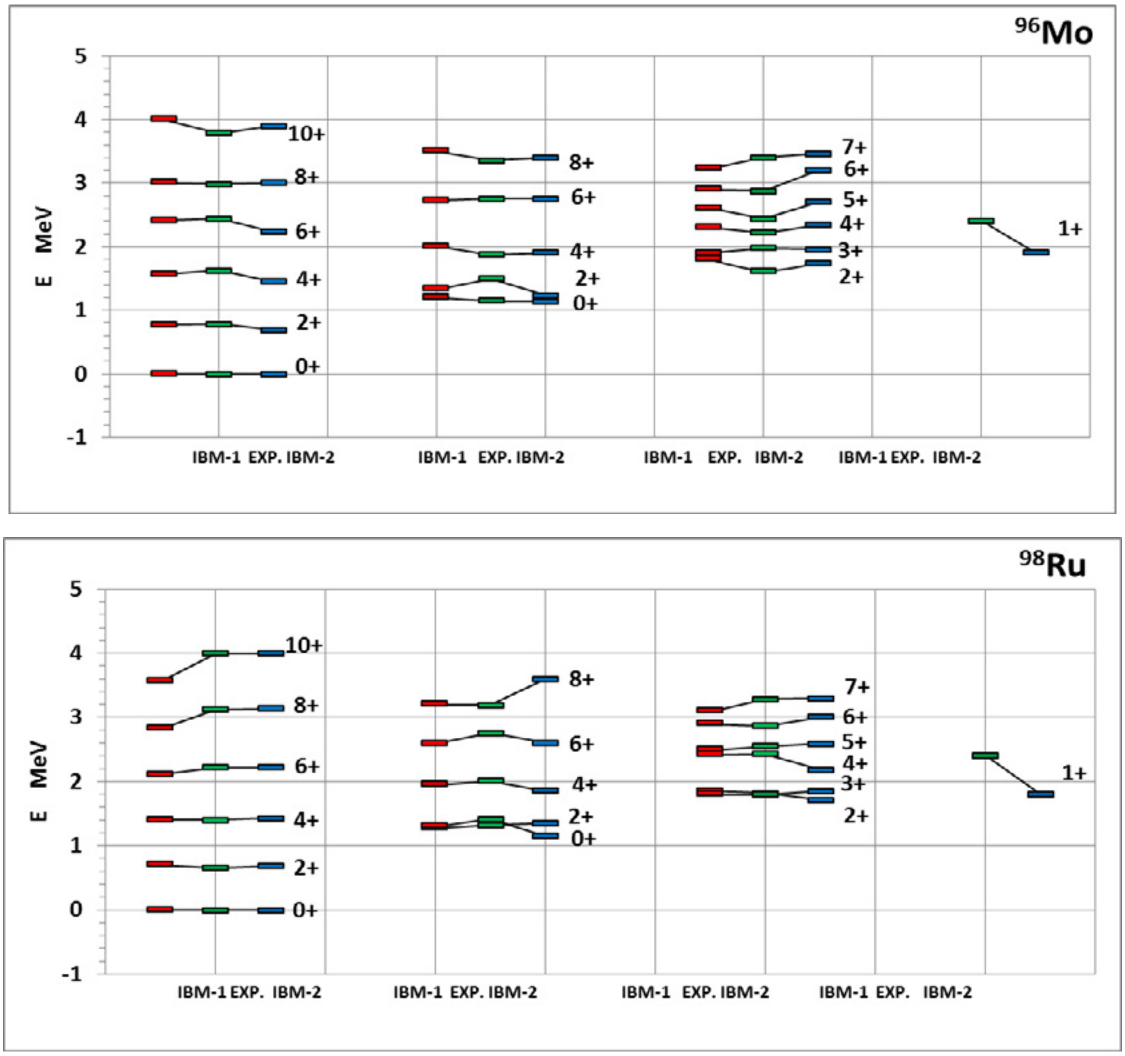

Figure 3 Comparison of the estimated and experimental[23-25] states for isotones,

For isotopes, the effect of Majorana parameters ($\zeta_{1,3}$, and ζ_2) on the levels of the calculated excitation energy for isotones has been conducted for all by vary the ζ_2 around the best-fitted with experimental data[23-25] for the states (2_3^+, 3_1^+ , $5_1^+$and 1_1^+). Figure (4) stated the variation of the energy of these states as the Majorana parameter function ζ_2.

