An Analysis of Optimal Tax Revenue Sharing for Mexico

Raúl Alberto Ponce Rodríguez[1] - Universidad Autónoma de Ciudad Juárez, México
Benito Alan Ponce Rodríguez[2] - Universidad Autónoma de Ciudad Juárez, México

Abstract

We develop an analysis that identifies the characteristics of an optimal system of shared tax collection and intergovernmental transfers. Mathematical optimization is used to find the level of taxes and intergovernmental transfers. Formulas for the optimal level of taxes and transfers to subnational governments are characterized. We suggest reforms to intergovernmental transfers to include the costs of tax inefficiency, some tax equalization transfer rules, and the marginal social benefits of local public spending. Future research could include local public spending with regional externalities, migration, and consider a dynamic model. This article proposes an original theoretical model of optimal tax coordination and transfers. The optimal level of taxes and transfers are identified. This paper proposes reforms to the participation formula for subnational governments.

JEL Classification: H2, H7, H72, H77.

Keywords: Tax revenue sharing, optimal taxation, state and local expenditures, fiscal federalism.

Un análisis de la participación óptima en los ingresos fiscales de México

Resumen

Se desarrolla un análisis que identifica las características de un sistema óptimo de recaudación tributaria compartida y transferencias intergubernamentales. Se utiliza la técnica de optimización para encontrar el nivel de impuestos y transferencias intergubernamentales. Se caracterizan fórmulas para el nivel óptimo de impuestos y transferencias a gobiernos subnacionales. Reformar las participaciones al incluir los costos de ineficiencia de los impuestos, algunas normas de transferencias de ecualización fiscal, y los beneficios sociales marginales del gasto público local. Para futuras investigaciones se podrían incluir el gasto público local con externalidades regionales, la migración, y considerar un modelo dinámico. Este artículo propone un modelo teórico original de coordinación de impuestos y transferencias óptimas. Se identifican el nivel óptimo de impuestos y transferencias. Este trabajo propone reformas a la fórmula de participaciones a gobiernos subnacionales.

Clasificación JEL: H2, H7, H72, H77.

Palabras clave: Ingresos tributarios compartidos, impuestos óptimos, gastos estatales y locales, federalismo fiscal.

---

[1] Corresponding Author. Av. Universidad y Av. Heroico Colegio Militar S/N Zona Chamizal C.P. 32300, Ciudad Juárez, Chihuahua, México. Phone: 6883837, email: rponce@uacj.mx. ORCID: https://orcid.org/0000-0003-0851-2687
[2] Email: alan.ponce@uacj.mx. ORCID: https://orcid.org/0000-0003-4550-2103

1. Introduction

One of the most distinctive characteristics of fiscal federalism in Mexico is its tax revenue sharing accord in which the federal government collects tax revenue from nationwide uniform tax rates in sales, personal income taxes and other taxes that are distributed to subnational governments though a formula of intergovernmental transfers. This tax revenue sharing accord is also known as the *law of fiscal coordination* established in Mexico in 1980 in which the federal and subnational governments seek to coordinate the fiscal system to establish rules for administrative collaboration between the federation, states and municipalities to collect tax revenue and distribute it among them.

In terms of tax collection, the law of fiscal coordination constitutes an agreement for the application of some taxes to collect public income. Some of the main taxes in the law of fiscal coordination include the special form of sales tax on production and services, also known as IEPS, the income tax from hydrocarbon exploration and extraction, wages and the provision of personal services, as well as other taxes.[3] In addition, the fund constituted by this law is financed by 80.29% of oil revenues from the federal government. In terms of the distribution of tax revenue, the law of fiscal coordination mandates the federal government to establish a set of intergovernmental transfers to be allocated to subnational governments. To do so, the federal government employs linear formulas for the distribution of intergovernmental transfers. For instance, the following formula describing the general fund of participations (which is one of several funds of "Ramo 28") in the last reform of the law of fiscal coordination published in 2018, is the following: [4,5]

$$P_{i,t} = P_{i,07} + \triangle FGP_{07,t}\left(0.6C1_{i,t} + 0.3C2_{i,t} + 0.1C3_{i,t}\right) \qquad (1)$$

Where $P_{i,t}$ is the transfer (or participation) from the federal government to state i in year t, $P_{i,07}$ is the transfer (or participation) from the federal government to state i in year 2007 and $\triangle FGP_{07,t}$ is the growth in the general participation fund between 2007 and year t. In addition, $C1_{i,t}$ is a distribution coefficient of the general participation fund related to the evolution of gross domestic product of state i, while $C2_{i,t}$ and $C3_{i,t}$ are distribution coefficients related to realized tax collections from state i.

The specific structure of the tax revenue sharing agreement in Mexico begs for the following question: Is the tax revenue sharing structure in Mexico optimal? Which are the main advantages and shortcomings from the current system? More importantly, how can we identify socially beneficial reforms to the current system of tax revenue sharing? In this paper we seek to provide answers to these questions. To do so, we develop a model of optimal tax revenue sharing. The purpose of this paper is to develop an in-depth analysis of the welfare calculus involved in designing a tax revenue sharing policy. Our model let us to develop a comparative analysis between socially optimal policies

---

[3] Originally, the law of fiscal coordination also considers a tax on new motor vehicles and fees applied to owners of mining concessions for the activities of exploration or exploitation.

[4] For more specific details see the document http://www.diputados.gob.mx/LeyesBiblio/pdf/31_300118.pdf

[5] We thank an anonymous referee for clarifying that equation (1) corresponds to a particular formula of the general participation fund, which is only one of several funds in "Ramo 28". Other participation funds have their own formulas.

and current policies implemented in the case of Mexico. Our purpose is to identify profitable avenues for fiscal policy reform to the current system of tax revenue sharing.

To do so, our paper characterizes the optimal structure of a tax revenue sharing by using some specific characteristics observed in the Mexican economy such as the tax structure. Hence our model provides specific formulas for the optimal level of taxation in the economy and we provide a new formula for the distribution of intergovernmental transfers known as participations. More importantly, our analysis provides several insights for possible fiscal policy reforms to the tax revenue sharing agreement in place in Mexico. In particular, our analysis prescribes the following insights: *first*, the formula of participations should include the regional distribution of inefficiency costs from taxation. Currently, the formula of participations does not consider any type of costs of taxation. However, including the inefficiency costs of taxation represents an important innovation for policy reform because one of the main sources of income in the tax revenue accord is derived from taxation which inevitably causes deadweight (or inefficiency) costs. Ignoring the inefficiency costs of taxes is likely to lead to suboptimal levels of taxation, tax revenue and ultimately participations.

Second, the formula for participations should include some elements of equalization transfers to improve the capacity of local governments to finance goods and services not currently considered under the law (specially in low income localities in which local governments might not be able to generate their own resources to finance such essential goods and services). *Third*, the formula for participations in each locality should incorporate indicators of the opportunity costs of allocating an extra $1 in the locality. In other words, the formula does not consider the marginal benefits of local spending associated with changing the size of the participation in the locality. This is an important shortcoming of the current system because the ultimate rationale of collecting taxes is to finance important goods and services provided by governments at the local level. The lack of indicators of marginal benefits of local public spending in the formula of participations is likely to lead to a suboptimal allocation of tax revenue in the economy.

The rest of the paper is structured as follows: section two includes a brief literature review. Section three considers the theoretical model in which we provide specific formulas for the optimal level of tax rates and the optimal level of participations for each subnational government. Section four includes a discussion of the implications of fiscal policy design of this paper. Section five concludes.

2. Literature Review

The literature of fiscal federalism has identified that one of the main advantages of a tax revenue sharing accord is that such policy helps to reduce problems associated with the misallocations of resources that result from uncoordinated tax and spending policies from different tiers of government. For instance, Keen (1998), Wilson (1999), and Devereux et al (2007) point out that each tier of government can design different tax and spending policies that could lead to vertical and horizontal tax externalities. Uncoordinated tax policies produce horizontal tax externalities when subnational governments do not recognize that their policies affect the tax base of neighborhood localities. In this case, subnational governments overestimate the marginal cost of public funds leading to sub-optimal levels of local taxation and spending (see Wilson, 1999).

In the event of vertical fiscal externalities, the central and subnational governments do not consider how their tax policies affect the tax bases of other tiers of government. Johnson (1988) and Boadway and Keen (1996) show that, in this case, all levels of government underestimate the marginal costs of public funds associated with raising tax revenue leading to too high taxation and spending. Since vertical and horizontal tax externalities arise because of differentiated and uncoordinated tax policies between different tiers of government, a tax revenue sharing accord could solve this problem by setting a unique set of uniform taxes applied across different regions of the economy.

However, a tax revenue sharing agreement also leads to more fiscal centralization which might reduce the nationwide welfare by setting uniform tax policies that might not recognize the regional heterogeneity of preferences (see Oates 1972). If the regional heterogeneity of preferences is high, it could be best to maintain fiscal decentralization (see Martinez-Vazquez et al 2015). Hankla et al (2019) show that policy coordination among different tiers of government can be reached not though a tax revenue agreement but by the right political incentives created in the electoral system of the economy. In this case, subnational governments maintain some autonomy in policy making but the cost of uncoordinated policies is mitigated.

For the particular case of the analysis of intergovernmental transfers in Mexico. Ponce and Ponce (2018) provide some estimates for the share of intergovernmental transfers. Their analysis is different to ours here since they analyze participations and contributions (which are discretionary transfer with specific purposes) while one of the purposes of this paper is to develop an analysis of the specific formula determining the transfers associated with the law of fiscal coordination currently in place in Mexico. In a different line of research, Broid and Timmons (2013) and Abbott et al (2015) develop an empirical analysis to study whether there is partisan effect in the allocation of intergovernmental transfers in Mexico. They find that the identity of the party in power matters to determine the size of intergovernmental transfers. Diaz-Cayeros, Magaloni and Weingast (2005) find that the allocation of federal transfers is consistent with short term electoral goals at the expense of providing public goods. All of these important papers are interested in the political economy of intergovernmental transfers in Mexico while our analysis is normative and interested in providing insights for policy reform.

In particular, our analysis seeks to develop a theoretical model that incorporates important stylized facts from the current tax revenue sharing agreement (such as the dependence of tax and the specific form of taxation included in the law of fiscal coordination) and to provide insights about the relevant social benefits and costs associated with taxation in the context of tax revenue sharing. Another important distinction of our analysis with recent literature, is that we provide socially optimal formulas for determining the size of participations for each subnational government. By so doing, in this paper we include an in-depth analysis of tradeoffs in policy design. In addition, we compare the policy that is currently in place with our analysis of optimal tax revenue sharing agreement for our economy and offer suggestions for policy reform. Finally, it is not less important to emphasize that, due to the need to make our analysis tractable, we did not include all relevant aspects of a tax revenue sharing system. From our analysis, it is absent the possibility of tax evasion and tax administration in the context of an economy with multi tiers levels of government. These two

issues are of great importance for the practice of fiscal policy in modern economies and, because of issues of space here, will be studied in future research.

3. The Model of Optimal Tax Revenue Sharing

In this section we develop a theory of an optimal tax revenue sharing system in which the federal government establishes a uniform tax, collects revenue and determines the formula for participation (or federal transfers) to all localities in the economy. In this economy there are several tiers of government. The fiscal structure is constituted by a federal government and bub-national governments for localities $i = 1,2, \dots I$. In each locality, there is a representative family with preferences given by $\mu_i = \mu_i(x_i, g_i) = (1 - \beta_i)ln(x_i) + \beta_i ln\,(g_i)$ where x_i and g_i are, correspondingly, private and public goods consumed by the representative household of locality i and $\beta_i > 0$ is a constant reflecting the intensity of preferences of the household for the local public good in locality i.We consider a Cobb-Douglas utility function because this function is widely used in the literature and it helps to compare our results with other works of related interest in the analysis of fiscal federalism. In addition, this utility function is considered for mathematical simplicity

In each locality there are n_i residents each of them with preferences given by μ_i. The budget constraint of the representative household is given by $x_i(1 + \tau) = y_i + T_i$ where y_i is the household's own income and T_i is an exogenous transfer received by the representative household while τ is a consumption tax. In this economy, the subnational distribution of income is not only determined by the localities' own resources but by the distribution of exogenous transfers (such as welfare programs of the government or private transfers such as remittances) in each locality. These transfers can affect the welfare calculus of policy makers about the optimal structure of taxation and intergovernmental transfers.

In this economy, there is a tax revenue sharing agreement between the federal and subnational governments in all localities. Under this accord, the federal government sets a uniform commodity tax to collect tax revenue from all localities. To develop our model, and to simplify the analysis, we consider only a sales tax. However, it is important to emphasize that a general sales tax is equivalent to a general income tax and therefore our analysis is also equivalent to the case in which we consider a broad income tax.[6] With the collected tax revenue, the federal government designs a rule for participation of this revenue to all localities. Hence the federal government transfers a participation of the tax revenue collected to each locality given by P_i for $i = 1,2 \dots I$. The indirect preferences of the representative household in locality i are given by v_i:[7]

$$v_i = Max \left\{ \begin{array}{c} \mu_i(x_i, g_i) = (1 - \beta_i)ln(x_i) + \beta_i ln\,(g_i) \\ s.t: \\ x_i(1 + \tau) = y_i + T_i \end{array} \right\} \qquad (2)$$

---

[6] To see this, note that in equation (2) we could replace the commodity tax τ for an income tax given by $t = 1 - \frac{1}{(1+\tau)}$. In this way the budget constraint of the household would look in the following way: $x_i = (y_i + T_i)(1 - t)$

[7] Because policy instruments are taxes and transfers the indirect utility function is useful for the analysis that follows. In addition, recall that, by definition, the operator "Max" in equation 2 says that the indirect utility is the maximum value that the direct utility can achieve for different values of income and prices.