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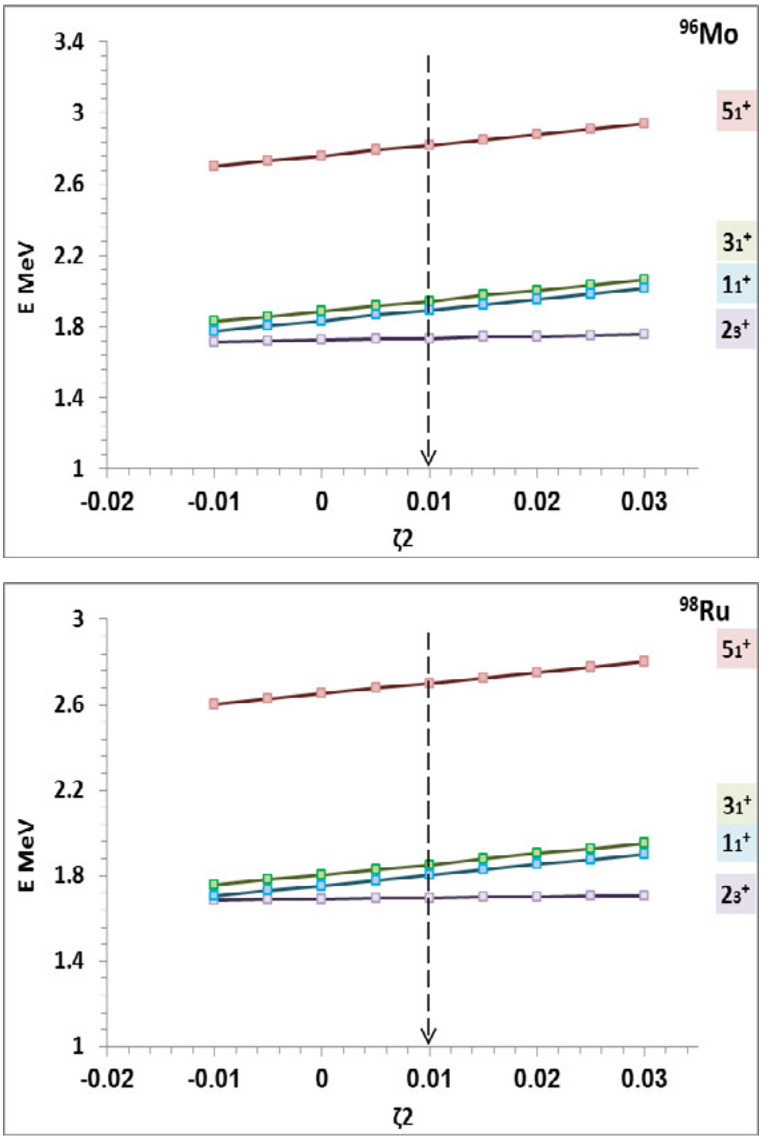

Figure 4 Mixed symmetry states MSS in even-even isotones.

DISCUSSION AND CONCLUSIONS

There are two approaches theoretical nuclear models $IBM1$ and $IBM2$which are used to predict the behavior of even-even isotones. The spectra of medium mass nuclei are usually characterized by the occurrence of low lying collective quadruple states. In this research, we discuss all results in term of dynamic symmetries, energy ratio, mixed symmetry states. The relatively medium mass nuclei nearly the mass number 100 which are located above the double closed $\mathrm{N}=50$ and $\mathrm{Z}=28$ have a few previous studies. The systematic excitation energy of low lying states mix the vibrational behavior and rotational behavior as illustrated by energy ratio $(\mathrm{E}4_1^+/\mathrm{E}2_1^+)$, $(\mathrm{E}6_1^+/\mathrm{E}2_1^+)$, $(\mathrm{E}8_1^+/\mathrm{E}2_1^+)$ and r in figure (2).