Equation (2) shows that the indirect utility of the representative household in locality i, $v_i = v_i(y_i, T_i, \tau, g_i)$ depends positively on the household's own income y_i, exogenous transfers T_i and local government spending g_i in the locality. The welfare of the household also depends negatively on the federal consumption tax τ.

The problem of policy design for subnational governments is to administer the transfers received from the federal government that are determined by the tax revenue agreement between the federal government and subnational governments. The subnational government in locality i seeks to provide a local public good, g_i, that is financed by the federal transfer or *participation* P_i to maximize the social welfare of residents of locality i given by $\Psi_i = n_i v_i(y_i, T_i, \tau, g_i)$. For our analysis, it is convenient to state the social welfare of residents of locality i as follows:

$$\Psi_i = n_i v_i(y_i, T_i, \tau, g_i) \quad \text{subject to: } P_i = g_i \quad \forall i = 1,2 \tag{3}$$

Note that by using the budget constraint of the subnational government, the indirect utility function of the representative household can be written as follows $v_i(y_i, T_i, \tau, P_i)$ and therefore the social welfare function of the subnational government in that locality is $\Psi_i = n_i v_i(y_i, T_i, \tau, P_i)$ for $i = 1,2$.

The problem of policy design of the federal government is to set an optimal tax rate τ^* to collect tax revenue from all localities and then share the corresponding tax revenue through a set of transfers to all localities defined by *participations* P_i^* for $i = 1,2\ldots I$. These policy tools, τ^* $P_1^*, P_2^*, \ldots P_I^*$ are chosen to maximize the nationwide social welfare of all residents in this economy denoted by $\Psi_f = \omega_1 \Psi_1 + \omega_2 \Psi_2 + \cdots \ldots \omega_I \Psi_I$ which is the weighted sum of the aggregate welfare of residents in all localities where Ψ_i for $i = 1,2,\ldots I$ is the aggregate welfare of all residents in locality i and ω_i, is a positive constant that reflect the social weight or importance of residents of locality i in the social welfare function of the policy maker at the federal government.

The budget constraint of the federal government is given by $P_1 + P_2 + \cdots P_I = \tau \sum_{i=1}^{2} n_i x_i^*$ where the expression $\tau \sum_{i=1}^{2} n_i x_i^*$ corresponds to the nationwide tax revenue collected by the federal government and $P_1 + P_2 + \cdots P_I$ corresponds to the aggregate outlays of the federal government that provides fiscal resources for subnational governments to exercise local public spending.[8] Therefore, the problem of policy design for the federal government is:

$$\underset{\{\tau^*, P_1^*, \ldots. P_I^*\}}{Max} \quad \Psi_f = \sum_{i=1}^{I} \omega_i \Psi_i = \sum_{i=1}^{I} \omega_i n_i v_i(y_i, T_i, \tau, P_i) \tag{4}$$

$$s.t: \quad \sum_{i=1}^{I} P_i = \tau \sum_{i=1}^{I} n_i x_i^* \tag{5}$$

To solve the problem, we state the following Lagrangian

---

[8] In the budget constraint of the federal government the tax is applied over the Marshallian demand function of the representative household x_i^*, in other words, $x_i^* \in argmax\ \mu_i(x_i, g_i) = (1 - \beta_i)ln(x_i,) + \beta_i ln\ (g_i)$ subject to $x_i(1 + \tau) = y_i + T_i$. The reason to use the Marshallian demand in the budget constraint of the federal government is that the commodity tax is applied over the realized purchases of households.

$$\delta = \sum_{i=1}^{I} \omega_i n_i v_i(y_i, T_i, \tau, P_i) + \lambda \left(\tau \sum_{i=1}^{I} n_i x_i^* - \sum_{i=1}^{I} P_i \right) \tag{6}$$

Where λ is a Lagrange multiplier. The first order conditions of this policy problem are:

$$\frac{\partial \delta}{\partial \tau} = \sum_{i=1}^{I} \omega_i n_i \frac{\partial v_i(y_i, T_i, \tau, P_i)}{\partial \tau} + \lambda^* \left(\sum_{i=1}^{I} n_i x_i^* + \tau^* \sum_{i=1}^{I} n_i \frac{\partial x_i^*}{\partial \tau} \right) = 0 \tag{7}$$

For the optimal distribution of participations $P_1^*, P_2^*, \dots P_I^*$, it is satisfied:

$$\frac{\partial \delta}{\partial P_i} = \omega_i n_i \frac{\partial v_i(y_i, T_i, \tau, P_i)}{\partial P_i} - \lambda^* = 0 \quad for \ i = 1,2 \dots . I \tag{8}$$

And

$$\frac{\partial \delta}{\partial \lambda} = \tau^* \sum_{i=1}^{I} n_i x_i^* - \sum_{i=1}^{I} P_i^* = 0 \tag{9}$$

The first order condition in (7) imply that the optimal tax rate τ^* must satisfy the following welfare calculus: an increase in the optimal tax rate τ^* brings a positive marginal tax revenue collection given by $\sum_{i=1}^{I} n_i x_i^* + \tau^* \sum_{i=1}^{I} n_i \frac{\partial x_i^*}{\partial \tau}$ which at the optimum must be equal to the social marginal welfare cost of raising revenue from taxation. This nationwide welfare cost can be expressed as follows $-\frac{1}{\lambda^*} \sum_{i=1}^{I} \omega_i n_i \frac{\partial v_i(y_i, T_i, \tau, P_i)}{\partial \tau}$. The first order condition in (8) says that the optimal allocation of transfers for each locality must be equalized, that is the government cannot increase the welfare of the economy by reallocating money across localities. This condition is equivalent to:

$$\omega_1 n_1 \frac{\partial v_1(y_1, T_1, \tau, P_1)}{\partial P_1} = \omega_2 n_2 \frac{\partial v_2(y_2, T_2, \tau, P_2)}{\partial P_2} = \cdots \dots = \omega_I n_I \frac{\partial v_I(y_I, T_I, \tau, P_I)}{\partial P_I} \tag{10}$$

And finally, the first order condition (9) says that the government cannot waste resources by not allocating all of its tax revenue across all localities.

In what follows, proposition 1 characterizes the optimal commodity tax rate $\frac{\tau^*}{1+\tau^*}$ to be applied by the federal government.

Proposition 1. *The optimal commodity tax rate* $\frac{\tau^*}{1+\tau^*}$ *is given by:*

$$\left(\frac{\tau^*}{1 + \tau^*} \right) = \frac{1 - \frac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\lambda^*}}{\xi_A} \tag{11}$$

Where α_i is the marginal utility of income of household in locality i and λ^* is the social marginal welfare of transferring resources through participations to locality i:

$$\lambda^* = \frac{\omega_i n_i \beta_i}{P_i^*} \quad for \ i = 1,2,\dots.I \tag{12}$$

Moreover, s_i is the share of the tax base of locality i in relation to the nationwide tax base which is equivalent to the ratio of full income in locality i, $y_i + T_i$, in relation to the economy's full income $\sum_{i=1}^{I} n_i(y_i + T_i)$. That is,

$$s_i = \frac{n_i x_i^*}{\sum_{i=1}^{I} n_i x_i^*} = \frac{n_i(y_i + T_i)}{\sum_{i=1}^{I} n_i(y_i + T_i)} \quad for \ i = 1,2,\dots I \tag{13}$$

In addition, we define $\xi_i > 0$ as the elasticity of private consumption of locality i with respect to commodity tax:

$$\xi_i = -\frac{\partial x_i^*}{\partial \tau} \frac{1 + \tau^*}{x_i^*} > 0 \quad for \ i = 1,2,\dots I \tag{14}$$

And the nationwide aggregate elasticity of consumption with respect to commodity tax, ξ_A, is a weighted average from elasticities in all localities while the weighting factor is $s_i \in [0,1]$. Hence ξ_A satisfies

$$\xi_A = \sum_{i=1}^{I} \xi_i s_i > 0 \tag{15}$$

Proof.
See the appendix

Proposition 1, says that the optimal tax rate $\frac{\tau^*}{1+\tau^*}$ depends negatively on the welfare costs of taxation given by $\sum_{i=1}^{2} \omega_i \alpha_i s_i$. The society's welfare falls as the commodity tax rate increases because the price of the private good increases which leads to a fall in private consumption and welfare. The optimal tax rate $\frac{\tau^*}{1+\tau^*}$ also depends positively from the social marginal utility of transferring resources to localities $i = 1,2,\dots I$ that is λ^*, and negatively from the deadweight (or inefficiency) costs of taxation caused by the distortion of relative prices in the economy. The higher is the weighted aggregate elasticity of private consumption with respect to commodity tax, ξ_A, the higher the deadweight costs from taxation and the lower should be the tax rate at equilibrium.

In what follows, proposition 2 characterizes the optimal distribution of *participations* or federal transfers to subnational governments in localities $i = 1,2,\dots I$ while we leave the formal proof for the interested reader to the appendix. In addition, propositions 3 explains the main determinants of the formula that determines tax revenue into federal transfers to subnational governments in all

localities. After the characterization of proposition 3, we explain intuitively the main determinants of participations in our economy.

Proposition 2. *The optimal distribution of tax revenue in locality i, P_i^* for $i = 1,2....I$, is given by:*

$$P_i^* = \frac{\left(\frac{1}{\xi_A}\right)\sum_{i=1}^{I} n_i(y_i + T_i)}{\left(\frac{1}{\psi_i} + \frac{1}{\xi_A}\left(\frac{\sum_{i=1}^{I}\omega_i\alpha_i n_i(y_i + T_i)}{\omega_i n_i \beta_i}\right)\right)} \quad for \ i = 1,2,...I \quad (16)$$

Where ψ_i is the relative social importance (or relative social marginal utility) of locality i in the economy.

$$\psi_i = \frac{\omega_i n_i \beta_i}{\sum_{i=1}^{2}\omega_i n_i \beta_i} \quad for \quad i = 1,2,....I \quad (17)$$

Proof
See the appendix.

Proposition 3. *The optimal size of the federal transfer through the tax revenue sharing agreement to subnational government i, $P_i^* \ \forall i = 12,...I$, depends*
3.i) Positively on full income of locality i if the size of population in locality i is sufficiently large.
3.ii) In an ambiguous way from increases in the inefficiency costs of taxation measured though the elasticity of consumption with respect to commodity tax ξ_A. That is, $dP_i^*/d\xi_A \gtrless 0$.

3.iii) In an ambiguous way from increases in the population of locality i n_i. That is, $dP_i^*/dn_i \gtrless 0$.

3.iv) Positively from the relative social importance (or relative social marginal utility) of locality i, ψ_i in the economy.

Proof.
See the appendix.

Proposition 3 says that the main determinants of federal *participations* to subnational governments are the per capita full income from the locality, $y_i + T_i$, the inefficiency costs of taxation measured though the elasticity of consumption with respect to commodity tax ξ_A, the population in each locality, and the relative social marginal utility of residents in each locality ψ_i. This last concept reflects the fact that policy makers might have preferences for the welfare of residents in different localities. This preference means that the social marginal utility of residents of certain locality could have a high weight in the social welfare function of the policy maker in the federal government.

In fact, the relative social marginal utility of residents of certain locality might be related with concerns of inter-regional equity. Policy makers might care about the regional inequality of income because the income of certain localities in the economy can be sufficiently low such that the local government does not have enough resources to provide basic public services such as health care, education, spending in infrastructure and anti-poverty programs. If the relative social marginal utility of this type of households is high in the social welfare function, the policy maker reveals that

the social marginal benefit of transferring an extra \$1 to that locality is high as well. As a result, the federal government will provide a high transfer to that locality to reduce the inter-regional inequality in the capacity of the local government in providing essential government services.

   Proposition 3 also says that:

i.   An increase in the full percapita income in a locality, $y_i + T_i$, increases the capacity of the central government to collect tax revenue and increases the size of transfers in all localities.

ii. An Increase in the deadweight costs of taxation leads to a fall of transfers in all localities. Higher deadweight costs increase the welfare costs of taxation which tends to reduce the optimal tax rate $\frac{\tau^*}{1+\tau^*}$ at the equilibrium and consequently reduces the size of transfers of all localities. However, an increase of the deadweight costs of taxation also changes the social marginal utility of participations. This effect tends to increase the size of optimal participations. Hence the net effect is ambiguous, that is to say, $dP_i^*/d\xi_A \gtrless 0$.

iii. An increase in the population of locality i has an ambiguous effect on transfers in all localities. Increases in population lead to three different effects on P_i^* that are in conflict:

   o   First, an increase in the population of locality i increases the economy´s full income (the size of the tax base) and the government's tax revenue. This effect tends to increase P_i^*.

   o   Second, an increase in the population of locality i increases the social marginal benefit of transferring \$1 through an intergovernmental transfer to locality i. This effect also tends to increase P_i^*.

   o   The third effect is that the welfare costs of taxation are a positive function of population of locality i.[9] Hence, the higher the population in locality i the higher the welfare costs from taxation and the lower should be the optimal transfer P_i^*.