In the context of the interacting boson model-1, the competition between the two parameters ($\varepsilon$and a_2) is observed in the isotones of, where increases inε are correlated with decreases in a_2. This indicates that the opposite of the rotational

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properties, the vibrational features are continuously increasing. They emerge as a transition between the limits of vibration and rotation. The variants $\varepsilon, \kappa,$ χ_{π} and χ_{ν} in the interacting boson model-2 represent the resemblance with the interacting boson model-1 anticipated as planned in table (1) and figure (1). The most significant element of the interactive boson paradigm is the potential to describe composite symmetrical conditions in even-even nuclei formed from a combination of the capabilities of the protons and neutrons waveforms. The lowest MS states are those with $J^{+} = 2^{+}$ in more vibrational and gamma soft nuclei, while they are detected as $J^{+} = 1^{+}$ states in rotational nuclei. By varying the Majorana parameter effect (ζ_2) around the optimum-fitted to experimental data, it has been possible to determine the calculated excitation energy level for the isotones. It has been discovered that the state $\mathrm{J}^{+} = {2_3}^{+}$ has the lowest mixing symmetry state that is still roughly constant in all isotones. The effect of raising $, a_2$ on mixing symmetry states is the same in all isotones but varied from state to state. isotones contain $1^{+}, 3^{+}{}_{1}, 5^{+}{}_{1}$ mixed symmetry states at that moment, which rapidly increase as ζ_2 increases. The collective states location of mixed proton-neutron symmetry is one of the most remarkable open experimental difficulties in the study of collective features of nuclei. The experimental value[23-25] of the first 1^{+} for isotones are$\left(2.794 \text{ and } 2.406\right)$ MeV represented spin and parity as $(1^{+}\text{and } (1^{+}, 2^{+}))$ respectively, somewhat higher than the values in the best fitting (1.9027and 1.8012) MeV in addition to energy levels values for the 3_1^{+} state have a clear mixed symmetry state (MSS) increasing with increase the Majorana parameters ζ_2, the experimental values[23-25] of the first 3_1^{+} for isotones are $(1.978 \text{ and } 1.797)\mathrm{MeV}$, very similar to the exact fitted values$(1.9539$ and$1.85)$ MeV, the same notification applies to 5_1^{+} state with an excellent affinity between practical values and the best fitted.

REFERENCES

(1) F. Iachello and A. Arima. (1987). **The Interacting Boson Model.** *The Syndicate Press of the University of Cambridge, England.*

(2) R. Casten and D. Warner. (1988). **The Interacting Boson Approximation.** *Rev. Mod. Phys.*, 60, 389.

(3) A. Arima and F. Iachello. (1975). **Collective Nuclear States as Representations of a Group.** *Physical Review Letters*, 35, 16.

(4) A. Arima and F. Iachello. (1976). **Interacting Boson Model of Collective States I. The Vibrational Limit.** *Ann. Phys. NY,* 99, 253.

(5) A. Arima and F. Iachello. (1978). **Interacting Boson Model of Collective Nuclear States II.** *The Rotational Limit, Ann. Phys.,* 111, 201.

(6) A. Arima and F. Iachello. (1979). **Interacting Boson Model of Collective Nuclear States IV. The Limit.** *Ann.Phys.*, 123, 468.

(7) A. Arima, T. Otsuka, F. Iachello and I. Talmi. (1977). **Collective nuclear states as symmetric couplings of proton and neutron excitations.** *Physics Letters, B* 66, 205.

https://doi.org/10.17993/3cemp.2023.120151.243-255

(8) F. Iachello. (1979). **Interacting Boson in Nuclear Physics.** *Plenum, New York.*

(9) H. Najy Hady and M. Kadhim. (2021). **Investigation of transition symmetry shapes of 160- 168Yb nuclei using IBM.** *Iraqi Journal of Science*, 62, 4.