   If the first and second effects of a positive change in population in locality i dominate the third effect then an increase in population should increase P_i^*, conversely (if the third effect dominates the first and second effects described above) then P_i^* should decrease.

iv. An increase in the relative social marginal utility of residents in locality i leads to a higher social marginal benefit of transferring an extra \$1 to that locality. Therefore P_i^* should be higher at the equilibrium.

   In what follows, corollary 4 presents a result that is relevant to compare with the current policy in place for the tax revenue sharing agreement used in Mexico.

Corollary 4. *The optimal formula for the allocation of participations P_i^* for $i = 1,2,...I$, can be stated as follows:*

---

[9] Recall that the welfare costs of taxation are associated with lower utility levels of residents of locality i due to the fact that taxes take away income from households.

$$P_i^* = \sigma_i \sum_{i=1}^{I} n_i(y_i + T_i) \qquad \forall i = 1,2 \qquad (18)$$

Where

$$\sigma_i = \frac{\Phi_i}{\xi_A} > 0 \quad for \quad i = 1,2,\dots I \qquad (19)$$

And

$$\Phi_i = \frac{1}{\frac{1}{\psi_i} + \frac{1}{\xi_A}\left(\frac{\sum_{i=1}^{I} \omega_i \alpha_i n_i(y_i + T_i)}{\omega_i n_i \beta_i}\right)} > 0 \quad for \quad i = 1,2,\dots I \qquad (20)$$

Corollary 4 shows that a nonlinear relationship between participations in any given locality *i* to changes in the percapita full income, population, and the deadweight costs from taxation in locality *i,* could be socially optimal. Corollary 4 shows that the marginal effect on participations in locality *i* due to increases in full income in locality *i*, $y_i + T_i$ is positive but marginally decreasing. It is also relevant to point out that the marginal effect of full income in locality *i*, $y_i + T_i$ on P_i^* *is not equivalent to* σ_i. The exact relationship between changes in full income in locality *i*, $y_i + T_i$, is more complicated and it is demonstrated in proposition 3 (see the appendix).

4. Policy Implications

The specific structure of the tax revenue sharing agreement in Mexico begs the following questions: Is the tax revenue sharing structure in Mexico optimal? Which are the main advantages and shortcomings from the current system? And more importantly: How can we identify guidelines for beneficial policy reforms to the current tax revenue sharing agreement? As we mentioned before, a tax revenue sharing system brings advantages (such as solving problems of horizontal and vertical tax externalities) and disadvantages (such as the increase of fiscal centralization in the economy and the welfare loss in the society associated with less diversity of tax and spending policies at local governments). In addition, it is important to discuss specific issues related with the current policy of tax revenue sharing in Mexico.

In particular, from the current formulas of distribution of intergovernmental transfers in Mexico (such as the one shown in condition 1), it is important to emphasize the following: *first*, the formula does not consider inefficiency costs from taxation as a main determinant of intergovernmental transfers. However, this couldbe an important element of fiscal policy design because one of the main sources of public revenue of this fund is derived from taxation which inevitably causes deadweight (or inefficiency) costs. Ignoring inefficiency costs from taxation is likely to lead to suboptimal tax policies and a suboptimal tax revenue sharing system.

Second, inter-regional equity considerations are absent in the current formula for participations. However, the literature in fiscal federalism has emphasized that one advantage of a fiscally centralized instrument, such as the tax revenue sharing accord in Mexico, is the establishment

of equalization transfers. In practice, policy makers might care about the regional inequality of income because subnational governments in low income localities might not have enough resources to provide basic public services such as health care, education, spending in infrastructure and anti-poverty programs. A system of equalization transfers can improve the inequality in the regional distribution of welfare by instituting transfers from rich to poor localities that improve the capacities of local governments with low income to provide fundamental goods and services.

Third, there are no considerations for the opportunity costs of intergovernmental transfers in the formula of participations. That is, the formula of participations does not consider the relative marginal benefit from allocating an additional $1 to locality i at the expense of allocating that transfer to some other locality. Hence, the formula of participations does not consider the marginal benefits of local spending associated with changing the size of intergovernmental transfers in each locality. Ignoring the benefits of spending financed by participations in localities is also likely to lead to suboptimal decisions which might reduce significantly the overall efficiency of a tax revenue sharing program. This, could be a significant shortcoming of the current system in Mexico.

Fourth, non-linear incentives from the localities' domestic product are excluded in the formula (that is to say, the formula of participations is linear). What this means is that marginal changes in participations in each locality due to changes in local GDP and tax effort are constant.

Our analysis of optimal tax revenue sharing systems can provide interesting avenues for policy reform. In particular, our analysis suggests the following insights:

I) *Incorporate the inefficiency Costs of Taxation in The Formula for Participations* The formula of participations should also include the regional distribution of inefficiency costs from taxation. In practice, these inefficiency costs can be incorporated by estimating the elasticity of consumption in each locality with respect to sales tax (or if it is the case in which the fund of tax revenue sharing is primarily determined by the income tax, then use the elasticity of the supply of labor with respect to wages).

II) *Incorporate Indicators of Regional Equity in The Formula for Participations.* The formula should include some elements of equalization transfers to improve the capacity of local governments to finance essential goods and services such as education, health services and anti-poverty programs. In practice, the formula would incorporate the objective of equalization transfers by designing an indicator of relative social importance (or relative social marginal utility) of locality i in the society. In our model, the parameter ψ_i is the relative social marginal utility of locality i and this parameter could be inversely proportional to the relative income of locality i in relation to the nationwide economy. By so doing, localities with lower than average income in the economy would receive higher than average transfers relative the transfers received in a formula that ignores the regional inequality in the distribution of income.

III) *Incorporate the Opportunity Cost of Allocating an Extra $1 in some given Locality in The Formula for Participations.* The tax revenue sharing accord should incorporate the social marginal benefits of local spending, that is to say, how allocating an extra $1 in some locality is beneficial not only to that locality but also to the society. Our analysis in this paper in

proposition 2 says that the optimal size of participations requires that the social marginal utility of allocating an extra $1 through the formula of participations must be the same across localities. What this means is that the nationwide social welfare in the economy can not improve by reallocating participations from some locality at the expense of other locality. In practice, one way to incorporate the opportunity costs of allocating $1 through the formula of participations is considering the regional heterogeneity of preferences of households for goods and services provided by the local government. In practice, the heterogeneity of preferences of households for public spending in localities can be estimated by considering the social-demographic characteristics of residents in each locality such as gender, median or average age, density of population, etc.