(10) H. Najy Hady. (2012). **Investigation of the low-lying energy levels structure of the rich neutron isotopes 96-104Mo**, *Journal of Kerbala University,* 10, 1, Scientific.

(11) Ch. Mu and Da. Zhang. (2018). **Description of Shape Coexistence and Mixed-Symmetry States in 96Mo Using IBM-2.** *Chin. Phys. Lett.* 35, 6.

(12) S. Thakur, P. Kumar, V. Thakur, V. Kumar and S. Dhiman. 2021. **Shape transitions and shell structure study in zirconium, molybdenum and ruthenium**, *Nuclear Physics A*, 1014, 122254.

(13) A. Yilmaz and and M. Kuruoglu, (2006). **Investigation of Even-Even Ru Isotopes in Interacting Boson Model-2.** *Commun. Theor. Phys. (Beijing, China)*, 46, 4.

(14) T. Thomasab, V. Wernerbc, J. Joliea, K. Nomurade, T. Ahnbf, N. Cooperb, H. Duckwitza A. Fitzlera, C. Fransena, A. Gadea1, M. Hintonb, G. Ilieb, K. Jessena, A. Linnemanna, P. Petkovagh, N. Pietrallac and D. Radecka. (2016). **Nuclear structure of 96,98Mo: Shape coexistence and mixed-symmetry states.** *Nuclear Physics A*, 947.

(15) P. Van Isacer and G. Puddu. (1980). **The Ru and Pd isotopes in the proton – neutron interacting boson model.** *Nuclear Physics,* A348.

(16) I. Ahmed, H. Abdullah, S. Ahmed, I. Hossain, M. Kasmin, M. Saeed and N. Ibrahim. (2012). **The evolution properties of even-even 100−110Pd nuclei**, *International Journal of Modern Physics E*, 21, 12.

(17) F. Iachello and A. Arima. (1974). **Boson symmetries in vibrational nuclei.** *Physical Letters*, 53, 4.

(18) F. Iachello, G. Puddu and O. Scholten. (1974). **A Calculation of low -lying collective state in even-even nuclei.** *Physical Letters*, 89B, 1.

(19) V. Hellemans, P. Van Isacker, S. De Baerdemacker and K. Heyde. (2007). **Phase transitions in the configuration mixed interacting boson model – mixing.** *Acta Phys. Polonica B*, 38, 4.

(20) J. Engel and F. Iachello. (1987). **Interacting boson model of collective octupole states (I). The rotational limit.** *Nuclear Physics A*, 472.

(21) R. Casten,and R. Cakirli. (2016). **The evolution of collectivity in nuclei and the proton–neutron interaction.** *Physica Scripta*, 91, 033004.

(22) A. Khalaf and A. Ismail. (2013). **Structure Shape Evolution in Lanthanide and Actinide Nuclei.** *Progress in physics*, 2.

(23) D. Abriola and A. Sonzogni. (2008). **Nuclear Data Sheets for A= 96.** *Nuclear Data Sheets*, 109(11), 2501-2655.

(24) K. Howard, U. Garg, M. Itoh, H. Akimune, M. Fujiwara, T. Furuno, Y. Gupta, M. Harakeh, K. Inaba, Y. Ishibashi, K. Karasudani, T. Kawabata, A. Kohda, Y. Matsuda, M. Murata, S. Nakamura, J. Okamoto, S. Ota, J. Piekarewicz, A. Sakaue, M. ¸Senyigit, M. Tsumura and Y. Yang. (2020). **Compressional-mode resonances in the molybdenum isotopes: Emergence of softness in open-shell nuclei near A = 90.** *Physics Letters B*, 807, 135608.

https://doi.org/10.17993/3cemp.2023.120151.243-255

(25) J. Chen and B. Singh. (2020). **Nuclear Data Sheets for A = 98**, *Nuclear Data Sheets*, 164(1), 1-477.