5. Conclusions

of the most distinctive characteristics of fiscal federalism in Mexico is its tax revenue sharing accord (the law of fiscal coordination) in which the federal government collects tax revenue from nationwide uniform tax rates in sales, income and other taxes that are distributed to subnational governments though a formula of intergovernmental transfers called participations ("*participaciones*"). These participations finance important goods and services provided by local governments. The formula of participations used in Mexico uses local gross domestic product and tax effort to determine the size of participations to be distributed to each state. The specific structure of the formula for participations begs for the following question: Is the tax revenue sharing system used in Mexico optimal? Which are the main advantages and shortcomings from the current system? And more importantly: How can we identify guidelines for beneficial policy reforms to the current tax revenue sharing accord?

In this paper we provide answers to these questions by developing a theoretical analysis that provide insights about the characteristics of an optimal tax revenue sharing system. Our analysis contributes to the literature by making a comparison between the current structure of the tax revenue sharing accord used in Mexico and our own analysis of an optimal tax revenue sharing system. Our analysis provides insights about feasible policy reforms to the current law of fiscal coordination: *first*, the tax revenue sharing accord should incorporate the inefficiency costs of taxation in the formula for participations. *Second*, the efficacy of the tax revenue sharing system in Mexico could improve if we incorporate indicators of regional equity in the formula for participations. *Third*, the formula of participations should incorporate the social marginal benefits of allocating resources to finance local public spending. A reform in this direction would link more closely the local costs of taxation with the local benefits of spending, improving this way, the net efficiency of the tax revenue sharing accord.

Some topics for future research on tax revenue sharing could be the effect of local public goods with regional spillovers and inter-state migration. Of interest for public policy design would be an analysis of the relative merits of direct transfers from the central governments to state and local governments compared with transfers from the central government to state governments which in turn make transfers to local governments. Another avenue of interest for future research could be to include a dynamic model to understand the role of non tax revenue such as income from oil.

References

[1]   Abbott, A., Cabral, R., Jones, P., & Palacios, R. (2015). Political pressure and procyclical expenditure: An analysis of the expenditures of state governments in Mexico. European Journal of Political Economy, 37, 195–206. https://doi.org/10.1016/j.ejpoleco.2014.12.001

[2]   Boadway, R., and Keen, M. (1996). Efficiency and the optimal direction for federal-state transfers". International Tax and Public Finance, 3(2): 137-155. https://doi.org/10.1920/wp.ifs.1996.9601

[3]   Broid, D. and J.F. Timmons, (2013). The political economy of municipal transfers: evidence from Mexico. Publius: The Journal of Federalism 43, Issue 4: 551–579, https://doi.org/10.1093/publius/pjt007

[4]   Devereux M.P., Lockwood, B., Redoano, M. (2007). Horizontal and vertical indirect tax competition: Theory and some evidence from the USA". Journal of Public Economics, 91, (3-4): 451-479. https://doi.org/10.1016/j.jpubeco.2006.07.005

[5]   Diaz-Cayeros, A., B. Magaloni, and B. Weingast. (2005). Tragic brilliance: equilibrium party hegemony in Mexico. Working Paper, Stanford University. https://doi.org/10.2139/ssrn.1153510

[6]   Hankla, C. Martínez-Vazquez J. and R.A Ponce-Rodríguez, (2019). Local accountability and national coordination in fiscal federalism: a fine balance, Edward and Elgard Publishing, United Kingdom. https://doi.org/10.4337/9781788972178

[7]   Johnson, W. R. (1988). Income redistribution in a federal system, American Economic Review, 78(3): 570-573. https://doi.org/10.1016/b978-0-12-781820-7.50011-x

[8]   Keen, M. (1998). Vertical tax externalities in the theory of fiscal federalism. *IMF Staff Papers*, 45,(3): 454-485. https://doi.org/10.5089/9781451977318.001

[9]   Martinez-Vazquez, J., Lago-Peñas, S. and Sacchi, A. (2017). The impact of fiscal decentralization: a survey. Journal of Economic Surveys, 31: 1095-1129. https://doi.org/10.1111/joes.12182

[10]   Oates, Wallace E. 1972. Fiscal Federalism. New York: Harcourt, Brace, Jovanovich. https://doi.org/10.2307/1959998

[11]   Ponce-Rodríguez, R.A and B.A. Ponce-Rodríguez (2019), Social Weights and the Regional Allocation of Inter-Governmental Transfers. Revista Mexicana de Economía y Finanzas, Vol. 14, Num 4, 745-767. https://doi.org/10.21919/remef.v14i4.389

[12]   Wilson, J.D. (1999). Theories of tax competition. National Tax Journal 52(2), 269-304. https://doi.org/10.1086/ntj41789394

Appendix

Proof of Proposition 1

From the first order conditions it is satisfied:

$$\frac{\partial \delta}{\partial \tau} = \sum\nolimits_{i=1}^{I} \omega_i n_i \frac{\partial v_i(y_i, T_i, \tau, P_i)}{\partial \tau} + \lambda^* \left(\sum\nolimits_{i=1}^{I} n_i x_i^* + \tau^* \sum\nolimits_{i=1}^{I} n_i \frac{\partial x_i^*}{\partial \tau} \right) = 0 \quad (A.1)$$

Which is equivalent to:

$$\frac{\partial \delta}{\partial \tau} = -\sum_{i=1}^{I} \omega_i \alpha_i n_i x_i^* + \lambda^* \left(\sum_{i=1}^{I} n_i x_i^* + \tau^* \sum_{i=1}^{I} n_i \frac{\partial x_i^*}{\partial \tau} \right) = 0 \qquad (A.2)$$

Equivalent to:

$$-\frac{\sum_{i=1}^{I} \omega_i \alpha_i \, n_i x_i^*}{\lambda^* \sum_{i=1}^{I} n_i x_i^*} + 1 + \left(\frac{\tau^*}{1+\tau^*} \right) \sum_{i=1}^{I} \left(\frac{\partial x_i^*}{\partial \tau} \frac{1+\tau^*}{x_i^*} \right) \frac{n_i x_i^*}{\sum_{i=1}^{I} n_i x_i^*} = 0 \qquad (A.3)$$

Define s_i as the share of consumption on the private good in locality i in relation to the aggregate consumption in the economy

$$s_i = \frac{n_i x_i^*}{\sum_{i=1}^{I} n_i x_i^*} \qquad \forall i = 1,2, \dots . I \qquad (A.4)$$

And we define the elasticity of private consumption of locality i and the commodity tax is:

$$\xi_i = -\frac{\partial x_i^*}{\partial \tau} \frac{1+\tau^*}{x_i^*} > 0 \quad \forall i = 1,2 \dots \dots . I \qquad (A.5)$$

And the nationwide aggregate elasticity of consumption and tax ξ_A is a weighted average from elasticities in all localities while the weighting factor is $s_i \in [0,1]$. *Hence*

$$\xi_A = \sum_{i=1}^{I} \xi_i s_i > 0 \qquad (A.6)$$

Use the last equations into the first order conditions in (A.3) to prove that

$$-\frac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\lambda^*} + 1 - \left(\frac{\tau^*}{1+\tau^*} \right) \xi_A = 0 \qquad (A.7)$$

Therefore, the optimal tax rate $\frac{\tau^*}{1+\tau^*}$ is given by:

$$\left(\frac{\tau^*}{1+\tau^*} \right) = \frac{1 - \dfrac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\lambda^*}}{\xi_A} \qquad (A.8)$$

Proof of Proposition 2.

From the budget constraint of the federal government

$$\sum_{i=1}^{I} P_i^* = \tau^* \sum_{i=1}^{I} n_i x_i^* \qquad (A.9)$$

From the first order condition

$$\omega_1 n_1 \frac{\partial v_1(y_1, T_1, \tau, P_1)}{\partial P_1} = \omega_2 n_2 \frac{\partial v_2(y_2, T_2, \tau, P_2)}{\partial P_2} = \cdots \ldots = \omega_I n_I \frac{\partial v_I(y_I, T_I, \tau, P_I)}{\partial P_I} \qquad (A.10)$$

Implying

$$\frac{\omega_1 n_1 \beta_1}{P_1^*} = \frac{\omega_2 n_2 \beta_2}{P_2^*} = \cdots \ldots \ldots \frac{\omega_2 n_2 \beta_2}{P_I^*} \qquad (A.11)$$

Therefore

$$P_2^* = \left(\frac{\omega_2 n_2 \beta_2}{\omega_i n_i \beta_i}\right) P_i^* \; ; \; P_3^* = \left(\frac{\omega_3 n_3 \beta_3}{\omega_i n_i \beta_i}\right) P_i^* \; ; \ldots \ldots \qquad P_I^* = \left(\frac{\omega_I n_I \beta_I}{\omega_i n_i \beta_i}\right) P_i^* \qquad (A.12)$$

Use the former conditions into the budget constraint to show that

$$P_i^* \left(\frac{\sum_{i=1}^{I} \omega_i n_i \beta_i}{\omega_i n_i \beta_i}\right) = \tau^* \sum_{i=1}^{2} n_i x_i^* \qquad \forall i = 1,2 \ldots \ldots I \qquad (A.13)$$

Define

$$\psi_i = \frac{\omega_i n_i \beta_i}{\sum_{i=1}^{2} \omega_i n_i \beta_i} \qquad \forall i = 1,2, \ldots \ldots I \qquad (A.14)$$

Therefore, the budget constraint is given by:

$$P_i^* \left(\frac{1}{\psi_i}\right) = \tau^* \sum_{i=1}^{I} n_i x_i^* \qquad (A.15)$$

Recall $x_i^* = \frac{y_i + T_i}{1 + \tau^*}$, therefore

$$P_i^* \left(\frac{1}{\psi_i}\right) = \left(\frac{\tau^*}{1 + \tau^*}\right) \sum_{i=1}^{I} n_i (y_i + T_i) \qquad (A.16)$$

Hence, the budget constraint of the federal government is

$$P_i^* \left(\frac{1}{\psi_i} \right) = \left(\frac{\tau^*}{1+\tau^*} \right) \sum_{i=1}^{I} n_i(y_i + T_i) \qquad (A.17)$$

Recall that

$$\left(\frac{\tau^*}{1+\tau^*} \right) = \frac{1 - \frac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\lambda^*}}{\xi_A} \qquad (A.18)$$

Therefore, the expression $\frac{\tau^*}{1+\tau^*}$ is equivalent to:

$$P_1^* \left(\frac{1}{\psi_i} \right) = \left(\frac{1 - \frac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\lambda^*}}{\xi_A} \right) \sum_{i=1}^{I} n_i(y_i + T_i) \qquad (A.19)$$

Recall

$$\lambda^* = \frac{\omega_i n_i \beta_i}{P_i^*} \qquad (A.20)$$

Therefore

$$P_i^* \left(\frac{1}{\psi_i} + \frac{1}{\xi_A} \left(\frac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\omega_i n_i \beta_i} \right) \sum_{i=1}^{I} n_i(y_i + T_i) \right) = \frac{\sum_{i=1}^{2} n_i(y_i + T_i)}{\xi_A} \qquad (A.21)$$

Hence

$$P_i^* \left(\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^{I} \omega_i \alpha_i s_i}{\omega_i n_i \beta_i \xi_A} \right) \sum_{i=1}^{I} n_i(y_i + T_i) \right) = \left(\frac{1}{\xi_A} \right) \sum_{i=1}^{I} n_i(y_i + T_i) \qquad (A.22)$$

Use the fact that

$$s_i = \frac{n_i x_i^*}{\sum_{i=1}^{I} n_i x_i^*} = \frac{n_i(y_i + T_i)}{\sum_{i=1}^{I} n_i(y_i + T_i)} \qquad \forall i = 1,2,\ldots\ldots I \qquad (A.23)$$

To reduce terms and express

$$P_i^* = \frac{\left(\frac{1}{\xi_A} \right) \sum_{i=1}^{I} n_i(y_i + T_i)}{\left(\frac{1}{\psi_i} + \frac{1}{\xi_A} \left(\frac{\sum_{i=1}^{I} \omega_i \alpha_i n_i(y_i + T_i)}{\omega_i n_i \beta_i} \right) \right)} \qquad (A.24)$$

Proof of proposition 3.i.

To develop our analysis state the budget constraint of the federal government as follows

$$\sum_{i=1}^{I} P_i^* = \tau^* \sum_{i=1}^{I} n_i x_i^* \tag{A.25}$$

Following the mathematical analysis of proposition 2, (in other words, using the definition of P_i^*), we define the variable $\Upsilon_i^* = 0$ to develop our comparative static analysis as follows:

$$\Upsilon_i^* = P_i^* \left(\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^{I} \omega_i \alpha_i n_i (y_i + T_i)}{\omega_i n_i \beta_i \xi_A} \right) \right) - \left(\frac{1}{\xi_A} \right) \sum_{i=1}^{I} n_i (y_i + T_i) = 0 \tag{A.26}$$

Note that the function Υ_i^* is an equilibrium condition that is a function of the following exogenous variables $\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A$, and ψ_i. Hence, we can state Υ_i^* as follows

$$\Upsilon_i^* = 0: \Upsilon_i^* = f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \psi_i) \tag{A.27}$$

To prove statement *3.i)* we obtain the total differential of Υ_i^* to calculate:

$$\Delta \Upsilon_i^* = \frac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial P_i^*} dP_i^*$$

$$+ \frac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial (y_i + T_i)} d(y_i + T_i) \tag{A.28}$$

Therefore, to show proposition (*3.i*), that is how changes in per capita full income in locality *i*, $y_i + T_i$, affect the size of transfers in locality *i*, P_i^*, we solve mathematically for $\frac{dP_i^*}{d(y_i+T_i)}$, therefore:

$$\frac{dP_i^*}{d(y_i + T_i)} = - \frac{\dfrac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial (y_i + T_i)}}{\dfrac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial P_i^*}} \tag{A.29}$$

Which is equivalent to:

$$\frac{dP_i^*}{d(y_i + T_i)} = \frac{\dfrac{1}{\xi_A} \left(n_i - \dfrac{\alpha_i P_i^*}{\beta_i} \right)}{\dfrac{1}{\psi_i} + \left(\dfrac{\sum_{i=1}^{I} \omega_i \alpha_i n_i (y_i + T_i)}{\omega_i n_i \beta_i \xi_A} \right)} \tag{A.30}$$

Note $\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^{I} \omega_i \alpha_i n_i (y_i+T_i)}{\omega_i n_i \beta_i \xi_A} \right) > 0$ and $\xi_A > 0$. Hence, if the size of population of locality *i* is large enough then

$$n_i > \frac{\alpha_i P_i^*}{\beta_i} \quad \text{which implies} \qquad \frac{dP_i^*}{d(y_i + T_i)} > 0 \tag{A.31}$$

Which proves that the optimal distribution of tax revenue in locality i, P_i^* for $i = 1,2, \ldots I$ depends positively from per capita full income in locality i, $y_i + T_i$, but the relationship is decreasing as percapita full income increases. From (A.37) it is obvious that $\frac{d^2 P_i^*}{d^2(y_i+T_i)} < 0$.

Proof of Proposition 3ii.

From the optimal allocation of P_i^* for $i = 1,2, \ldots I$ it can be shown that

$$\frac{dP_i^*}{d\xi_A} = -\frac{\frac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial \xi_A}}{\frac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial P_i^*}} \tag{A.32}$$

Recall

$$\Upsilon_i^* = P_i^* \left(\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I \omega_i \alpha_i n_i (y_i + T_i)}{\omega_i n_i \beta_i \xi_A} \right) \right) - \left(\frac{1}{\xi_A} \right) \sum_{i=1}^I n_i(y_i + T_i) - \Omega = 0 \tag{A.33}$$

Therefore

$$\frac{dP_i^*}{d\xi_A} = \frac{\left(\frac{1}{\xi_A} \right)^2 \left(P_i^* \frac{\sum_{i=1}^I \omega_i \alpha_i n_i (y_i + T_i)}{\omega_i n_i \beta_i} - \sum_{i=1}^I n_i(y_i + T_i) \right)}{\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I \omega_i \alpha_i n_i (y_i + T_i)}{\omega_i n_i \beta_i \xi_A} \right)} \tag{A.34}$$

Since $\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I \omega_i \alpha_i n_i(y_i+T_i)}{\omega_i n_i \beta_i \xi_A} \right) > 0$ and $\left(\frac{1}{\xi_A} \right)^2 > 0$ then

$$P_i^* \frac{\sum_{i=1}^I \omega_i \alpha_i n_i (y_i + T_i)}{\omega_i n_i \beta_i} \underset{<}{\overset{>}{=}} \sum_{i=1}^I n_i(y_i + T_i) \quad \Rightarrow \frac{dP_i^*}{d\xi_A} \underset{<}{\overset{>}{=}} 0 \tag{A.35}$$

Proof of Proposition 3.iii.

In this section, we seek to calculate $\frac{dP_i^*}{dn_i}$. From the optimal allocation of P_i^* for $i = 1,2, \ldots I$, it can be shown that

$$\frac{dP_i^*}{dn_i} = -\frac{\frac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial n_i}}{\frac{\partial f(\omega_i, \alpha_i, n_i, y_i + T_i, \xi_A, \Omega, \psi_i)}{\partial P_i^*}} \tag{A.36}$$

From(A.26), it is satisfied that

$$\frac{dP_i^*}{dn_i} = \frac{-P_i^*\left(\frac{\alpha_i(y_i+T_i)}{n_i\beta_i\xi_A}\right) + P_i^*\frac{\sum_{i=1}^I\omega_i\alpha_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\left(\frac{1}{n_i}\right)}{\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I\omega_i\alpha_in_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\right)}$$

$$+ \frac{\left(\frac{1}{\xi_A}\right)\sum_{i=1}^I(y_i+T_i)}{\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I\omega_i\alpha_in_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\right)} \overset{>}{\underset{<}{}} 0 \qquad (A.37)$$

Since

$$\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I\omega_i\alpha_in_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\right) > 0 \qquad (A.38)$$

And

$$\left|\left(\frac{1}{\xi_A}\right)\sum_{i=1}^I(y_i+T_i) + P_i^*\frac{\sum_{i=1}^I\omega_i\alpha_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\left(\frac{1}{n_i}\right)\right| \overset{>}{\underset{<}{}} \left|P_i^*\left(\frac{\alpha_i(y_i+T_i)}{n_i\beta_i\xi_A}\right)\right| \qquad (A.39)$$

Then

$$\frac{dP_i^*}{dn_i} \overset{>}{\underset{<}{}} 0 \qquad (A.40)$$

Proof of Proposition 3.iv.

This proposition is proved by calculating $\frac{dP_i^*}{d\psi_i}$. As we have done before, consider the optimal allocation of P_i^* for $i = 1,2,\ldots I$, it can be shown that

$$\frac{dP_i^*}{d\psi_i} = -\frac{\frac{\partial f(\omega_i,\alpha_i,n_i,y_i+T_i,\xi_A,\Omega,\psi_i)}{\partial\psi_i}}{\frac{\partial f(\omega_i,\alpha_i,n_i,y_i+T_i,\xi_A,\Omega,\psi_i)}{\partial P_i^*}} \qquad (A.41)$$

Use (A.26) to show

$$\frac{dP_i^*}{d\psi_i} = \frac{\left(\frac{1}{\psi_i}\right)^2 P_i^*}{\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I\omega_i\alpha_in_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\right)} > 0 \qquad (A.42)$$

Since $\frac{1}{\psi_i} + \left(\frac{\sum_{i=1}^I\omega_i\alpha_in_i(y_i+T_i)}{\omega_in_i\beta_i\xi_A}\right) > 0$, it follows that $\frac{dP_i^*}{d\psi_i} > 0$.